Games and Economic Behavior 137 (2023) 211–227

Contents lists available at ScienceDirect

Games and Economic Behavior

journal homepage: www.elsevier.com/locate/geb

A theory of progressive lending [☆]

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A R T I C L E   I N F O

Article history:
Received 20 October 2021
Available online 16 November 2022

JEL classification:
O16
G21
D86

Keywords:
Dynamic contracts
Borrowing constraints
Moral hazard
Progressive lending
Microfinance

A B S T R A C T

We characterize Pareto efficient long term 'relational' lending contracts with one-sided lender commitment in a context where the borrower can accumulate wealth, has intertemporal consumption smoothing preferences, and the lender has some sanctioning power following default. We show the negative results of Bulow and Rogoff (1989) do not apply irrespective of the extent of sanctions, the borrower's preferences for smoothing, initial wealth or relative welfare weight. Borrowing, investment and wealth grow and converge to the first-best. Optimal allocations can be implemented by backloaded 'progressive' lending: a sequence of one period loans of growing size.

1. Introduction

This paper studies Pareto efficient lending contracts between a lender that can commit, and a borrower who cannot commit to a long term relationship and is thereby subject to an *ex post* moral hazard problem. The borrower can accumulate wealth via saving and investing in a concave productive technology, discounts the future at the rate of interest, and has a strictly concave utility function over current consumption that create a preference for intertemporal consumption smoothing. Our framework differs from existing literature in two aspects – by allowing the lender to impose *some* sanctions (which lower the borrower's productivity, or appropriate part of her capital stock) but not restricting their magnitude, and by imposing no additional restrictions on borrower utility. We show that lending is feasible without inducing any default, and optimal contract outcomes converge to the first-best levels. The optimum can be implemented via a sequence of one period loans that grow in size, conditional on no past default. The model is applicable to the contexts of microcredit or sovereign debt where default risk is endogenous owing to the *ex post* moral hazard problem.

Bulow and Rogoff (1989) showed that if the borrower does not face any sanction following default (except for the suspension of future access to borrowing), there does not exist any incentive compatible loan contract that enables the

☆ We acknowledge useful comments from two anonymous referees, Mausumi Das, Parikshit Ghosh, Hugo Hopenhayn, Debraj Ray, Prabal Roy Chowdhury, Rohini Somanathan and participants of Delhi Theory Workshop at Indian Statistical Institute and seminar participants at Delhi School of Economics. Dyotona Dasgupta acknowledges funding from the Robert S. McNamara Fellowship provided by the World Bank, and a postdoctoral fellowship provided jointly by Centre for Development Economics, Delhi School of Economics and Institute of Economic Growth. She also thanks the Institute for Economic Development, Boston University for hosting her visit during 2016-17.

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https://doi.org/10.1016/j.geb.2022.11.005
0899-8256/

lender to break-even. Rosenthal (1991) extended this result when the default sanction included suspension of future access to saving as well,[1] while continuing to assume the lender lacked any capacity to lower the borrower's productivity. In the subsequent literature, many papers (Thomas and Worrall (1994), Aguiar et al. (2009) for example) analyze optimal debt contracts under the assumption of strong sanctions imposed by the lender following defaults, which remove the borrower's access to the production technology altogether (while the borrower is still able to spread resources into the future via saving).

Such an assumption of strong sanctions may perhaps be appropriate for small sovereign countries that rely entirely on a multinational firm for access to technology as well as credit. However it does not seem realistic for most sovereign debt settings. For example, Kuvshinov and Zimmermann (2019) estimate the cost of sovereign default to be 2.9% of GDP on impact, 4.3% at peak (five years later), which dissipates after ten years.[2] In the microcredit context relevant to most developing countries, borrowers are typically poor individuals who own some productive assets (such as land, equipment or earning capacity) that cannot be seized by formal lending institutions owing to lack of secure titles and/or weak legal institutions. Indeed, these weak legal institutions are the very reason that these borrowers cannot meet collateral requirements for access to formal credit channels.[3] Hence defaulting borrowers are able to engage in subsistence production even if they are denied access to microcredit loans (besides other benefits bundled with these loans such as training, access to peer groups and marketing channels). This motivates the need to consider contexts where default sanctions include both denial of future access to borrowing and of complementary inputs and knowhow that lower the borrower's productivity, but do not eliminate their productive capacity entirely. Indeed, an important question is how loans, investment and welfare of the borrower vary with the extent of sanctions available to the lender.

