Computable General Equilibrium Models for the Analysis of Economy-Environment Interactions

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1 Introduction and Motivation

This chapter is an introduction to the subject of computable general equilibrium (CGE) modeling in environmental and resource economics. CGE models are a widely-used tool for the quasi-empirical analysis of environmental externalities—and policies for mitigating them—which are large enough to influence prices across multiple markets in the economy. We provide a simple, rigorous, and practically-oriented exposition that develops the framework of a CGE model from microeconomic fundamentals, outlines how the resulting algebraic structure may be numerically calibrated using the economic and environmental data and then solved for the equilibrium values of economic variables, and illustrates how CGE simulations may be used to analyze the economy-wide impacts of environmental externalities and associated mitigation policies.

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Walrasian general equilibrium prevails when supply and demand are equalized across all of the interconnected markets in the economy. A CGE model is an algebraic representation of the abstract Arrow-Debreu general equilibrium structure which is calibrated on economic data. The resulting numerical problem is solved for the supplies, demands and prices which support equilibrium across a specified set of markets, which can range from a single sub-national region to multiple groups of countries interacting within the global economy. Notwithstanding this, every economy represented in these models typically has the same basic structure: a set of producers, consumers and governments whose activities are linked by markets for commodities and factors as well as taxes, subsidies and perhaps other distortions.

[Figure 1 about here]

The conceptual framework at the core of every CGE model is the familiar circular flow of the economy, which is illustrated in Figure 1. The circular flow stems from the market interactions among three institutions: (A) households, which are assumed to own the factors of production, (B) firms, which rent factors from the households for use in the production of commodities subsequently purchased by households and other firms, and (C) the government, which provides government goods that are consumed by households and firms. These inter-institutional transfers of goods and factors are indicated by solid black arrows, each of which represents a distinct market. In equilibrium, the value of goods and services traded in each market is balanced by a compensating financial transfer, indicated by a dashed black arrow. Factor rentals generate a stream of income for households (D), which finances their expenditure on commodity purchases. The latter in turn generates revenue for firms (E1). Likewise, an individual firm's purchase of intermediate inputs generates revenue for the other firms which produce those commodities (E2), and public provision is financed out of taxes on firms and households levied by the government (F). In general, most economies will also include a foreign sector, with which firms and households will exchange flows of goods and payments. For the sake of brevity we do not go into detail about open economy CGE modeling, as that constitutes an entire area of research all its own.

The chapter's principal motivation is to elaborate the interactions between the circular flow of the economy and the environment (G). Diminished ecological functioning or increases in the scarcity rent of nominally unpriced natural resources occurs as a result of firms' and households' emissions of residuals or consumption of environmental services as inputs (H). These effects constitute classic negative externalities, as environmental services which assimilate pollution or provide inputs to production tend neither to be traded within markets nor tabulated in the economy's accounts, resulting in a divergence between the private and social costs of the goods and factors with whose quantity and quality they are linked. While earlier surveys by Conrad (2002), Bergman (2005) and Böhringer and Loschel (2006) have sought to organize the broad and diverse literature on environmental CGE modeling along a variety of functional and topical lines, Figure 1 suggests a simple structural classification based on the scope and character of the environment-economy interactions represented within the model of the relevant externality.

Most CGE studies in the past decade either (i) quantify how changes in the circular flow due to endogenous processes, policy changes or exogenous shocks affect the level of environmental quality,[1] or (ii) shed light on the consequences for the circular flow of policies that seek to increase the level of environmental quality by altering the allocations of polluting residuals or the consumption of environmental resources.[2] As the solid gray

[1]This body of work includes investigations of consequences of trade liberalization—principally in developing countries, its environmental and economic costs, and the effectiveness of various policy options for managing them (Dessus and Bussolo, 1998; Abler et al., 1999; Jansen, 2001; Faehn and Holmoy, 2003; He, 2005; Li, 2005; Vennemo et al., 2008), examinations of the interactions among environmental externalities and economic growth, income distribution and labor markets (Abler et al., 1998; Berck and Hoffmann, 2002; Nugent and Sarma, 2002; Coxhead and Jayasuriya, 2004; Taylor et al., 2009), and studies of resource management practices in the areas of water quality and allocation (Seung et al., 2000; Roe et al., 2005; Diao et al., 2005, 2008; Brouwer et al., 2008; Strzepek et al., 2008; van Heerden et al., 2008) as well as agricultural and forest land—with particular emphasis on the impacts of macroeconomic shocks deforestation, land degradation (Abdelgalil and Cohen, 2001; Bashaasha et al., 2001; Cattaneo, 2001, 2005; Ianchovichina et al., 2001; Wiig et al., 2001; Fraser and Waschik, 2005).

[2]The bulk of the policy literature lies in this area, focusing on the general equilibrium economic costs

arrows (H1) imply, these analyses typically specify a one-way linkage from the economy to the environment, treating the latter as a more or less exogenous boundary condition on economic activity.

Qualitatively similar linkages in the reverse direction can be found in a less common but rapidly growing arena of application: (iii) assessments of the economic impacts of changes in environmental quality (including disasters of anthropogenic or natural origin), indicated by the dashed gray arrows (H2).[3] Lastly, comparatively few analyses (iv) bring the environment fully within the ambit of the circular flow, simultaneously capturing H1 and H2 by representing both the impacts of economic activity on the environment and the simultaneous feedback effects on production, consumption and welfare.[4] The reason

of policies to abate pollution in various media. There are comparatively few papers on air (Morris, 1999; O'Ryan et al., 2003, 2005; Cao et al., 2009) and water (Xie and Saltzman, 2000) pollution, the vast majority focus on the problem of climate change, in particular the consequences of greenhouse gas (GHG) emission limits for energy markets and the implications for aggregate welfare. This topic has itself spawned several lines of research beyond standard single-economy analyses of the growth and welfare effects of abatement measures (e.g., Garbaccio et al., 1999; Kamat et al., 1999; Nwaobi, 2004; Fisher-Vanden and Ho, 2007; Telli et al., 2008), in particular examinations of the economic effects of environmentally-motivated taxes, especially their interactions with pre-existing market distortions (Parry et al., 1999; Bye, 2000; Bovenberg and Goulder, 2001; Babiker et al., 2003; Andre et al., 2005; Bovenberg et al., 2005; Conrad and Loschel, 2005), the environmental and economic consequences of long run increases in R&D and factor productivity, energy-saving technological progress or the emergence of GHG-abatement or alternative energy technologies with the potential for radical savings in compliance costs (Dellink et al., 2004; Grepperud and Rasmussen, 2004; Schafer and Jacoby, 2005; Jacoby et al., 2006; McFarland and Herzog, 2006; Allan et al., 2007; Berg, 2007; Otto et al., 2007; Fisher-Vanden and Sue Wing, 2008), the nexus between GHG abatement, the global trading system, and leakage of emissions from abating to non-abating developing countries (Böhringer and Rutherford, 2002, 2004; Babiker, 2005; Babiker and Rutherford, 2005), the conditions for endogenous emergence of coalitions of abating regions in a game theoretic setting (Babiker, 2001; Carbone et al., 2009), the interactions between emission reduction measures such as tradable permit schemes and imperfections in energy markets (Klepper and Peterson, 2005; Hagem et al., 2006), and the distributional impacts of schemes to share the burden of GHG emission reductions, such as allowance allocations in tradable permit systems, across countries and regions (Böhringer, 2002; Böhringer et al., 2003; Böhringer and Welsch, 2004; Kallbekken and Westskog, 2005; Klepper and Peterson, 2006; Nijkamp et al., 2005; Böhringer et al., 2007; Böhringer and Helm, 2008), firms and industries (Bovenberg et al., 2005, 2008) and income groups within regions (Rose and Oladosu, 2002; van Heerden et al., 2006; Oladosu and Rose, 2007; Ojha, 2009).

[3]This literature utilizes the outputs of global climate models to simulate the impacts of global warming on various sectors of the economy as economic shocks (Winters et al., 1998; Dessus and O'Connor, 2003; Bosello et al., 2006, 2007; Aunan et al., 2007; Hubler et al., 2008; Vennemo et al., 2008; Boyd and Ibarrarán, 2009). Similar types of assessments are also used to quantify the impacts of large-scale natural hazards on regional economies (Rose and Liao, 2005).

[4]These include quasi-benefit-cost analyses of pollution control policies when the environmental externalities in question affect consumers' health (Williams, 2002, 2003; Matus et al., 2008; Mayeres and Van Regemorter, 2008), an assessment of the impacts of non-separability between the environment and commodities or factors—especially labor, with the goal of developing new model calibration techniques (Carbone and Smith, 2008), and, in a particularly exciting development, the coupling of a CGE model of the economy with

is the difficulty involved in representing physical processes in such a way that they can be integrated within CGE models' abstract equilibrium structure.[5]

The chapter's secondary motivation is pedagogic. Despite their popularity as a research tool, CGE models continue to be unfairly dismissed by some in the field as "black boxes", whose complex internal workings obfuscate the linkages between their outputs and features of their input data, algebraic structure, or method of solution, and worse, allow questionable assumptions to be hidden within them that end up driving their results. The problem can largely be traced to a lack of familiarity with CGE models on the part of economic researchers, a situation which is exacerbated by the dearth of graduate methods courses on, and rigorous yet accessible article-length introductions to, the subject. Although descriptions of CGE models' underlying structure, calibration and solution methods, and techniques of application abound, they tend to be divided across different literatures, each of which focuses on its own aspect of the subject.[6] The aim of this chapter is to remedy this lacuna by unifying these disparate elements to provide a systematic, practical and intuitive explanation of the methods by which CGE models are constructed,

a general equilibrium simulation of an ecosystem (Finnoff and Tschirhart, 2008), where the latter models predator-prey interactions as arising out of the net energy-maximizing behavior of a variety of representative organisms (Tschirhart, 2000, 2002, 2003).

[5]Many studies have found it easier to employ a wholly separate physical impacts model in an iterative manner, using the precursors of environmental damage generated by a CGE model for its inputs, and then using its outputs to specify shocks to the CGE model's simulated economy. But this solution often proves computationally challenging. For example, the MIT Integrated Global System Model (IGSM) is a system of "soft-linked" models in which the MIT EPPA CGE simulation generates future emission trajectories which then drive large-scale global climate and terrestrial ecosystem simulations in an open-loop fashion, without feedback effects on the economy (Sokolov et al., 2005). The challenges involved in closing this loop are illustrated by Böhringer et al.'s (2007) recasting of Nordhaus's (1994) DICE model of global warming as an intertemporal CGE model, in which a highly stylized physical model of atmospheric GHG accumulation and climate change is approximated using the jacobian of the global mean temperature response to the simulated path of emissions.

[6]Of the numerous articles cited above, the majority merely discuss only those attributes of their models that are pertinent to the application at hand, or present their model's equations with little explanation to accompany them. Books and manuals devoted to modeling techniques (e.g., Ginsburgh and Keyzer, 1997; Lofgren et al., 2002) tend to be exhaustively detailed, and articles focused on the operations research aspects of modeling (e.g., Rutherford, 1995; Ferris and Pang, 1997) often involve a high level of mathematical abstraction, neither of which makes it easy for the uninitiated to quickly grasp the basics. Finally, while pedagogic papers (e.g., Devarajan et al., 1997; Rutherford, 1999; Paltsev, 2004) often provide a lucid introduction to the fundamentals, they tend to focus either on models' structural descriptions or the details of the mathematical software packages used to build them, while glossing over CGE models' theoretical basis or procedures for calibration. For an exception, see Kehoe (1998).

calibrated, solved, and employed in the analysis of economy-environment interactions. By laying bare the simple yet elegant structural underpinnings common to the multitude of the studies cited above, I hope to make CGE models more accessible to the broader community of natural resource and environmental economists, and to encourage their use as a tool for research in the discipline.

The rest of the chapter is organized as follows. Section 2 begins by exploiting the circular flow to algebraically derive the equilibrium conditions at the core of every CGE model. Section 3 demonstrates how a CGE model of the economy is constructed by imposing the axioms of consumer and producer optimization on this framework to generate a system of nonlinear equations which represents an economy of arbitrary dimension. Section 4 outlines the techniques of using an economic database known as a social accounting matrix to numerically calibrate the foregoing algebraic structure. The meat of the chapter is in section 5, which introduces economy-environment interactions and illustrates several techniques for modeling problems which arise in environmental and resource economics. Section 6 concludes with a summary and brief remarks on key directions for future research.

