Risk, Returns, and Multinational Production[*]

José L. Fillat[†]   Stefania Garetto[‡]

Federal Reserve Bank of Boston   Boston University

July 9, 2015

Abstract

This paper starts by unveiling a strong empirical regularity: multinational corporations exhibit higher stock market returns and earning yields than non-multinational firms. Within non-multinationals, exporters exhibit higher earning yields and returns than firms selling only in their domestic market. To explain this pattern, we develop a real option value model where firms are heterogeneous in productivity, and have to decide whether and how to sell in a foreign market where demand is risky. Selling abroad is a source of risk exposure to firms: following a negative shock, they are reluctant to exit the foreign market because they would forgo the sunk cost that they paid to enter. Multinational firms are the most exposed due to the higher costs they have to pay to invest. The calibrated model is able to match both aggregate US export and foreign direct investment data, and the observed cross-sectional differences in earning yields and returns.

[†]Contact: Federal Reserve Bank of Boston, 600 Atlantic Avenue, Boston MA 02210. E-mail: Jose.Fillat@bos.frb.org. The views expressed in this paper are those of the authors and not necessarily those of the Federal Reserve Bank of Boston or Federal Reserve System.

[‡]Corresponding author. Contact: Department of Economics, Boston University, 270 Bay State Road, Boston MA 02215. E-mail: garettos@bu.edu.

I.   Introduction

Multinational firms tend to exhibit higher stock market returns and earning yields than non-multinational firms. Among non-multinationals, exporters tend to exhibit higher returns and earning yields than firms selling only in their domestic market. Many studies in the new trade literature have documented features distinguishing firms that sell into foreign markets from firms that do not.[1] However, none of this literature has studied whether the international status of the firm matters for its investors. Similarly, in the financial literature, explanations of the cross section of returns overlooked the role of the international status of the firm.[2]

In this paper we attempt to fill this gap in the literature. We develop a real option value model where aggregate uncertainty and fixed and sunk costs of production provide the missing link between firms' international status and their stock market returns. Following standard finance theory, we interpret differences in average realized returns as differences in the expected returns required by the investors because of differences in risk.

The purpose of our structural model is to identify a plausible channel delivering differential exposure to risk of domestic firms, exporters and multinational corporations (henceforth, MNCs). It might at first seem puzzling that exporters and (even more so) multinational firms, which are typically large, established corporations, are riskier than domestic firms. In particular, exposure to foreign markets makes them naturally more diversified than domestic firms. We argue that, due to the large fixed and sunk costs of accessing foreign markets, exporters and multinational firms are actively engaged in risky strategies that make their profits more sensitive to the state of the global economy. We also argue that the risk of a global downturn is the main

---

[1]Bernard, Jensen, and Schott (2009), among others, show that exporters and multinational firms tend to be larger, more productive, to employ more workers, and sell more products than firms that sell only domestically. The higher yields and returns of exporters and multinational firms hold even once controlling for size, leverage, and many other firm-level characteristics.

[2]One notable exception is Fatemi (1984).

risk investors worry about, so that diversification brings limited benefits.

The mechanism of the model is simple: suppose a firm decides whether to enter a foreign market where aggregate demand is subject to fluctuations, and entry involves a sunk cost. In "good times", when prospects of growth make entry profitable, a firm may decide to pay the sunk cost and enter. If – after entry – the shock reverses, the firm may experience losses due to the necessity of covering fixed operating costs. In this case, the firm will be reluctant to exit immediately because of the sunk cost it paid to enter, and may prefer to bear losses for a while, hoping for better times to come again. Hence risk exposure increases with the costs of foreign activities: higher fixed costs make firms more leveraged and more likely to incur losses, while higher sunk costs make firms more willing to bear those losses for a longer time. If the costs of establishing and operating a foreign affiliate are larger than the costs of exporting, then the exposure to demand fluctuations and possible negative profits will be higher for multinational firms than for exporters, and will command a higher return in equilibrium. The presence of sunk costs alters the timing of the cashflows compared to a frictionless model: exporters and multinationals may be willing to take losses in certain bad states to avoid repaying the sunk entry cost. For instance, in a recession a firm may chose to make losses overseas even though those losses are costly to its owners. By altering the covariance of cash flows with the aggregate economy the model can explain the empirical findings.

The choice of whether to serve the foreign market and how (via export or foreign investment, henceforth FDI) is endogenous, and we model it following Helpman, Melitz, and Yeaple (2004). Exports are characterized by low fixed and sunk costs and high variable costs, due to the necessity of shipping goods every period, while FDI entails high fixed and sunk costs of setting up a plant and producing abroad, but low variable costs, since there is no physical separation between production and sales. The model in Helpman, Melitz, and Yeaple (2004) is static, hence the value of a firm coincides with its profits and earning yields are constant across firms. A dynamic but deterministic model, or a dynamic and stochastic model with idiosyncratic shocks share the same feature, with earnings-to-price ratios simply given by the discount rate. The same is true for the returns, which are given by the earning yields plus the expected change in the valuation of the firm (this last term being zero in the static

framework). To generate heterogeneity in these variables across firms, we extend the basic framework to a dynamic and stochastic environment characterized by persistent shocks, using Dixit (1989) as a benchmark to model entry and exit decisions under uncertainty. In the model, firms choose whether to export or invest abroad based on their productivity and on prospects of growth of foreign demand. Larger costs of foreign investment compared to export imply that, compared to exporters, multinational firms may experience larger losses for longer periods of time if the economy is hit by a negative shock.

How does this behavior generate heterogeneity in earning yields and returns? First, persistent aggregate shocks to demand imply that agents discount operating profits based on their expectations and risk aversion, while they discount deterministic fixed costs at the risk free rate. Hence profit flows and firm value are not proportional due to these different discounts. Second, sunk costs of exports and FDI can be interpreted as the premia to be paid to exercise the option of entering the foreign market. Hence profit flow and firm value are not proportional also due to the option value of entering/exiting the market, which differs across firms. To generate heterogeneity in stock market returns, we nest the heterogeneous firms framework into an aggregate endowment economy, in the spirit of consumption-based asset pricing models à la Lucas (1978). Risk-averse consumers own shares of the firms, and discount future consumption streams with a stochastic discount factor dependent on the aggregate shocks. Firms' heterogeneity and endogenous status choices imply that different firms will differ in the covariance of their cash flows with the aggregate uncertainty, which affects consumers' marginal utility. As a result, the model endogenously determines cross-sectional differences in earnings-to-price ratios and returns, and provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view.

As is well known, it is difficult to generate significant risk premia without assuming a very large risk aversion. The literature has settled on a few models that generate plausible risk premia.[3] To depict the risk of a global downturn, and in the interest

---

[3]Campbell and Cochrane (1999) use internal habits specifications which impose non-separability of preferences over time. Yogo (2006) and Piazzesi, Schneider, and Tuzel (2007) are examples of non-separable goods in the utility function. Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008) use Epstein and Zin (1989)'s recursive preferences coupled with persistent shocks to the endowment and cash flow dynamics to generate long run risk. Finally, Rietz (1988), Barro (2006),

of simplicity, we chose to follow Rietz (1988), Barro (2006), and Gabaix (2012) and use the disaster risk model. However, our key mechanism is not dependent on this assumption.

The model can be parameterized to be consistent with aggregate data on export and FDI participation, and with the financial data on earning yields and returns. We present our quantitative results starting from a simpler version of the model that emphasizes the role of trade frictions for returns, but disregards endogenous selection into international status and aggregation. We then move to the simulation of the full industry equilibrium. Quantitatively, the model is able to explain between 33% and 100% of the differences in earning yields across statuses, and between 35% and 75% of the differences in returns, depending on the calibration. Our sensitivity analysis reveals that the inclusion of disaster risk is crucial to obtain a good quantitative fit: the calibrated economy with no disasters generates differences in returns across statuses whose magnitude is only 11% of what we observe in the data. Disaster risk generates a larger spread in returns across firms with different international statuses compared to a world with only i.i.d. shocks to the growth rates: disasters are more severe and more persistent than i.i.d. shocks, hence they induce higher sensitivity of the valuation of firms' cash flow to the fixed and sunk cost mechanism described above.

Besides being successful at replicating both the trade data and the financial data, our quantitative analysis sheds light on the nature of the costs of trade and FDI for large, publicly listed firms. Consistent with the assumptions typically put forward in trade models, both the fixed and the sunk costs of FDI needed to match the data are higher than the costs of exports. Overall, trade and FDI costs are very large, indicating that large gains from trade can be obtained from their removal.

I. A.   Literature

There is a large body of literature that investigates cross-sectional differences in stock returns and earnings-to-price ratios. Fama and French (1996) provide comprehensive evidence about returns differentials across portfolios formed according to particular

---

and Gabaix (2012) add jump shocks to the endowment and cash flow dynamics (with and without recursive preferences) to generate more realistic risk premia.

characteristics (like size and book-to-market). In this paper we cut the data along an unexplored dimension, addressing the risk-return trade-off of firms serving foreign markets. We focus on the cash flow dynamics of the firm and on how these are determined by endogenous decisions and exogenous risk.

The existing finance literature that focuses on cross-sectional differences in earnings-to-price ratios and returns abstracts from the international organization of the firm. There are numerous attempts to explain risk premia and cross sectional differences in expected returns that generalize the canonical power utility consumption-based model. These attempts entail different specifications of preferences, different specifications of the cash flow dynamics, or both. Our paper is aligned with the production-based models that link asset prices to firms' decisions, like Gomes, Kogan, and Zhang (2003), Zhang (2005), Gourio (2011), and Gala (2012). We contribute to the finance literature by endogenizing the exposure of firms' cash-flows to fundamental shocks. Exposure is directly linked to the decision of whether and how to serve the foreign market, which is ultimately driven by the interaction between productivity and cost structure. Several papers have documented the importance of operating leverage and/or fixed costs to generate heterogeneity in stock returns (see Berk, Green, and Naik (1999), Cooper (2006), Novy-Marx (2011)), but this mechanism has not been applied to explain returns' heterogeneity across firms with different international status.[4]

Most importantly, our paper is closely related to a growing literature that explains the cross-section of returns using exposure to disaster risk, starting with Rietz (1988), and more recently Barro (2006), Barro and Ursúa (2008), Gabaix (2012), and Gourio (2012). Particularly, we see our empirical regularity (which we could name "higher equity premia of international firms") as an additional puzzle that can be added to the list of ten in Gabaix (2012), and solved by incorporating disaster risk into the analysis.

This paper is also related to a strand of literature in corporate finance, studying the linkages between international activity and stock market variables. Our empirical evidence is consistent with the analysis in Denis, Denis, and Yost (2002), who find

---

[4]Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), and Cooper (2006) also find that book-to-market is a proxy for exposure to non-diversifiable risk due to option values. Ai and Kiku (2013) analyze the role of growth options in explaining the value premium.

that multinational corporations trade at a discount, and with Desai and Hines (2008) and Baker, Foley, and Wurgler (2009), in linking empirically market valuations, returns, and FDI activity. Our approach departs from these contributions in explicitly acknowledging the endogeneity of the variables of interest and in using a structural model to understand the economic forces behind the correlations that we find in the data.

Our work is related to the literature on trade and FDI under uncertainty, mainly to Rob and Vettas (2003), Russ (2007), Ramondo and Rappoport (2010), and Ramondo, Rappoport, and Ruhl (2013). Rob and Vettas (2003) developed a model of trade and FDI with uncertain demand growth. In their framework FDI is irreversible, so it can generate excess capacity, but has lower marginal cost compared to export. Our work generalizes their model to one with many heterogeneous firms and a more general process for demand growth. Russ (2007) also formulates a problem of foreign investment under uncertainty to study the response of FDI to exchange rate fluctuations.[5] Her model features firm heterogeneity, but does not allow trade as a way to serve foreign markets. Ramondo and Rappoport (2010) introduce idiosyncratic and aggregate shocks in a model where firms can locate plants both domestically and abroad. Multinational production allows firms to match domestic productivity and foreign shocks, and works as a mechanism for risk sharing. Ramondo, Rappoport, and Ruhl (2013) extend their setting to a model featuring also exports. Our framework allows for risk sharing and diversification in addition to the risk exposure driven by the combination of aggregate shocks and fixed and sunk costs. We allow for country-specific shocks with various correlation patterns.

Finally, this paper is related to a growing body of literature on trade dynamics with sunk costs. Particularly, Alessandria and Choi (2007) and Impullitti, Irarrazabal, and Opromolla (2013) model entry and exit into the export market in a world with idiosyncratic productivity shocks and sunk costs. Our model is closer to the framework in Impullitti, Irarrazabal, and Opromolla (2013) for the use of the real option value analogy in solving the firm's optimization problem. While these papers concentrate their attention on the impact of idiosyncratic productivity shocks for firm dynamics, we model aggregate demand shocks that affect firms differently only

[5]Goldberg and Kolstad (1995) study the effect of exchange rate fluctuations on the location choices of multinational firms.

through their endogenous choice of international status.[6] Moreover, both papers analyze the decision to export, but do not consider the possibility of FDI sales. Ghironi and Melitz (2005) develop a general equilibrium model of trade and macroeconomic dynamics in an environment with aggregate productivity shocks, sunk costs of domestic entry and fixed export costs. Our framework shares with Ghironi and Melitz (2005) the emphasis on the effect that aggregate shocks have on the behavior of heterogeneous firms, but overlooks the possible effect of domestic entry and endogenous wages on firms' outcomes. Roberts and Tybout (1997) and Das, Roberts, and Tybout (2007) empirically address the issue of market participation for export. Our model has similar predictions for both exports and FDI sales, and can be calibrated using information from trade and FDI data. In general, we contribute to the trade dynamics literature by developing a dynamic model of entry and exit into foreign markets where the mode of entry (*i.e.*, the decision between export and FDI sales) is also a choice variable.

While individual elements of our framework are found in other work, to our knowledge this paper is the first to propose a dynamic industry equilibrium model where risk affects firms' international strategies and their financial variables in the stock market. The remainder of the paper is organized as follows. Section II. presents empirical evidence establishing the ranking in earning yields and returns. Section III. develops the model and characterizes the equilibrium. Section IV. brings the theory to the data: we calibrate the model to match aggregate trade and FDI data and financial data. Section V. concludes.

II.   Empirical Evidence

In this section we document a novel empirical regularity linking firms' international activities to stock market data. We find that multinational firms exhibit higher annual stock returns and earning yields than exporters. In turn, exporters exhibit higher annual stock returns and earning yields than firms selling only in their domestic market. This pattern holds controlling for a broad set of firms' characteristics.

---

[6]Aggregate shocks are necessary to generate variation in expected returns across firms. The model can be extended to feature also idiosyncratic shocks, at the cost of additional complications and with little impact on the financial variables of interest.

II. A.   The Data

The data used in this paper are from a sample of manufacturing firms that are publicly traded in the US stock market. Financial data is available from Compustat.[7] Matched stock market data, like stock prices, dividends and returns, are obtained from the Center for Research in Security Prices (CRSP). We restrict our sample to firms incorporated in the US whose headquarters are also located in the US.

In addition to the financial data, Compustat contains information about the geographical segments where the firms operate.[8] For every fiscal year of the sample period, the segments information allows us to classify the firms in three groups: multinationals, exporters, and domestic firms. Firms that report the existence of a foreign geographical segment associated with positive sales are classified as multinationals.[9] Firms that do not report any foreign segment with positive sales but report positive exports are classified as exporters. All other firms are classified as domestic.[10] The information contained in the segments data restricts our sample period to 1979-2009.

Table 1 shows firm-level descriptive statistics of the data by type of firm. Consistent with the empirical trade literature, multinational firms are overwhelmingly larger than exporters and domestic firms, both in terms of sales and number of employees. They also have much higher earnings and market capitalization. In turn, exporters are larger than domestic firms according to all measures, even if in this sample differences between exporters and domestic firms are more nuanced than what other papers report (see for example Bernard, Jensen, and Schott (2009) or Bernard et al. (2007)). This feature of our data is due to sample selection: Compustat is a sample of publicly listed, large firms. Many small (mostly domestic) firms are not included. As a result, since the sample includes the largest domestic firms only, domestic firms and exporters are less different than in a sample representative of the entire firm size

---

[7]Firms report their financial data to the Securities and Exchange Commission (SEC) on the annual 10K files.

[8]Multinational and exporter dummies are constructed based on Compustat geographic and operating segments data. Appendix A-I contains a summary of data reporting criteria from the Financial Accounting Standards (FAS) Statement, and details about the construction of the sample.

[9]Denis, Denis, and Yost (2002) also use information from Compustat Segments to identify multinational firms in the data.

[10]Most multinational firms have also positive exports. For reasons that will become clear when we present our structural model, we believe that MNCs that also export are exposed to at least the same risks affecting non-exporting MNCs, and they are hence classified as multinationals.

9

Table 1: **Descriptive Statistics.** All statistics except the number of firms are averages across firm-year observations with year fixed effects. The reported number of firms is the time series average of the number of firms in each category every year.

	Domestic	Exporter	Multinational
Domestic sales (millions $)	228.79	243.56	1681.21
Export sales (millions $)	0	37.30	143.63
FDI sales (millions $)	0	0	663.28
Number of employees (thousands)	1.91	2.09	11.99
Capital/labor ratio (millions $ per worker)	0.14	0.11	0.14
Market capitalization (millions $)	174.30	195.06	1600.70
Book-to-market ratio	1.15	1.08	2.55
Total earnings (millions $)	5.04	8.17	87.67
Annual earnings-to-price ratio (%)	3.96	4.58	6.33
Annual returns (%)	5.68	9.59	10.28
Number of firms	679.34	351.74	814.56

distribution. Most importantly for our purposes, a pecking order in earnings-to-price ratios and stock returns appears in the summary statistics at the firm level: multinational firms have earnings-to-price ratios and returns on average above those of exporters, and in turn exporters have earnings-to-price ratios and returns on average above those of firms selling only in their domestic market. In particular, we find that multinationals exhibit average returns of 10.28%, while those for exporters and domestic firms are 9.59% and 5.68%, respectively.[11] Annual earnings-to-price ratios are defined as annual earnings per share divided by the end-of-year price per share. The ranking in earnings-to-price ratios is consistent with the observation in Denis, Denis, and Yost (2002) that multinational corporations trade at a discount. Stock returns are defined as one-year capital gains plus dividend yields: $R_{t+1} = \frac{p_{t+1}+d_t}{p_t}$ where p_t denotes the price of a share and d_t the dividends per share at time t. We identify firm-level returns with the returns of the firm's common equity. Since data on returns are available at the monthly level, we annualize them for the corresponding firm fiscal year.

Figure 1 presents more evidence on the ranking of earning yields by status. Over a 30-years time period, on average multinational firms have earnings-to-price ratios

---

[11]To put the numbers of Table 1 into perspective, Mehra and Prescott (2003) report that the average return on a market index in the US in the time period 1946-2000 is 9.03%, corresponding to an equity premium of 8.36%. Volatility of returns in the three groups of firms ranges between 14% and 16%, in line with the volatility of the aggregate equity premium in the US.
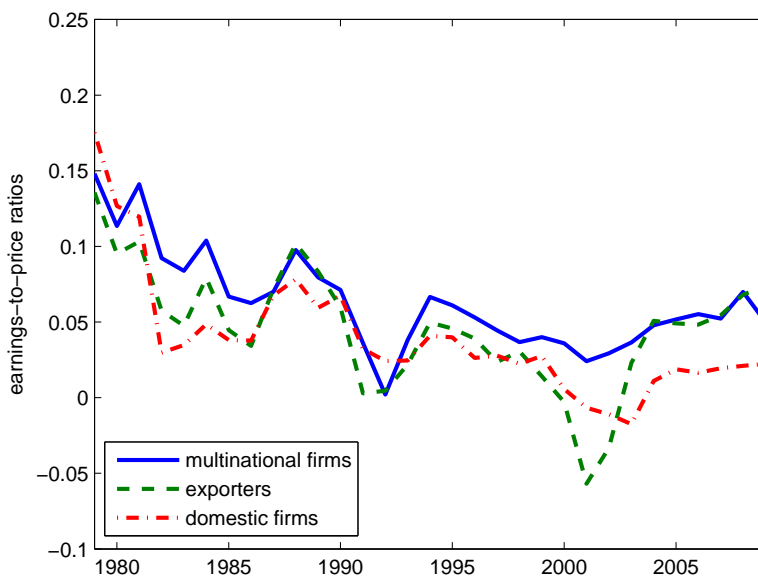
Figure 1: **Earnings-to-Price Ratios.** Portfolios formed yearly based on the international status of each firm. Data source: Compustat and CRSP, 1979-2009.

consistently above those of exporters, and in turn exporters have earnings-to-price ratios on average above those of firms selling only in their domestic market.[12]

Figure 2 shows a similar, albeit noisier, pattern for the stock returns: multinational firms have returns on average above those of exporters, and in turn exporters have returns on average above those of firms selling only in their domestic market.

II. B.   Firm-level Regressions

In the previous section we have shown the ordering of the returns and earnings-to-price ratios in the raw data. However, simple averages across observations by type may hide other underlying characteristics not necessarily related to international status. To address this concern, we run firm level regressions of the financial variables of interest on a set of firm characteristics – financial and non-financial – which could be correlated with cross sectional differences in returns and earning yields.