Albuquerque and Hopenhayn (2004) allow sanctions to be less extreme, but assume a linear utility function for the borrower. This simplifies the analysis considerably, as optimal contracts are then fully backloaded – the borrower is not permitted to consume until investment distortions disappear. However, when the borrower's utility is strictly concave, such maximal backloading is no longer optimal. Optimal contracts then require spreading of consumption across time owing to a higher marginal value of current over future consumption, thereby aggravating default incentives. The limited scope for backloading then reduces the scope of loans to generate asset growth, raising the possibility that debt traps could emerge with a sufficiently high degree of curvature of the borrowers utility when default sanctions are weak. Specifically, one needs to analyze the nature of credit frictions that result and how these vary with parameters such as maximal sanctions, curvature of the borrower's utility and initial wealth.

Models of dynamic lending with two-sided commitment problems have shown how debt traps can emerge under suitable parameter ranges (e.g., Mookherjee and Ray (2002), Liu and Roth (2022)). However, many financial institutions (such as the IMF or large microfinance institutions) are long-lived, lend to large numbers of borrowers and have established reputations. A model of one-sided commitment therefore seems more relevant for such contexts, which motivates the model studied in this paper.

Allowing an arbitrary concave borrower utility function raises a number of technical problems, e.g., the borrower's value function need not be concave. These motivated Thomas and Worrall (1994) and Aguiar et al. (2009) to impose restrictions on the extent of concavity of the borrower's utility. Our paper develops a different method of analysis which does not rely on concavity of the borrowers value function, and is thus able to investigate the consequences of high degrees of concavity of the utility function in conjunction with arbitrary default sanctions.

We start by showing that borrowers with initial wealth above some threshold w^* can achieve first-best investment via borrowing, in a steady state with stationary consumption and wealth. The borrower borrows and repays a stationary loan at the threshold, so w^* is lower than first-best steady state wealth in the absence of any borrowing or lending. Default is costly to the borrower owing to the sanctions subsequently imposed which lower productivity. As *some* sanctions are possible, this permits *some* lending to be sustainable and help borrowers with wealth at least w^* attain first-best investment.

The future prospect of attaining the w^*-steady state can then be used to sustain lending for borrowers with initial wealth below w^*. We show that net wealth, investment, consumption and borrowing grow over time. Though allocations are distorted at every date, these distortions eventually vanish: the allocation converges to the first-best w^* steady state, irrespective of initial wealth, borrowers utility function or ex ante welfare weight. The dynamic is qualitatively similar to that in a Ramsey model of autarkic self-financing, except that the steady state involves higher welfare, consumption and output at every date. Comparative dynamics with respect to decrease in the lender's bargaining power or profit target yield short-run increases in investment and consumption, leaving the long-run allocation unaffected. Hence the benefits accruing to the borrower are entirely front-loaded, with no long-term consequences.

Our approach bypasses the need to establish concavity of the borrower's value function. The underlying arguments are simple and can be explained intuitively as follows. A recursive representation of the optimal contracting problem is used to show that the loan contracts and the optimal investment are conditional on a single state variable – a measure of the (net) wealth of the borrower equal to value of current output, less debt repayments due. Next, we show that the sequence

[1] For a borrower whose future discount factor is no greater than the interest rate on savings.

[2] Their paper summarizes the findings of a number of other studies that have estimated the cost of default using different data and methodologies, with broadly consistent results.

[3] Djankov et al. (2008), in a study of 88 countries, find a strong correlation between the efficiency of debt enforcement and per capita income of a country.

of wealths is monotonically increasing or decreasing over time, and thus must converge. This argument relies only on a single-crossing property that follows from the concavity of the utility function, implying that wealthier borrowers face a lower marginal cost of investing, and thus cannot invest less than poorer borrowers. Further, the recursive representation implies that as net wealth converges, so must consumption — for large enough t, the borrower's consumption must be smoothed nearly perfectly. The investment distortion must also vanish, since the borrower can always self-finance some extra investment. Hence a first-best allocation must be attained in the limit.

However a first-best allocation cannot be achieved in finite time, because this would result in a consumption distortion without a co-existing investment distortion. Hence the borrower's wealth must be strictly less than w^* at all dates. This is only possible if wealth is rising and converging to w^*. Finally, the argument for optimality of progressive lending (i.e., loan sizes are increasing in net wealth) is based on showing that consumption grows faster on the equilibrium path than in the counter-factual event of default. While the detailed argument is somewhat complicated, it is essentially driven by the observation that the technology available to the borrower on the equilibrium path has a higher rate of return than the autarkic technology.