2 The Algebra of General Equilibrium

For expositional clarity we consider a hypothetical closed economy made up of an unspecified number of firms which are grouped into distinct industry sectors—each of which is assumed to produce a single commodity, and an unspecified number of households which are assumed to jointly own an endowment of distinct primary factors of production. When the economy is in equilibrium, the circular flow embodies the physical principle of material balance, whereby the quantity of a factor with which households are endowed, or of a commodity produced by firms, must be completely absorbed by firms or households (respectively) in the rest of the economy. It also embodies the accounting principle of budgetary balance. Firms' expenditures on inputs must be balanced by the value of the revenue generated by the sale of the resulting product, households' expenditures on goods must be balanced by their income, and each unit of expenditure has to purchase some amount of some type of commodity or factor. These principles are the reflection of the Karush-Kuhn-Tucker complementary slackness conditions for the optimal allocation of commodities and factors and the distribution of activities in the economy (Mathiesen, 1985a,b). Indeed, we build on Paltsev's (2004) intuitive exposition to demonstrate that CGE models are at their core the mathematical expression of this fact.

Before we proceed, it should be noted that the financial transfers in Figure 1 may be deduced from the equilibrium price and quantity allocation of goods and services. For this reason it suffices to model the equilibrium of an economy at a particular instant of time in terms of barter trade in commodities and factors, without explicitly keeping track of money as a commodity. However, by the same token, it is necessary to denominate the relative values of the different commodities and factors in terms of some common unit of account. This is accomplished by expressing the flows of goods in units of a single commodity (the so-called numeraire good) whose price is taken to be a fixed reference point. CGE models embody this device by solving only for relative prices, a point about which more will be said below.

An additional feature of our exposition is that it holds government as an institution offstage while capturing the distortionary impacts of its activity on private markets. On one hand, government's role in the circular flow is often passive—to levy taxes and disburse the resulting revenues to firms and households as subsidies and transfers, subject to exogenous rules of budgetary balance. On the other, its impact on private markets is critical, as every tax (subsidy) distorts the supply-demand balance for the commodity in question and simultaneously generates a stream of revenue that increments (decrements) the income of consumers. The essence of this process may be captured by assuming that tax receipts (subsidy payments) are immediately transferred to (assessed upon) households in a lump-sum fashion, which allows these flows to be represented as income transfers from (to) the institution being taxed (subsidized).

Our first step is to introduce some notation. Let the indices $i = \{1,\ldots,\mathcal{N}\}$, $j = \{1,\ldots,\mathcal{N}\}$ and $f = \{1,\ldots,\mathcal{F}\}$ denote the set of commodities, industry sectors and primary factors, respectively. We assume that households' use of commodities satisfies $d = \{1,\ldots,\mathcal{D}\}$ types of demands, which for simplicity we lump into two broad categories, $d = \{C, O\}$: consumption, C, from which households derive utility, and all other final uses, O, which are assumed to be exogenous. With this infrastructure in hand we may specify the key variables in the economy which need to be kept track of: p_i and w_f, which are the equilibrium prices of good i and factor f; y_j and $x_{i,j}$, the quantities of industry j's output and its use of sector i's output as an intermediate input ; V_f and $v_{f,j}$, households' aggregate endowment of factor f and industry j's use of that factor; $g_{i,d}$, households' use of commodity i to satisfy final demand d; and u and μ, the indexes of households' utility and aggregate expenditure per unit of utility. We also define the output-normalized unit quantities of intermediate goods and primary factors, $\widetilde{x}_{i,j} = x_{i,j}/y_j$ and $\widetilde{v}_{f,j} = v_{f,j}/y_j$, the utility-normalized unit quantity of consumption of the i^{th} good, $\widetilde{g}_{i,C} = g_{i,C}/u$, and the unit expenditure-normalized prices of commodities and factors, $\widetilde{p}_i = p_i/\mu$ and $\widetilde{w}_f = w_f/\mu$.

Turning to taxes and subsidies, it is typical in CGE models for these instruments to be specified in an ad-valorem fashion, which determines the fractional change in the price of the commodity or factor to which it is being applied. Thus, a tax rate τ on the output of industry j drives a wedge between the producer price of output, p_j, and the consumer price, $(1+\tau)p_j$, in the process generating revenue for the government from the y_j units of output in the amount of $\tau p_j y_j$. Note that a subsidy which lowers the consumer price relative to the producer price may also be modeled in this way, by specifying $\tau < 0$. Taxes and subsidies may appear in any number of markets, enumerated by the index $h = \{1,\ldots,\mathcal{H}\}$. For simplicity we consider only two of these in our exposition: the markets for the output of the industry sectors (indicated by the superscript Y), and the markets for factor inputs to industries (indicated by the superscript F), using τ_j^Y and τ_f^F to indicate the distortions associated with the outputs of the various industries and households' endowments of various factors.

We now formally analyze the firms (B). Budgetary balance implies that the value of a unit of each commodity must equal the sum of the values of all the inputs used to produce it, which is simply the unit cost of the inputs of intermediate materials as well as the payments to the primary factors employed in its production. The guiding principle is the complete accounting of the sources of economic value, which simultaneously reflects constancy of returns to scale in production and perfectly competitive markets for firms' inputs and outputs, and ensures that in equilibrium producers make *zero profit*. Recognizing that producers value their outputs on a net-of-tax basis and their inputs in on a gross-of-tax basis, we have:

$$p_j y_j = \sum_{i=1}^{\mathcal{N}} (1 + \tau_i^Y) p_i x_{i,j} + \sum_{f=1}^{\mathcal{F}} (1 + \tau_f^F) w_f v_{f,j}$$

$$\Leftrightarrow \; p_j y_j = \sum_{i=1}^{\mathcal{N}} (1 + \tau_i^Y) p_i \widetilde{x}_{i,j} y_j + \sum_{f=1}^{\mathcal{F}} (1 + \tau_f^F) w_f \widetilde{v}_{f,j} y_j. \quad (8.1)$$

A conceptual sleight-of-hand makes it possible to apply similar reasoning to the households (A). Consumption of commodities may be thought of as the production of a "utility good" whose value, given by the product of the utility level and the unit expenditure index or "utility price", must equal the sum of the gross-of-tax values of the commodities consumed by the household (i.e., the inputs to utility production):

$$\mu u = \sum_{i=1}^{\mathcal{N}} (1 + \tau_i^Y) p_i g_{i,C} \; \Leftrightarrow \; \mu u = \sum_{i=1}^{\mathcal{N}} (1 + \tau_i^Y) p_i \widetilde{g}_{i,C} u. \quad (8.2)$$

We now turn to the product market (E). Under the assumption that commodities are not freely disposable, firms' outputs are fully consumed by households, and households' endowment of primary factors is in turn fully employed by firms. The implication is that, for a given commodity, the quantity produced must equal the sum of the quantities that are demanded by the other firms and households in the economy. Analogously, in the market for a given factor (D), the aggregate quantity demanded by the various firms must completely exhaust the aggregate supply with which the households are endowed. This is the familiar condition of *market clearance*, which implies that in each market in equilibrium the value of sellers' receipts must equal the value of purchaser's outlays on a net-of-tax basis:

$$y_i = \sum_{j=1}^{\mathcal{N}} x_{i,j} + \sum_{d=1}^{\mathcal{D}} g_{i,d} \Leftrightarrow p_i y_i = \sum_{j=1}^{\mathcal{N}} p_i x_{i,j} + \sum_{d=1}^{\mathcal{D}} p_i g_{i,d}. \tag{8.3}$$

$$V_f = \sum_{j=1}^{\mathcal{N}} v_{f,j} \Leftrightarrow w_f V_f = \sum_{j=1}^{\mathcal{N}} w_f v_{f,j}. \tag{8.4}$$

In equilibrium the flows of revenue in the markets for commodities and factors are fundamentally linked. The return to households' endowments of primary factors, which is equal to the value of their factor rentals to producers, constitutes consumers' primary income, which, along with recycled tax revenue, they exhaust on purchases of commodities. The implication is that the revenue gained by renting out primary factors, plus the value of tax receipts transferred to households by the government (shown in parentheses below), balances the gross expenditure on the satisfaction of demands, reflecting the principle of balanced-budget accounting known as *income balance*:

$$\mathscr{I} = \sum_{f=1}^{\mathcal{F}} w_f V_f + \left(\sum_{f=1}^{\mathcal{F}} \tau_f^F w_f V_f + \sum_{j=1}^{\mathcal{N}} \tau_j^Y p_j y_j \right) = \sum_{i=1}^{\mathcal{N}} \sum_{d=1}^{\mathcal{D}} p_i g_{i,d}. \tag{8.5}$$

This expression highlights the fundamental equivalence between aggregate disposable income (given by $\mathscr{I}^D = \mathscr{I} - \sum_{i=1}^{\mathcal{N}} (1 + \tau_y^Y) p_i g_{i,O}$) and the value of utility defined by (8.2), which implies the following the supply-demand balance condition for utility goods:

$$\mu u = \mathscr{I}^D \Leftrightarrow \mu u = \sum_{f=1}^{\mathcal{F}} \widetilde{w}_f \mu V_f + \left(\sum_{f=1}^{\mathcal{F}} \tau_f^F \widetilde{w}_f \mu V_f + \sum_{j=1}^{\mathcal{N}} \tau_j^Y \widetilde{p}_j \mu y_j \right) - \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) \widetilde{p}_i \mu g_{i,O}. \quad (8.6)$$

The algebraic framework of a CGE model is derived from equilibrium conditions (8.3)-(8.2) and (8.6). The key feature of each condition is that the residual equation which results from eliminating the common factor exhibits complementary slackness with respect to the variable that is common to both sides. The economic intuition behind this is straightforward (Paltsev, 2004). Looking first at the zero profit conditions (8.1) and (8.2), firms earning negative profit will shut down with an output of zero, while those earning zero profit can continue to produce a positive quantity of output. Similarly, households whose spending on consumption of goods exceeds the value of utility generated by consumption will cease activity with utility level of zero, while households whose consumption expenditure exactly matches the value of utility thus generated can continue to consume with a positive level of utility. In both cases the common factor is the activity level of the firm or household, with unit profit in goods production complementary to the relevant producer's level of output, and unit expenditure complementary to the level of utility:

$$p_j < \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) p_i \widetilde{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} (1+\tau_f^F) w_f \widetilde{v}_{f,j},\ y_j = 0$$

$$\text{or} \quad p_j = \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) p_i \widetilde{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} (1+\tau_f^F) w_f \widetilde{v}_{f,j},\ y_j > 0. \quad (8.7a)$$

$$\mu < \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) p_i \widetilde{g}_{i,C},\ u = 0 \quad \text{or} \quad \mu = \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) p_i \widetilde{g}_{i,C},\ u > 0 \quad (8.7b)$$

Turning to the market clearance conditions (8.3)-(8.4), any commodity or factor which is in excess supply will have a price of zero, while a good or factor which is neither in excess demand nor excess supply will have a positive price. Therefore the balance between supply and demand for each of these inputs is complementary to the corresponding price level:

$$y_i > \sum_{j=1}^{\mathcal{N}} x_{i,j} + g_{i,C} + g_{i,O},\ p_i = 0 \quad \text{or} \quad y_i = \sum_{j=1}^{\mathcal{N}} x_{i,j} + g_{i,C} + g_{i,O},\ p_i > 0 \tag{8.7c}$$

$$V_f > \sum_{j=1}^{\mathcal{N}} v_{f,j},\ w_f = 0 \quad \text{or} \quad V_f = \sum_{j=1}^{\mathcal{N}} v_{f,j},\ w_f > 0 \tag{8.7d}$$

Finally, market clearance in utility (8.6) exhibits complementary slackness with respect to the unit expenditure index. The intuition is that if the supply of utility goods exceeds the demand generated by consumption out of disposable income then unit expenditure on commodities will be zero, while if the supply just matches the demand then unit expenditure is positive:

$$u > \mathscr{I}^D/\mu,\ \mu = 0, \quad \text{or} \quad u = \mathscr{I}^D/\mu,\ \mu > 0 \tag{8.7e}$$

We note that disposable income does not exhibit complementary slackness with respect to any of its constituent variables, and moreover is made redundant by (8.6), which relegates it to the simple role of an accounting identity. This is often made explicit by specifying an additional expression in which $\mathscr{I}^D$ exhibits complementary slackness with respect to its own definition, and by designating the unit expenditure index as the numeraire price by fixing $\mu = 1$. Doing so has the effect of dropping eq. (8.7e) by re-casting the utility index as the value of aggregate consumption, $u = \mathscr{I}^D$.

The core of a CGE model is given by the system of equations (8.7c)-(8.7e). However, in order to actually compute the general equilibrium of an economy it is necessary to elaborate this structure by specifying the primal quantity allocation of commodity and factor uses in terms of the dual commodity and factor prices. It is this task which we tackle next.

3 From Equilibrium Conditions to a CGE Model: The CES Economy

The algebraic specification of a CGE model in terms of the dual results from the imposition of the axioms of producer and consumer maximization on the market-clearance, zero-profit and income balance conditions derived above. In so doing, the key simplifying assumption is to model the economic decisions of the disparate households collectively, by treating them as a representative agent, and to model the economic decisions of the disparate competitive firms in each industry collectively, by treating sectors as representative producers. To provide a concrete illustration of how this may be done we employ the device of a "constant elasticity of substitution (CES) economy" in which the representative agent has CES preferences and each industry has CES production technology.