---

[12]The average earning yields shown in Figure 1 are computed as follows. For each firm i, determine its status S ($S = D, X, I$) at the end of year $t-1$, and collect data on earnings (e_t^i) and market capitalization (p_t^i) in year t. Average earnings E_t^S and average value P_t^S are constructed as simple averages of individual firm-level values. Average earning yields are given by E_t^S/P_t^S.
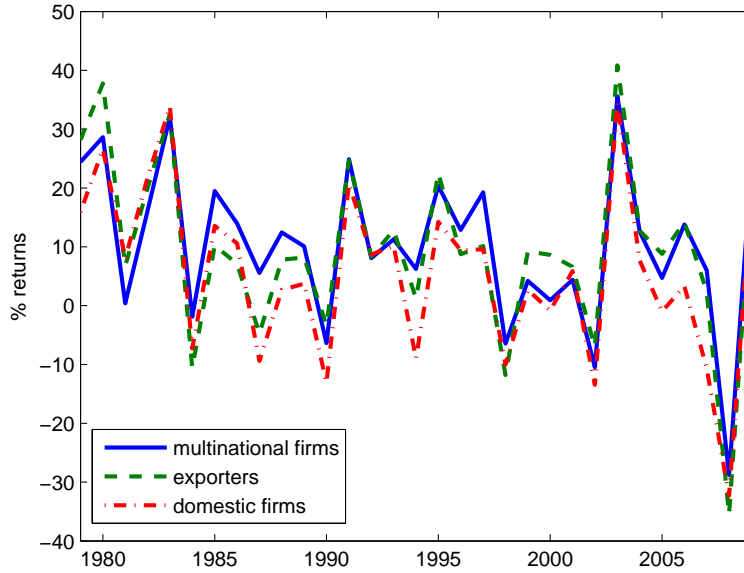
Figure 2: **Returns.** Portfolios formed yearly based on the international status of each firm. Data source: Compustat and CRSP, 1979-2009.

Tables 2 and 3 display the results of the following firm-level regressions:

$$(1) \qquad Y_{it} = \alpha + \gamma_1 D_{it}^I + \gamma_2 D_{it}^X + \gamma_3 X_{it} + \delta_{NAICSt} + \varepsilon_{it}$$

where Y_{it} is the financial variable of interest for firm i at time t: earnings-to-price ratios in Table 2 and returns in Table 3. D_{it}^I and D_{it}^X are dummies that take the value of 1 when firm i is a multinational or an exporter in year t, respectively. X_{it} is a set of controls, including capital/labor ratio, sales per employee (our measure of productivity), profitability, book-to-market ratio, leverage, total revenues and market capitalization (measures of size), and the firm *beta*.[13] δ_{NAICSt} are 4-digit industry-year

---

[13]A large literature in finance has shown the existence of a positive relationship between stock returns and profitability (see Haugen and Baker (1996), Fama and French (2006), and Novy-Marx (2013)). We defined profitability like in Novy-Marx (2013), as profits over total assets. Leverage is defined as the ratio of firm debt over firm book equity. Book-to-market ratio and leverage enter the regressions only separately as they both give information on the relationship between a firm's own resources and its borrowed resources. The market *beta* of the primary security of firm i captures the comovement of the firm's returns with the aggregate market returns. The market *betas* have been computed by running a regression of individual security returns on the market aggregate returns (NYSE, AMEX, and Nasdaq) for the entire sample period. The purpose of adding the market *betas* is to control for each firm's individual exposure to aggregate market risk and to highlight the

12

Table 2: **Earnings-to-Price Regressions.** Firm-level regressions of earnings-to-price ratios on multinational and exporter dummies and other controls, with industry-year fixed effects. Standard errors are clustered by firm and status. (Top and bottom one percent of sample excluded. All dollar values are expressed in billions).

	(1)	(2)	(3)	(4)
multinational dummy	.061	.073	.063	.079
	(.008)***	(.008)***	(.008)***	(.009)***
exporter dummy	.019	.016	.019	.016
	(.004)***	(.006)**	(.004)***	(.006)**
$beta$	.008	.010	.008	.009
	(.003)**	(.003)***	(.003)**	(.003)***
sales per emp.	.040	.041	.043	.049
	(.015)***	(.017)**	(.017)***	(.020)**
K/L	.0003	.022	-.00002	.023
	(.009)	(.018)	(.009)	(.018)
profitability	.161	.169	.158	.165
	(.016)***	(.016)***	(.015)***	(.016)***
total revenue	.004	.006		
	(.0006)***	(.0009)***		
market cap.			2.82e-06	2.35e-06
			(4.84e-07)***	(5.18e-07)***
book/market	.058		.058	
	(.005)***		(.005)***	
leverage		.005		.005
		(.002)**		(.002)**
Prob > F:				
H_0: $\gamma_1 = \gamma_2$	1.62e-17	4.44e-11	3.19e-16	3.18e-11
No. of Obs.	55687	54855	55687	54855
adj. R^2	.201	.106	.2	.103

fixed effects, and ε_{it} is an orthogonal error term.

Table 2 shows the results for the earnings-to-price ratios. The coefficients associated with export and multinational status dummies are positive and significant in all specifications. Moreover, the coefficient associated with multinational status is significantly larger than the one associated with export status, identifying a further difference between these two groups. We reject the null hypothesis that the coefficients of the two dummies are the same, confirming the difference in the earning

---

contribution of the international status to the magnitude of earning yields and returns once market risk is accounted for. Any cross-sectional differences in returns generated by exposure to aggregate risk is captured by cross sectional differences in their market *betas*. Hence, the significant coefficients on the multinational and exporter dummies identify a separate source of higher returns.

Table 3: **Returns Regressions.** Firm-level regressions of log-stock returns on multinational and exporter dummies and other controls, with industry-year fixed effects. Standard errors are clustered by firm and status. (Top and bottom five percent of sample excluded. All dollar values are expressed in billions).

	(1)	(2)	(3)	(4)
multinational dummy	.055	.054	.054	.054
	(.009)***	(.009)***	(.009)***	(.009)***
exporter dummy	.020	.020	.019	.020
	(.009)**	(.009)**	(.009)**	(.009)**
beta	.022	.023	.022	.022
	(.009)**	(.009)**	(.009)**	(.009)**
sales per emp.	.023	.022	.023	.023
	(.011)**	(.010)**	(.011)**	(.010)**
K/L	.021	.006	.021	.006
	(.008)***	(.003)*	(.008)***	(.003)*
profitability	.149	.147	.146	.144
	(.014)***	(.014)***	(.014)***	(.014)***
total revenue	.003	.003		
	(.0005)***	(.0006)***		
market cap			3.39e-06	3.49e-06
			(7.19e-07)***	(7.24e-07)***
book/market	-.0003		-.0003	
	(.0001)**		(.0001)**	
leverage		.005		.005
		(.002)***		(.002)***
Prob > F:				
H_0: $\gamma_1 = \gamma_2$	.00007	.0001	.0001	.0002
No. of Obs.	51454	50661	51454	50661
adj. R^2	.128	.128	.129	.129

yields of multinationals *versus* exporters.

Table 3 reports the results of regression (1) with annual firm-level returns as the dependent variable. The coefficients on the multinational and exporter dummies are positive and significant, which confirms that firms selling in foreign markets tend to have higher returns than firms selling only domestically. The coefficient on the multinational dummy is significantly higher than the one on the exporter dummy, indicating even larger excess returns for multinational firms. The ranking and significance of the coefficients are preserved across specifications.

By looking at these regressions one could argue that differences among multinationals, exporters and domestic firms are not necessarily driven by intrinsic differences

14

related to the status itself, but by unobservable firm characteristics. A more conservative specification of these regressions would include firm fixed effects. However, this would identify variation across groups by using only the information of the firms that change status at least once during the sample period. In our sample, on average every year 95% of firms do not change status, so the fixed effects specification looses most of the information contained in the sample. This problem is particularly acute for multinational firms, which tend to change status even less than other firms (only 2% of MNCs change status every year). As a result, the returns regressions with firm fixed effects show positive and significant coefficients on the export status dummy, but non-significant coefficients on the multinational status dummy, while the earning yields regressions with firm fixed effects display positive and significant coefficients for the two dummies, but whose difference is not significant.

Alternatively, one could argue that the results of our baseline regressions are driven by changes over time and correlations across years in the sample. The year fixed effects address this concern. For robustness, we also run regression (1) as a cross-sectional specification where every firm-level variable is calculated as an average over the sample period. This specification suffers of the opposite problem of the firm fixed effect specification: since we have to define the status dummies over the entire sample period, we lose all the information coming from firms that switch status at least once in the sample. However, the results are robust: also in the cross-sectional specification, the status dummies are positive and significant, and the MNC dummy is significantly higher than the export dummy.

Appendix A-II contains additional portfolio-level asset pricing tests that illustrate that differences in returns across firms with different international statuses are not explained by any of the standard Fama-French factors that the empirical finance literature has used to explain the cross section of returns. Finally, to support the interpretation of cross-sectional differences in returns as differences in risk, in the Appendix we also report evidence on the heterogeneous exposure of returns to consumption growth fluctuations.

III. Model

The model we develop in this section is designed to provide a structural explanation of the cross-section of earning yields and returns by international status. At the aggregate level, the model is specified as an endowment economy, consistently with consumption-based asset pricing models. At the micro level, heterogeneous firms' optimal choices determine how aggregate consumption is allocated into domestic goods, imported goods, and goods produced by affiliate plants of foreign multinationals. Firms' decisions endogenously determine profit flows. Through the first-order conditions of the agent's problem, firms' valuations and the covariance of their profits with the agents' intertemporal rate of substitution drive the returns.

III. A. Consumer Behavior and Aggregate Uncertainty

The economy is composed of two countries, Home and Foreign. Variables related to consumers and firms from the foreign country are marked with an asterisk (*). Both countries are populated by infinitely lived, risk-averse agents with preferences:

$$U = \int_0^\infty e^{-\vartheta t} \frac{Q(t)^{1-\gamma}}{1-\gamma} dt$$

where $\vartheta > 0$ is the subjective discount factor, and $\gamma > 1$ denotes risk aversion. $Q(t)$ is a constant elasticity of substitution (CES) aggregate of differentiated varieties:

$$Q(t) = \left(\int q_i(t)^{1-1/\eta} di \right)^{\eta/(\eta-1)}$$

where $\eta > 1$ denotes the elasticity of substitution across varieties.

In each country, aggregate consumption is hit by random shocks. Time is continuous, and Q and Q^* evolve according to combined geometric Brownian motion and

jump processes:[14]

$$(2) \qquad \frac{dQ}{Q} = \mu dt + \sigma dz + dj$$

$$(3) \qquad \frac{dQ^*}{Q^*} = \mu^* dt + \sigma^* dz^* + dj^*$$

where $\mu, \mu^* \geq 0$, $\sigma, \sigma^* > 0$ and dz, dz^* are the increments of two standard Wiener processes with correlation $\rho \in [-1, 1]$. The term dj (dj^*) is the increment of a Poisson process with arrival rate λ. If the event occurs, Q (Q^*) falls of a deterministic percentage ϕ (ϕ^*) with probability 1:[15]

$$(4) \qquad dj = \begin{cases} -\phi; & \text{with prob. } \lambda dt \\ 0; & \text{with prob. } (1 - \lambda dt) \end{cases}$$

$$(5) \qquad dj^* = \begin{cases} -\phi^*; & \text{with prob. } \lambda dt \\ 0; & \text{with prob. } (1 - \lambda dt). \end{cases}$$

We assume the jump processes to be independent from the Wiener processes: $E(dzdj) = E(dzdj^*) = E(dz^*dj) = E(dz^*dj^*) = 0$. The occurrence of a jump mimics the realization of a worldwide disaster. When the disaster happens, consumption growth in the Home (Foreign) country drops of a percentage ϕ (ϕ^*). The assumption of a global disaster that hits the entire world economy at the same time (even if with different

---

[14]It is well accepted that equilibrium consumption growth can be represented with a random walk since Hall (1978). To the risk of sounding redundant, we report all the equations for both the Home and the Foreign market to make clear the instances where Home market variables affect Foreign market ones and viceversa.

[15]Our framework builds directly on Rietz (1988) and Barro (2006), where shocks are aggregate and the size and probability of the disaster are deterministic. It is straightforward to extend the model to the case where the size of the jump is stochastic. As Gabaix (2012) explains, variable-sized disasters are essential to resolve puzzles related to second moments, but not for puzzles related to first moments, like ours is. For simplicity, we also confine our analysis to CRRA utility. There are two main differences between our framework and the one in Gabaix (2012). First, and most substantially, consistent with a large literature in asset pricing, Gabaix (2012) assumes an exogenous process for the dividend stream of the firms, which also depends (exogenously) on the jump. We model firms' decisions explicitly, hence cash flows are endogenous to the profit-maximizing decisions of the firms and so depend endogenously on the jump. Second, we decided to keep the formulation of the model in continuous time, which makes easier the solution of the real option problem.

intensities) seems appropriate in the context of an open economy. On the technical side, assuming one "worldwide" Poisson process maximizes the effect that disaster risk has on asset prices while preserving entirely the tractability of the problem.

International markets are incomplete: consumers in the Home (Foreign) country consume the stochastic endowment Q (Q^*), without any possibility of consumption smoothing over time. In our framework, asset prices, as opposed to goods' quantities, reflect the agents' willingness to transfer wealth from period to period. Consumers in the Home (Foreign) country own the firms incorporated in the Home (Foreign) country. As such, there is no possibility of international portfolio diversification because the model features perfect home bias in equity portfolios.[16]

In equilibrium, utility maximization implies that agents in each country discount future consumption with stochastic discount factors described by the following mixed processes:[17]

$$(6) \qquad \frac{dM}{M} = -\left[\vartheta + \gamma\mu - \gamma(\gamma+1)\frac{1}{2}\sigma^2\right]dt - \gamma\sigma dz + dj_M$$

$$(7) \qquad \frac{dM^*}{M^*} = -\left[\vartheta + \gamma\mu^* - \gamma(\gamma+1)\frac{1}{2}\sigma^{*2}\right]dt - \gamma\sigma^* dz^* + dj_M^*$$

where dj_M (dj_M^*) is the jump in the stochastic discount factor M (M^*) implied by a jump in Q (Q^*):

$$(8) \qquad dj_M = \begin{cases} (1-\phi)^{-\gamma} - 1; & \text{with prob. } \lambda dt \\ 0; & \text{with prob. } (1-\lambda dt) \end{cases}$$

$$(9) \qquad dj_M^* = \begin{cases} (1-\phi^*)^{-\gamma} - 1; & \text{with prob. } \lambda dt \\ 0; & \text{with prob. } (1-\lambda dt). \end{cases}$$

---

[16]Tesar and Werner (1998) provide evidence of an extreme home bias in equity portfolios: about 90% of US equity was invested in the US stock market in the mid-1990s. Hamano (2015) explains the existence of the large home bias in equity holdings as a result of the "extensive margin risk" induced by entry and exit of firms across markets. Atkeson and Bayoumi (1993), Sorensen and Yosha (1998), and Crucini (1999) present evidence supporting the assumption of international market incompleteness.

[17]The stochastic discount factor is equal to the intertemporal marginal rate of substitution. The marginal utility of consumption is: $M = e^{-\vartheta t}Q(t)^{-\gamma}$. By applying Ito's Lemma to M and M^* one obtains (6) and (7).

The risk-free rates in the two countries are given by:

$$(10) \quad rdt \quad \equiv \quad -E\left[\frac{dM}{M}\right] = \left[\vartheta + \gamma\mu - \gamma(\gamma+1)\frac{1}{2}\sigma^2 - \lambda\left[(1-\phi)^{-\gamma} - 1\right]\right] dt$$

$$(11) \quad r^*dt \quad \equiv \quad -E\left[\frac{dM^*}{M^*}\right] = \left[\vartheta + \gamma\mu^* - \gamma(\gamma+1)\frac{1}{2}\sigma^{*2} - \lambda\left[(1-\phi^*)^{-\gamma} - 1\right]\right] dt.$$

This is a partial equilibrium model where labor is the only factor of production and is supplied inelastically. As in Lucas (1978), we do not model how labor endowments produce the aggregate consumption levels Q, Q^*. We use preferences to derive an expression for the stochastic discount factor and to find equilibrium goods and asset prices.[18]

III. B.   Technology and Firms' Behavior

Each country is populated by a continuum of firms, which operate under a monopolistically competitive market structure. Each firm produces a differentiated variety q_i taking the demand function as given. Firms produce with a linear technology defined by a firm-specific unit labor requirement a, which is a random draw from a distribution $G(a)$ $(G^*(a))$. Differentiated varieties are tradeable: a firm may sell its own variety only in its domestic market or both in the domestic and in the foreign market.

For simplicity we assume that there are no fixed costs associated with production for the domestic market, so every firm makes positive profits from domestic sales, and always sells in its domestic market.[19] On the contrary, sales to the foreign market involve fixed operating costs, to be paid every period, and sunk costs of entry. If a

---

[18]Consistent with partial equilibrium, wages are taken as exogenous and do not adjust to changes in trade and FDI volumes (we do not impose trade balance in the model). An alternative way of setting up the model would have been to specify exogenous, country-specific productivity shocks and solve the model in general equilibrium. Shocks to aggregate productivity would imply equilibrium shock processes for Q and Q^*, making the behavior of the model qualitatively similar to our partial equilibrium one. Under such specification, we would need the full solution of the model to recover the processes ruling the stochastic discount factors, making the analysis much less transparent than in the current setting. For this reason we confine ourselves to a partial equilibrium analysis and model aggregate consumption as exogenous.

[19]We could have introduced positive fixed costs of domestic production, and modeled the initial decision of entry in the domestic market, like in Helpman, Melitz, and Yeaple (2004) and Impullitti, Irarrazabal, and Opromolla (2013). This would have introduced additional complications in solving for the firms' value functions, without any gains for our empirical analysis. Compustat includes only publicly listed firms, so when a firm enters or exits Compustat we do not have any information about whether the firm is in fact entering or exiting the market.

firm decides to sell in the foreign market, it can do so either via exports or via foreign direct investment. We call multinationals those firms that decide to serve the foreign market through FDI sales.

We model the choice between trade and FDI along the lines of Helpman, Melitz, and Yeaple (2004).[20] Both exports and FDI entail sunk entry costs (which we denote with F_X and F_I for exporters and MNCs, respectively) and fixed operating costs to be paid every period (which we denote with f_X and f_I for exporters and MNCs, respectively). In addition, exporters need to pay a per-unit iceberg transportation cost τ, also to be paid every period.[21] Exporters from the Home country hire labor in their domestic market and pay a wage w, while MNCs from the Home country produce abroad, hence they hire labor in the Foreign country and pay a wage w^*. Similar to Helpman, Melitz, and Yeaple (2004), we assume that exports entail relatively small fixed and sunk costs, but higher variable costs due to the iceberg trade costs. Instead, FDI is associated to larger fixed and sunk cost, but there are no transportation costs to be covered every period, as both production and sales happen in the foreign market. We summarize these assumptions in the following restriction on the parameters, which ensures selection of more (less) productive firms into FDI sales (exports): $w^{*\eta-1}(f_I + \vartheta F_I) > (\tau w)^{\eta-1}(f_X + \vartheta F_X)$. Notice also that the cost structure and the nature of uncertainty imply that if a firm decides to enter the foreign market, it will do so either as an exporter or as a multinational firm, but it will never adopt the two strategies at the same time.[22]

---

[20] Our modeling choice restricts the characterization of MNCs to firms that engage in horizontal FDI. The information included in the Compustat Segments database does not allow us to distinguish between horizontal and vertical FDI. In a follow-up paper (Fillat, Garetto, and Oldenski (2015)) we merge Compustat/CRSP with the affiliate-level data on the Operations of Multinational Corporations from the Bureau of Economic Analysis (BEA). The BEA data includes information on the destination country of affiliate sales, allowing us to identify horizontal and vertical investment. We examined our stylized fact comparing domestic firms in Compustat with the set of multinational firms that appear both in Compustat and in the BEA data, and MNCs exhibit higher returns regardless of the type of foreign investment they are mostly engaged into: the fact holds for both a smaller sample of firms that do only horizontal investment and for the full sample. The merged dataset also confirms the fact that the majority of sales of foreign affiliates can be classified as horizontal FDI: about 94% of the firms have at least some horizontal sales, and about 64% percent of all sales by foreign affiliates are of the horizontal type.

[21] $\tau > 1$ units of good need to be shipped for one unit of good to arrive to the destination country.

[22] This feature of the model is the same as in Helpman, Melitz, and Yeaple (2004). Rob and Vettas (2003) obtain the existence of an equilibrium where firms can optimally choose to adopt simultaneously the two strategies because in their model firms choose the amount of the foreign investment, and given the structure of demand there may be the possibility of over-investment. In

After entering in the foreign market, a firm can exit at no cost. However, if it decides to re-enter, it will have to pay the sunk cost again.[23] Sunk and fixed costs and stochastic demand imply that firms decide to enter when their expected profits are well above zero, and are reluctant to exit even in case of losses due to negative shocks. We refer to the set of realizations of the shocks such that a domestic firm is not willing to enter and an international firm is not willing to exit the foreign market as the band of inaction. Dixit and Pindyck (1994) show that the band of inaction is wider the larger the sunk cost of entry.