On the demand side of the economy, the representative agent derives utility from the consumption of commodities according to a CES utility function with technical coefficients $\alpha_{i,C}$ and elasticity of substitution ω. The agent's problem is to maximize utility by allocating expenditure over commodities subject to the constraint of disposable income, taking goods prices parametrically. We re-cast this problem in terms of the dual, and model the agent as allocating the input quantities of consumption goods necessary to generate a unit of utility so as to minimize unit expenditure subject to the constraint of her consumption technology:

$$\max_{g_{i,C}} \left\{ u = \left[\sum_{i=1}^{\mathcal{N}} \alpha_{i,C} g_{i,C}^{(\omega-1)/\omega} \right]^{\omega/(\omega-1)} \middle| \sum_i (1+\tau_i^Y) p_i g_{i,C} \leq \mathscr{I}^D \right\}$$

$$\Leftrightarrow \min_{\widetilde{g}_{i,C}} \left\{ \mu = \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) p_i \widetilde{g}_{i,C} \middle| 1 = \left[\sum_{i=1}^{\mathcal{N}} \alpha_{i,C} \widetilde{g}_{i,C}^{(\omega-1)/\omega} \right]^{\omega/(\omega-1)} \right\}.$$

The solution to this problem is the instantaneous unit demand for commodities and associated conditional final demands:

$$\widetilde{g}_{i,C} = \alpha_{i,C}^{\omega}\left((1+\tau_i^Y)p_i\right)^{-\omega}\mu^{\omega}, \quad g_{i,C} = \alpha_{i,C}^{\omega}\left((1+\tau_i^Y)p_i\right)^{-\omega}\mu^{\omega}u. \tag{8.8}$$

On the supply side, the representative firm in industry j generates output by combining the instantaneous uses of intermediate goods and factors according to a CES production technology with technical coefficients $\beta_{i,j}$ and $\gamma_{f,j}$, and elasticity of substitution σ_j. The producer's problem is to maximize profit π_j by allocating expenditure among i intermediate inputs and f factor inputs, taking the prices of inputs and output parametrically. Re-casting this problem in terms of the the dual, we treat the firm as allocating input quantities of intermediate goods and factors necessary to generate a unit of output so as to minimize unit production cost subject to the constraint of its production technology:

$$\begin{aligned}
&\max_{x_{i,j}, v_{f,j}} \left\{ \pi_j = p_j y_j - \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) p_i x_{i,j} - \sum_{f=1}^{\mathcal{F}} (1+\tau_f^F) w_f v_{f,j} \right| \\
&\qquad\qquad y_j = \left[\sum_{i=1}^{\mathcal{N}} \beta_{i,j} x_{i,j}^{(\sigma_j-1)/\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j} v_{f,j}^{(\sigma_j-1)/\sigma_j} \right]^{\sigma_j/(\sigma_j-1)} \Bigg\} \\
&\Leftrightarrow \min_{\widetilde{x}_{i,j}, \widetilde{v}_{f,j}} \left\{ p_j = \sum_{i=1}^{\mathcal{N}} (1+\tau_i^Y) p_i \widetilde{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} (1+\tau_f^F) w_f \widetilde{v}_{f,j} \right| \\
&\qquad\qquad 1 = \left[\sum_{i=1}^{\mathcal{N}} \beta_{i,j} \widetilde{x}_{i,j}^{(\sigma_j-1)/\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j} \widetilde{v}_{f,j}^{(\sigma_j-1)/\sigma_j} \right]^{\sigma_j/(\sigma_j-1)} \Bigg\}.
\end{aligned}$$

The solution to this problem is the instantaneous unit demands for commodities and factors and associated conditional demands:

$$\widetilde{x}_{i,j} = \beta_{i,j}^{\sigma_j}\left((1+\tau_i^Y)p_i\right)^{-\sigma_j} p_j^{\sigma_j}, \qquad x_{i,j} = \beta_{i,j}^{\sigma_j}\left((1+\tau_i^Y)p_i\right)^{-\sigma_j} p_j^{\sigma_j} y_j, \tag{8.9a}$$

$$\widetilde{v}_{f,j} = \gamma_{f,j}^{\sigma_j}\left((1+\tau_j^F)w_f\right)^{-\sigma_j} p_j^{\sigma_j}, \qquad v_{f,j} = \gamma_{f,j}^{\sigma_j}\left((1+\tau_f^F)w_f\right)^{-\sigma_j} p_j^{\sigma_j} y_j. \tag{8.9b}$$

[Table 1 about here]

The crucial step is to substitute the demands (8.8) and (8.9) into the equilibrium conditions (8.7a)-(8.7e). Doing this yields the system of $2\mathcal{N}+\mathcal{F}+3$ nonlinear inequalities, each paired with one of the $2\mathcal{N}+\mathcal{F}+3$ unknown variables, shown in Table 1. This is what is referred to as "a CGE model". Letting $\Xi(\cdot)$ denote the pseudo-excess demand correspondence of the economy made up of the stacked vector of equations (8.10a)-(8.10f), and using $\mathbf{z}=\{\mathbf{p},\mathbf{w},\mathbf{y},\mu,u,\mathscr{I}^D\}$ to indicate the stacked vector of unknowns, the system may be expressed compactly as

$$\Xi(\mathbf{z})\geq 0,\quad \mathbf{z}\geq 0,\quad \mathbf{z}'\Xi(\mathbf{z})=0, \tag{8.10}$$

or, using the shorthand notation "$\perp$" to indicate the complementarity between a variable and its matched equation, $\Xi(\mathbf{z})\ \perp\ \mathbf{z}$.

The mathematical problem defined by eq. (8.10) is highly non-linear, with the result that it is not possible to obtain a closed-form solution for $\mathbf{z}$. This is the reason for the "C" in CGE models: to find the general equilibrium of an economy with realistic utility and production functions, the system of equations that describes equilibrium must be transformed into a numerical problem that can be solved using optimization techniques. In the operations research literature, the square system of numerical inequalities (8.10) is known as a mixed complementarity problem or MCP (Ferris and Pang, 1997; Ferris and Kanzow, 2002), which is easily solved using algorithms that are now routinely embodied in modern, commercially-available software systems for optimization (Dirkse and Ferris, 1995; Ferris and Munson, 2000; Ferris et al., 2000).[7] All that remains to be demonstrated is how to generate a numerical problem by taking the foregoing algebraic framework to the data.

[7] The interested reader is referred to Sue Wing (2009) for a sketch of the basic computational approach. In modern software implementations, the solution to the MCP is computed to numerical tolerances that are routinely six orders of magnitude smaller than the value of the aggregate income level.

4 Social Accounting Matrices and Numerical Calibration

The circular flow of an economy in instantaneous equilibrium can be completely characterized by four data matrices. The market for firms' intermediate inputs (E2) is described by an $\mathcal{N} \times \mathcal{N}$ input-output matrix of industries' uses of commodities, $\overline{\mathbf{X}}$, the factor market (D) is described by an $\mathcal{F} \times \mathcal{N}$ matrix of primary factor inputs to industries, $\overline{\mathbf{V}}$, the product market (E1) is described by an $\mathcal{N} \times \mathcal{D}$ matrix of commodity uses by final demand activities, $\overline{\mathbf{G}}$, and tax and subsidy distortions are summarized by the $\mathcal{H} \times \mathcal{N}$ matrix $\overline{\mathbf{T}}$.

[Figure 2 about here]

Arranging these matrices in the manner shown in Figure 2 results in an accounting tableau known as a social accounting matrix, or SAM. A SAM is the cash-flow statement for an economy in equilibrium, which gives a snapshot of the inter-industry and inter-activity financial flows over an interval of time—typically one year. Each cell element is an input-output account that is denominated in the units of value of the period for which the flows in the economy are recorded, typically the value of the currency in the year in question. Each account is uniquely defined by its row and column, and records the payment from the account of a column to the account of a row. Thus, an account's components of income of (e.g., the value of receipts from the sale of a commodity or rental of a factor) appear along its row, and the components of its expenditure (i.e., the values of households' purchases or the production of a good) appear along its column (King, 1985; Reinert and Roland-Holst, 1997).

The structure the SAM reflects the principle of double-entry book-keeping, which requires that for each account, total revenue—the row total—must equal total expenditure—the column total. This is apparent from Figure 2, where the sum down any column of the left-hand matrices $\overline{\mathbf{X}}$, $\overline{\mathbf{V}}$ and $\overline{\mathbf{T}}$ is equivalent to the expression for zero-profit in production (8.1), the sum across any row in the uppermost matrices $\overline{\mathbf{X}}$ and $\overline{\mathbf{G}}$ is equivalent to the expression for goods market clearance (8.3), and the sum across any row in the middle array $\overline{\mathbf{V}}$ is equivalent to the expressions for factor market clearance (8.4). Furthermore, once these conditions hold, the sums of the elements of $\overline{\mathbf{G}}$ on one hand and $\overline{\mathbf{V}}$ and $\overline{\mathbf{T}}$ on the other should equal one another, reflecting the fact that in a closed economy GDP (the sum of the components of expenditure) equals value added (the sum of the components of income), which is equivalent to the income balance condition jointly specified by eqs. (8.5) and (8.6). Formally:

$$\bar{y}_j = \sum_{i=1}^{\mathcal{N}} \bar{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} \bar{v}_{f,j} + \sum_{h=1}^{\mathcal{H}} \bar{t}_{h,j}, \tag{8.11a}$$

$$\bar{y}_i = \sum_{j=1}^{\mathcal{N}} \bar{x}_{i,j} + \sum_{d=1}^{\mathcal{D}} \bar{g}_{i,d}, \tag{8.11b}$$

$$\overline{V}_f = \sum_{j=1}^{\mathcal{N}} \bar{v}_{f,j}, \tag{8.11c}$$

$$\sum_{f=1}^{\mathcal{F}} \overline{V}_f + \sum_{h=1}^{\mathcal{H}} \overline{T}_h = \sum_{i=1}^{\mathcal{N}} \sum_{d=1}^{\mathcal{D}} \bar{g}_{i,d}. \tag{8.11d}$$

To numerically calibrate our example CES economy, it is necessary to establish equivalence between the systems of equations (8.10) and (8.11). Dawkins et al. (2001) describe a variety of approaches to addressing this problem depending on what kind of information is available in addition to the SAM. For example, Kehoe (1998) describes a procedure when data exist on benchmark prices, however, more often than not such information is simply unavailable. In a typical data-constrained environment the simplest method to fit eq. (8.10) to the benchmark equilibrium in the SAM is to treat the price variables as indices with benchmark values of unity: $p_i = w_f = \mu = 1$, and treat the activity and income variables as real values which are set equal to the entries in the SAM.

To establish the latter equivalence, observe that the simplified tax structure in our CES economy partitions $\overline{\mathbf{T}}$ into a vector of payments for indirect taxes by industry, $\bar{t}_{Y,j} = \tau_j^Y p_j y_j$, and a matrix of tax payments associated with each industry's use of the f factors, $\bar{t}_{f,j} = \tau_f^F w_f v_{f,j}$. We may therefore write the zero profit condition (8.11a) as:

$$\bar{y}_j - \bar{t}_{Y,j} = \sum_{i=1}^{\mathcal{N}} \bar{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} (\bar{v}_{f,j} + \bar{t}_{f,j}).$$

in which the left-hand side gives j's net-of-tax value of output, and terms on the right-hand side are the gross-of-tax value of j's uses of intermediate and factor inputs. At benchmark prices these imply that $y_j = \overline{y}_j - \overline{t}_{Y,j}$, $(1+\tau_i^Y)x_{i,j} = \overline{x}_{i,j}$ and $(1+\tau_f^F)v_{f,j} = \overline{v}_{f,j} + \overline{t}_{f,j}$, with benchmark tax rates on output and factors given by $\overline{\tau}_j^Y = \overline{t}_{Y,j}/\overline{y}_j$ and $\overline{\tau}_f^F = \overline{T}_f/(\overline{V}_f + \overline{T}_f)$. Moreover, inspection of $\overline{\mathbf{G}}$ and $\overline{\mathbf{V}}$ reveals that $(1+\tau_i^Y)g_{i,d} = \overline{g}_{i,d}$, $u = \overline{G}_C$ and $V_f = \overline{V}_f + \overline{T}_f$. The technical coefficients of the cost and expenditure equations are then easily found by substituting these conditions into the demand functions (8.8) and (8.9):

$$\alpha_{i,C} = (\overline{g}_{i,C}/\overline{G}_C)^{1/\omega}, \tag{8.12a}$$

$$\beta_{i,j} = (\overline{x}_{i,j})^{1/\sigma_j}(\overline{y}_j - \overline{t}_{Y,j})^{-1/\sigma_j} \tag{8.12b}$$

$$\gamma_{f,j} = (\overline{v}_{f,j} + \overline{t}_{f,j})^{1/\sigma_j}(\overline{y}_j - \overline{t}_{Y,j})^{-1/\sigma_j}. \tag{8.12c}$$

We draw attention to the fact that the calibrated values of the technical coefficients are not independent of the values of the elasticities of substitution. The fact that the SAM contains no information on the values of $\boldsymbol{\sigma}$ and ω makes the calibration problem (8.12) under-determined, with more free parameters than there are model equations or observations of benchmark data. This is a problem which is magnified in CGE models which specify industries' cost functions and consumers' expenditure functions using hierarchical CES functions, each of which contains multiple elasticity parameters. For this reason, elasticity parameters are almost always exogenously specified by the analyst, but because relevant empirical estimates are frequently lacking, modelers all too often resort to selecting values based on a mix of judgment and assumptions.

The ad-hoc character of this procedure has come in for criticism by some empirical economists (e.g., Jorgenson, 1984; McKibbin and Wilcoxen, 1998; McKitrick, 1998) who advocate an econometric approach to CGE modeling in which the pseudo-excess demand correspondence is built up from statistically estimated cost and expenditure functions.

However, the main drawback of this alternative is that the econometric estimations are data-intensive, requiring time-series of input-output matrices which are a challenge to construct and therefore rarely available. In response, sophisticated limited-information methods have been developed to calibrate the substitution elasticities and technical coefficients based on more widely available ancillary data in addition to the SAM (e.g., Arndt et al., 2002), but so far these techniques have not seen widespread adoption.

In any event, specifying values for σ and ω makes it possible to calibrate the technical coefficients and substitute these two sets of parameters into the expressions in Table 1 to generate a system of numerical inequalities which constitutes the actual CGE model. We emphasize that to satisfy the resulting expressions with equality, all one has to do is simply set the price variables equal to unity and the quantity variables equal to the corresponding values in the SAM. This procedure, known as *benchmark replication*, permits the analyst to verify that the calibration is correct. The intuition is that since a balanced SAM represents the initial benchmark equilibrium of the economy, plugging the values in the SAM back into the calibrated numerical pseudo-excess demand correspondence should reproduce that equilibrium.

All our calculations thus far have assumed a realistic setting in which the economy under consideration is initially ridden with tariffs and/or subsidies. But it is worth noting that in the absence of benchmark distortions ($\overline{\mathbf{T}} = \mathbf{0}, \overline{\tau}_j^Y = \overline{\tau}_f^F = 0$), eq. (8.12) replaces the terms $\alpha_{i,C}^{\omega}$, $\beta_{i,j}^{\sigma_j}$ and $\gamma_{f,j}^{\sigma_j}$ in eq. (8.10) with coefficients given by the ratio of the relevant cells of the SAM and the corresponding column totals, that is, the value shares of the inputs to consumption and production. The key implication is that the values selected for the substitution elasticities have no practical impact on the benchmark equilibrium, which makes intuitive sense because the SAM completely specifies the model's initial equilibrium, which in turn is consistent with an infinite number of potential values for σ and ω. The corollary is that the substitution possibilities in the economy—i.e., the degree of adjustment of economic quantities in response to changes in prices, both within and between sectors—are fundamentally determined by the SAM.

5 Modeling Applications: Integrating the Environment

The astute reader will perhaps find the analysis so far unsatisfying, because all that has been done is to demonstrate how a CGE model can be constructed to replicate the economic dataset used for its calibration, without mentioning the environment. This section redresses this imbalance by elucidating several techniques for incorporating various kinds of economy-environment interactions within CGE models.

We begin with a note about data. As mentioned in the introduction, social accounting matrices compiled by statistical agencies generally do not record either the value of environmental resources or the environmental damage costs of the economy's residuals. Although there is a literature on the construction of so-called "environmentally-extended SAMs" which seek to integrate flows of resources and residuals into the traditional economic input-output framework,[8] the majority of CGE studies employ satellite accounts of pollution or the use of environmental factors. This is the approach that we use below.

As a general matter, to use CGE model for any kind of research or policy analysis, the analyst must first capture the initial effect of a policy shock by perturbing one or more of the exogenous parameters of the economy, and then compute a counterfactual equilibrium based on the new parameters. To evaluate the impacts of the shock, the analyst compares the initial and counterfactual equilibrium vectors of prices and activity levels, and the level of utility, and uses the results to draw inferences about the shock's effects in the real world, subject to the caveats of the accuracy and realism of the model's assumptions.

CGE models' principal advantage is their ability to measure the ultimate impact of the shock on consumers' aggregate well-being in a theoretically consistent way, by quan-

[8]See, e.g., Alarcón et al. (2000); Xie (2000); Lenzen and Schaeffer (2004); Martínez de Anguita and Wagner (2010).

tifying the changes in the consumption of the representative agent which result from the myriad supply-demand adjustments across the various markets in the economy. In particular, given an initial level of utility $\overline{u}$ implied by the SAM, a counterfactual level of utility u', and unit expenditure set as the numeraire, the welfare change $(u'/\overline{u} - 1)$ measures the impact of the shock in terms of equivalent variation, expressed as a percentage of initial expenditure.

But ironically, it is this very functionality which exposes the kernel of truth to the black box criticism articulated in the introduction. CGE models' comprehensive representation of the economy, combined with their popularity as a tool for prospective policy analysis, has earned them a reputation as a sort of economic crystal ball. Table 1 emphasizes that the non-linearity and dimensionality of the pseudo-excess demand correspondence often make it difficult to intuitively understand the net impact of a change in a single parameter, even in models that are structurally simple and represent only a modest number of sectors and/or households. A rigorous accounting of the myriad interactions that determine the character of the counterfactual equilibrium can therefore entail a substantial amount of sensitivity analysis and testing, with the result that ex-post evaluations of the veracity of model projections are rare. Moreover, the few such assessments point to a disturbingly poor forecasting record,[9] which suggests that CGE models' usefulness as a research tool owes less to their predictive accuracy than their ability to elucidate the mechanisms responsible for the transmission of price and quantity adjustments among markets. CGE models should therefore be properly regarded as computational laboratories within which to study the dynamics of the economy-environment interactions and the manner in which they ultimately give rise to welfare impacts.

With this admonition, we outline how the environment may be integrated into the CES economy in Table 1. We consider two types of externalities, pollution and the depletion of nominally unpriced non-market environmental goods and services used as inputs

[9] e.g., Panagariya and Duttagupta (2001); Kehoe (2005); Valenzuela et al. (2007).

to production. Our first step is to specify the link between economic activity and each externality. Pollution is easily represented, with the most common modeling device being exogenously specified emission coefficients, which, when multiplied by (for example) the outputs of dirty industries represented in the model, yield the sectoral and aggregate pollution loads associated with economic activity. Using $e \subset j$ and ϕ_e to indicate the polluting subset of sectors and their associated emission coefficients implies sectoral emissions $\varepsilon_e = \phi_e y_e$ and an overall pollution level $\mathcal{E} = \sum_e \varepsilon_e$. The use of non-market environmental factors is more challenging to represent because the aggregate demand for such inputs—say, ϱ—must be specified as a function of endogenous variables within the model. In particular, if firms are able to substitute market inputs for non-market factors, then it becomes necessary to adjust the model's the core algebraic framework and its calibration procedures. An example below provides a detailed illustration of the kinds of changes involved.

The second step is to capture the initiating impulse of shocks. It is common to represent these through exogenous parameters whose values may be specified by the analyst. By far the most important of these is the endowment of primary non-reproducible factors, V_f, which is nominally determined by the row totals of the factor supply matrix in the SAM and defines the overall scale of economic activity, thereby playing the role of a boundary condition in the resulting equilibrium. Other parameters are policy variables which may be price-based—taxes and subsidies, as we have seen, or quantity-based—e.g., constraints on commodity and factor demand and/or supply. They can also take the form of technology parameters which represent improvements in productivity, changes in the rate and/or bias of technical progress, or even structural shifts in consumers' preferences.

The third step is to propagate the shock through the economy. In most cases the equilibrium in the SAM implies benchmark values for the types of shock parameters discussed above, so that a counterfactual scenario may be simulated by simply changing the values of these parameters and solving the CGE model for a new equilibrium. The impact of the various shocks on environmental quality may then be computed in an open-loop fashion. In the case of pollution the emission coefficients may be applied to the new industry activity levels to make an ex-post calculation of the pollution load, while the change in demand for non-market inputs will typically be computed as part of the counterfactual simulation.

The final step in the analysis is to evaluate the welfare impact of the shock in question. The disutility of pollution and depletion of environmental factors may be represented through the specification of an environmental damage function, $\mathscr{D}$, denominated over $\mathcal{E}$ or ϱ, which is exogenous to the general equilibrium problem. This device permits aggregate welfare to be computed as $(u - \mathscr{D})$, and then compared across benchmark and counterfactual scenarios. Fundamental to this approach is the implicit treatment of environmental damage as separable from other economic variables, principally consumption. However, Espinosa and Smith (1995) and Carbone and Smith (2008) demonstrate that this assumption, which is ubiquitous in the literature is not innocuous—alternative assumptions of complementarity or substitutability between environmental quality and consumption or leisure can result in a given shock having dramatically different welfare impacts.

In line with our focus on CGE models as a research tool we go on to provide step-by-step examples of model formulation, specification and calibration in the four archetypical research areas surveyed in the introduction:

(i). The consequences of liberalization and expansion of the economy for pollution and the demand for unpriced environmental resources (section 5.1),

(ii). The economic impacts of policies for environmental resource conservation (section 5.2) and pollution control (sections 5.4-5.7),

(iii). The impacts of changes in environmental conditions and availability of resources on the equilibrium of the economy (section 5.3), and

(iv). Full integration of an environmental amenity within the general equilibrium framework (section 5.8).

5.1 Environmental consequences of liberalization and growth

Our first example examines the consequences of economic liberalization and expansion for pollution and the consumptive use of environmental resources. Given the present closed-economy setting, it is only possible to simulate the effects of trade liberalization in the most heuristic of ways. In the case where the exogenous commodity demands $g_{i,O}$ represent net exports, the partial impacts of balance-of-payments shocks on goods market clearance and aggregate income can be modeled by applying augmentation factors $A_i^O > 0$ to the benchmark values of these demands in the SAM, i.e., $g_{i,O} = A_i^O \overline{g}_{i,O}$. Exogenous reductions in import demand and increases in export supply can therefore be modeled by setting $A_i^O \gtrless 1$ if $\overline{g}_{i,O} \gtrless 0$. Liberalization of the domestic economy is easily simulated by reducing the tax rates τ^Y and τ^F from their benchmark calibrated values—or, where these values are negative, setting them to zero to model the elimination of subsidies. Expansion of the economy is typically captured through increases in the supplies of labor and capital (indicated by the subscripts L and K), implemented by augmenting these components of the endowment vector relative to the values given by the SAM. This is easily accomplished by introducing aggregate productivity growth parameters $A_L^V, A_K^V > 1$, which relax the boundary condition for the economy: $V_f = A_f^V \overline{V}_f$ $(f = L, K)$. Efficiency-improving technical progress which reduces the industries' unit production costs can be modeled in the same fashion, by multiplying the sectoral cost functions by neutral productivity parameters $A_j^Y < 1$.

As mentioned earlier, the equilibrium in the SAM implies benchmark values of unity for the elements of $\mathbf{A}^O$, $\mathbf{A}^V$ and $\mathbf{A}^Y$. Perturbing these parameters and solving the model yields counterfactual values of industry outputs from which the new level of pollution and associated environmental damage, $\mathscr{D}(\mathcal{E})$, may be computed. Modeling the impacts of shocks on the use of unpriced environmental resources is more complicated because the failure of the SAM to record the value of these inputs implies that their use by firms entails an unmeasured cost of production. Although the symmetry of the SAM indicates that the latter constitutes missing factor returns, these neither redound to the households nor exert income effects on consumption. CRTS in market inputs therefore suggests that industries' production technology is subject to diminishing returns once non-market inputs are accounted for. Our exposition deals with the simplest case of a single non-market factor, whose effect on equilibrium may be captured by including an additional term in the sectoral cost functions. Then, assuming that the resource has a "virtual price" ϖ and a production coefficient δ_j, the industry zero profit and commodity market clearance conditions become:[10]

$$p_j \leq A_j^Y \left(\sum_i \beta_{n,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{1-\sigma_j} + \sum_f \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{1-\sigma_j} + \delta_j^{\sigma_j} \varpi^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \quad \perp y_j \qquad \text{(8.13a)}$$

$$y_i \geq \sum_j A_j^Y \beta_{i,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{-\sigma_j} p_j^{\sigma_j} y_j + \alpha_{i,C}^{\omega} \left((1+\tau_i^Y) p_i \right)^{-\omega} \mu^{\omega} u + g_{i,O} \quad \perp p_i \qquad \text{(8.13c)}$$

Now, by Shepard's Lemma, eq. (8.13a) implies the additional market clearance condition:

$$\varrho \geq \sum_j \delta_j^{\sigma_j} \varpi^{-\sigma_j} p_j^{\sigma_j} y_j \qquad \perp \varpi \qquad \text{(8.13g)}$$

In a manner analogous to eq. (8.10d), this expression casts the variable ϱ in the role of an