Firms' activities abroad are both subject to risk and to diversification potential. On the one hand, the correlation of the consumption processes in the two countries implies that the presence of foreign activities is a source of diversification. On the other hand, fixed costs of trade and FDI imply that fluctuations of Q and Q^* can induce negative profits. The band of inaction generated by sunk costs implies that firms may be willing to bear losses for a while, in order not to forego the sunk cost that they paid to enter. These potential protracted losses are a source of cash flow risk associated with foreign activities. In this sense, trade barriers limit the potential to diversify risk. In a frictionless world ($f_X = f_I = F_X = F_I = 0$) there would be no risk associated with foreign activities, but only diversification possibilities. Notice that this is true from the perspective of the firms, so different extents of risk/diversification have effects on firms' choices, but not on individuals' consumption levels.

For a given realization of (Q, Q^*), in equilibrium, a firm with productivity $1/a$ chooses its optimal status S ($S \in \{D, X, I\}$, *i.e.* domestic, exporter, or multinational), the current selling price $p_S(a)$, and an updating rule (how to change the optimal price and status following changes in aggregate demand).

---

their framework, FDI can be adopted to cover certain demand, while exports are used to serve the additional random excess demand without incurring the cost of a larger investment, which could be under-utilized. In the data we do observe firms that both export and have FDI sales (about 13% of the total). This fact can be rationalized within our framework by having multiproduct firms choosing different strategies for different product lines, or in a multi-country model where firms choose different strategies to enter different countries. Unfortunately, there is not enough information in the Compustat Segments data to check whether any of these is the case. Explaining the choice of firms to adopt both entry strategies would need a differently tailored framework, and is beyond the scope of this paper.

[23]Roberts and Tybout (1997) show empirically that previous exporting experience matters as long as firms do not exit the foreign market. They find that the entry costs for first-time exporters are not statistically different from the entry costs for second-time exporters, *i.e.* firms that were once selling in the foreign market, exited, and decided to re-enter.

The CES aggregation over individual varieties implies that optimal prices are independent of (Q, Q^*). From the firm's intratemporal first order condition: $p_S(a) = \frac{\eta}{\eta - 1} MC_S(a)$, where $MC_S(a)$ denotes the marginal cost of a firm with productivity $1/a$ in status S. The marginal cost of production varies with the status of the firm. For Home country firms, the marginal cost of domestic production is given by the labor requirement times the domestic wage, $MC_D = aw$. The marginal cost of exporting is augmented by the iceberg transportation cost: $MC_X = \tau a w$. When the firm serves the foreign market through FDI, firm-specific productivity is transferred to the foreign country and the firm employs foreign labor: $MC_I = aw^*$. The prices charged by firms from the Foreign country are determined in the same way.

Let $\pi_D(a; Q)$, $\pi_X(a; Q^*)$ and $\pi_I(a; Q^*)$ denote the maximal flow profits from domestic sales, from exports and from FDI sales abroad, respectively, for a Home country firm with productivity $1/a$, given a realization of the aggregate quantity demanded (Q, Q^*):

$$
\begin{align}
\pi_D(a; Q) &= H(aw)^{1-\eta} P^\eta Q \tag{12} \\
\pi_X(a; Q^*) &= H(\tau a w)^{1-\eta} P^{*\eta} Q^* - f_X \tag{13} \\
\pi_I(a; Q^*) &= H(aw^*)^{1-\eta} P^{*\eta} Q^* - f_I \tag{14}
\end{align}
$$

where $H \equiv \eta^{-\eta}(\eta - 1)^{\eta-1}$, and P (P^*) is the price index in the Home (Foreign) country, which firms take as given while solving their maximization problem.

III. C. Industry Equilibrium

The state of the economy is described by the vector $\Sigma = (Q, Q^*, \Omega, \Omega^*)$, where $\Omega = (\omega_X, \omega_I)$ ($\Omega^* = (\omega_X^*, \omega_I^*)$) describes the distribution of firms from the Home (Foreign) country into the three statuses.[24] Let $\mathcal{V}_S(a, Q, Q^*)$ denote the expected net present value of a Home country firm whose productivity is $1/a$, starting in status S ($S = D, X, I$) when the realization of aggregate demand is (Q, Q^*), and following optimal policy. A firm's value coincides with the price at which its ownership is traded in the assets market.

Definition 1. *An industry equilibrium for this economy is defined by a set of value*

---

[24]$\omega_D = 1 - \omega_X - \omega_I$.

functions $(\mathcal{V}_S(a, Q, Q^*), \mathcal{V}_S^*(a, Q, Q^*))$, *for* $S = D, X, I$, *policy functions, price indexes* (P, P^*), *and laws of motion of the distributions of firms into statuses* (Ω, Ω^*) *such that:*

  i. *the stochastic discount factors* M *and* M^* *are given by the consumers' intertemporal marginal rate of substitution;*

  ii. *firms' maximize their lifetime profits;*

  iii. *goods and assets markets clear.*

We solve the model along the lines of Dixit (1989). In the following, we omit the dependence of the value functions on Ω and Ω^* to ease the notation. Firms are active in their domestic market and make positive profits $\pi_D(a; Q)$ from domestic sales. Domestic activities do not depend directly on the realization of foreign demand Q^*. Similarly, the decision of whether to sell in the foreign market does not depend directly on the realization of domestic demand Q. For this reason, we can express the value function as:

(15) $$\mathcal{V}_S(a, Q, Q^*) = \mathcal{S}(a, Q) + V_S(a, Q^*)$$

where $\mathcal{S}(a, Q)$ is the expected present discounted value of profits from domestic sales, which is independent on firm status, and $V_S(a, Q^*)$ is the expected present discounted value of profits from foreign sales for a firm in status S.

Over a generic time interval Δt, the two components of the value function for a firm that is currently selling only in its domestic market can be expressed as:

(16) $\mathcal{S}(a, Q) = \pi_D(a, Q) M \Delta t + E[M \Delta t \cdot \mathcal{S}(a, Q') | Q]$

(17)
$$V_D(a, Q^*) = \max \left\{ E[M \Delta t \cdot V_D(a, Q^{*\prime}) | Q^*] \; ; \; V_X(a, Q^*) - F_X \; ; \; V_I(a, Q^*) - F_I \right\}.$$

While (16) simply tracks the evolution of domestic profits, the right hand side of (17) expresses the firm's possible choices. If it sells only domestically, it gets the continuation value from not changing status, equal to the expected discounted value of the firm conditional on the current realization of foreign demand Q^*. If it decides

23

to switch to exports (FDI) it gets the value of being an exporter, V_X (multinational, V_I) minus the sunk entry cost F_X (F_I). Similarly, the present discounted value of profits from foreign sales for an exporter is:

(18)
$$V_X(a, Q^*) = \max \left\{ \pi_X(a, Q^*) M \Delta t + E[M \Delta t \cdot V_X(a, Q^{*\prime}) | Q^*] \; ; \; V_D(a, Q^*) \; ; \; V_I(a, Q^*) - F_I \right\}$$

and for a multinational:

(19)
$$V_I(a, Q^*) = \max \left\{ \pi_I(a, Q^*) M \Delta t + E[M \Delta t \cdot V_I(a, Q^{*\prime}) | Q^*] \; ; \; V_D(a, Q^*) \; ; \; V_X(a, Q^*) - F_X \right\}.$$

Notice that the continuation value of an exporter (a multinational) also includes the profit flow from sales to the foreign market. There are no costs of exiting the foreign market: if a firm decides to exit, its value is simply that of a domestic firm.

In Appendix A-III we show that the value functions take the form:

(20) $\quad \mathcal{S}(a, Q) = \dfrac{\pi_D(a, Q)}{r - \mu + \gamma \sigma^2 + \lambda \phi (1 - \phi)^{-\gamma}}$

(21) $\quad V_D(a, Q^*) = A_D(a) Q^{*\alpha} + B_D(a) Q^{*\beta}$

(22) $\quad V_X(a, Q^*) = A_X(a) Q^{*\alpha} + B_X(a) Q^{*\beta} + \dfrac{H(\tau a w)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma \rho \sigma \sigma^* + \lambda \phi^* (1 - \phi)^{-\gamma}} - \dfrac{f_X}{r}$

(23) $\quad V_I(a, Q^*) = A_I(a) Q^{*\alpha} + B_I(a) Q^{*\beta} + \dfrac{H(a w^*)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma \rho \sigma \sigma^* + \lambda \phi^* (1 - \phi)^{-\gamma}} - \dfrac{f_I}{r}$

where $\alpha < 0$ and $\beta > 1$ are the roots of:

(24) $\dfrac{1}{2} \sigma^{*2} \xi^2 + (\mu^* - \gamma \rho \sigma \sigma^* - \dfrac{1}{2} \sigma^{*2}) \xi - (r + \lambda (1 - \phi)^{-\gamma}) + \lambda (1 - \phi)^{-\gamma} (1 - \phi^*)^\xi = 0.$

$A_S(a)$ and $B_S(a)$ ($S \in \{D, X, I\}$) are firm-specific, time-varying parameters to be determined.[25]

Since there are no fixed or sunk costs of domestic production, there is no option value associated with future profits from domestic sales. The value function $\mathcal{S}(a, Q)$ is simply equal to the discounted flow of domestic profits. Conversely, the option value of changing status (the term $A_S(a) Q^{*\alpha} + B_S(a) Q^{*\beta}$, for $S = D, X, I$) is a component

---

[25] The parameters $A_S(a)$ and $B_S(a)$ are time-varying because they also depend on the distribution of firms in the three statuses, which we are not making explicit in the value functions.

of the expected present discounted value of foreign profits. In particular, the option value is the only component of the present discounted value of foreign profits for domestic firms. For exporters and multinationals, the value is given by the sum of the discounted foreign profit flow from never changing status plus the option value of changing status. The terms $\mu - \gamma\sigma^2 - \lambda\phi(1-\phi)^{-\gamma}$ and $\mu^* - \gamma\rho\sigma\sigma^* - \lambda\phi^*(1-\phi)^{-\gamma}$ in the discount terms are the risk-adjusted drifts, result of taking expectations of the value functions under the risk-neutral measure.

Equations (21)-(23) describe the value of foreign profits in the firms' continuation regions. We still need to solve for the policy function, which consists of thresholds in the realizations of Q^* that induce firms to change status. Let $Q^*_{RS}(a)$ denote the quantity threshold at which a firm with productivity $1/a$ switches from status R to status S, for $R, S \in \{D, X, I\}$.[26] In order to find the six quantity thresholds $Q^*_{SR}(a)$ and the six value function parameters $A_S(a)$, $B_S(a)$, for $S \in \{D, X, I\}$, we impose value-matching and smooth-pasting conditions.

For each firm with unit cost a, the value-matching and smooth-pasting conditions define a system of twelve equations in twelve unknowns: the six quantity thresholds $Q^*_{SR}(a)$ and the six parameters $A_S(a)$, $B_S(a)$, for $S, R \in \{D, X, I\}$. To get an economically sensible solution, we follow Dixit (1989) and impose a series of restrictions on the parameters $A_S(a)$, $B_S(a)$, $S \in \{D, X, I\}$. Since $\alpha < 0$, $\beta > 1$, the terms in $Q^{*\alpha}$ ($Q^{*\beta}$) are large for low (high) realizations of Q^*. For low realizations of Q^*, entry is a remote possibility for a firm selling only in its domestic market, hence the value of the option of entering must be nearly worthless: $A_D(a) = 0$, $\forall a$. It must then be that $B_D(a) \geq 0$ to insure non-negativity of $V_D(a, Q^*)$.

Similarly, for high realizations of Q^*, the option of quitting FDI for another strategy is nearly worthless, hence $B_I(a) = 0$. Moreover, a multinational firm has expected value $\frac{H(aw^*)^{1-\eta}P^{*\eta}Q^*}{r-\mu^*+\gamma\rho\sigma\sigma^*+\lambda\phi^*(1-\phi)^{-\gamma}} - \frac{f_I}{r}$ from the strategy of never changing status, hence the optimal strategy must yield a no lesser value: $A_I(a) \geq 0$.

Finally, an exporter has expected value $\frac{H(\tau aw)^{1-\eta}P^{*\eta}Q^*}{r-\mu^*+\gamma\rho\sigma\sigma^*+\lambda\phi^*(1-\phi)^{-\gamma}} - \frac{f_X}{r}$ from the strategy of never changing status, hence its optimal strategy must yield a no lesser value for any realization of Q^*: $A_X(a), B_X(a) \geq 0$.

As a result of these restrictions, the value function of a domestic firm is increasing

---

[26]The quantity thresholds $Q^*_{RS}(a)$ also depend on the distribution of firm statuses Ω^*, which affects the equilibrium price index, and are hence time-varying.

on its entire domain, indicating that as Q^* increases, the value of the option of entering the foreign market (either through trade or FDI) increases. The value functions of an exporter and of a multinational are U-shaped: for low levels of Q^*, the term with the negative exponent α dominates, and the value is high due to the option of leaving the market. Conversely, for high levels of Q^*, the value is high due to the profit stream that the firm derives from staying in the market and, for exporters, due to the additional option value of becoming a multinational firm (the term with the positive exponent β). Details about the numerical solution of the value-matching and smooth-pasting system are contained in Appendix A-IV.

The system of value-matching and smooth-pasting conditions includes among its variables the aggregate price index P^*. P^* is an endogenous variable that depends on the realization of Q^*. For this reason, one should write each condition taking into account the equilibrium price at that specific realization of Q^* (*i.e.*, if $Q^* = Q^*_{DX}$, then $P^* = P^*(Q^*_{DX})$). However, we appeal to the result developed in Leahy (1993) and Dixit and Pindyck (1994), Chapters 8-9, whereby a firm can ignore the effects of the actions of other firms when solving for the optimal thresholds triggering investment, and use the market equilibrium price P^* in all the value-matching and smooth-pasting equations. The intuition behind this result is as follows. The actions of other firms affect the problem of an individual firm via the price index in two ways: more firms entering the foreign market reduce the profit flows from foreign sales and also the option value of waiting to start selling abroad. It can be shown that these two effects exactly offset each other; hence, taking into account the effect of the actions of other firms on the price index is immaterial for the determination of the thresholds.[27] In the interest of space, we relegate the description of the remaining features of the

---

[27]Dixit and Pindyck (1994) and Leahy (1993) show this results for a perfectly competitive industry with free entry and CRS production technologies, where the shocks follow a general diffusion process. Leahy (1993) also shows that free entry is unnecessary to obtain the result. Our economy differs from the ones they study in that firms' technologies exhibit increasing returns to scale and the market structure is monopolistically competitive. However, we argue that the result still applies for the following reasons. The potential problem with increasing returns is that they may induce "too large" investment by the firms. This does not apply to our framework, where firms only decide whether to entry or not, and not the amount to invest. Imperfect competition in turn may invalidate the result if firms display some type of strategic or interdependent behavior. In our setting, monopolistic competition with a continuum of firms rules out strategic behavior. Moreover, Dixit-Stiglitz preferences over a continuum of products imply that individual prices are independent of the price index, ruling out any interdependence in firms' decisions.

industry equilibrium to Appendix A-III, where we report the solution of the price indexes (P, P^*) and the law of motion of the status distributions (Ω, Ω^*).

Equipped with the solutions of the value functions, we now move to the computation of the earnings-to-price ratios and returns generated by the model.

III. D.   Earnings-to-Price Ratios and Returns

The solution of the model delivers quasi-closed form solutions (up to multiplicative parameters) for the value functions $\mathcal{V}_S(a, Q, Q^*)$ $(S \in \{D, X, I\})$, and allows us to compute the earnings-to-price ratios and returns generated by the model.

Our earning yields measure in the model is given by the ratio $\pi_t / \mathcal{V}_t$, where π_t represents flow profits and $\mathcal{V}_t$ is the market value of the firm. Let $ep_S(a, Q, Q^*)$ denote the earning yields of a firm with productivity $1/a$ in status S when the realization of aggregate demand is (Q, Q^*). Earning yields in the model are given by:

$$(25) \qquad ep_D(a, Q, Q^*) = \frac{\pi_D(a, Q)}{\mathcal{V}_D(a, Q, Q^*)}$$

$$(26) \qquad ep_X(a, Q, Q^*) = \frac{\pi_D(a, Q) + \pi_X(a, Q^*)}{\mathcal{V}_X(a, Q, Q^*)}$$

$$(27) \qquad ep_I(a, Q, Q^*) = \frac{\pi_D(a, Q) + \pi_I(a, Q^*)}{\mathcal{V}_I(a, Q, Q^*)}.$$

The empirical evidence presented in Section II. suggests the following ordering in aggregate earning yields across groups:

$$\int_{\omega_D(Q^*)} ep_D(a, Q, Q^*) dG(a) < \int_{\omega_X(Q^*)} ep_X(a, Q, Q^*) dG(a) < \int_{\omega_I(Q^*)} ep_I(a, Q, Q^*) dG(a).$$

While is not possible to prove analytically that the model generates this ordering, the results of our numerical simulations confirm that the calibrated model is consistent with it.

Expected stock returns in the model are given by the earning yields plus the expected change in the valuation of the firm:

$$(28) \ ret_S(a, Q, Q^*) = ep_S(a, Q, Q^*) + \frac{E[d\mathcal{V}_S(a, Q, Q^*)]}{\mathcal{V}_S(a, Q, Q^*)} \ , \ for \ S \in \{D, X, I\}.$$

Also in this case, the model does not have clear-cut analytical predictions for the ordering of $\frac{E[d\mathcal{V}_S(a,Q,Q^*)]}{\mathcal{V}_S(a,Q,Q^*)}$. The value of this object depends on the curvature of the value functions and it also critically depends on selection, since different firms exhibit different value functions and respond differently to the same realizations of the shocks.

To gain intuition on the main elements of the model at work, it is instructive to notice that returns can be expressed as:[28]

$$
\begin{aligned}
ret_S(a,Q,Q^*) \;=\; & r + \frac{\gamma\sigma^2\mathcal{S}(a,Q) + \gamma\rho\sigma\sigma^*Q^*V_S'(a,Q^*)}{\mathcal{V}_S(a,Q,Q^*)} - \dots \\
& \lambda[(1-\phi)^{-\gamma} - 1]\frac{\mathcal{V}_S[a,(1-\phi)Q,(1-\phi^*)Q^*] - \mathcal{V}_S(a,Q,Q^*)}{\mathcal{V}_S(a,Q,Q^*)}.
\end{aligned}
\tag{29}
$$

Returns depend on the covariance of shocks across countries ($\rho\sigma\sigma^*$) and on the responsiveness of the value function with respect to changes in Q^*. The sensitivity of the value function to changes in Q^* appears in the elasticity of the value function with respect to continuous changes in Q^*, and in the percentage change in the value function following a discrete jump in Q^*. This illustrates how the presence of disaster risk increases the impact of the aggregate shocks on returns through its effect on the value function.

Heterogeneity in returns across groups depends on firm-specific productivity and on the fixed costs associated with different international exposures. Holding productivity (and hence revenues) constant, larger fixed costs result in larger percentage changes in values with respect to changes in demand. As a result, the presence of fixed costs associated with foreign activities increases the returns of internationally engaged firms compared to domestic firms.

The model reproduces the ordering in returns across groups found in the data if:

$$
\int_{\omega_D(Q^*)} ret_D(a,Q,Q^*)dG(a) < \int_{\omega_X(Q^*)} ret_X(a,Q,Q^*)dG(a) < \int_{\omega_I(Q^*)} ret_I(a,Q,Q^*)dG(a).
$$

Our quantitative analysis below illustrates that the calibrated model is consistent with this ranking.

---

[28]The derivation of equation (29) is contained in Appendix A-III.

IV.   Quantitative Analysis

In this section we assess the quantitative performance of our model in replicating the data on earning yields and returns of firms with different international status, and with aggregate data on trade and FDI participation.

There are many elements in the model, and the lack of fully analytical expressions may prevent the reader from gaining intuition of the model's workings. For this reason, our quantitative analysis proceeds in steps, starting from a simpler version of the model that disregards endogenous selection into status and aggregation, but delivers closed form solutions for the value functions and the returns. After the channels of the model are made more transparent in this simpler case, we move to the calibration and computation of the full industry equilibrium.

IV. A.   A Simple Example: Exogenous Participation in Foreign Markets

In this section we use a special case of the model to illustrate the relationship between returns and fixed costs of production in a more transparent way. For the time being, we assume that sunk costs of export and FDI are extremely large: $F_X, F_I \to \infty$. Extremely high sunk costs imply extremely wide bands of inaction, so that firms do not move across statuses.