[10] These expressions highlight the need for care when modifying the equilibrium conditions of the model, as consistency among the conditions typically requires that a change in any one equation be matched by adjustments in one or more of the other model equations.

exogenously-determined endowment of the environmental resource. But our purposes require a different sort of boundary condition: to investigate how the use of the resource adjusts endogenously to shocks, we need to transform ϱ from a fixed factor into one that is in perfectly elastic supply at a constant virtual price. This price level is a fundamental unknown: zero is incompatible with the computation of general equilibrium because it permits the supply of the environmental factor—and therefore the size of the economy—to grow arbitrarily large, while economic theory provides no guidance on what the appropriate positive level should be. A reasonable assumption is that the private opportunity cost of a nominally unpriced input will be one or more orders of magnitude smaller than the prices of market goods. We therefore assume an arbitrary target price, denoted by the exogenous parameter $\underline{\varpi} \ll 1$, which constitutes a price constraint with respect to which ϱ exhibits complementary slackness. The assumption is that a virtual price higher than $\underline{\varpi}$ chokes off demand for the resource, so that either $\varpi > \underline{\varpi},\ \varrho = 0$ or $\varpi = \underline{\varpi},\ \varrho > 0$, expressed as the additional auxiliary equation:

$$\varpi \geq \underline{\varpi} \qquad \perp \varrho \qquad (8.13\text{h})$$

The CGE model with non-market resources is made up of the three conditions (8.13a), (8.13c), (8.13g) and (8.13h) in conjunction with eqs. (8.10b) and (8.10d)-(8.10f), which we re-label (8.13b) and (8.13d)-(8.13f). The resulting algebraic structure can be calibrated by applying the techniques in section 4 to a standard SAM, setting $\varrho = \underline{\varpi} = 0$. The technical coefficient on the non-market input may be calibrated using the share of the non-market factor in the true (i.e., market + non-market) cost of industry j's production, given by $\delta_j^{\sigma_j}/(1+\delta_j^{\sigma_j})$, whose value can be assumed or imputed from the results of valuation studies. The key numerical trick is to calibrate the implicit baseline quantity of resource use, $\overline{\varrho}$, while approximating the benchmark equilibrium in the SAM. This is done by solving the calibrated model with $\underline{\varpi}$ set close to zero (say 0.001). Counterfactual scenarios may then be simulated using the procedure outlined above, with ex-post evaluation focusing on the impact of shocks on ϱ relative to its baseline value. A damage function, $\mathscr{D}(\varrho)$, which is sufficiently steeply sloped admits the possibility that economic expansion facilitated by increased use of non-market environmental factors is welfare-diminishing.

5.2 Environmental resources: scarcity and conservation

Our second topical example is resource scarcity and management, which the model in eq. (8.13) is eminently suited to examine. Scarcity of the environmental factor is easily introduced by replacing the target price level in the auxiliary constraint (8.13h) with an upward-sloping supply curve, Ψ:

$$\varpi \geq \Psi(\varrho) \qquad\qquad \perp \varrho \qquad (8.13\text{h}')$$

The function Ψ may be specified is different ways to model a diverse array of economy-environment interactions, in particular the economic implications of critical thresholds in resource use. We may want to examine situations where exceeding such a threshold, indicated by the parameter $\varrho^{\star}$, causes the private opportunity cost of the environmental factor to rise steeply enough to exert a non-negligible drag on economic activity. A classic example is a fishery where rapid declines in fish stocks warrant increasing expenditures on search and harvesting effort. Parameterizing the effect of the threshold as

$$\Psi = \underline{\varpi} \left[1 + \psi_1 (\varrho/\varrho^{\star})^{\psi_2}\right], \quad \text{with} \quad 0 < \psi_1 \ll 1, \psi_2 \gg 1$$

leads to increases in ϖ inducing substitution of capital, labor and intermediate goods for fish stocks in fish production, and, ultimately, choking off demand for ϱ. The crucial piece of information provided by the model is how far in excess of the threshold this equilibrium lies.

Policies to manage resources may be modeled in an analogous fashion. A pigovian tax that narrows the gap between the private and social opportunity costs of the non-market resource can be introduced on the right-hand side of both eqs. (8.13h) and (8.13h′) through the addition of a tax parameter, τ^{ϱ}, whose value is specified by the analyst. This change necessitates a corresponding adjustment to the definition of income to account for the effects of the revenue raised by the tax on the consumption of the representative agent:

$$\mathscr{I}^D = \sum_f w_f V_f - \sum_i (1+\tau_i^Y) p_i g_{i,O} + \sum_f \tau_f^F w_f V_f + \sum_j \tau_j^Y p_j y_j + \tau^{\varrho} \varrho \quad \perp \mathscr{I}^D \qquad (8.13\text{f}')$$

The resource management CGE model is made up of (8.13a)-(8.13e), (8.13f′) and (8.13g), along with either (8.13h) or (8.13h′) inclusive of τ^{ϱ}. By solving for the counterfactual equilibria associated with a range of values of τ^{ϱ} and tracing out the resulting sectoral and aggregate reductions in the environmental factor, one can construct marginal abatement cost (MAC) curves which not only identify which industries respond most elastically to the tax but also indicate the direct cost of conservation.[11] The same procedure also makes it possible to characterize the dual welfare impacts of bringing non-market environmental factors within the ambit of the price system. In addition to the intuitive beneficial effect of reduced environmental demand on $\mathscr{D}$, there is the income effect of recycled tax revenue in eq. (8.13f′), whose impact is ambiguous. Resource tax payments constitute a new stream of income which offsets the rising price of market goods due to higher costs of environmental inputs, and attenuate the reduction in u. Simultaneously, however, increases in the unit production costs and declines in the output of relatively resource-intensive industries are likely to reduce revenue from pre-exisiting taxes on output, amplifying welfare losses.

This framework also allows us to compute the optimal pigovian tax, which should

[11]The total direct cost of conservation is approximated by the area under the MAC curve up to the observed quantity of resource reduction.

equal the marginal environmental damage of resource extraction, $\mathscr{D}'(\varrho)$. Somewhat heuristically, it is convenient to think of the tax itself as exhibiting complementary slackness with respect to this condition, which can be exploited to transform τ^{ϱ} into an endogenous variable through the following auxiliary equation:[12]

$$\tau^{\varrho} = \mathscr{D}'(\varrho) \qquad \perp \tau^{\varrho} \tag{8.13i}$$

In principle, including this condition as an additional component of the pseudo-excess demand correspondence allows the optimal value of τ^{ϱ} to be computed along with the other economic variables. However, in practice introducing such a constraint can make a CGE model difficult to solve.[13] A less elegant put potentially more effective alternative is to keep τ^{ϱ} exogenous, use brute-force computational power to repeatedly solve the model over a grid of values of this parameter, and finally select the value of the tax at which $(u - \mathscr{D})$ is maximized.

5.3 Climate impacts and environmental disasters

In contrast to the bi-directional interaction in the previous example, environmental changes are generally be modeled as exerting a one-way influence on the economy. These kinds of shocks can operate through several channels, including induced changes in consumer expenditure patterns (e.g., increased demand for space conditioning and residential energy use due to greater temperature extremes arising from climatic change), changes in the

[12]Of course, the real problem we wish to solve is $\max_{\varrho}(u - \mathscr{D})$, but the resulting first-order condition $u'(\varrho) = \mathscr{D}'(\varrho)$ cannot be used in eq. (8.13i) because no closed-form expression exists for the marginal utility of the resource.

[13]The underlying problem is $\max_{\tau^{\varrho}}(u - \mathscr{D})$ subject to (8.13a)-(8.13h'), which is known as a mathematic program with equilibrium constraints, or MPEC. Optimal tax problems of this kind pose a challenge for numerical optimization algorithms because the correspondence between u and τ^{ϱ} may not be smooth or even locally monotonic in the presence of pre-existing distortions $\boldsymbol{\tau}^{Y}$ and $\boldsymbol{\tau}^{F}$. One important reason is discrete jumps in the values of the tax revenue components of income induced by a marginal change in τ^{ϱ} as firms' and households' substitution patterns determine the economy's price and quantity allocations, which can seriously degrade the performance of gradient-based iterative schemes like Newton's method. The development of methods for the specification and solution of these sorts of problems is an active area of computational economic research.

productivity of primary factors in various industries (e.g., increases or decreases in crop yields due to shifting temperature or precipitation regimes), and reductions in the aggregate endowments of capital and labor (e.g., disaster-related property losses and morbidity or mortality). We capture these three types of influences within the model by introducing shock parameters A_i^C, $A_{f,j}^F$ and A_f^V into the representative agent's expenditure function, industry sectors' cost functions, and the market clearance condition for factors, respectively.

The second and third channels are important for modeling large shocks such as extreme climate impacts, the incidence of which is likely to be concentrated in a small number of vulnerable activities. The CES economy's treatment of inputs to production and consumption as fungible, combined with the assumption of perfect intersectoral factor mobility, suggests that a significant adverse shock to industrial productivity or factor supplies may generate only modest increases in the unit costs of exposed industries. By contrast, concerns about irreversible climate impacts center on *specific* inputs with limited substitution possibilities.[14] A convenient way of implementing the latter within a CGE model is to introduce sector-specific factors which are subject to environmental shocks. By designating a subset of primary factors as intersectorally immobile through the index $s \subset f$, whose elements may be assigned to sectors, $j'(s)$, we may perturb the endowments of specific factors in the same way as in the model of economic growth: $V_s = A_s^V \overline{V}_s$, except with $A_s^V < 1$. Letting $s' = j \backslash s$ indicate intersectorally mobile factors, the unit cost

[14]e.g., sites of special cultural, historical, ecological or aesthetic significance in the tourism sector, and specific combinations of climate and arable land in the production of various kinds of crops.

and expenditure functions and the factor market clearance condition become:

$$p_j \leq \left(\sum_i \beta_{n,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{1-\sigma_j} + \sum_f A_{f,j}^F \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \qquad \perp y_j \quad (8.14a)$$

$$\mu \leq \left(\sum_i A_i^C \alpha_{i,C}^{\omega} \left((1+\tau_i^Y) p_i \right)^{1-\omega} \right)^{1/(1-\omega)} \qquad \perp u \quad (8.14b)$$

$$y_i \geq \sum_j \beta_{i,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{-\sigma_j} p_j^{\sigma_j} y_j + A_i^C \alpha_{i,C}^{\omega} \left((1+\tau_i^Y) p_i \right)^{-\omega} \mu^{\omega} u + g_{i,O} \qquad \perp p_i \quad (8.14c)$$

$$V_f \geq \begin{cases} A_{s,j}^F \gamma_{s,j'}^{\sigma_{j'}} \left((1+\tau_s^F) w_s \right)^{-\sigma_{j'}} p_{j'}^{\sigma_{j'}} y_{j'} & s \subset f, j' = j'(s) \\ \sum_j A_{s',j}^F \gamma_{s',j}^{\sigma_j} \left((1+\tau_{s'}^F) w_{s'} \right)^{-\sigma_j} p_j^{\sigma_j} y_j & s' \subset f \end{cases} \qquad \perp w_f \quad (8.14d)$$

The environmental impact CGE model is made up of eqs. (8.14a)-(8.14d), with remaining conditions (8.14e) and (8.14f) identical to (8.10e) and (8.10f), and can be calibrated in the usual way with the parameters $\mathbf{A}^C$, $\mathbf{A}^F$ and $\mathbf{A}^V$ set to unity. In cases where the factor account in the SAM does not record the payments to sector-specific factors, it is customary for the latter to be imputed as a proportion of the benchmark capital remuneration in the relevant industries, often based on a combination of judgment and ancillary data. The productivity-reducing impacts of environmental shocks are simulated as increases in the values of various elements of A_i^C and $A_{f,j}^F$, and reductions in A_f^V, with the analyst controlling the sectoral specificity of the shock through the values of the parameters A_s^V. As this type of application typically does not consider damage to the environment, the welfare impact is easily calculated as the decline in u that results from the income effects of diminished factor supplies and the substitution effects of commodity price changes.

5.4 Pollution control: pigovian taxation

We now turn to a model of the aggregate costs of reducing pollution, using the classic case of a pigovian tax τ^{ε} as a starting point. Under the assumption that emissions are linked to output, τ^{ε} translates into a vector of industry-specific output tariffs $\phi_e \tau^{\varepsilon}$, each element of which constitutes an additional markup on the gross-of-tax consumer price of dirty good e, and generates a stream of tax receipts to households in the total amount $\sum_e \phi_e \tau_e^\varepsilon y_e$.

To incorporate the tax into the model in a way that minimizes notational clutter we introduce an additional variable, $\widehat{p}_j$, to denote the pigovian tax-inclusive consumer price of commodity j. For polluting goods, $\widehat{p}_e = (1+\tau_e^Y)p_e + \phi_e \tau^\varepsilon$, while for the subset of clean goods which emit no pollution, indexed by $e' = i \backslash e$, the consumer price is the same as in eq. (8.10): $\widehat{p}_{e'} = (1+\tau_{e'}^Y)p_{e'}$. In the same way that each industry's activity level exhibits complementary slackness with respect to its zero-profit condition, we must introduce an additional variable, $\widehat{y}_j$, which indicates the aggregate demand for j's output and is complementary to the definition of $\widehat{p}_j$. Furthermore, in the same way that each commodity's price exhibits complementary slackness with respect to its market-clearance condition, we must explicitly state that $\widehat{p}_i$ is complementary to the aggregate supply-demand balance for good i: $y_i = \widehat{y}_i$. Intuitively, we can think of $\widehat{y}_e$ as representing the activity level of a vacuous "toll-booth" sector whose sole purpose is to convert each unit of untaxed dirty good at a price p_e into the same quantity of taxed dirty good at a price $(1+\tau_e^Y)p_e + \phi_e \tau^\varepsilon$. These adjustments imply the new pseudo-excess demand correspondence:

$$p_j \leq \left(\sum_i \beta_{i,j}^{\sigma_j} \widehat{p}_i^{1-\sigma_j} + \sum_f \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \quad \perp y_j \tag{8.15a}$$

$$\mu \leq \left(\sum_i \alpha_{i,C}^{\omega} \widehat{p}_i^{1-\omega} \right)^{1/(1-\omega)} \quad \perp u \tag{8.15b}$$

$$\widehat{y}_i \geq g_{i,O} + \sum_j \beta_{i,j}^{\sigma_j} \widehat{p}_i^{-\sigma_j} p_j^{\sigma_j} y_j + \alpha_{i,C}^{\omega} \widehat{p}_i^{-\omega} \mu^\omega u \quad \perp p_i \tag{8.15c}$$

$$\mathscr{I}^D = \sum_f w_f V_f - \sum_i \widehat{p}_i g_{i,O} + \sum_f \tau_f^F w_f V_f + \sum_j \tau_j^Y p_j y_j + \sum_e \phi_e \tau^\varepsilon \widehat{y}_e \quad \perp \mathscr{I}^D \tag{8.15f}$$

with the remaining conditions (8.15d) and (8.15e) identical to eqs. (8.10d) and (8.10e), and the two additional equations:

$$\widehat{p}_j \leq \begin{cases} (1+\tau_e^Y)p_e + \phi_e \tau^\varepsilon & e \subset j \\ (1+\tau_{e'}^Y)p_{e'} & e' \subset j \end{cases} \qquad \perp \widehat{y}_j \qquad (8.15g)$$

$$y_i = \widehat{y}_i \qquad \perp \widehat{p}_i \qquad (8.15h)$$

which together make up the pigovian pollution control CGE model. Per the discussion in section 5.1, τ^ε may be varied over a range of values to generate equilibria which can be employed to compute MAC curves for pollution as well as the optimal tax on emissions.

5.5 Pollution control: quantitative emission targets

A simple modification transforms the previous model into a test-bed for the analysis of quantitative pollution control measures such as performance standards and tradable permit systems. We introduce the primal variable $\underline{\mathcal{E}}$ to indicate the constrained aggregate pollution load associated with the dual pigovian tax. This allows us interpret the tax as the Lagrange multiplier on a limit on emissions, $\sum_e \phi_e \widehat{y}_e \leq \underline{\mathcal{E}}$, which is essentially a market clearance condition for pollution with respect to which τ^ε exhibits complementary slackness:

$$\sum_e \phi_e \widehat{y}_e \leq \underline{\mathcal{E}} \qquad \perp \tau^\varepsilon \qquad (8.15i)$$

This expression enables the analyst to specify the emission limit $\underline{\mathcal{E}}$ as a policy parameter, which in turn transforms the tax into an endogenous variable, the implicit "quota-equivalent" price of pollution. Finally, notice that the performance target could as easily have been specified in terms of the pollution intensity of aggregate income by dividing the left-hand side of (8.15i) by $\mathscr{I}^D$.

The variable τ^ε can also be interpreted as the endogenous market-clearing price of emission allowances in an economy-wide cap-and-trade scheme. Interestingly, in the present representative-agent setting the two main methods for allocating allowances—auctioning by the government and grandfathering to firms—are identical in their algebraic expression and economic impacts. Grandfathering allowances is equivalent to defining a new factor of production, which, while notionally improving firms' profitability, is actually owned by their shareholders, so that the returns accrue as income to the representative agent. Likewise, auctioning allowances generates additional government revenue which is then immediately recycled to the representative agent in a lump sum. This is the fundamental symmetry of general equilibrium: a given value of τ^{ε}, or the constrained pollution level $\underline{\mathcal{E}}$ consistent with this value, will generate identical general equilibrium allocations of emissions, production activity, prices, and welfare.

5.6 Pollution control: elastic factor supply and the double dividend

Up until this point in the chapter taxes on primary factors have played a minor role in the determination of equilibrium. The reason is that all of the models thus far assume that factors of production are in perfectly inelastic supply, with the result that factor taxes have no income effects, and only serve to distort industries' demands for different factors in accordance with the relative magnitudes of the elements of $\boldsymbol{\tau}^F$.[15] This is highly unrealistic, especially in the case of labor, a fact which is the key to the voluminous literature on the "double dividend" of environmental taxation. Payroll taxes distort labor supply decisions by inducing households to over-allocate their endowment of time to the consumption of leisure. However, the revenue raised by taxing pollution may be used to partially replace payroll tax receipts, facilitating revenue-neutral reductions in payroll taxes which generate welfare improvements that offset the macroeconomic costs of abating pollution. In this setting the double dividend refers to the dual benefits of reduced environmental

[15] Full employment is known as a neoclassical macro closure rule. The alternative Keynesian closure rule assumes perfectly elastic labor supply with the wage which fixed at its benchmark level. See, e.g., Robinson (2006).

damage and the efficiency gain from labor-market liberalization. The magnitude of the latter depends crucially on the tax elasticity of labor supply, which in a CGE model is determined by the coefficient on leisure and the substitutability of leisure for goods in consumers' utility functions.

To simulate the choice between leisure and supplying labor into the framework of the pigovian CGE model, we assume that the representative agent possesses an exogenous endowment of time, $\mathcal{T}$, of which an endogenous quantity ℓ is consumed as leisure. This implicitly defines the economy's aggregate labor endowment as an endogenous variable, $\mathcal{T} \geq V_L + \ell$. Leisure substitutes for the consumption of commodities in the generation of utility, which we model by specifying the representative agent as having a two-tiered nested utility function whose lower tier is a CES sub-utility function denominated over goods consumption similar to eq. (8.15b), and whose upper tier is a CES aggregation of consumption of commodities and leisure. The upper tier's indexes of activity and unit expenditure are u and μ, as in (8.15), and we assume that leisure has a technical coefficient λ an elasticity of substitution ξ with the composite of consumed goods. The lower tier's indexes of activity and unit expenditure are $\widehat{u}$ and $\widehat{\mu}$. The upper and lower tiers of the corresponding dual expenditure function are complementary to the activity levels u and $\widehat{u}$, while the supply-demand balance for $\widehat{u}$ is complementary to $\widehat{\mu}$, the demand for utility u is the same as eq. (8.15e), and the supply-demand balance for leisure is complementary to its own activity level. The distorting effect of a payroll tax (τ_L^F) is the wedge that it drives between the consumer prices of labor and leisure: the representative agent demands leisure at the pre-tax wage while firms demand labor at the tax-inclusive wage. The implication is that the value of the representative agent's endowment of leisure is $w_L \ell$, while her income from wages and recycled payroll tax receipts are $w_L(\mathcal{T} - \ell)$ and $\tau_L^F w_L(\mathcal{T} - \ell)$.

The CGE model with a pigovian pollution tax and labor-leisure choice is given by the following equations:

$$\widehat{\mu} \leq \left(\sum_i \alpha_{i,C}^{\omega} \widehat{p}_i^{1-\omega} \right)^{1/(1-\omega)} \qquad \perp \widehat{u} \qquad (8.16b)$$

$$\left. \begin{aligned} V_f &\geq \sum_j \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{-\sigma_j} p_j^{\sigma_j} y_j \qquad f \neq L \\ \mathcal{T} &\geq \sum_j \gamma_{L,j}^{\sigma_j} \left((1+\tau_L^F) w_L \right)^{-\sigma_j} p_j^{\sigma_j} y_j + \ell \end{aligned} \right\} \qquad \perp w_f \qquad (8.16d)$$

$$\begin{aligned} \mathscr{I}^D = \sum_{f \neq L} w_f V_f - \sum_i \widehat{p}_i g_{i,O} + \sum_{f \neq L} \tau_f^F w_f V_f + \sum_j \tau_j^Y p_j y_j \\ + w_L \mathcal{T} + \tau_L^F w_L (\mathcal{T} - \ell) + \sum_e \phi_e \tau^{\varepsilon} \widehat{y}_e \end{aligned} \qquad \perp \mathscr{I}^D \qquad (8.16f)$$

$$\mu \leq \left(\lambda^{\xi} w_L^{1-\xi} + (1-\lambda)^{\xi} \widehat{\mu}^{1-\xi} \right)^{1/(1-\xi)} \qquad \perp u \qquad (8.16i)$$

$$\widehat{u} \geq (1-\lambda)^{\xi} \widehat{\mu}^{-\xi} \mu^{\xi} u \qquad \perp \widehat{\mu} \qquad (8.16j)$$

$$\ell \geq \lambda^{\xi} w_L^{-\xi} \mu^{\xi} u \qquad \perp \ell \qquad (8.16k)$$

along with eqs. (8.15a), (8.15c), (8.15e) and (8.15g)-(8.15h), which we re-label (8.16a), (8.16c), (8.16e) and (8.16g)-(8.16h). The double dividend is modeled by reducing the labor tax burden to the point where the additional pollution tax revenue just offsets the loss in payroll tax revenue, so that the sum of pollution and labor tax receipts remains at the benchmark level, $\overline{T}_L$. As this condition is satisfied by appropriately decrementing the labor tax rate for a given value of τ^{ε}, we model τ_L^F as an endogenous variable which exhibits complementary slackness with respect to the revenue neutrality constraint, which is implemented as an auxiliary equation:

$$\overline{T}_L = \tau_L^F w_L (\mathcal{T} - \ell) + \sum_e \phi_e \tau^{\varepsilon} \widehat{y}_e \qquad \perp \tau_L^F \qquad (8.16l)$$

The critical uncalibrated parameters in this model are the benchmark time endowment, the coefficient on leisure and the goods-leisure substitution elasticity, whose values are traditionally calculated from ancillary data on the uncompensated labor supply elasticity, η_L^S, and the ratio of the representative agent's time endowment to her labor supply, $\mathcal{R} = \mathcal{T}/(\mathcal{T} - \ell)$. Multiplying the numerator and denominator of $\mathcal{R}$ by the wage yields the benchmark values of the time endowment and leisure demand, $\overline{\mathcal{T}} = \mathcal{R}\overline{V}_L$ and $\overline{\ell} = (\mathcal{R} - 1)\overline{V}_L$. Using eqs. (8.16d) and (8.16k), the labor supply elasticity can be expressed as:

$$\eta_L^S = \frac{w_L}{\mathcal{T} - \ell} \frac{\partial(\mathcal{T} - \ell)}{\partial w_L} = \xi \left(\frac{\ell}{\mathcal{T} - \ell} \right) \left[\frac{(1-\lambda)^\xi \widehat{\mu}^{1-\xi}}{\lambda^\xi w_L^{1-\xi} + (1-\lambda)^\xi \widehat{\mu}^{1-\xi}} \right],$$

in which the fraction in parentheses is $\mathcal{R} - 1$, the ratio of the benchmark values of leisure and labor income, and the term in square braces is the share of commodities in the value of total (leisure plus non-leisure) consumption, which at benchmark prices is simply $\overline{G}_C/(\overline{G}_C + w_L\ell) = \overline{G}_C/\left(\overline{G}_C + (\mathcal{R} - 1)\overline{V}_L\right)$. The elasticity of goods-leisure substitution and the coefficient on leisure may then be calibrated as:

$$\xi = \eta_L^S \frac{\overline{G}_C + (\mathcal{R} - 1)\overline{V}_L}{(\mathcal{R} - 1)\overline{G}_C} \quad \text{and} \quad \lambda = \left[\frac{(\mathcal{R} - 1)\overline{V}_L}{\overline{G}_C + (\mathcal{R} - 1)\overline{V}_L} \right]^{1/\xi}.$$

We close our discussion of this topic with two observations. First, the time endowment ratio is a key uncertain parameter, so that given its importance for the value of the calibrated parameters—and, ultimately, the magnitude of the double dividend—it is frequently the object of sensitivity analysis.[16] Second and more substantively, implicit in the foregoing analysis is the strong assumption of no *in*voluntary unemployment, which in CGE models is typically the result of wage rigidities that drive a wedge between households' time endowment and their labor supply. Crucially, there is no utility benefit associated with these non-leisure, non-work hours, which we denote $\mathcal{T}^U$.