We start by considering an economy where firms are homogeneous (the unit cost distribution $G(a)$ is degenerate at $\bar{a}$) and are exogenously assigned to the three international statuses. In this way, we abstract from the role of productivity differences and selection and study the effect of international status on returns keeping constant firms' characteristics. We refer to this special case of the model as the exogenous participation case, since participation in foreign markets is exogenously given and does not depend on firm characteristics.

When firms do not move across statuses, the option value components of the value functions tend to zero (the value of an option is negligible if the probability of exercising it tends to zero). Hence the value of the firm converges to the present discounted value of profits from never changing status. Let $V_S^e(a, Q^*)$ denote the value of foreign sales for a firm in status S when participation is exogenous and status is

constant over time:

$$
\text{(30)} \qquad V_D^e(a, Q^*) \;=\; 0
$$

$$
\text{(31)} \qquad V_X^e(a, Q^*) \;=\; \frac{H(\tau \bar{a} w)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma \rho \sigma \sigma^* + \lambda \phi^* (1 - \phi)^{-\gamma}} - \frac{f_X}{r}
$$

$$
\text{(32)} \qquad V_I^e(a, Q^*) \;=\; \frac{H(\bar{a} w^*)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma \rho \sigma \sigma^* + \lambda \phi^* (1 - \phi)^{-\gamma}} - \frac{f_I}{r}.
$$

Using (30)-(32) in equation (29), we can express the returns under exogenous participation as follows:

$$
ret_D^e \;=\; r + \gamma \sigma^2 + \lambda \phi [(1 - \phi)^{-\gamma} - 1]
$$

$$
ret_X^e \;=\; r + \frac{\left[\gamma \sigma^2 + \lambda \phi [(1 - \phi)^{-\gamma} - 1]\right] \frac{\pi_D}{r - \delta} + \left[\gamma \rho \sigma \sigma^* + \lambda \phi^* [(1 - \phi)^{-\gamma} - 1]\right] \frac{H(\tau \bar{a} w)^{1-\eta} P^{*\eta} Q^*}{r - \delta^*}}{\frac{\pi_D}{r - \delta} + \frac{H(\tau \bar{a} w)^{1-\eta} P^{*\eta} Q^*}{r - \delta^*} - \frac{f_X}{r}}
$$

$$
ret_I^e \;=\; r + \frac{\left[\gamma \sigma^2 + \lambda \phi [(1 - \phi)^{-\gamma} - 1]\right] \frac{\pi_D}{r - \delta} + \left[\gamma \rho \sigma \sigma^* + \lambda \phi^* [(1 - \phi)^{-\gamma} - 1]\right] \frac{H(\bar{a} w^*)^{1-\eta} P^{*\eta} Q^*}{r - \delta^*}}{\frac{\pi_D}{r - \delta} + \frac{H(\bar{a} w^*)^{1-\eta} P^{*\eta} Q^*}{r - \delta^*} - \frac{f_I}{r}}
$$

where $\delta \equiv \mu - \gamma \sigma^2 - \lambda \phi (1 - \phi)^{-\gamma}$ and $\delta^* \equiv \mu^* - \gamma \rho \sigma \sigma^* - \lambda \phi^* (1 - \phi)^{-\gamma}$ are the risk-adjusted drifts of the operating profits processes for domestic and foreign sales, respectively. It is apparent from these equations that returns are increasing in $\rho \sigma \sigma^*$ (the covariance of shocks across countries reduces diversification and increases returns) and in the fixed costs f_X, f_I (operating leverage increases returns). Moreover, we can easily and reasonably parameterize the model to replicate exactly the magnitude of the returns observed in the data across the three groups.

We present a bilateral calibration exercise which is meant to describe export and FDI activity between the US and an aggregate set of trading partners. We assume that preferences, shock processes, and productivity are identical in the US and in the other country, and that the trade and FDI cost parameters are also the same across countries.

We start by normalizing the common unit cost to $\bar{a} = 1$. For the time being, we disregard aggregation and assume that prices and wages satisfy: $w^{1-\eta} P^{\eta} = w^{*1-\eta} P^{*\eta} = 1$.[29] As the expressions above show, the returns of domestic firms (ret_D) depend

---

[29]The calibration of the full model in the next section shows the results for the industry equilibrium where P and P^* are endogenous.

only on the parameters ruling preferences and the evolution of the shocks. We take values of γ, λ and ϕ from the calibration in Barro and Ursúa (2008): the risk aversion parameter is chosen equal to 4, the probability of a disaster is 0.036, and – when a disaster happens – aggregate consumption in both countries drops of 22%. The Brownian motion parameters are set to be equal to the mean and standard deviation of aggregate consumption growth in the US over the sample period: $\mu = \mu^* = 0.02$, and $\sigma = \sigma^* = 0.022$. We then choose the subjective discount factor ϑ to match ret_D perfectly: with a reasonable value $\vartheta = 0.0275$, we obtain $ret_D = 5.68\%$.

The returns of exporters and multinational firms also depend on η, ρ, Q, Q^*, τ, f_X, and f_I. Like many papers focusing on long run macroeconomic predictions, we use a standard value of $\eta = 2$ for the elasticity of substitution. To select a value for ρ, we computed correlations of GDP growth rates between the US and its main trading partners, and took the median value: $\rho = 0.45$.[30] We set the average values of aggregate consumption to $Q = 5$ and $Q^* = 20$ to ensure that profits are positive on average and to capture the fact that the US economy accounts for approximately 20% of world GDP. The iceberg cost of export is set to $\tau = 1.3$, consistent with a medium-range estimate in Eaton and Kortum (2002).

Finally, f_X, and f_I are chosen to match ret_X and ret_I perfectly: $f_X = 3.44$, and $f_I = 4.5$. The fixed costs needed to reconcile the model with the data are very large, at 67% and 72% of operating profits, respectively. Fixed costs are chosen to perfectly match returns, but we are not choosing any parameter to match the earnings-to-price ratios. The earning yields generated by the model are 4.47% for domestic firms, 5.03 % for exporters, and 5.19% for multinational firms. The model is consistent with the ordering that we found in the data, and generates numbers that are of the right order of magnitude, but produces less variation across the three groups compared to what we see in the data. The calibrated parameters and implied financial moments are summarized in the third column of Table 4.

This example also showcases the nature of the effect of sunk costs: the presence

---

[30]We use data from OECD Statistics for 28 countries: 26 OECD countries plus China and South Africa (all the countries for which data are available for the entire sample period). This is a large sample of countries in terms of the destinations of US foreign sales, as it accounts for approximately 75% of total US exports. GDP growth correlations range from 0.177 between the US and Portugal and 0.858 between the US and Canada. For this reason, the choice of $\rho = 0.45$ is conservative: if one had to construct an export-weighted measure of correlation, for example, the value of ρ would increase to 0.81.

Table 4: **Exogenous Participation Case: Calibration.**

Parameter	Definition	Value	
		$\bar{a} = 1$	$\bar{a}_D = 1$ $\bar{a}_X = 1/1.209$ $\bar{a}_I = 1/1.44$
μ, μ^*	average consumption growth	0.02	0.02
σ, σ^*	st. dev. of consumption growth	0.022	0.022
ρ	median correlation of consumption growth	0.45	0.45
λ	probability of disaster	0.036	0.036
ϕ, ϕ^*	size of disaster (% drop in consumption)	0.22	0.22
γ	risk aversion	4	4
ϑ	subjective discount factor	0.0275	0.0275
η	elasticity of substitution	2	2
τ	iceberg export cost	1.3	1.3
f_X	fixed export cost	3.44	4.16
f_I	fixed FDI cost	4.5	6.48
Q, Q^*	average aggregate demand levels	10, 20	10, 20
Implied returns (targeted)			
ret_D (%)		5.68	5.68
ret_X (%)		9.29	9.29
ret_I (%)		10.28	10.28
Implied earning yields (not targeted)			
ep_D (%)		4.47	4.47
ep_X (%)		5.03	5.03
ep_I (%)		5.19	5.19

of sunk costs is what forces a firm into a status and makes it pay the operating fixed cost. Higher fixed costs, in turn, are associated with higher returns. This mechanism takes an extreme form in this example, where extremely large sunk costs imply infinite persistence into status.

Finally, this example shows a simple and transparent way to generate the levels and differences in returns across the three groups of firms by international status, but it is clearly not a realistic description of the economy. In the real world firms are not assigned randomly to international status, there is selection by productivity, and entry and exit are endogenous. To bring back all these elements into the analysis, the next section will show a calibration that uses the full structure of the model. Before that, we argue here that the full model has a harder time to fit the data exactly because of the effect of selection on returns.

Suppose that we still abstract from endogenous selection, but for some exogenous reason there are productivity differences between domestic firms, exporters, and multinationals. We quantify these productivity differences by setting them equal to the average sales per employee within group in our sample, after normalizing the productivity of domestic firms to 1: $\bar{a}_D = 1$, $\bar{a}_X = 1/1.209$ and $\bar{a}_I = 1/1.44$.

When we compute returns in the exogenous participation case using these unit cost levels, we obtain: $ret_D = 5.68\%$, $ret_X = 7.72\%$, and $ret_I = 7.24\%$. Returns of domestic firms are not affected by productivity in this scenario, but the higher productivity of exporters and multinationals has the effect of decreasing their returns. Moreover, the higher productivity of MNCs over exporters reverts the ordering of the returns between these two groups: within group, the model predicts that more productive firms have lower returns. In order to bring back the returns to the target values, larger fixed costs are required, equal to 4.16 and 6.48 for exports and FDI, respectively (equivalent to 68% and 72% of exporters and MNCs' operating profits, respectively). This is feasible if participation is exogenous, but when we compute the full model with endogenous selection into status, fixed costs affect selection and if they are too high there will be too little firms (or no firms at all) that are exporters or multinationals in equilibrium. These results are summarized in the fourth column of Table 4.

With these observations in mind, we now move to the calibration and computation

of the full model.

IV. B.   Quantitative Analysis of the Full Model

The calibration exercise presented in this section is designed to evaluate the ability of the full model to match qualitatively and quantitatively the cross-sectional differences in earnings-to-price ratios and returns of firms according to their international status, and aggregate moments related to trade and FDI participation. Also here we present a bilateral calibration exercise which describes export and FDI activity between the US and an aggregate set of trading partners. Due to data availability, we impose a series of symmetry assumptions: we assume that preferences and productivity distributions are identical in the US and in the other countries, and that the trade and FDI cost parameters are also common across countries.[31]

Given that the model does not admit closed-form solutions, we choose parameter values such that – when we simulate the model for a time horizon as long as our sample period in the data (30 years), the cross-sectional moments that the model delivers are as close as possible to the ones in the data.

The calibration of most parameters is unchanged with respect to the simple example illustrated in the previous section: the values of μ, μ^*, σ, σ^*, ρ, λ, ϕ, ϕ^*, γ, ϑ, η are the ones reported in Table 4. As we mentioned above, we consider an economy with positive growth ($\mu = \mu^* = 0.02$). We impose that wages in the two countries grow at the exogenous and deterministic rate $\mu_w \equiv (\eta - 1)\mu$, to prevent the economy from converging to a long-run scenario with all multinational firms. For simplicity, we also assume that wages are equal across countries: $w/w^* = 1$.

Since in the full model selection into status is endogenous, we need to parameterize the cost distributions $G(a)$, $G^*(a)$. Several studies document that the tail of the empirical firm size distribution is well approximated by a Pareto distribution (see for example Luttmer (2007)). Since firm size (sales) is determined by productivity in the model, we assume that firms' productivities $1/a$ are distributed according to a Pareto law with location parameter b and shape parameter k.[32] We set $b = 1$ to facilitate the

---

[31]Compustat records data of firms with activities in the US, among which there are both US-based firms and foreign firms. However, only data of foreign firms with activities in the US are reported (in other words, we have no data about foreign firms with activities only in their domestic market), which implies that we cannot construct shares of foreign firms in each status.

[32]The Pareto distribution is also a convenient choice for computational reasons, since it allows

comparison with the exogenous participation version of the model, while we calibrate k jointly with the remaining parameters.

Together with k, it remains to calibrate the trade and FDI costs τ, f_X, f_I, F_X, F_I, and the initial values of the aggregate demand levels Q_0, Q_0^*. We choose these parameters so that our calibrated economy: i) is consistent with the fact that the US account for approximately 20% of world GDP, ii) the shares of exporters and MNCs and their average labor productivities in equilibrium are consistent with the ones in our sample, and iii) the earnings-to-price ratios and the returns that the model generates are ranked and quantitatively as close as possible to the data.

To simulate the economy, we generate an artificial dataset of 1000 firms with productivities drawn from a Pareto distribution with parameters $(b, k) = (1, 4)$, and we simulate a 30-period economy 100 times.[33] In each simulation, we initialize the firms' distribution into status to look like the one in the data and we generate a sample process for (Q, Q^*). When simulating the paths of Q and Q^* we impose that no disaster occurs, consistently with the fact that our sample period (1979-2009) is disaster-free. The possibility of disasters affects asset prices only by affecting how agents discount future cash flows. Given the process of the shocks, we simulate the economy, recording the distributions of firms into status in each period. For each firm and period, we compute earnings, equilibrium values, and returns. For each year we create three portfolios of domestic firms, exporters, and multinationals, and we compute portfolio earnings-to-price ratios and returns. For each simulation, we compute the mean of earning yields and returns over time. We repeat this process for the 100 Monte Carlo simulations, and we average the results across simulations.

We use a grid search to select the parameters that best match the moments in the data. The resulting calibrated parameters are reported in Table 5. In order to match the data, the model requires a relatively low iceberg trade cost ($\tau = 1.2$), but very high fixed and sunk costs. Fixed costs of export and FDI are equal to 4.88 and 6.92, respectively. These costs imply that, according to the simulated model, an exporter must spend, on average, 46.6% of its revenues in fixed costs, while a multinational

---

to solve explicitly for the aggregate prices P, P^* as functions of the productivity thresholds that induce firms to transition across statuses and of the other parameters of the model.

[33]The entire computation of the model for a given parametrization takes about 40 hours on a cluster of 10 CPUs. Details about the algorithm used are provided in Appendix A-IV.

Table 5: **Calibration of the Full Model.**

Parameter	Definition	Value
b	lower bound of the productivity distribution	1
k	shape parameter of the productivity distribution	4
τ	iceberg export cost	1.2
f_X	fixed export cost	4.88
f_I	fixed FDI cost	6.92
F_X	sunk export cost	20
F_I	sunk FDI cost	24
Q, Q^*	average aggregate demand level	3, 10

must spend, on average, about 39.5% of its revenues in fixed costs. Sunk costs of export and FDI are much higher, equal to 20 and 24, respectively. These numbers imply that a domestic firm must spend on average 12 times its per-period revenue to enter the foreign market as an exporter, and about 15 times its per-period revenue to start FDI operations there.[34] Aggregate demand parameters are set at $Q(0) = 3$ and $Q^*(0) = 10$.

Our calibration sheds light on the nature of trade and FDI costs. In order to match selection, limited participation in export and FDI, and the financial moments, the barriers to trade and FDI that our model requires are much higher than previous estimates. Targeting US data, the calibration in Alessandria and Choi (2007) produces sunk costs of export equivalent to 12.6% of the average sales of a non-exporter, and fixed export costs equal to about 2.1% percent of the average sales of an exporter. Using Colombian data, Das, Roberts, and Tybout (2007) estimate that sunk export costs account for between 18.4 and 41.2% of the annual value of a firm's exports while fixed export costs are on the order of 1% of the annual value of exports. To our knowledge, the only paper that estimates the fixed costs of horizontal FDI is Tintelnot (2014), who estimates that the fixed costs of operating foreign plants for German multinationals range between 6 and 9 million Euros, equivalent to 2.7-4% of the av-

---

[34]Since entry and exit decisions are taken comparing sunk and fixed costs with operating profits, it is also instructive to present the size of these costs relative to firms' operating profits. The sunk costs to start exporting (doing FDI) are equal to 17 (20) times the operating profits of the average domestic firm. The fixed operating costs of continuing exporting (doing FDI) in turn are equal to 94% (80%) of the operating profits of the average exporter (multinational). These numbers confirm that a huge portion of firms' resources are invested in fixed and sunk costs.

erage revenues of a multinational firm. We are not aware of any paper estimating the sunk costs of FDI.

Despite the large difference in levels between our calibrated frictions and previous estimates in the literature, our calibrated model generates a ratio of sunk to fixed costs of exports that is similar to the one obtained by Alessandria and Choi (2007): $F_X/f_X \approx 4.1$ in our paper, while it is about 6 in theirs. It has to be noted that the way in which we identify trade and FDI frictions is very different from previous work: while Alessandria and Choi (2007) and Das, Roberts, and Tybout (2007) quantify trade frictions by matching transitions into and out of export status, our calibration targets the financial moments of firms with different international status.

The larger frictions that our model requires also imply larger gains from trade to be obtained from their removal. The partial equilibrium structure of the model does not make it well-suited for welfare calculations. However, we can get some idea of the magnitude of the relative welfare gains induced by fixed versus sunk costs. Removing the fixed costs of production ($f_X = f_I = 0$) implies reductions in the price indexes P and P^* of 15% and 35%, respectively, while removing the sunk costs of production ($F_X = F_I = 0$) implies reductions in the price indexes P and P^* of 10% and 3%, respectively, suggesting that much larger gains can be obtained from reducing fixed operating costs than sunk entry costs.

Table 6 jointly displays the moments computed from the data and the moments generated by the calibrated model. The model successfully replicates the participation in exports and FDI that we observe in the data, with 19% of firms being exporters and 44% of firms being MNCs in equilibrium. The ranking by labor productivity across groups is also consistent with the data: the productivity premium of exporters is below the one in the data (about 10% in the model versus 20% in the data), while the productivity premium of MNCs is slightly higher in the model than in the data (58% in the model versus 44% in the data). The model is successful in generating the observed ordering in earning yields and returns. The variation across groups is less than what we see in the data, but the overall levels are in the right range: the model produces average earning yields of 4.41% for multinational firms, 3.89% for exporters, and 3.63% for firms selling only domestically, and average stock returns of 7.98% for multinational firms, 7.34% for exporters, and 6.35% for firms selling only

Table 6: **Moments.** Comparison of the moments, model *versus* data.

	Data	Model ($\phi^* = \phi$)	Model ($\phi^* = 2\phi$)
share of exporters (%)	18.24	19.1	19.03
share of MNCs (%)	45.14	43.96	43.96
sales per employee X/D	1.209	1.098	1.098
sales per employee I/D	1.44	1.583	1.583
ep_D (%)	3.96	3.63	4.28
ep_X (%)	4.58	3.89	4.74
ep_I (%)	6.33	4.41	6.77
ret_D (%)	5.68	6.35	5.9
ret_X (%)	9.59	7.34	7.12
ret_I (%)	10.28	7.98	9.33

domestically.[35]

The calibrated model generates a "multinational premium" (the difference between ret_I and ret_D) which is about 35% of what we observe in the data. This result can be improved further by removing a restriction on the parameters that we imposed in order to present a conservative baseline calibration. In our baseline calibration, we impose that the size of disasters in the Home and Foreign country is the same: $\phi = \phi^* = 0.22$. In the third column of Table 6 we present the results of a calibration where we allow the size of the disasters in the two countries to be different. Specifically, we set $\phi^* = 2\phi$. Higher disaster risk in the Foreign market has the effect of increasing the returns of MNCs, so that this alternative parameterization generates a "multinational premium" which is about 75% of what we observe in the data.

---

[35]In terms of non-targeted moments, one could wonder about the performance of the model in matching international real business cycle moments. The model generates tiny movements of the price indexes over time, so that changes in the real exchange rate (defined as P/P^*) are virtually zero, and so is the correlation of changes in the real exchange rate with consumption growth. Since exports and FDI sales are linear in Q^*, the volatility of export growth is close to the volatility of dQ^*/Q^* (calibrated to 0.022). For the same reason, the correlations of export growth and FDI sales growth with consumption growth are close to the correlation of consumption growth in the two countries (calibrated to 0.45). In the data, aggregate export sales growth is more volatile that changes in the real exchange rate. While the correlation of export growth with consumption growth is slightly higher than the correlation of changes in the real exchange rate with consumption growth, the correlation of FDI sales growth with consumption growth is much lower. We believe that the failure of the model to match international real business cycle moments is mostly due to its partial equilibrium structure: the changes in the equilibrium wages that drive most of the action in related work (see Ghironi and Melitz (2005), for example) do not play a role here.

Given that all the parameters jointly contribute to determine the equilibrium moments, it is hard to talk about identification. This said, the sensitivity analysis reveals that sunk costs are the main determinants of the extent of participation into exports and FDI. Through their effect on participation, higher sunk costs also increase the spread among returns in the three groups. The initial levels of Q, Q^* are important drivers of the levels of the earnings-to-price ratios, while adjustments in the iceberg cost help with the ordering of the earnings-to-price ratios (τ is the only parameter that affects only the profits of exporters). The next section reports the results of the sensitivity analysis in a systematic way, to illustrate the individual contributions of the various elements of the model to the results.