Perhaps the simplest way of operationlizing involuntary unemployment is to spec-

[16] Ballard et al. (1985, p. 135) set $\mathcal{R} = 1.75$ "...to reflect that individuals typically work a forty-hour, out of a possible seventy-hour week." Ballard (2000) shows that in CGE studies this parameter can range from as low as 1.5 to as high as 5, and develops a calibration technique which uses econometrically-estimated values of compensated and uncompensated labor supply elasticities in place of $\mathcal{R}$.

ify the quantity of "dead time" as the dual of a minimum wage, $\underline{w}_L$, via the auxiliary complementarity condition:[17]

$$w_L \geq \underline{w}_L \qquad \perp \mathcal{T}^U \qquad (8.16\text{m})$$

Integrating this with our previous formulation necessitates an adjustment of the market clearance condition for hours (8.16d) to accommodate unemployed time:

$$\mathcal{T} \geq \sum_j \gamma_{L,j}^{\sigma_j} \left((1+\tau_L^F)w_L\right)^{-\sigma_j} p_j^{\sigma_j} y_j + \ell + \mathcal{T}^U \qquad \perp w_f \qquad (8.16\text{d}')$$

and also a change in the calibration formulas. The attractive feature of this approach is that ancillary information on the level of unemployment, $\mathscr{U}$, may be used to approximate the unobserved quantity of unemployed time in the benchmark as $\overline{\mathcal{T}}^U = \mathscr{U}/(1-\mathscr{U})\overline{V}_L$. From there, a redefinition of the time-endowment ratio to include unemployed time, $\mathcal{R} = \mathcal{T}/(\mathcal{T} - \mathcal{T}^U - \ell)$, allows the representative agent's endowment of time to remain the same, with a reduced initial leisure endowment: $\overline{\ell} = (\mathcal{R} - \frac{\mathscr{U}}{1-\mathscr{U}} - 1)\overline{V}_L$. Updated values for the goods-leisure elasticity and the coefficient on leisure may then be derived by following the calibration procedure outlined above.

5.7 Pollution control: technology policies

Another shortcoming of the canonical model of pollution control is its implicit assumption of a Leontief (fixed-coefficients) relationship between pollution and output. While this formulation correctly represents the stoichiometric relationship between carbon dioxide and fossil fuels in analyses of climate policy (e.g., Goulder et al., 1999), is not generally applicable to most pollutants, which are often capable of being at least partially abated through investments in pollution control. Availability of the control option means that the e^{th} polluting industry faces the choice of either abating pollution and associated tax

[17] For an alternative specification, see Balistreri (2002).

liabilities on residual emissions by reducing output—and revenue, or installing pollution control capital. Cost minimization implies that the industry will choose to undertake the latter up to the point where the marginal cost of installation equals the marginal revenue savings from the ability to expand output.

Representing the outcome of this process requires us to specify the pollution control technology, which for the sake of simplicity we model as a CRTS Leontief transformation technology which combines inputs of abatement capital and the dirty good to produce "remediated", i.e., pollution-free, output. Abatement generated by tax-induced output reductions and by pollution control are assumed to be perfect substitutes. Therefore the control activity is modeled as demanding inputs at their ad-valorem tax-inclusive consumer prices while supplying output at the pigovian tax-inclusive price of the dirty good in question. Assuming a production coefficient χ on the use of capital for pollution control, this formulation implies the zero profit condition $\widehat{p}_e \leq \chi(1+\tau_K^F)w_K + (1-\chi)(1+\tau_e^Y)p_e$. As in prior models, this condition must be specified as complementary to a primal activity level. We therefore introduce the variable $\widehat{q}_e$ to indicate the operating activity of sectors' pollution controls, which in turn determines the input demands for capital and the dirty good, χq_e and $(1-\chi)q_e$.

A few points about this formulation are worth noting. The most important is that the pollution control activity competes with the dummy toll-booth activity mentioned above, as they both demand the output of dirty sector e as an input while their outputs jointly satisfy aggregate demand for dirty good e. The effect of pollution control will be to shift industry e's MAC curve downward over some portion of its domain, but the specifics depend on the capital intensity of the control technology (determined by χ) as well as ancillary information, without which it is not possible to make *a priori* predictions about the extent to which $\widehat{q}_e$ will displace $\widehat{y}_e$. The latter is found by simulating the model, which is easiest to do in the case where pollution is uncontrolled in the benchmark equilibrium. This sidesteps the potentially significant complications that attend the calibration of pollution control technology as a sub-component of the dirty sectors.[18] But in either situation care must be exercised in accounting for emissions: here the toll-booth sector should be thought of as the source of pollution, in the form of residual emissions released after each dirty industry abates up to the point where the marginal cost of doing so equals the pigovian tax.[19]

We therefore specify the model with the pollution control activity as a counterfactual alternative, by replacing eqs. (8.15c), (8.15d) and (8.15h) with

$$\left.\begin{array}{lll} \widehat{q}_e + \widehat{y}_e & \geq g_{e,O} + \sum_j \beta_{e,j}^{\sigma_j} \widehat{p}_e^{-\sigma_j} p_j^{\sigma_j} y_j + \alpha_{e,C}^{\omega} \widehat{p}_e^{-\omega} \mu^{\omega} u & e \subset i \\ \widehat{y}_{e'} & \geq g_{e',O} + \sum_j \beta_{e',j}^{\sigma_j} \widehat{p}_{e'}^{-\sigma_j} p_j^{\sigma_j} y_j + \alpha_{e',C}^{\omega} \widehat{p}_{e'}^{-\omega} \mu^{\omega} u & e' \subset i \end{array}\right\} \perp p_i \quad (8.17c)$$

$$V_f \geq \begin{cases} \gamma_{K,j}^{\sigma_j} \left((1+\tau_K^F) w_K\right)^{-\sigma_j} p_j^{\sigma_j} y_j + \sum_e \chi \widehat{q}_e & \\ \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f\right)^{-\sigma_j} p_j^{\sigma_j} y_j & f \neq K \end{cases} \perp w_f \quad (8.17d)$$

$$y_j = \begin{cases} \widehat{y}_e + (1-\chi)\widehat{q}_e & e \subset j \\ \widehat{y}_{e'} & e' \subset j \end{cases} \perp \widehat{p}_j \quad (8.17h)$$

and including the remaining equations (8.15a), (8.15b), (8.10e), (8.15f), and (8.15g)—which we relabel (8.17a), (8.17b), (8.17e), (8.17f) and (8.17g), along with the additional condition of zero profit in pollution control

$$\widehat{p}_e \leq \chi(1+\tau_K^F) w_K + (1-\chi)(1+\tau_e^Y) p_e \perp \widehat{q}_e \quad (8.17i)$$

The present structure opens up a wealth of possible applications. Especially relevant in the context of U.S. environmental policy is the general equilibrium benefit-cost assessment of controlling pollution through technology mandates. This instrument is modeled

[18] See, e.g., the methodology developed by Hyman et al. (2003) to calibrate technologies for non-CO_2 GHG emission control using marginal abatement cost curves derived from bottom-up engineering analyses.

[19] This is the reason for *not* specifying the left-hand side of eq. (8.15i) as $\sum_e \phi_e y_e$, which in the present setting overstates emissions.

by replacing eq. (8.15i) with a constraint that specifies an upper bound, say ϑ, on the fraction of each dirty industry's output which may be produced without control technology. The associated complementary variable is the shadow price on the standard, $\tau_e^{\widehat{q}}$, which may be interpreted as an implicit subsidy on the (more expensive) production with pollution control. The associated rents represent a transfer of revenue from dirty production to the pollution control activity, and are revenue-neutral with respect to both the government and the income balance of households. We therefore drop the terms in τ^{ε} from the tax-inclusive goods price and income definition equations (8.15g and 8.15f), and add the cost-reducing effect of the subsidy to the pollution control activity's zero-profit condition (8.17h). The resulting technology mandate model is made up of eqs. (8.17a)-(8.17e) and (8.17h), along with the following equations:

$$\mathscr{I}^D = \sum_f w_f V_f - \sum_i \widehat{p}_i g_{i,O} + \sum_f \tau_f^F w_f V_f + \sum_j \tau_j^Y p_j y_j \qquad \perp \mathscr{I}^D \qquad (8.17\text{f}')$$

$$\widehat{p}_j \leq (1 + \tau_j^Y) p_j \qquad \perp \widehat{y}_j \qquad (8.17\text{g}')$$

$$\widehat{p}_e \leq \chi(1 + \tau_K^F) w_K + (1 - \chi)(1 + \tau_e^Y) p_e - \tau_e^{\widehat{q}} \qquad \perp \widehat{q}_e \qquad (8.17\text{i}')$$

$$\widehat{q}_e \geq \vartheta(\widehat{q}_e + \widehat{y}_e) \qquad \perp \tau_e^{\widehat{q}} \qquad (8.17\text{j}')$$

Lastly, it is also possible to extend the basic pollution control model to a encompass a range of discrete technology options, some of which may be operated in the benchmark equilibrium, some of which may be unprofitable and inactive—as in the present example—but may become active in response to changes in relative prices induced by policy or other kinds of shocks. While the latter are comparatively easy to model, and indeed are a mainstay of prospective technological analyses (e.g., Sue Wing, 2006), the former are more difficult to calibrate because of the need to reconcile the economic accounts in the SAM with often incommensurate engineering descriptions of technologies' operating characteristics and performance (e.g., Sue Wing, 2008; Böhringer and Rutherford, 2008).

5.8 A non-separable environmental amenity

Our final example illustrates the challenges which attend modeling of both the benefits as well as the costs of pollution control when the latter are non-separable from consumption. Following Carbone and Smith (2008), we introduce an environmental amenity as a public good within the general equilibrium framework of (8.10). The amenity (in Carbone and Smith, air quality) has an activity level $\mathscr{A}$, defined as the inverse of the aggregate pollution load ($1/\mathcal{E}$), and a dual marginal willingness to pay, or virtual price, θ. To keep the exposition simple we assume that the amenity is a substitute for aggregate goods consumption, and enters the utility function in the same manner as leisure in the model of eq. (8.16). In the resulting two-level nested CES utility function, we use ζ to denote the elasticity of substitution between goods and the amenity and ν to denote technical coefficient on the amenity. Finally, the value of the amenity to the representative agent is reflected as an additional term in the income definition equation, which points to the interpretation of $\mathscr{I}^D$ as "virtual income", i.e., market income plus the value of environmental quality.

The non-separable amenity CGE model is made up of the following equations,

$$\widehat{\mu} \leq \left(\sum_i \alpha_{i,C}^{\omega} \left((1+\tau_i^Y) p_i \right)^{1-\omega} \right)^{1/(1-\omega)} \qquad \perp \widehat{u} \qquad (8.18\text{b})$$

$$\mathscr{I}^D = \sum_f w_f V_f - \sum_i (1+\tau_i^Y) p_i g_{i,O} + \sum_f \tau_f^F w_f V_f + \sum_j \tau_j^Y p_j y_j + \theta \mathscr{A} \qquad \perp \mathscr{I}^D \qquad (8.18\text{f})$$

$$\mu \leq \left(\nu^{\zeta} \theta^{1-\zeta} + (1-\nu)^{\zeta} \widehat{\mu}^{1-\zeta} \right)^{1/(1-\zeta)} \qquad \perp u \qquad (8.18\text{g})$$

$$\widehat{u} \geq (1-\nu)^{\zeta} \widehat{\mu}^{-\zeta} \mu^{\zeta} u \qquad \perp \widehat{\mu} \qquad (8.18\text{h})$$

$$\mathscr{A} \geq \nu^{\zeta} \theta^{-\zeta} \mu^{\zeta} u \qquad \perp \theta \qquad (8.18\text{i})$$

$$\mathscr{A} = \left(\sum_e \phi_e y_e \right)^{-1} \qquad \perp \mathscr{A} \qquad (8.18\text{j})$$

in addition to eqs. (8.10a) and (8.10c)-(8.10e), which we relabel (8.18a) and (8.18c)-(8.18e). In contrast to the double-dividend model, calibrating the present structure requires that the analyst make a number of ad-hoc assumptions. Chief among these is the substitutability of the amenity for goods consumption, which is currently a wide open empirical question that renders the value of ζ a matter of judgment.[20] The public good character of the amenity implies that its benchmark virtual price $\overline{\theta}$ is the value of marginal willingness to pay for the amenity (e.g., as estimated by econometric studies), summed over the number of individuals or households in the economy, while the benchmark quantity of the amenity, $\overline{\mathscr{A}}$, is simply the inverse of the ambient pollution load estimated for the benchmark year. The technical coefficient on the amenity may then be calibrated as $\nu = \left(\overline{\theta \mathscr{A}} / (\overline{G}_C + \overline{\theta \mathscr{A}})\right)^{1/\zeta}$.