IV. C.   Sensitivity Analysis

In this section we illustrate the workings of the model by performing sensitivity analysis on its main parameters. We study the role of disaster risk (through changes in ϕ, ϕ^*), of trade frictions (through changes in f_X, f_I, F_X, F_I), and of diversification potential (through changes in ρ) for the numerical results. For each exercise, we report results for earnings-to-price ratios, returns, and shares of firms in each group, since changes in the parameters also affect selection into statuses. Some parameter changes have very large effects on selection, so that the implied changes in financial variables are driven both by the direct effect of the changed parameter and by the selection effect (*i.e.* averages within group are computed using different firms). For those exercises that imply large changes in selection, we also report the results imposing that the shares of firms in each group are the same ones of the baseline calibration: these "constant shares" results isolate the direct effect of parameter changes on financial variables, and disregard the indirect effect through selection.

We start by assessing the role of disaster risk for our quantitative results. Table 7 reports the results of three different versions of the calibrated economy with no disaster shocks ($\phi = \phi^* = 0$). We report the results in terms of excess returns (returns minus the risk free rate) for comparison purposes, since ϕ also affects the value of the risk-free rate (equal to $\vartheta + \gamma\mu - \gamma(\gamma + 1)\frac{1}{2}\sigma^2 - \lambda\left[(1 - \phi)^{-\gamma} - 1\right]$ in the model).

In the second column of Table 7 we report the results obtained by setting $\phi =$

Table 7: **Sensitivity analysis: the role of disaster risk.**

	$\phi = \phi^* = 0.22$ (baseline)	$\phi = \phi^* = 0$ (endogenous shares)	$\phi = \phi^* = 0$ (constant shares)	$\phi = \phi^* = 0$ (endogenous shares, constant r)
ep_D (%)	3.63	8.23	8.31	2.81
ep_X (%)	3.89	4.51	-30.19	3.00
ep_I (%)	4.41	6.08	5.17	3.31
$ret_D - r$ (%)	2.21	0.20	0.20	0.14
$ret_X - r$ (%)	3.2	0.54	0.73	0.13
$ret_I - r$ (%)	3.84	0.28	0.66	0.09
share of exp. (%)	19.1	2.84	19.1	24.89
share of MNCs (%)	43.96	24.99	43.96	42.56

$\phi^* = 0$ and leaving all the other parameter values unchanged. This specification generates much lower export and FDI participation compared to the baseline economy: removing disaster risk increases the risk-free rate, which in turns reduces the value of entry into foreign markets. The economy without disaster risk does not display the ranking in earning yields and returns. Moreover, excess returns and their differences across the three groups are tiny, at less than a percentage point. This result comes from the direct effect of disaster risk on returns and not from selection, since it is present also in the calibration where we keep selection constant (the third column of the table). The fourth column of Table 7 reiterates this point by presenting the results of the simulation of the model without disaster risk and with the risk-free rate kept at the same value as in the baseline calibration.[36] Removing disaster risk while keeping the risk-free rate constant generates export and FDI participation close to the baseline scenario, but again generates very small excess returns, exhibiting the opposite ordering compared to the one in the data.

Overall, the results of Table 7 illustrate that the presence of disaster risk in the model is crucial for the quantitative fit of the financial data.

Table 8 illustrates the effect of trade and FDI frictions in the model. We report the results of the calibrated economy with no fixed costs ($f_X = f_I = 0$), with no sunk

---

[36]We set $\phi = \phi^* = 0$ and choose $\mu = \mu^*$ such that the implied risk-free rate is the same as in the baseline calibration. This implies $\mu = \mu^* = 0.0047$.

Table 8: **Sensitivity analysis: the role of fixed and sunk costs.**

	baseline calibration	$f_X = f_I = 0$ (constant shares)	$F_X = F_I = 0$ (constant shares)	$f_X = f_I = F_X = F_I = 0$ (constant shares)
ep_D (%)	3.63	0.12	2.37	4.27e-05
ep_X (%)	3.89	4.39	3.77	1.73e-04
ep_I (%)	4.41	4.39	3.83	4.39
ret_D (%)	6.35	9.17	7.36	9.26
ret_X (%)	7.34	5.69	7.33	9.26
ret_I (%)	7.98	5.69	6.12	5.6

costs ($F_X = F_I = 0$) and of a frictionless economy where all trade and FDI costs are removed ($f_X = f_I = F_X = F_I = 0$). Since the lack of trade and FDI frictions affects selection enormously (in most cases, all firms become MNCs) we present here the results keeping the shares of firms in each group constant, so that we can isolate the direct effect of trade and FDI frictions on the financial variables.[37]

Removing either friction does not affect the ranking in earnings-to-price ratios, but delivers the opposite ranking in returns: with no costs of trade or FDI, the diversification potential that foreign sales give make exporters and multinationals safer than domestic firms, and hence commanding lower returns.[38] Hence Table 8 shows that trade and FDI frictions are essential to obtain both selection and high returns of international firms.

Next, we investigate the role of the correlation of shocks across countries. Table 9 reports the results of sensitivity with respect to the correlation of consumption growth across countries (ρ). Our baseline calibration uses $\rho = 0.45$ (the median correlation between the US GDP growth rate and the ones of its main trading partners). We report the results using perfectly correlated shocks ($\rho = 1$, $\rho = -1$) and uncorrelated shocks ($\rho = 0$).

The correlation coefficient ρ affects selection minimally. As expected, both earnings-

---

[37]The results with endogenous shares, together with all other non reported sensitivity results, are available upon request to the authors.

[38]The high returns of exporters in the frictionless case can be rationalized by thinking that these firms would become MNCs if we were not holding the shares of firms constant, so their value function is highly convex due to the high option value of becoming a MNC.

Table 9: **Sensitivity analysis: the role of the correlation of consumption growth across countries.**

	$\rho = 1$	$\rho = 0.45$ (baseline)	$\rho = 0$	$\rho = -1$
ep_D (%)	3.78	3.63	3.47	3.23
ep_X (%)	4.01	3.89	3.79	3.47
ep_I (%)	4.6	4.41	4.26	3.91
ret_D (%)	6.36	6.35	6.35	6.18
ret_X (%)	7.54	7.34	7.18	6.79
ret_I (%)	8.37	7.98	7.68	7.14
share of exporters (%)	19	19.1	19.14	19.16
share of MNCs (%)	43.97	43.96	43.88	44

to-price ratios and returns are increasing in ρ: the more the shocks are correlated across countries, the less the diversification benefits of foreign sales, so that firms that operate in foreign markets are riskier the higher the value of ρ. The ranking in earnings-to-price ratios and returns across groups is preserved across all values of ρ, and the spread decreases for lower values of ρ.

At a first read, it might seem surprising that exporters and MNCs in the model require higher returns than domestic firms even when $\rho = -1$, and the diversification potential is maximized. This happens because in the model there are two different shocks: i.i.d. shocks to consumption growth rate (with correlation ρ across countries) and disaster risk shocks (that we assume to be perfectly correlated across countries). To see the results of an economy where the diversification benefits are more apparent, we report in Table 10 below the results of the calibrated model with $\rho = 0$ and no disaster shocks ($\phi = \phi^* = 0$). We report the results in terms of excess returns (returns minus the risk free rate) for comparison purposes, since ϕ also affects the value of the risk-free rate.

The economy with zero correlation of shocks and no disaster features very different selection of firms into status compared to the baseline economy: for this reason we report also the results keeping the shares of firms in each group constant, to isolate the direct effect of the parameters on earning yields and returns. In the economy with uncorrelated shocks and no disaster, earning yields and returns are not ranked,

Table 10: **Sensitivity analysis: the role of the correlation of consumption growth across countries in a no-disaster economy.**

	$\rho = 0.45,$ $\phi = \phi^* = 0.22$ (baseline)	$\rho = 0,$ $\phi = \phi^* = 0$ (endogenous shares)	$\rho = 0$, $\phi = \phi^* = 0$ (constant shares)
ep_D (%)	3.63	8.2	8.27
ep_X (%)	3.89	3.7	7.47
ep_I (%)	4.41	5.93	3.29
$ret_D - r$ (%)	2.21	0.19	0.19
$ret_X - r$ (%)	3.2	-0.22	-0.44
$ret_I - r$ (%)	3.84	0.1	0.14
share of exporters (%)	19.1	2.53	19.1
share of MNCs (%)	43.96	24.36	43.96

and the heterogeneity in returns across groups is minimal. Since the diversification benefits of foreign activities are maximized in this scenario, domestic firms display higher excess returns than international firms. Among international firms, MNCs display higher returns than exporters, consistent with the fact that they are more exposed to risk due to the higher fixed and sunk costs of production.

We conclude our analysis by briefly mentioning sensitivity results with respect to risk aversion (γ) and elasticity of substitution (η). As expected, excess returns and their spread across groups increase with γ. Increasing η generates returns that are ranked but exhibit less spread across groups compared to the baseline model, while earnings-to-price ratios are not sorted. This happens because a higher η reduces firms' monopoly power and their profits, and generates much lower shares of exporters and MNCs.

V.   Conclusions

This paper started by presenting a novel fact distinguishing multinational firms from exporters and from firms selling only in their domestic market. Multinational corporations tend to exhibit higher stock market returns and earning yields than non-multinational firms. Within non-multinationals, exporters tend to have higher stock

market returns and earning yields than firms selling only in their domestic market. To explain this fact, we developed a real option value model where firms' heterogeneity, aggregate uncertainty (in the form of disaster risk) and fixed and sunk costs provide a link between firms' choice of international status, risk exposure, and financial variables. We endogenized the exposure of these firms to sources of systematic risk.

The model is based on a very simple mechanism: firms decide to enter a risky foreign market when prospects of growth make entry profitable, and entry involves a sunk cost. If after entry firms are subject to negative shocks, they will be reluctant to exit immediately because of the sunk cost they paid to enter, and may prefer to bear losses for a while. These losses, generated by the existence of fixed operating costs, increase the risk perceived by the firms' stockholders. Moreover, if the costs of establishing and operating a foreign affiliate are larger than the ones of exporting, the exposure to fluctuations and possible losses are higher for multinational firms than for exporters, commanding a higher return in equilibrium. While being consistent with data about export and FDI participation, the model endogenously determines cross-sectional differences in financial variables and provides a complementary explanation for the cross section of returns exploiting the international dimension of the data.

We see this paper as the first step in a novel research agenda linking trade and FDI activities to asset pricing. The structural framework that we developed can be used to analyze the responses of heterogeneous agents (firms and investors) to different types of shocks: idiosyncratic, firm-specific, country-specific or aggregate, both in terms of real and financial variables. We think this is a promising avenue for research in finance and international trade, which we plan to pursue in future work.

References   *

Ai, Hengjie, and Dana Kiku. 2013. "Growth to Value: Option Exercise and the Cross-Section of Equity Returns." *Journal of Financial Economics* 107 (2): 325–349.

Alessandria, George, and Horag Choi. 2007. "Do Sunk Costs of Exporting Matter for Net Exports Dynamics?" *Quarterly Journal of Economics* 122 (1): 289–336.

Atkeson, Andrew, and Tamim Bayoumi. 1993. "Do private capital markets insure

regional risk? Evidence from the United States and Europe." *Open Economies Review* 4:303–324.

Baker, Malcolm, C. Fritz Foley, and Jeffrey Wurgler. 2009. "Multinationals as Arbitrageurs? The Effect of Stock Market Valuations on Foreign Direct Investment." *Review of Financial Studies* 22 (1): 337–369.

Bansal, Ravi, and Amir Yaron. 2004. "Risks for the Long-Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance* 59 (4): 1481–1509.

Barro, Rober J. 2006. "Rare Disasters and Asset Markets in the Twentieth Century." *The Quarterly Journal of Economics* 121 (3): 823–866.

Barro, Rober J., and José F. Ursúa. 2008. "Macroeconomic Crises since 1870." *Brookings Papers on Economic Activity*, pp. 255–350.

Berk, Jonathan B., Richard C. Green, and Vasant Naik. 1999. "Optimal investment, growth options, and security returns." *Journal of Finance* 54 (5): 15531607.

Bernard, Andrew B., J. Bradford Jensen, Stephen Redding, and Peter K. Schott. 2007. "Firms in International Trade." *Journal of Economic Perspectives* 21 (3): 105–130.

Bernard, Andrew B., J. Bradford Jensen, and Peter K. Schott. 2009. "Importers, Exporters, and Multinationals: a Portrait of Firms in the U.S. that Trade Goods." In T. Dunne, J.B. Jensen and M.J. Roberts (eds.)*, Producer Dynamics: New Evidence from Micro Data*. Chicago: University of Chicago Press.

Campbell, John Y., and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107 (2): 205–251.

Carlson, Murray, Adlai Fisher, and Ron Giammarino. 2004. "Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns." *The Journal of Finance* 59 (6): 2577–2603.

Cooper, Ilan. 2006. "Asset Pricing Implications of Nonconvex Adjustment Costs and Irreversibility of Investment." *Journal of Finance* 61 (1): 139–170 (2).

Crucini, Mario. 1999. "On international and national dimensions of risk sharing." *Review of Economics and Statistics* 8 (1): 73–84.

Das, Sanghamitra, Mark J. Roberts, and James R. Tybout. 2007. "Market Entry Costs, Producer Heterogeneity, and Export Dynamics." *Econometrica* 75 (3): 837–873.

Denis, David J., Diane K. Denis, and Keven Yost. 2002. "Global Diversification, Industrial Diversification, and Firm Value." *The Journal of Finance* 57 (5): 1951–1979.

Desai, Mihir A., and James R. Hines. 2008. "Market reactions to export subsidies." *Journal of International Economics* 74:459474.

Dixit, Avinash K. 1989. "Entry and Exit Decisions under Uncertainty." *Journal of Political Economy* 97 (3): 620–638.

Dixit, Avinash K., and Robert S. Pindyck. 1994. *Investment under Uncertainty.* Princeton, NJ: Princeton University Press.

Eaton, Jonathan, and Samuel Kortum. 2002. "Technology, Geography and Trade." *Econometrica* 70 (5): 1741–1779.

Epstein, Larry G., and Stanley E. Zin. 1989. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57 (4): 937–69.

Fama, Eugene F., and Kenneth R. French. 1993. "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics* 33:3–56.

———. 1996. "Multifactor Explanations of Asset Pricing Anomalies." *Journal of Finance* 51 (1): 55–84.

———. 2006. "Profitability, Investment and Average Returns." *Journal of Financial Economics* 82 (3): 491–518.

Fatemi, Ali M. 1984. "Shareholder Benefits from Corporate International Diversification." *Journal of Finance* 39 (5): 1325–1344.

Fillat, José L., Stefania Garetto, and Lindsay Oldenski. 2015. "Diversification, Cost Structure, and the Risk Premium of Multinational Corporations." *Journal of International Economics,* Forthcoming.

Gabaix, Xavier. 2012. "Variable Rare Disasters: an Exactly Solved Framework for

Ten Puzzles in Macro-Finance." *The Quarterly Journal of Economics* 127(2), p. 645-700. (2): 645–700.

Gala, Vito. 2012. "Irreversible Investment and the Cross-Section of Stock Returns in General Equilibrium." Mimeo, London Business School.

Ghironi, Fabio, and Marc J. Melitz. 2005. "International Trade and Macroeconomic Dynamics with Heterogeneous Firms." *The Quarterly Journal of Economics* 120 (3): 865–915.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken. 1989. "A Test of the Efficiency of a Given Portfolio." *Econometrica* 57 (5): 1121–1152.

Goldberg, Linda S., and Charles D. Kolstad. 1995. "Foreign Direct Investment, Exchange Rate Variability and Demand Uncertainty." *International Economic Review* 36 (4): 855–73.

Gomes, Joao, Leonid Kogan, and Lu Zhang. 2003. "Equilibrium Cross Section of Returns." *Journal of Political Economy* 111 (4): 693–732.

Gourio, Francois. 2011. "Putty-Clay Technology And Stock Market Volatility." *Journal of Monetary Economics* 58 (2): 117–131.

———. 2012. "Disaster Risk and Business Cycles." *American Economic Review* 102 (6): 2734–2766.

Hall, Robert E. 1978. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy* 86 (6): 971–987.

Hamano, Masashige. 2015. "International equity and bond positions in a DSGE model with variety risk in consumption." *Journal of International Economics*, Forthcoming.

Hansen, Lars Peter, John C. Heaton, and Nan Li. 2008. "Consumption Strikes Back? Measuring Long-Run Risk." *Journal of Political Economy* 116 (2): 260–302.

Haugen, Robert A., and Nardin L. Baker. 1996. "Commonality in the Determinants of Expected Stock Returns." *Journal of Financial Economics* 41:401–439.

Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple. 2004. "Exports Versus

FDI with Heterogeneous Firms." *The American Economic Review* 94 (1): 300–316.

Impullitti, Giammario, Alfonso A. Irarrazabal, and Luca D. Opromolla. 2013. "A Theory of Entry and Exit into Exports Markets." *Journal of International Economics* 90 (1): 75–90.

Leahy, John V. 1993. "Investment in Competitive Equilibrium: the Optimality of Myopic Behavior." *Quarterly Journal of Economics* 108 (4): 1105–1133.

Lucas, Robert E., Jr. 1978. "Asset Prices in an Exchange Economy." *Econometrica* 46 (6): 1429–1445.

Luttmer, Erzo G.J. 2007. "Selection, Growth, and the Size Distribution of Firms." *Quarterly Journal of Economics* 122 (3): 1103–1144.

Mehra, Rajnish, and Edward C. Prescott. 2003. "The equity premium in retrospect." *Handbook of the Economics of Finance* 1:889–938.

Novy-Marx, Robert. 2011. "Operating leverage." *Review of Finance* 15, no. 1.

———. 2013. "The Other Side of Value: The Gross Profitability Premium." *Journal of Financial Economics* 108, no. 1.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel. 2007. "Housing, Consumption, and Asset Pricing." *Journal of Financial Economics* 83:531–569.

Ramondo, Natalia, and Veronica Rappoport. 2010. "The Role of Multinational Production in a Risky Environment." *Journal of International Economics* 81:240–252.

Ramondo, Natalia, Veronica Rappoport, and Kim J. Ruhl. 2013. "The Proximity-Concentration Tradeoff under Uncertainty." *Review of Economic Studies* 80 (4): 1582–1621.

Rietz, Thomas A. 1988. "The equity risk premium: a solution." *Journal of Monetary Economics* 22 (1): 117–131.

Rob, Rafael, and Nikolaos Vettas. 2003. "Foreign Direct Investment and Exports with Growing Demand." *Review of Economic Studies* 70 (3): 629–648.

Roberts, Mark J., and James R. Tybout. 1997. "The Decision to Export in Colombia: An Empirical Model of Entry with Sunk Costs." *The American Economic Review* 87 (4): 545–564.

Russ, Katheryn N. 2007. "The Endogeneity of the Exchange Rate as a Determinant of FDI: A Model of Entry and Multinational Firms." *Journal of International Economics* 71:344–372.

Sorensen, Bent E., and Oved Yosha. 1998. "International risk sharing and European monetary unification." *Journal of International Economics* 45:211–238.

Tesar, Linda L., and Ingrid Werner. 1998. "The internationalization of securities markets since the 1987 crash." In R. Litan and A. Santomero (eds.) , *Brookings-Wharton Papers on Financial Services*. Washington: The Brookings Institution.

Tintelnot, Felix. 2014. "Global Production with Export Platforms." Mimeo, University of Chicago.

Yogo, Motohiro. 2006. "A Consumption-Based Explanation of Expected Stock Returns." *Journal of Finance* 61 (2): 539–580.

Zhang, Lu. 2005. "The Value Premium." *Journal of Finance* 60 (1): 67–103.

"Risk, Returns, and Multinational Production", by
José L. Fillat and Stefania Garetto

Online Appendix

A-I   Accounting Standards and Data Selection

The empirical analysis contained in this paper is based on annual firm-level data. Our sample is the universe of publicly traded, US-based manufacturing firms included in the Standard & Poor's Compustat Segments Database.[1] We use data from both the Center for Research in Security Prices (CRSP) and Compustat, obtained via Warton Research Data Services (WRDS). Our sample starts in 1979 and ends in 2009.

We use CRSP data on prices, earnings per share, numbers of shares outstanding, and returns of ordinary common stocks that are the primary security for each firm. Compustat data is comprised of key components from annual SEC regulatory filings (10K) and provides the link to Compustat Segments, which contains information on firms' foreign operations. Segments data categorize a firm's operations along a particular business division and report sales, assets, and other information. The four segment classifications are business, geographic, operating, and state. Multinational and export status dummies are constructed based on Compustat geographic and operating segments data.

The Financial Accounting Standards Board (FASB), in its Statement No. 131, sets the standards for the way in which public businesses report information about operating segments in their annual financial statements. Operating segments are defined by the FASB as *"components of an enterprise about which separate financial information is available that is evaluated regularly by the chief operating decision maker in deciding how to allocate resources and in assessing performance"*. The Statement of Financial Accounting Standards (FAS) No. 131 determines that firms should report data about revenues derived from the firm's *products or services*, *countries* in which they earn revenues and hold assets, and about major *customers* regardless of whether that information is used in making operating decisions. However, the statement does

---

[1] The NAICS codes for manufacturing firms contain the 2-digit prefixes 31, 32, and 33.

not require firms to disclose the information on all the different segment types if it is not prepared for internal use and reporting would be impracticable. Therefore, the firms decide how to report the data, disaggregated in several different ways: either by product, geography, legal entity, or by customer, but they do not necessarily have to report all of them. This method is referred to as the management approach. The statement establishes a minimum threshold to report separately information about an operating segment: either revenues of the segment are 10% or more of the combined revenue of all operating segments, or profits or losses are 10% or more of the combined reported profit or losses, or its assets are 10% or more of the combined assets of all operating segments. Hence, if a given firm considers best practice to aggregate the information upstream to the management level by customer, it may elect not to disclose geographical segments information.

According to the FAS 131, when a firm reports the existence of a geographical segment, it must report revenues and holdings of long-lived assets held in foreign countries. The FAS is not explicit in defining an ownership threshold for reporting, but the existence of accounting standards for the segments themselves leads us to think that the parent (US-based) firm must have a control stake in the foreign entity. One of the Financial Accounting Standards Board (FASB)'s roles is to *"require significant disclosures about the separate operating segments of an entity's business so that investors can evaluate the differing risks in the diverse operations"*. Moreover, this information may or may not be disaggregated by individual foreign countries.

The relevant segment for our classification of firms by status is the geographic segment. Faced with the potential measurement problems associated with the loose reporting requirements of Compustat Segments, we had two options to select our dataset: 1) include in the dataset only those firms that reported the existence of operating segments and drop all the others, or 2) include all firms in Compustat and impute as Domestic the status for those firms that did not report the existence of operating segments. The data analysis reported in Section II. corresponds to the second selection criterium, which we prefer, because it generates a richer data set.[2] 96% of the firms that reported the existence of operating segments also report the existence of a geographic segment. For firms that report a geographic segment in a

---

[2]For robustness, we run all the regressions also using the data set constructed with the second selection criterium, and the results on the ranking of earning yields and returns are unchanged

given year, all non-geographic segment observations were excluded. For the remaining firms, we aggregate data from the business segment and assume the firm's operations are entirely domestic. For these firms, all non-business segments are excluded from the sample. There are three types of geographic segments: (1) total, (2) domestic, and (3) non-domestic. A firm is classified as domestic in a given year if there is a missing or zero value for exports. A firm is classified as an exporter in a given year if the value for exports is non-missing and greater than zero. This includes firms that reported exports as insignificant or firms that reported exports in other data items.[3] A firm is classified as multinational in a given year if its segments have a maximum Segment Geographic Type of 3 for a given year and if foreign sales are non-missing and greater than zero. Due to reporting errors, misclassifications, and multiple classifications, a few notes are required.

As is typical when a data point is unreliable, unreported, or otherwise a break from the traditional definition, the provider will report codes in place of an interpretable value. Compustat employs a similar methodology. In these instances we assume the segment to be of negligible importance and consider associated sales and exports to be null. As mentioned above our implementation of segment data relies entirely on the classification provided in the data. However, in a few instances sales for the non-domestic segments indicate the market of operation as the United States. In these cases we assume the reported classification was in error and re-classify the segment as domestic.

The data is aggregated by firm-year. For many firms this aggregation requires combining multiple segments and may result in competing classifications for a firm in a particular year. In these instances we classify the firm by the most "globally engaged" reported segment (for example domestic firms with exports are classified as exporters, while exporters with foreign sales are considered multinationals).

Examining a firm's international classification over time reveals what we believe to be reporting errors. These cases are characterized by a one-year "downward" status change, which results in a return to the original status. We believe this transient status change is a result of inaccurate segment reporting. As such we re-classify the observation to ensure continuity in the series. However, the opposite is not true: when

---

[3]Compustat specifically provides code values for *"value reported in another data item"*.

3

a firm enters into an international market only to exit the following year, the firm retains the reported classification. The logic for this is evident: it is far easier to omit classification in a given year than to report segment details that were nonexistent.

Another dimension of selecting the data involves which criteria to use to establish the unit of observation and to eliminate outliers. The data span 31 years, from 1979 to 2009, but many firms have observations only for a part of this time interval.[4] We drop firms that are active for less than one year and duplicate observations after a merge. The firms' classification is based on the fiscal year of each firm, as annual reports correspond to fiscal years and not annual years. As a result, annual returns are the result of compounding firm level monthly returns based on their fiscal year period. We disregard years for which we do not have 12 months of returns.

A-II   Empirical Analysis: Robustness

This section complements the analysis of Section II. by performing additional portfolio-level asset pricing tests. We explore empirically the source of higher returns of firms selling in foreign markets. Higher average returns do not constitute a puzzle per se: they simply indicate that multinational firms and exporters are riskier than domestic firms. Research in finance has interpreted riskiness of a stock either as a higher covariance with financial market factors (i.e., the aggregate market portfolio) or as a higher covariance with consumption-based factors embedded in the agents' intertemporal marginal rate of substitution (IMRS). We report results from the CAPM and Fama-French regressions, and a simple version of the consumption-based CAPM.

The CAPM model explains higher returns of certain assets as being generated by a larger covariance with systematic risk, represented by the returns on the aggregate market portfolio.[5] We run one time-series CAPM regression for each of the three portfolios of firms characterized by the same international status.[6] The results are displayed in the first column of Table A-I. The risk to which multinationals and exporters are exposed, and the corresponding higher returns they provide to investors,

---

[4]All variables have been deflated to constant 1984 dollars.

[5]The return on the market is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate.

[6]Every year portfolios are formed by equally-weighting the returns of firms belonging to each of the three categories.

are partially explained by higher *betas*: the portfolio formed by multinational corporations exhibits a higher *beta* than the one of exporters, and the domestic firms' portfolio exhibits the lowest.

The exposure to the returns on the market portfolio only partially explains the higher reward that multinationals' and exporters' stocks provide, which is also reflected in the pricing errors of the model. In fact, the *alpha* of the portfolio of multinational firms is significantly higher than the one of the exporters' portfolio, which in turn is higher than the *alpha* of the portfolio of domestic firms.

Fama and French (1993) introduced a multi-factor extension of the original CAPM. They argued that a unique source of risk is not able to explain the cross section of returns. Instead, a three-factor model explains a higher portion of the variation in expected returns. Higher returns must be explained by higher exposure to either of these three factors: market excess returns, high-minus-low book-to-market, or small-minus-big portfolio, as these characteristics seem to provide independent information about average returns.[7] Therefore, any asset is represented as a linear combination of the three Fama-French factors. These regressions, though, are not informative about why exposures are different across portfolios. The purpose of our model is to endogenize these exposures.

The second column of Table A-I shows the results of running one time-series Fama-French regression for each of the three portfolios of firms characterized by the same international status. The risk to which multinationals and exporters are exposed, and the corresponding higher returns that they provide to investors, are not explained by the three existing Fama-French factors. On the contrary, we find that the market *betas* of exporters and multinationals are lower than the ones of domestic firms. Multinationals' exposure to the SMB factor, related to size, and multinationals and exporters' exposure to the HML factor, related to the value premium, are also lower than the exposure of domestic firms to the same factors. Differences in returns are not

---

[7]The small-minus-big (SMB) and high-minus-low (HML) factors are constructed upon 6 portfolios formed on size and book-to-market. The portfolios are the intersection of 2 portfolios formed on size (small and big) and 3 portfolios formed on book equity to market equity (from higher to lower: value, neutral, and growth.) This generates 6 portfolios: small-value, small-neutral, small-growth, big-value, big-neutral, and big-growth. SMB is the average returns on the three small portfolios minus the average returns on the three big portfolios. HML is the average return on the two value portfolios minus the average return on the two growth portfolios. For more details see Fama and French (1993).

Table A-I: **Portfolio Regressions: CAPM, Fama-French, 4-Factors Fama-French.** Time-series coefficient estimates of CAPM and Fama-French regressions for the three equally-weighted portfolios based on international status. The α coefficients capture the pricing errors.

	CAPM			Fama-French			4-Factors FF (international market)		
	DOM	EXP	MN	DOM	EXP	MN	DOM	EXP	MN
α	-0.047 (0.020)**	-0.010 (0.023)	-0.003 (0.017)	-0.081 (0.012)***	-0.034 (0.016)**	-0.025 (0.013)*	-0.083 (0.012)***	-0.036 (0.017)**	-0.031 (0.013)**
R^{mkt}	0.643 (0.120)***	0.665 (0.137)***	0.670 (0.097)***	0.710 (0.066)***	0.688 (0.091)***	0.706 (0.073)***	0.673 (0.085)***	0.659 (0.117)***	0.622 (0.091)***
R^{SMB}				0.734 (0.092)***	0.850 (0.128)***	0.509 (0.101)***	0.743 (0.094)***	0.858 (0.131)***	0.530 (0.100)***
R^{HML}				0.267 (0.075)***	0.083 (0.106)	0.164 (0.083)*	0.264 (0.076)***	0.079 (0.108)	0.156 (0.081)*
R^{INT}							0.041 (0.060)	0.033 (0.083)	0.095 (0.065)
GRS (p-value) H_0: $\alpha_D = \alpha_X = \alpha_I$	0.791 (0.5097)			21.12 (4.92E-07)***			20.622 (7.89E-07)***		
No of Obs.	31	31	31	31	31	31	31	31	31
adj. R^2	0.479	0.429	0.610	0.867	0.779	0.810	0.864	0.771	0.818

fully explained by the three factors, and are hence reflected in the pricing errors. Also in this specification, the *alpha* of the portfolio of multinational firms is significantly higher than the one of the exporters' portfolio, which in turn is higher than the *alpha* of the portfolio of domestic firms. We use the GRS[8] statistic to test the null hypothesis that the *alphas* from separate time series regressions are jointly equal to zero. The test strongly rejects the hypothesis.

In the third column of Table A-I we enlarge the set of factors by considering the excess returns on an international market portfolio that serves as a market benchmark for firms with foreign operations. Data on the excess returns on this global market portfolio are obtained from Kenneth French's data library on international indexes.[9] The coefficients on the international market portfolio are not significant, while the ordering and differences in the *alphas* are preserved also in this specification.

Recently, the empirical finance literature has explored the explanatory power of a fourth factor, in addition the the market portfolio, SMB, and HML. In particular, the addition of portfolios based on previous performance has been successful at explaining additional cross sectional differences and improve upon the three-factor model. To ensure that the differences that we find between the three types of firms are not explained by previous performance or reversal of previous performance, we also run the three portfolios on three different versions of the four-factor models. In particular, we use separately the *momentum*, *short-term reversal*, and *long-term reversal* portfolios as an additional factor. We show in Table A-II the results for these four-factor specifications. The estimates of the pricing errors indicate that the international status of firms has an effect on expected returns that it is not captured by any of these augmented factor models either.

To conclude, we validate empirically the relationship of the returns of multinational, exporting, and domestic firms with a simple stochastic discount factor derived from a power utility representative agent framework. In equilibrium, the following pricing equation must be satisfied for the returns on all assets of the economy (the

---

[8]See Gibbons, Ross, and Shanken (1989).
[9]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/int_index_port_formed.html.

Table A-II: **Portfolio Regressions: 4-Factors Fama-French.** Time-series coefficient estimates of Fama-French regressions for the three equally-weighted portfolios based on international status. The α coefficients capture the pricing errors.

	4-Factors FF (ST reversals)			4-Factors FF (LT reversals)			4-Factors FF (momentum)		
	DOM	EXP	MN	DOM	EXP	MN	DOM	EXP	MN
α	-0.081 (0.012)***	-0.032 (0.016)*	-0.025 (0.013)*	-0.0806 (0.012)***	-0.0327 (0.017)*	-0.0213 (0.013)	-0.073 (0.013)***	-0.030 (0.018)	-0.010 (0.014)
R^{mkt}	0.711 (0.068)***	0.704 (0.094)***	0.703 (0.076)***	0.709 (0.070)***	0.680 (0.096)***	0.677 (0.075)***	0.695 (0.067)***	0.682 (0.094)***	0.678 (0.070)***
R^{SMB}	0.734 (0.096)***	0.871 (0.131)***	0.506 (0.106)***	0.737 (0.105)***	0.869 (0.144)***	0.576 (0.112)***	0.716 (0.093)***	0.842 (0.132)***	0.473 (0.097)***
R^{HML}	0.268 (0.081)***	0.119 (0.115)	0.159 (0.090)*	0.268 (0.077)***	0.0893 (0.109)	0.181 (0.083)**	0.246 (0.077)***	0.0732 (0.110)	0.121 (0.080)
R^{ST}	-0.004 (0.103)	-0.120 (0.144)	0.018 (0.114)						
R^{LT}				-0.007 (0.096)	-0.040 (0.132)	-0.137 (0.102)			
R^{mom}							-0.056 (0.050)	-0.026 (0.071)	-0.114 (0.052)**
GRS (p-value) $H_0: \alpha_D = \alpha_X = \alpha_I$	21.256 (6.09E-07)***			20.973 (6.83E-07)***			14.347 (1.46E-05)***		
No of Obs.	31	31	31	31	31	31	31	31	31
adj. R^2	0.862	0.776	0.803	0.862	0.771	0.816	0.868	0.771	0.834

three types of firms and the risk free asset):

$$(A\text{-}1) \qquad E\left[\vartheta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R^i\right] = 1 \ \ \text{for} \ \ i = I, X, D, f.$$

We construct the sample counterpart of (A-1) using quarterly data on aggregate consumption and quarterly returns on the three types of firms. We estimate ϑ and γ via the generalized method of moments. Finally, we compare the returns predicted by the consumption based model with those observed in the data. Predicted returns are computed as:

$$(A\text{-}2) \qquad E(R_{t+1}) = R^f - R^f \cdot \text{Cov}\left(\vartheta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R_{t+1}\right)$$

The results, depicted in figure A-I, corroborate the existence of a relationship between consumption risk and cross sectional differences in stock returns among the three types of firms. We report three sets of results: predicted returns weighting the four moments in (A-1) equally (variables with the superscript $1s$, for "first stage" weighting matrix), predicted returns using the optimal weights (variables with the superscript $*$), and predicted returns calibrating the model to the parameters used in our simulation ($\gamma = 4$ and $\vartheta = 0.98$). On the x axis, we plot the returns realized in the data. On the y axis, we plot the returns predicted by (A-2). The sensible parameters that are used in our calibration generate very small cross sectional differences in returns. Higher γ increases the spread among the three types of firms, and the largest spread is given by the unrealistic value of $\gamma = 47$ that corresponds to the estimation with the optimal weighting matrix. The main takeaway is that the simple consumption based model captures different exposures of the three types of firms to aggregate risk but only generates large differences in exposure with unrealistic values of the parameters. The goal of our model is to formalize the firm's problem in order to provide a plausible and quantitatively accurate explanation for the cross sectional differences in exposure to consumption and disaster risk.
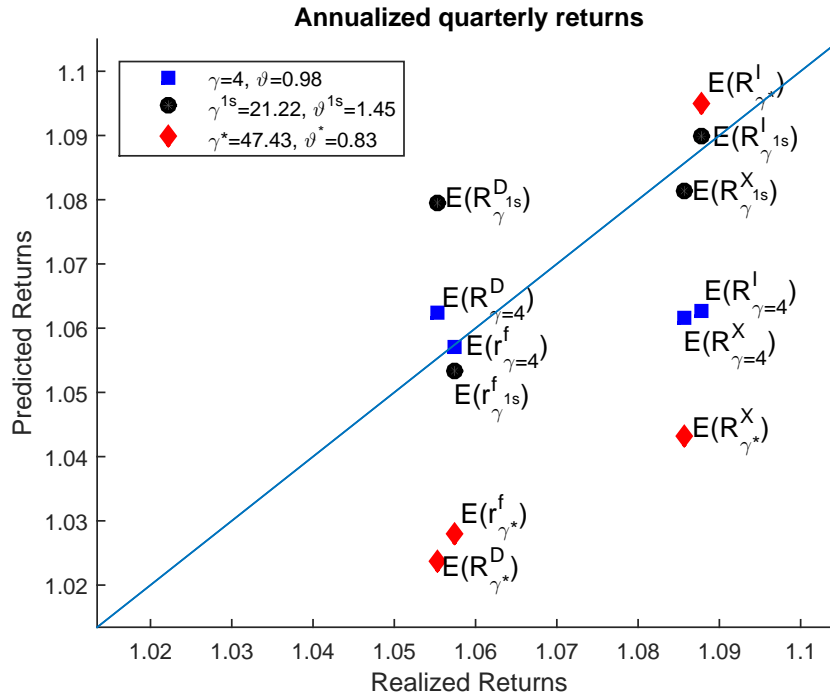
Annualized quarterly returns

Figure A-I: **CCAPM.** Realized returns on the x axis against returns predicted by the CCAPM model on the y axis for the portfolios of multinational firms, exporting firms, domestic firms, and the risk free. The circles depict results using the identity weighting matrix, the diamonds depict results using the optimal weighting matrix, and the squares show predicted returns for a calibrated model using $\gamma = 4$ and $\vartheta = 0.98$ (no estimation).

10

A-III   Derivations, Proofs, and Properties of the Industry Equilibrium

In this section we present the details of the derivation of the value functions and of the expression for model-based returns presented in Section III. , and some comparative statics properties. We also report features of the industry equilibrium that we did not include in the main text in the interest of space, like the solution of the price indexes and the law of motion of the distribution of firms across statuses.

A-III.1   Value Functions

In the continuation region, the value of a firm selling only in its domestic market is:

$$\pi_D(a,Q)M\Delta t + E[M\Delta t \cdot \mathcal{S}(a,Q')|Q] - \mathcal{S}(a,Q) + E[M\Delta t \cdot V_D(a,Q^{*\prime})|Q^*] - V_D(a,Q^*) = 0.$$

For $\Delta t \to 0$:

$$\pi_D(a,Q)Mdt + E[d(M \cdot \mathcal{S}(a,Q))] + E[d(M \cdot V_D(a,Q^*))] = 0.$$

The terms in the expectations can be written as:

$$
\begin{aligned}
E[d(M \cdot \mathcal{S})] &= E[dM \cdot \mathcal{S} + M \cdot d\mathcal{S} + dM \cdot d\mathcal{S}] \\
&= M \cdot S \cdot E\left[\frac{dM}{M} + \frac{d\mathcal{S}}{\mathcal{S}} + \frac{dM}{M} \cdot \frac{d\mathcal{S}}{\mathcal{S}}\right] \\
&= M \cdot S\left[-rdt + E\left(\frac{d\mathcal{S}}{dS}\right) + E\left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{\mathcal{S}}\right)\right] \\
\text{(A-3)} \qquad &= Mdt\left[-r\mathcal{S} + E\left(\frac{d\mathcal{S}}{dt}\right) + E\left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dt}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
E[d(M \cdot V_D)] &= E[dM \cdot V_D + M \cdot dV_D + dM \cdot dV_D] \\
&= M \cdot V_D \cdot E\left[\frac{dM}{M} + \frac{dV_D}{V_D} + \frac{dM}{M} \cdot \frac{dV_D}{V_D}\right] \\
&= M \cdot V_D\left[-rdt + E\left(\frac{dV_D}{V_D}\right) + E\left(\frac{dM}{M} \cdot \frac{dV_D}{V_D}\right)\right] \\
&= Mdt\left[-rV_D + E\left(\frac{dV_D}{dt}\right) + E\left(\frac{dM}{M} \cdot \frac{dV_D}{dt}\right)\right]
\end{aligned}
$$

(A-4)

where the dependence of $\mathcal{S}$ on (a, Q) and the dependence of V_D on (a, Q^*) have been suppressed to ease the notation. Plugging (A-3) and (A-4) into the no-arbitrage condition:

(A-5) $\pi_D - r\mathcal{S} + E\left(\dfrac{d\mathcal{S}}{dt}\right) + E\left(\dfrac{dM}{M} \cdot \dfrac{d\mathcal{S}}{dt}\right) - rV_D + E\left(\dfrac{dV_D}{dt}\right) + E\left(\dfrac{dM}{M} \cdot \dfrac{dV_D}{dt}\right) = 0.$

By applying Ito's Lemma and using the expressions for the Brownian motions ruling Q and Q^*, we can derive expressions for some of the terms in (A-5):

$$
\begin{aligned}
d\mathcal{S} &= \left[\mu Q \mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''\right] dt + \sigma Q \mathcal{S}' dz + \mathcal{S} dj_{\mathcal{S}} \\
dV_D &= \left[\mu^* Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D''\right] dt + \sigma^* Q^* V_D' dz^* + V_D dj_{V_D}
\end{aligned}
$$

where:

$$
dj_{\mathcal{S}} = \begin{cases} \frac{\mathcal{S}[(1-\phi)Q]}{\mathcal{S}(Q)} - 1; & \text{with prob. } \lambda dt \\ 0; & \text{with prob. } (1 - \lambda dt). \end{cases}
$$

$$
dj_{V_D} = \begin{cases} \frac{V_D[(1-\phi^*)Q^*]}{V_D(Q^*)} - 1; & \text{with prob. } \lambda dt \\ 0; & \text{with prob. } (1 - \lambda dt). \end{cases}
$$

Hence:

$$
\begin{aligned}
E[d\mathcal{S}] &= \left[\mu Q \mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''\right] dt + \lambda\left[\mathcal{S}[(1 - \phi)Q] - \mathcal{S}(Q)\right] dt \\
E[dV_D] &= \left[\mu^* Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D''\right] dt + \lambda\left[V_D[(1 - \phi^*)Q^*] - V_D(Q^*)\right] dt.
\end{aligned}
$$

12

Using these results we can rewrite (A-5) as:

$$\pi_D dt - r\mathcal{S}dt + \left[\mu Q\mathcal{S}'dt + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + \lambda\left[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)\right]\right] dt + \dots$$

$$E\left\{\left[-(\vartheta + \gamma\mu - \gamma(\gamma+1)\frac{1}{2}\sigma^2)dt - \gamma\sigma dz + dj_M\right] \cdot \left[\left(\mu Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''\right)dt + \sigma Q\mathcal{S}'dz + \mathcal{S}dj_{\mathcal{S}}\right]\right\} + \dots$$

$$- rV_D dt + \left[\mu^* Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D'' + \lambda\left[V_D[(1-\phi^*)Q^*] - V_D(Q^*)\right]\right] dt + \dots$$

$$E\left\{\left[-(\vartheta + \gamma\mu - \gamma(\gamma+1)\frac{1}{2}\sigma^2)dt - \gamma\sigma dz + dj_M\right] \cdot \left[\left(\mu^* Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D''\right)dt + \sigma^* Q^* V_D' dz^* + V_D dj_{V_D}\right]\right\} =$$

$$= 0.$$

(A-6)

The terms in expectation can be reduced to:

$$E\left\{\left[-(\vartheta + \gamma\mu - \gamma(\gamma+1)\frac{1}{2}\sigma^2)dt - \gamma\sigma dz + dj_M\right]\left[\left(\mu Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''\right)dt + \sigma Q\mathcal{S}'dz + \mathcal{S}dj_{\mathcal{S}}\right]\right\} =$$

$$- \gamma\sigma^2 Q\mathcal{S}'dt + E[\mathcal{S}dj_{\mathcal{S}} \cdot dj_M] = -\gamma\sigma^2 Q\mathcal{S}'dt + [(1-\phi)^{-\gamma} - 1]\cdot[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)]\lambda dt$$

$$E\left\{\left[-(\vartheta + \gamma\mu - \gamma(\gamma+1)\frac{1}{2}\sigma^2)dt - \gamma\sigma dz + dj_M\right]\left[\left(\mu^* Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D''\right)dt + \sigma^* Q^* V_D' dz^* + V_D dj_{V_D}\right]\right\} =$$

$$- \gamma\rho\sigma\sigma^* Q^* V_D' dt + E[V_D dj_{V_D} \cdot dj_M] = -\gamma\rho\sigma\sigma^* Q^* V_D' dt + [(1-\phi)^{-\gamma} - 1]\cdot[V_D[(1-\phi^*)Q^*] - V_D(Q^*)]\lambda dt$$

where the last equality relies on the assumption that the Poisson process determining the occurrence of the jump is the same in the two countries.