In this example, increasing taxes on the output of dirty industries $e \subset i$ from their benchmark levels (say to some $\tau_e^{Y'} > \tau_e^Y$) will by (8.18j) and the last term in (8.18f) generate an extra positive income effect that offsets the increases in dirty commodities' consumer prices, potentially creating a net welfare gain. The payoff to this integrated approach is that it obviates an external environmental damage function. Indeed, just about any analogue of $\mathscr{D}(\mathcal{E})$ can be used to construct eq. (8.18j). Moreover, studies such as Böhringer et al. (2007) demonstrate the potential for replacing this expression in its entirety with the output of a detailed natural-science simulation run in tandem with the CGE model, in a scheme where the two models pass values of $\mathcal{E}$ and $\mathscr{A}$ back and forth to one another in an iterative fashion until their solutions converge. The environment in Figure 1 is represented by the natural-science simulation, which computes the physically-derived quantity of the amenity based on inputs of the pollution load generated by the CGE model (H1). The quantity of the amenity thus computed is then used as an input to the next iteration of the CGE model (H2). Such a scheme is made necessary by the feedback of the income and substitution effects of changes in $\mathscr{A}$ on the demand for dirty goods, the activity levels of dirty industries, and, ultimately, the level of pollution.

[20] It seems reasonable to assume $0 < \zeta < 1$, which ensures that goods and the amenity are both necessary inputs to consumption.

6 Summary, Caveats and Future Research Directions

This chapter has provided a lucid, rigorous and hands-on introduction to the fundamentals of computable general equilibrium models and their application in natural resource and environmental economics. The objective has been to de-mystify CGE models by developing a simple, transparent and comprehensive framework within which to conceptualize their structural underpinnings, numerical parameterization, mechanisms of solution and techniques of application. The circular flow of the economy was used as the starting point to develop the equilibrium conditions of a CGE model, and it was demonstrated how imposing the axioms of producer and consumer maximization on this conceptual edifice facilitate the complete algebraic specification of an economy of arbitrary sectoral dimension. There followed a description of the techniques of numerical calibration and an overview of the general procedures for the use of the resulting numerical model as an analytical tool.

The meat of the chapter focused on techniques of application, illustrating a range of structural modifications that enable CGE models to project the implications of changes in the circular flow for environmental quality or resource use, as well as to analyze the economy-wide impacts of environmental change and resource scarcity, and to evaluate the incidence of policies for limiting pollution and conserving environmental resources. Throughout, the exposition has been kept deliberately simple for the sake of practicality: with a only minimal data gathering beyond the SAM presented in Sue Wing (2009, Fig. 14.6), it is a straightforward task to specify all of the models discussed herein a high-level language such as GAMS (Brooke et al., 1998) and and solve the resulting numerical problems as an MCP. Moreover, different elements of the template models showcased above may be easily combined, facilitating the analysis of a range of problems arising in natural resource and environmental economics.

But in spite of the broad swath of intellectual territory covered by this chapter, space constraints have precluded discussion of many important topics. The chapter's closed-economy focus has paid scant attention to the specification and calibration of multi-region models which combine SAMs for individual economies with data on trade flows, and their application to transboundary environmental issues such as policy coordination, abatement coalition formation, and emission leakage. We have also given short shrift to the *intra*-regional distribution of the burden of environmental protection or resource conservation, which can be implemented by generalizing our simple CES economy to incorporate multiple households with different levels of income and endowment of traditional and environmental factors. As well, our models' maintained assumption of frictionless competitive markets has precluded consideration of the important role of market imperfections—particularly imperfect competition—in influencing both externalities and the level and distribution of the economy-wide costs of internalizing them. The multiplicity of methods available for representing imperfect competition within CGE models (Conrad, 2002), coupled with the divergence of the results generated by different formulations (Roson, 2006), highlights the need for methodological reconciliation. An even less adequately served area of study is the analysis of natural resource depletion using forward-looking CGE models specified in the complementarity format outlined here (cf. Lau et al., 2002), into which the Hotelling model has not yet been incorporated. Calibrating such a model is especially challenging, so much so that prior analyses have relied either on recursive-dynamic simulations (e.g., Bailey and Clarke, 2000), or dynamic models with alternative specifications of the depletion process (Babiker et al., 2009). A final research frontier which lies well beyond the scope of this review is the application of the general equilibrium framework to environmental science, in particular the study of trophic interactions within ecosystems (Tschirhart, 2000), which raises the exciting prospect of the development of integrated economic-ecological CGE models (e.g., Tschirhart, 2003). Hopefully, the base of practical and theoretical knowledge developed here helps lay the groundwork for the reader to go on to apply CGE models in these and other areas of environmental and resource economics.

Figure 1: The Circular Flow of the Economy, with Environmental Interactions

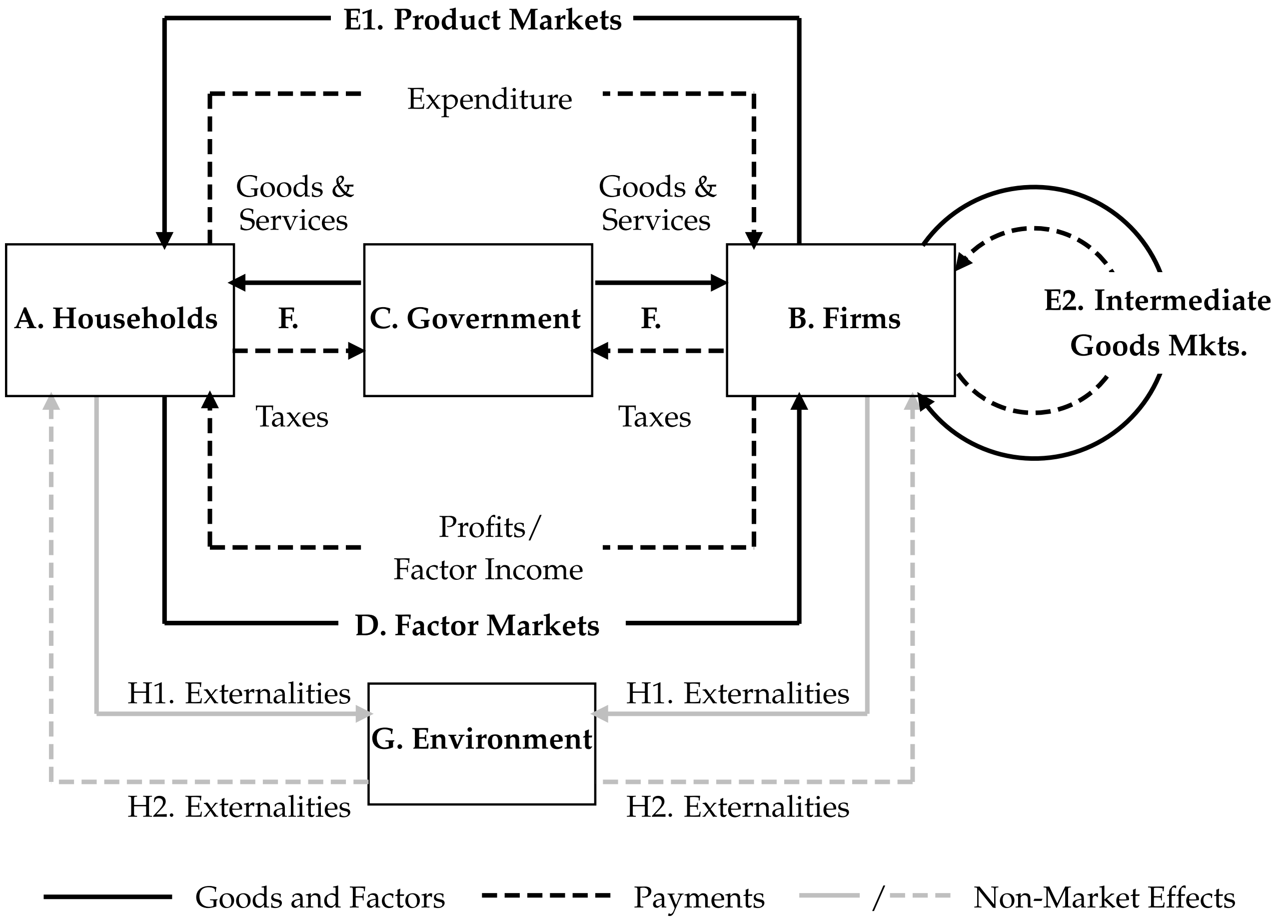

Figure 2: A Social Accounting Matrix

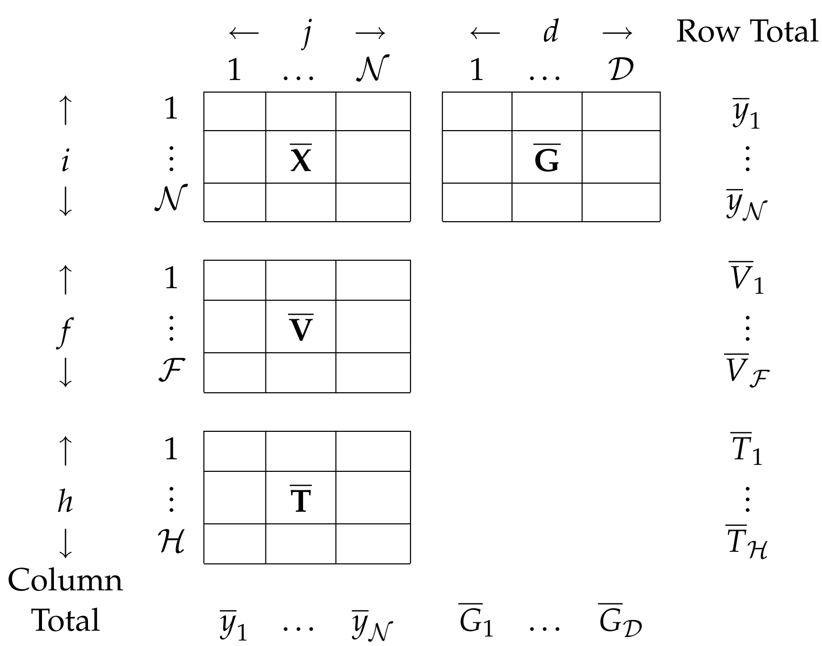

Table 1: The Equations of the CGE Model

Zero Profit:

$$p_j \leq \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{1-\sigma_j} \right)^{1/(1-\sigma_j)}, \quad y_j \geq 0, \quad y_j \left[p_j - \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \right] = 0 \tag{8.10a}$$

$$\mu \leq \left(\sum_{i=1}^{\mathcal{N}} \alpha_{i,C}^{\omega} \left((1+\tau_i^Y) p_i \right)^{1-\omega} \right)^{1/(1-\omega)}, \quad u \geq 0, \quad u \left[\mu - \left(\sum_{i=1}^{\mathcal{N}} \alpha_{i,C}^{\omega} p_i^{1-\omega} \right)^{1/(1-\omega)} \right] = 0 \tag{8.10b}$$

Market Clearance:

$$y_i \geq \sum_{j=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{-\sigma_j} p_j^{\sigma_j} y_j + \alpha_{i,C}^{\omega} \left((1+\tau_i^Y) p_i \right)^{-\omega} \mu^{\omega} u + g_{i,O}, \quad p_i \geq 0, \quad p_i \left[y_i - \sum_{j=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} \left((1+\tau_i^Y) p_i \right)^{-\sigma_j} p_j^{\sigma_j} y_j - \alpha_{i,C}^{\omega} \left((1+\tau_i^Y) p_i \right)^{-\omega} \mu^{\omega} u - g_{i,O} \right] = 0 \tag{8.10c}$$

$$V_f \geq \sum_{j=1}^{\mathcal{N}} \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{-\sigma_j} p_j^{\sigma_j} y_j, \quad w_f \geq 0, \quad w_f \left[V_f - \sum_{j=1}^{\mathcal{N}} \gamma_{f,j}^{\sigma_j} \left((1+\tau_f^F) w_f \right)^{-\sigma_j} p_j^{\sigma_j} y_j \right] = 0 \tag{8.10d}$$

$$u \geq \mathscr{I}^D / \mu, \quad \mu \geq 0, \quad \mu \left[u - \mathscr{I}^D / \mu \right] = 0. \tag{8.10e}$$

Income Definition:

$$\mathscr{I}^D = \sum_{f=1}^{\mathcal{F}} w_f V_f - \sum_{i=1}^{\mathcal{N}} p_i g_{i,O} + \sum_{f=1}^{\mathcal{F}} \tau_f^F w_f V_f + \sum_{j=1}^{\mathcal{N}} \tau_j^Y p_j y_j, \quad \mathscr{I}^D \geq 0, \quad \mathscr{I}^D \left[\mathscr{I}^D - \sum_{f=1}^{\mathcal{F}} w_f V_f + \sum_{i=1}^{\mathcal{N}} p_i g_{i,O} - \sum_{f=1}^{\mathcal{F}} \tau_f^F w_f V_f - \sum_{j=1}^{\mathcal{N}} \tau_j^Y p_j y_j \right] = 0. \tag{8.10f}$$

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