The no arbitrage condition (A-6) becomes:

$$\pi_D - r\mathcal{S} + (\mu - \gamma\sigma^2)Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + \lambda(1-\phi)^{-\gamma}[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)] + \dots$$

$$- rV_D + (\mu^* - \gamma\rho\sigma\sigma^*)Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D'' + \lambda(1-\phi)^{-\gamma}[V_D[(1-\phi^*)Q^*] - V_D(Q^*)] = 0.$$

One possible solution of this equation is:

(A-7) $$\pi_D - r\mathcal{S} + (\mu - \gamma\sigma^2)Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + \lambda(1-\phi)^{-\gamma}[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)] = 0$$

(A-8)

$$- rV_D + (\mu^* - \gamma\rho\sigma\sigma^*)Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D'' + \lambda(1-\phi)^{-\gamma}[V_D[(1-\phi^*)Q^*] - V_D(Q^*)] = 0.$$

13

We guess that the solution of (A-7) takes the form: $\mathcal{S}(a, Q) = Q^\chi + c_\mathcal{S}Q$. By substituting this expression into (A-7), we find that χ is the root of:

$$\frac{1}{2}\sigma^2\chi^2 + (\mu - \gamma\sigma^2 - \frac{1}{2}\sigma^2)\chi - (r + \lambda(1-\phi)^{-\gamma}) + \lambda(1-\phi)^{-\gamma}(1-\phi)^\chi = 0,$$

while the linear parameter $c_\mathcal{S}$ is given by:

$$c_\mathcal{S} = \frac{\pi_D/Q}{r - \mu + \gamma\sigma^2 + \lambda\phi(1-\phi)^{-\gamma}}.$$

Hence the value function describing the expected present discounted value of domestic profits takes the form:

$$\mathcal{S}(a, Q) = A_\mathcal{S}(a)Q^{\alpha_s} + B_\mathcal{S}(a)Q^{\beta_s} + \frac{\pi_D}{r - \mu + \gamma\sigma^2 + \lambda\phi(1-\phi)^{-\gamma}}$$

where α_s and β_s are the negative and positive value of χ, respectively, and $A_\mathcal{S}(a)$ and $B_\mathcal{S}(a)$ are firm-specific parameters to be determined. Since there is no option value associated with domestic profits, we can impose: $A_\mathcal{S}(a) = B_\mathcal{S}(a) = 0$, so that the solution is simply given by the value of profits discounted with the risk-adjusted measure:

(A-9) $$\mathcal{S}(a, Q) = \frac{\pi_D}{r - \mu + \gamma\sigma^2 + \lambda\phi(1-\phi)^{-\gamma}}.$$

Similarly, we guess that the solution of (A-8) takes the form: $V_D(a, Q^*) = Q^{*\xi}$. By substituting this expression into (A-8), we find that ξ is the root of:

(A-10) $$\frac{1}{2}\sigma^{*2}\xi^2 + (\mu^* - \gamma\rho\sigma\sigma^* - \frac{1}{2}\sigma^{*2})\xi - (r + \lambda(1-\phi)^{-\gamma}) + \lambda(1-\phi)^{-\gamma}(1-\phi^*)^\xi = 0.$$

Hence the value function describing the expected present discounted value of foreign profits of a domestic firm takes the form:

(A-11) $$V_D(a, Q^*) = A_D(a)Q^{*\alpha} + B_D(a)Q^{*\beta}$$

where α and β are the negative and positive value of ξ, respectively, and $A_D(a)$ and $B_D(a)$ are firm-specific parameters to be determined.

14

The value of domestic profits is independent of status, hence the value of domestic profits for an exporter is also given by (A-9). The value of foreign profits of an exporter solves:

(A-12)

$$\pi_X - rV_X + (\mu^* - \gamma\rho\sigma\sigma^*)Q^*V_X' + \frac{1}{2}\sigma^{*2}Q^{*2}V_X'' + \lambda(1-\phi)^{-\gamma}[V_X[(1-\phi^*)Q^*] - V_X(Q^*)] = 0.$$

We guess that the solution of (A-12) takes the form: $V_X(a, Q^*) = Q^{*\xi} + cQ^* + d$. By substituting this expression into (A-12), we find that ξ is the root of (A-10), while the parameters c and d are given by:

(A-13)
$$c = \frac{H(\tau aw)^{1-\eta}P^{*\eta}}{r - \mu + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}}$$

(A-14)
$$d = -\frac{f_X}{r}.$$

Hence the value function describing the expected present discounted value of foreign profits of an exporter takes the form:

(A-15) $$V_X(a, Q^*) = A_X(a)Q^{*\alpha} + B_X(a)Q^{*\beta} + \frac{H(\tau aw)^{1-\eta}P^{*\eta}Q^*}{r - \mu + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}} - \frac{f_X}{r}$$

where α and β are the negative and positive value of ξ, respectively, and $A_X(a)$ and $B_X(a)$ are firm-specific parameters to be determined.

Also for a multinational the value of domestic profits is independent of status and is given by (A-9). The value of foreign profits of a multinational solves:

(A-16)

$$\pi_I - rV_I + (\mu^* - \gamma\rho\sigma\sigma^*)Q^*V_I' + \frac{1}{2}\sigma^{*2}Q^{*2}V_I'' + \lambda(1-\phi)^{-\gamma}[V_I[(1-\phi^*)Q^*] - V_I(Q^*)] = 0.$$

Notice that the functional form of (A-16) is identical to the one of (A-12), hence: $V_I(a, Q^*) = Q^{*\xi} + c'Q^* + d'$, where ξ is given by (A-10), and:

(A-17)
$$c' = \frac{H(aw^*)^{1-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}}$$

(A-18)
$$d' = -\frac{f_I}{r}.$$

15

Hence the value function describing the expected present discounted value of foreign profits of a multinational takes the form:

$$\text{(A-19)} \quad V_I(a, Q^*) = A_I(a)Q^{*\alpha} + B_I(a)Q^{*\beta} + \frac{H(aw^*)^{1-\eta}P^{*\eta}Q^*}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}} - \frac{f_I}{r}$$

where α and β are the negative and positive value of ξ, respectively, and $A_I(a)$ and $B_I(a)$ are firm-specific parameters to be determined.

In order to find the six value function parameters $A_S(a)$, $B_S(a)$, for $S \in \{D, X, I\}$, together with the six quantity thresholds $Q^*_{SR}(a)$, which are the policy function of a firm with unit cost a, we impose the following value-matching and smooth-pasting conditions:

$$\text{(A-20)} \quad V_D(a, Q^*_{DX}(a)) \;=\; V_X(a, Q^*_{DX}(a)) - F_X$$

$$\text{(A-21)} \quad V_D(a, Q^*_{DI}(a)) \;=\; V_I(a, Q^*_{DI}(a)) - F_I$$

$$\text{(A-22)} \quad V_X(a, Q^*_{XD}(a)) \;=\; V_D(a, Q^*_{XD}(a))$$

$$\text{(A-23)} \quad V_X(a, Q^*_{XI}(a)) \;=\; V_I(a, Q^*_{XI}(a)) - F_I$$

$$\text{(A-24)} \quad V_I(a, Q^*_{ID}(a)) \;=\; V_D(a, Q^*_{ID}(a))$$

$$\text{(A-25)} \quad V_I(a, Q^*_{IX}(a)) \;=\; V_X(a, Q^*_{IX}(a)) - F_X$$

$$\text{(A-26)} \quad V'_R(a, Q^*_{RS}(a)) \;=\; V'_S(a, Q^*_{RS}(a)) \,, \;\; for \; S, R \in \{D, X, I\} \,, \; S \neq R.$$

For each firm, the value-matching and smooth-pasting conditions define a system of twelve equations in twelve unknowns: the six quantity thresholds $Q^*_{SR}(a)$ and the six parameters $A_S(a)$, $B_S(a)$, for $S, R \in \{D, X, I\}$. Details about the numerical solution of this system are contained in Appendix A-IV.

A-III.2   Comparative Statics: Value and Productivity

We show here qualitative properties of the value functions that are key to the solution of the model. Both the quantity thresholds and the parameters of the value functions depend on the productivity level $1/a$. Figure A-II shows the value of foreign profits of a domestic firm as a function of the aggregate quantity demanded in the foreign market Q^* and of productivity $1/a$. V_D is increasing in Q^*, as the option value of entering the foreign market is increasing in the quantity demanded. V_D is also
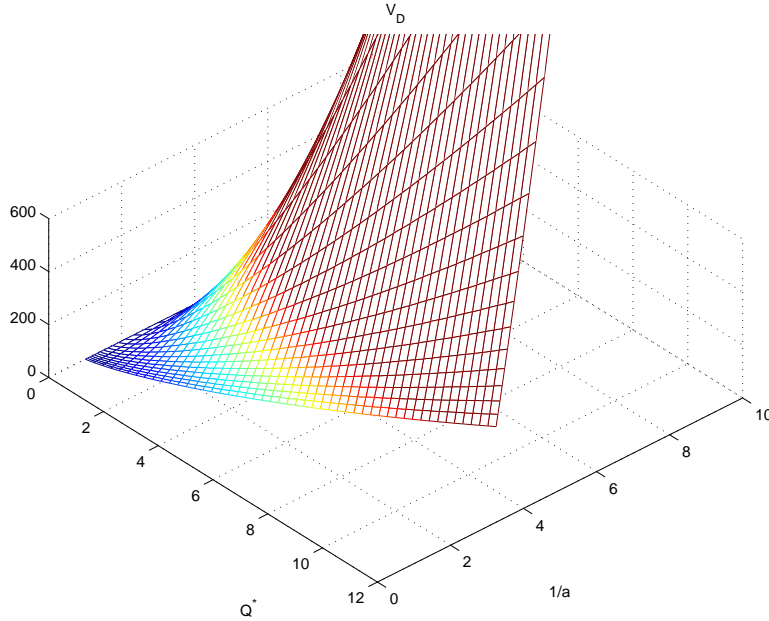
16

Figure A-II: **Value Functions.** Value of foreign profits of a domestic firm.

increasing in firm's productivity, as more productive firms can get higher profits from entering the foreign market.

Figure A-III shows the value of foreign profits of an exporter and of a multinational firm as functions of Q^* and $1/a$. V_X and V_I are U-shaped functions of Q^*, indicating the high option value of exiting for low realizations of Q^* and the high option value of not changing status for high realizations of Q^*. For $Q^* \to \infty$, the value function of an exporter is steeper than the one of a multinational, because the exporter gets high value both from staying in the market as an exporter and from the option value of becoming a multinational. The behavior of the value functions for $Q^* \to 0$ does not vary much across the productivity dimension: when Q^* is low, the value is high as firms of all productivity levels associate a high value to the option of exiting. Conversely, the behavior of the value functions when Q^* is "large" varies with individual productivity: the value function is steeper for higher productivity firms, indicating that more productive firms obtain higher returns from staying in the foreign market when the realized aggregate demand is high.

From Figure A-III, the qualitative behavior of V_X and V_I appears very similar. Figure A-IV plots the difference between the value of foreign profits of firms serving
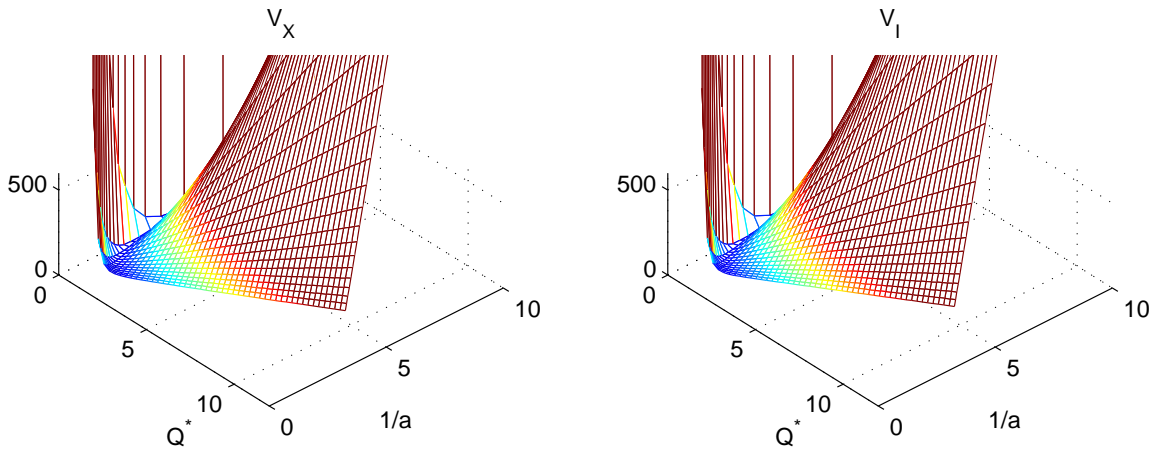
Figure A-III: **Value Functions.** Value of foreign profits of an exporter and of a multinational firm.
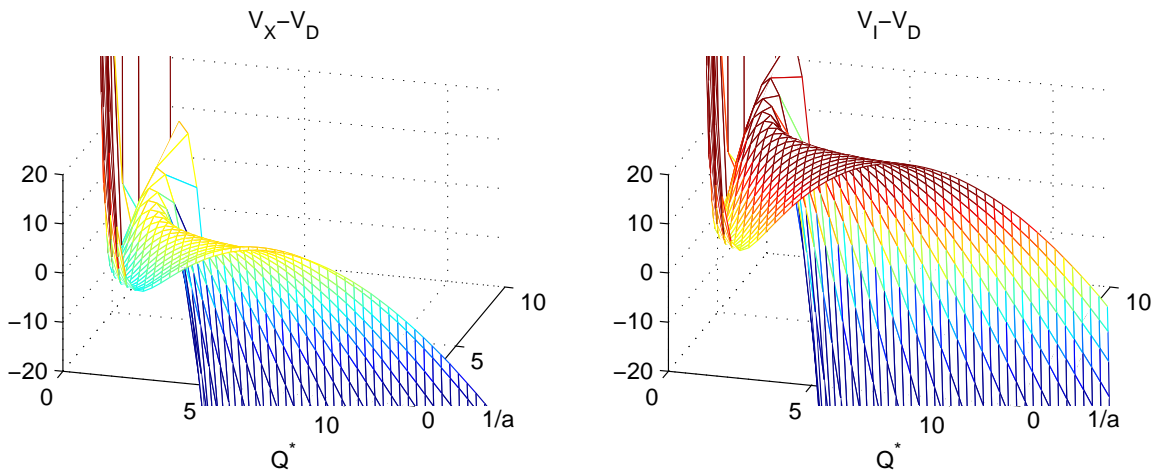

Figure A-IV: **Value Functions.** Difference between the value of foreign profits of exporters and multinationals and that of domestic firms.

the foreign market and of firms selling only domestically, $V_X - V_D$ and $V_I - V_D$. For each productivity level $1/a$, each plot has two stationary points, a local maximum and a local minimum. The value matching and smooth pasting conditions imply that the local maxima correspond to the "entry" thresholds (Q^*_{DX} and Q^*_{DI} in the left and right plot respectively), while the local minima correspond to the "exit" thresholds (Q^*_{XD} and Q^*_{ID}). Consistently with Theorem 1 below, both entry and exit thresholds are decreasing in $1/a$, indicating that more productive firms enter the foreign market for lower realizations of aggregate demand Q^* with respect to less productive firms. Similarly, more productive firms need larger negative shocks to demand to be induced to exit the foreign market with respect to less productive firms. Notice that for $Q^* \to 0$, $V_X - V_D$ and $V_I - V_D$ tend to infinity, because the option value of exiting the foreign market is extremely high for very low realizations of Q^* (and irrespective of firm's productivity). Conversely, for $Q^* \to \infty$, $V_X - V_D$ and $V_I - V_D$ tend to negative infinity, because the domestic firms' option value of entering the foreign market is extremely high compared to the flow profits of staying for firms that are already selling there. The difference between the value of foreign profits of a multinational firm and of an exporter displays similar properties.

A-III.3 Price Indexes and Transitions Across Statuses

We discuss here the computation of the price indexes and of the status distribution in the two countries.

The price indexes P and P^* are the solution of the following system:

$$P^{1-\eta} = n \int \left(\frac{\eta a w}{\eta - 1} \right)^{1-\eta} dG(a) + ...$$

(A-27) $$...n^* \left[\int_{\omega^*_X(Q)} \left(\frac{\eta \tau a w^*}{\eta - 1} \right)^{1-\eta} dG^*(a) + \int_{\omega^*_I(Q)} \left(\frac{\eta a w}{\eta - 1} \right)^{1-\eta} dG^*(a) \right]$$

$$(P^*)^{1-\eta} = n^* \int \left(\frac{\eta a w^*}{\eta - 1} \right)^{1-\eta} dG^*(a) + ...$$

(A-28) $$...n \left[\int_{\omega_X(Q^*)} \left(\frac{\eta \tau a w}{\eta - 1} \right)^{1-\eta} dG(a) + \int_{\omega_I(Q^*)} \left(\frac{\eta a w^*}{\eta - 1} \right)^{1-\eta} dG(a) \right].$$

Each price index is an aggregate of prices of domestic sales, prices of imports, and

prices of FDI sales of multinational firms from the other country. We denote by n (n^*) the exogenous mass of firms from the Home (Foreign) country, and by $\omega_X(Q^*)$, $\omega_I(Q^*)$ ($\omega_X^*(Q)$, $\omega_I^*(Q)$) the shares of these firms that export or have multinational sales when the realization of aggregate demand is (Q, Q^*).

The law of motion of the status distribution is given by:

$$\omega_D' = \omega_D \cdot [1 - G(\max\{a_{DX}, a_{DI}\})] + \omega_X \cdot [1 - G(a_{XD})] + ...$$

(A-29) $$...\omega_I \cdot \{[1 - G(a_{ID})]\mathbf{1}_{a_{ID} \geq a_{IX}} + [G(a_{IX}) - G(a_{ID})]\mathbf{1}_{a_{ID} < a_{IX}}\}$$

$$\omega_X' = \omega_D \cdot \{[G(a_{DX}) - G(a_{DI})]\mathbf{1}_{a_{DX} \geq a_{DI}} + G(a_{DX})\mathbf{1}_{a_{DX} < a_{DI}}\} + ...$$

$$...\omega_X \cdot [G(a_{XD}) - G(a_{XI})] + ...$$

(A-30) $$...\omega_I \cdot \{[1 - G(a_{IX})]\mathbf{1}_{a_{ID} < a_{IX}} + [G(a_{ID}) - G(a_{IX})]\mathbf{1}_{a_{ID} \geq a_{IX}}\}$$

$$\omega_I' = \omega_D \cdot \{G(a_{DI})\mathbf{1}_{a_{DX} \geq a_{DI}} + [G(a_{DI}) - G(a_{DX})]\mathbf{1}_{a_{DX} < a_{DI}}\} ...$$

(A-31) $$...\omega_X \cdot G(a_{XI}) + \omega_I \cdot G(\min\{a_{ID}, a_{IX}\})$$

where $a_{RS}(Q^*)$, for $R, S \in \{D, X, I\}$ is the productivity threshold that induces a Home country firm to switch status from R to S when the realization of foreign demand is Q^*.[10] For a given initial distribution Ω_0, equations (A-29)-(A-31) describe its evolution depending on the productivity thresholds ruling firms' allocation into statuses.

The system of value-matching and smooth-pasting conditions delivers firm-specific thresholds in Q^*: $Q_{RS}^*(a)$. To recover the productivity thresholds driving selection we need to invert the $Q_{RS}^*(a)$ thresholds. Theorem 1 warrants this possibility.

Theorem 1.
$$\frac{\partial Q_{RS}^*(a)}{\partial a} > 0, \quad for \quad R, S \in \{D, X, I\}, \ \forall a.$$

Proof: The proof closely follows Appendix B of Dixit (1989). We show the result for Q_{DX} only; the proof for the other thresholds follows the same steps.

---

[10] We omitted the dependence of ω_S and a_{RS} on Q^* to ease the notation.

The value-matching conditions for Q^*_{DX}, Q^*_{XD} are:

$$A_X Q^*_{DX}{}^\alpha + (B_X - B_D)Q^*_{DX}{}^\beta + \frac{H(\tau a w)^{1-\eta}P^{*\eta}Q^*_{DX}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}} - \frac{f_X}{r} = F_X$$

$$A_X Q^*_{XD}{}^\alpha + (B_X - B_D)Q^*_{XD}{}^\beta + \frac{H(\tau a w)^{1-\eta}P^{*\eta}Q^*_{XD}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}} - \frac{f_X}{r} = 0.$$

Differentiating and using the smooth-pasting conditions:

$$(\text{A-32}) \quad q Q^*_{DX}{}^\alpha dA_X + Q^*_{DX}{}^\beta d(B_X - B_D) + \frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}Q^*_{DX}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}}da = 0$$

$$(\text{A-33}) \quad Q^*_{XD}{}^\alpha dA_X + Q^*_{XD}{}^\beta d(B_X - B_D) + \frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}Q^*_{XD}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}}da = 0.$$

Dividing (A-32) by Q^*_{DX} and (A-33) by Q^*_{XD} and combining them:

$$(\text{A-34}) \qquad dA_X = \left(\frac{Q^*_{DX}{}^{\beta-1} - Q^*_{XD}{}^{\beta-1}}{Q^*_{XD}{}^{\alpha-1} - Q^*_{DX}{}^{\alpha-1}} \right) d(B_X - B_D).$$

Plugging (A-34) into (A-32):

(A-35)
$$d(B_X - B_D) = \left(\frac{Q^*_{XD}{}^{\alpha-1} - Q^*_{DX}{}^{\alpha-1}}{Q^*_{DX}{}^{\beta-1}Q^*_{XD}{}^{\alpha-1} - Q^*_{DX}{}^{\alpha-1}Q^*_{XD}{}^{\beta-1}} \right) \cdot \left(-\frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}}da \right)$$

and plugging (A-35) into (A-34):

(A-36)
$$dA_X = \left(\frac{Q^*_{DX}{}^{\beta-1} - Q^*_{XD}{}^{\beta-1}}{Q^*_{DX}{}^{\beta-1}Q^*_{XD}{}^{\alpha-1} - Q^*_{DX}{}^{\alpha-1}Q^*_{XD}{}^{\beta-1}} \right) \cdot \left(-\frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}}da \right).$$

The smooth-pasting condition for Q^*_{DX} is:

$$\alpha A_X Q^*_{DX}{}^{\alpha-1} + \beta(B_X - B_D)Q^*_{DX}{}^{\beta-1} + \frac{H(\tau a w)^{1-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}} = 0.$$

Let $G_{DX}(\cdot) = V_X(\cdot) - V_D(\cdot)$. Differentiating the condition above:

(A-37)
$$G''_{DX}(\cdot)dQ^*_{DX} + \alpha Q^*_{DX}{}^{\alpha-1}dA_X + \beta Q^*_{DX}{}^{\beta-1}d(B_X - B_D) + \frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^* + \lambda\phi^*(1-\phi)^{-\gamma}}da = 0.$$

Let $\Delta \equiv Q^*_{DX}{}^{\beta-1}Q^*_{XD}{}^{\alpha-1} - Q^*_{DX}{}^{\alpha-1}Q^*_{XD}{}^{\beta-1}$. Substituting in the expressions for

dA_X and $d(B_X - B_D)$, equation (A-37) can be rewritten as:

$$\underbrace{-G''_{DX}(\cdot)dQ^*_{DX}}_{>0} = \underbrace{\frac{(\eta-1)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{\Delta[r-\mu^*+\gamma\rho\sigma\sigma^*+\lambda\phi^*(1-\phi)^{-\gamma}]}}_{>0}da\cdot...$$

$$...\cdot\left[\alpha\left(Q^{*}_{DX}{}^{\alpha+\beta-2}-Q^{*}_{DX}{}^{\alpha-1}Q^{*}_{XD}{}^{\beta-1}\right)+\beta\left(Q^{*}_{DX}{}^{\alpha+\beta-2}-Q^{*}_{DX}{}^{\beta-1}Q^{*}_{XD}{}^{\alpha-1}\right)+\Delta\right].$$

In order to show that $\frac{\partial Q^*_{DX}(a)}{\partial a} > 0$, we must show that the last term of the expression above is positive:

$$\left[\alpha\left(Q^{*}_{DX}{}^{\alpha+\beta-2}-Q^{*}_{DX}{}^{\alpha-1}Q^{*}_{XD}{}^{\beta-1}\right)+\beta\left(Q^{*}_{DX}{}^{\alpha+\beta-2}-Q^{*}_{DX}{}^{\beta-1}Q^{*}_{XD}{}^{\alpha-1}\right)+\Delta\right] = ...$$

$$\frac{1}{Q^{*}_{DX}{}^{\alpha+\beta-2}}\left[\alpha\left(1-\left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\beta-1}\right)+\beta\left(\left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\alpha-1}-1\right)-\left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\alpha-1}+\left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\beta-1}\right] = ...$$

$$\underbrace{\frac{1}{Q^{*}_{DX}{}^{\alpha+\beta-2}}}_{>0}\left[\alpha\left(1-\left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\beta}\right)+\beta\left(\left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\alpha}-1\right)-\left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\alpha}+\left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\beta}\right].$$

Let $z \equiv \frac{Q^*_{DX}}{Q^*_{XD}} > 1$ and let $\phi(z) \equiv \left[\alpha\left(1-z^{1-\beta}\right)+\beta\left(z^{1-\alpha}-1\right)-z^{1-\alpha}+z^{1-\beta}\right]$. Then $\phi(1) = 0$ and $\phi'(z) = (1-\alpha)(\beta-1)(z^{-\alpha}-z^{-\beta}) > 0$, which proves the result. □

Theorem 1 establishes that the six thresholds $Q^*_{RS}(a)$ are decreasing in firm productivity $1/a$, indicating that more productive firms need smaller positive shocks to demand to enter the foreign market, and larger negative shocks to exit. The one-to-one correspondence between productivities and quantity thresholds established by Theorem 1 implies that the functions $Q^*_{RS}(a)$ are invertible, hence for each realization of aggregate foreign demand Q^* we can compute six productivity thresholds $a_{RS}(Q^*)$, for $R, S \in \{D, X, I\}$, that determine the selection of heterogeneous firms into the three statuses and their likelihood of switching across statuses. This redefinition of the thresholds in terms of productivity is essential to compute the price indexes in (A-27)-(A-28). Figure A-V illustrates Theorem 1.

The equations describing aggregate prices depend – via the integration limits – on the distribution of firms into statuses, which in turn depends on the quantity thresholds Q^*_{RS}. On the other hand, quantity thresholds themselves depend on aggregate prices (as evident from the value functions). In solving the firm's problem,
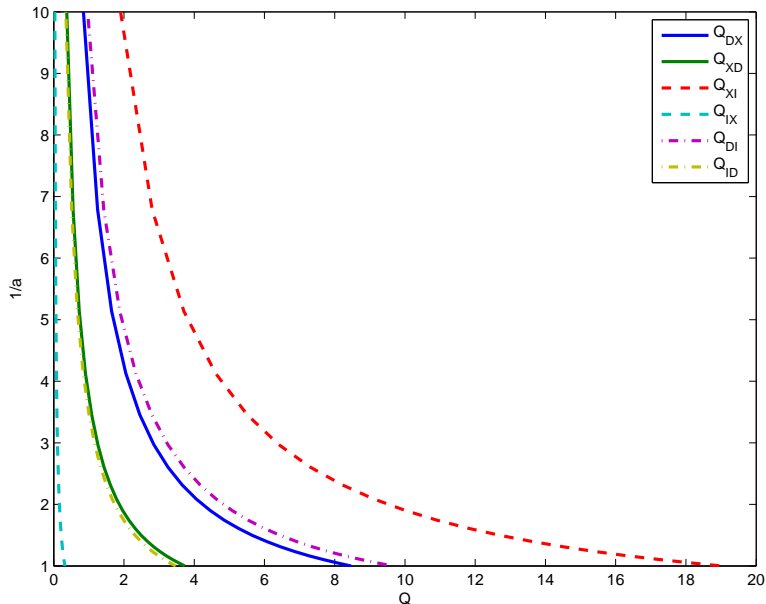
Figure A-V: **Monotonicity of the thresholds.** Quantity thresholds as functions of firm's productivity.

we appeal to the equivalence result shown in Leahy (1993): when finding the quantity thresholds, each firm takes aggregate prices and the firms' distribution into statuses as given, and does not take into account the effect of its own entry and exit decisions on these variables. This result simplifies considerably the computation of the equilibrium, which we describe in Appendix A-IV. Notice that the sets ω_S vary with the realization of Q^*, as firms may switch status, but only depend on the firms' status in the previous period, due to the Markov property of Brownian motions.[11]

Since we abstract from the problem of entry in the domestic market, we take the mass of firms in the two countries as given. We normalize them to one, and present the results in terms of shares of the total number of firms.

---

[11]The tractability of the law of motion of the status distribution hinges on the fact that there are only aggregate shocks in our economy. Adding persistent idiosyncratic shocks generates more realistic dynamics but implies that the entire status distribution becomes a state variable of the problem, requiring more sophisticated solution methods. The resulting additional complication, moreover, has no effect on asset prices, since idiosyncratic shocks are insurable and hence not priced in equilibrium.

We show here how to derive the expression for the returns in equation (29). Starting from the no arbitrage condition (without loss of generality, for an exporter):

$$\pi_D - r\mathcal{S} + (\mu - \gamma\sigma^2)Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + \lambda(1-\phi)^{-\gamma}[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)] + ...$$

$$+\pi_X - rV_X + (\mu^* - \gamma\rho\sigma\sigma^*)Q^* V_X' + \frac{1}{2}\sigma^{*2}Q^{*2}V_X'' + \lambda(1-\phi)^{-\gamma}[V_X[(1-\phi^*)Q^*] - V_X(Q^*)] = 0$$

and add and subtract the term $+\lambda[[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)] + [V_X[(1-\phi^*)Q^*] - V_X(Q^*)]]$:

$$\pi_D - r\mathcal{S} - \gamma\sigma^2 Q\mathcal{S}' + \underbrace{\mu Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + \lambda[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)]}_{E(d\mathcal{S})} + ...$$

$$+\lambda[(1-\phi)^{-\gamma} - 1][\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)] + ...$$

$$+\pi_X - rV_X - \gamma\rho\sigma\sigma^* Q^* V_X' + \underbrace{\mu^* Q^* V_X' + \frac{1}{2}\sigma^{*2}Q^{*2}V_X'' + \lambda[V_X[(1-\phi^*)Q^*] - V_D(Q^*)]}_{E(dV_X)} + ...$$

$$+\lambda[(1-\phi)^{-\gamma} - 1][V_X[(1-\phi^*)Q^*] - V_X(Q^*)] = 0$$

where $E(d\mathcal{S}) + E(dV_X) = E[d(\mathcal{S} + V_X)] = E[d\mathcal{V}_X]$. Hence:

$$\pi_D + \pi_X + E[d\mathcal{V}_X] = rV_X + \gamma\sigma^2 Q\mathcal{S}' + \gamma\rho\sigma\sigma^* Q^* V_X' - ...$$
$$\lambda[(1-\phi)^{-\gamma} - 1][[\mathcal{S}[(1-\phi)Q] - \mathcal{S}(Q)] + [V_X[(1-\phi^*)Q^*] - V_X(Q^*)]]$$
$$ret_X \equiv \frac{\pi_D + \pi_X + E[d\mathcal{V}_X]}{\mathcal{V}_X} = r + \frac{\gamma\sigma^2\mathcal{S} + \gamma\rho\sigma\sigma^* Q^* V_X'}{\mathcal{V}_X} - ...$$
$$\lambda[(1-\phi)^{-\gamma} - 1]\frac{\mathcal{V}_X[(1-\phi)Q, (1-\phi^*)Q^*] - \mathcal{V}_X(Q, Q^*)}{\mathcal{V}_X}$$

since $\mathcal{S}$ is linear in Q.

A-IV Computation Algorithm

Since the model features aggregate shocks and does not admit closed-form solutions, we simulate the economy and compute the variables of interest for a large number of simulations. We then average the results across simulations to obtain the model-generated moments. Each simulation proceeds as follows.

1. Define the exogenous parameters of the model (trade and FDI costs, preference parameters, parameters entering the shock processes). We simulate the economy for 1000 firms and 30 periods.

2. Simulate the shocks $Q(t)$, $Q^*(t)$ by discretizing the Brownian motions in equations (2)-(3).

3. Draw the productivities of 1000 firms in each country from Pareto distributions with parameters (b, k).

4. Initialize the firm distribution into statuses at the equilibrium $\Omega_0 = \Omega_0^* = (0.18, 0.45)$ (18% of firms are exporters, and 45% of firms are MNCs). Compute P_0, P_0^* from (A-27)-(A-28).[12]

5. For $t = 1, ...30$:

   (a) For each firm in each country, solve system (A-20)-(A-26) to find the quantity thresholds and the parameters of the value functions. More details about the solution of this system are contained in the next subsection.

   (b) Establish firm status at the end of the period by comparing the demand thresholds $Q_{RS}(a)$, $Q_{RS}^*(a)$ with the realizations of the shocks (Q, Q^*). Compute the new distributions of firms into statuses Ω_t, Ω_t^*. Compute P_t, P_t^* from (A-27)-(A-28).

   (c) Compute the profit and the value of each firm using equations (12)-(14) and (20)-(23), respectively. Compute returns using equation (29).

We repeat this procedure 100 times. The results of the simulations are aggregated as follows.

I. **Share of firms in each status.**

The distribution Ω_t gives the share of firms in each status for every year and simulation. The moments in the right panel of Table 6 are obtained by averaging the shares across years and across simulations.

---

[12]Since the data does not include information on the shares of foreign firms that export and have affiliate sales in the US, we assume that those shares are symmetric to the ones of US firms abroad. We initialize the price indexes by assuming that all firms are domestic, and we exclude these initial, off-equilibrium price indexes from the calculation of the moments.

II. **Earning yields.**

For each simulation and year, and consistently with the construction of earning yields in the data that we documented in Section II. , we sum the profits of firms in each group to obtain portfolio profits. Similarly, we sum the values within group to obtain portfolio values. Earning yields are given by the ratio of portfolio profits and portfolio value, then averaged across years and across simulations.

III. **Returns.**

For each simulation and year, we compute firm-level returns using equation (29). Portfolio returns are constructed by computing the average returns of all firms in the same status for each year and simulation, then averaged across years and across simulations.

A-IV.1   Solving the System of Value-Matching and Smooth-Pasting Conditions

System (A-20)-(A-26) is a non-linear system of 12 equations in 10 unknowns: the six thresholds inducing a firm to change status, for every status pair, and the four parameters entering the value functions (since we impose $A_D(a) = B_I(a) = 0$). We discipline the numerical solution of this system by a) splitting it in two perfectly identified, smaller systems, and b) choosing a "good" initial condition.

(a) **Splitting the system into two perfectly identified subsystems.**

To gain intuition about how to "separate" the system in two, consider the following example of two firms, call them y and z. Firm y is less productive than firm z, and following a positive shock to the quantity demanded abroad, it decides to start exporting. With the same realization of the shock, firm z decides to start FDI. As a result, the entry threshold Q^*_{DI} is irrelevant for firm y, and the entry threshold Q^*_{DX} is irrelevant for firm z. In other words, the fact that firms are heterogeneous and the shock process is continuous imply that there are firms that move directly from domestic sales only to FDI and viceversa, while there are firms that gradually move from domestic sales only to exports and then (eventually) to FDI.

According to this reasoning, we split system (A-20)-(A-26) in two subsystems:

(A-38) $$V_D(a, Q^*_{DX}(a)) = V_X(a, Q^*_{DX}(a)) - F_X$$

(A-39) $$V_X(a, Q^*_{XD}(a)) = V_D(a, Q^*_{XD}(a))$$

(A-40) $$V_X(a, Q^*_{XI}(a)) = V_I(a, Q^*_{XI}(a)) - F_I$$

(A-41) $$V_I(a, Q^*_{IX}(a)) = V_X(a, Q^*_{IX}(a)) - F_X$$

(A-42) $$V'_D(a, Q^*_{DX}(a)) = V'_X(a, Q^*_{DX}(a))$$

(A-43) $$V'_X(a, Q^*_{XD}(a)) = V'_D(a, Q^*_{XD}(a))$$

(A-44) $$V'_X(a, Q^*_{XI}(a)) = V'_I(a, Q^*_{XI}(a))$$

(A-45) $$V'_I(a, Q^*_{IX}(a)) = V'_X(a, Q^*_{IX}(a))$$

and:

(A-46) $$V_D(a, Q^*_{DI}(a)) = V_I(a, Q^*_{DI}(a)) - F_I$$

(A-47) $$V_I(a, Q^*_{ID}(a)) = V_D(a, Q^*_{ID}(a))$$

(A-48) $$V'_D(a, Q^*_{DI}(a)) = V'_I(a, Q^*_{DI}(a))$$

(A-49) $$V'_I(a, Q^*_{ID}(a)) = V'_D(a, Q^*_{ID}(a)).$$

System (A-38)-(A-45) is a system of eight equations in eight unknowns, that can be solved for the four quantity thresholds $Q^*_{DX}(a)$, $Q^*_{XD}(a)$, $Q^*_{XI}(a)$, $Q^*_{IX}(a)$, and the parameters of the value functions $B_D(a)$, $A_X(a)$, $B_X(a)$, and $A_I(a)$. Similarly, system (A-46)-(A-49) is a system of four equations in four unknowns, that can be solved for the two quantity thresholds $Q^*_{DI}(a)$, $Q^*_{ID}(a)$, and the parameters of the value functions $B_D(a)$ and $A_I(a)$. The two systems uniquely identify the six thresholds and the parameters $A_X(a)$, $B_X(a)$. To identify the remaining parameters $B_D(a)$, $A_I(a)$, we determine if the firm moves directly from domestic sales only to FDI or gradually moves from domestic sales only to exports and then (eventually) to FDI. If $Q^*_{DX}(a) < Q^*_{DI}(a)$, then $B_D(a)$ is identified by system (A-38)-(A-45). Conversely, if $Q^*_{DX}(a) > Q^*_{DI}(a)$, $B_D(a)$ is identified by system (A-46)-(A-49). Similarly, if $Q^*_{ID}(a) > Q^*_{IX}(a)$, then $A_I(a)$ is identified by system (A-46)-(A-49). Conversely, if $Q^*_{ID}(a) < Q^*_{IX}(a)$, $A_I(a)$ is identified by system

(A-38)-(A-45).

(b) **Choosing the initial condition.**

It is possible to show analytically (see Dixit (1989)) that the "entry" thresholds $Q_{DX}^*(a)$, $Q_{DI}^*(a)$, $Q_{XI}^*(a)$ are higher than the corresponding thresholds under certainty, while the "exit" thresholds $Q_{XD}^*(a)$, $Q_{ID}^*(a)$, $Q_{IX}^*(a)$ are lower than the corresponding thresholds under certainty. Accordingly, we solve the system for the absolute value of the differences between the equilibrium thresholds and the thresholds under certainty, with a vector of zeros as the initial condition.