The last section extends the model to incorporate productivity shocks. The recursive representation continues to apply, and investments increase in net wealth. Moreover, conditional on stationary (or nondecreasing) productivity shocks, the borrower's net wealth, investment and output increase over time, with underinvestment disappearing once net wealth crosses some threshold. Of course in this setting wealth could also contract, owing to the possibility of declining productivity shocks. If wealth falls from one period to the next owing to an adverse shock, it continues to fall thereafter if the shock persists or becomes worse over time. Hence the wealth dynamic continues to be qualitatively similar to the stochastic version of the neoclassical growth model.[4]

In summary, we show the possibility of lending that leads asymptotically to first-best outcomes, irrespective of the level of default sanctions or borrower smoothing preferences, in contrast to previous literature which either assumes extreme sanctions or near-linear utility. Moreover, this model provides a rationale for progressive lending both in a positive and normative sense. Loan sizes increase, conditional on past repayment, and loans are repaid on the equilibrium path, broadly consistent with observed practice of MFIs.[5] These strategies provide repayment incentives efficiently, ensure that the lender's profit targets are met, and enable poor borrowers to escape poverty and accumulate wealth (conditional on absence of adverse shocks).[6]

Section 2 discusses relation to existing literature in more detail. Sections 3 and 4 provide analyses for the deterministic and uncertainty contexts respectively. Proofs are collected in Appendix A.

2. Related literature

As explained in the Introduction, this paper differs from the literature on dynamic lending by exploring a larger range of ex post default productivity-lowering sanctions and borrower utility functions. Thomas and Worrall (1994) and Aguiar et al. (2009) assume sanctions include denial of access to any production, and confine attention to linear or nearly-linear utility functions. Albuquerque and Hopenhayn (2004) allow weaker sanctions but limit the analysis to linear utility for the borrower which removes any preference for current consumption relative to future consumption. Our analysis applies to contexts with arbitrarily low sanctions and arbitrary preferences for intertemporal consumption smoothing.

Acemoglu et al. (2008) study a dynamic political economy agency setting and show that distortions disappear asymptotically without imposing any restrictions on the degree of concavity of agent utility. However they assume the agent does not have the capacity to spread resources into the future following a default via savings or storage. Translating this into a borrowing-lending setting, this amounts to lenders having an even stronger capacity to sanction borrowers than allowed by Thomas and Worrall (1994) and Aguiar et al. (2009). We show that distortions disappear asymptotically even when the lender has arbitrarily small sanctioning capacity and the borrower has an arbitrary concave utility function.

Ray (2002) considers a general model of constrained Pareto efficient self-enforcing contracts in the context of a repeated game between a principal and agent with limited transferability of utility, where the agent can neither save nor commit. He shows that all such contracts back-load in the sense the allocation of surplus tends progressively to the agent's favor at later dates, converging to the one that maximizes the agent's continuation payoff, which may or may not involve a distortion.

[4] We conjecture that results concerning ergodicity of wealth dynamics also continue to hold, but do not pursue this further.

[5] All MFIs in India, Bangladesh and Vietnam listed in MIX (2017) utilize progressive lending strategies. Moreover, MFIs experience remarkably low default rates: less than 10% of MFI portfolios worldwide are at risk (defined by proportion of loans outstanding for more than 30 days). See Giné et al. (2010) for an illustration of the role of progressive lending strategies in microfinance. Another related practice in microfinance is installment lending, where loans are repaid in weekly or monthly payments rather than a single balloon payment at a particular point of time. This is also consistent with the contract structure in our model, where borrowers make a repayment at every date, on which additional lending is conditioned. In effect, actual loan sizes are smaller than advertised. However, MFI lending often includes other features such as group loans subject to joint liability, that do not arise in our model owing to the single borrower assumption.

[6] This is in an 'ideal' setting where borrowers have dynamically consistent preferences, convex technologies, and lenders commit to exclusive long-term credit contracts. Exclusivity clauses prevent borrowers from switching to a new lender after defaulting. These can arise owing to lender's coordinating on strategies, or borrowers being institutionally prevented from borrowing simultaneously from multiple lenders (see Pauly (1974), Arnott and Stiglitz (1986), Bizer and DeMarzo (1992) or Kahn and Mookherjee (1998)). In order to implement this exclusivity, however, the borrowers need to be identified uniquely. Giné et al. (2012) utilize a randomized field experiment in Malawi to illustrate the problems arising when such exclusivity clauses cannot be enforced.

Our model imposes more structure on preferences and technology, but incorporates investment and wealth accumulation, and shows that distortions disappear eventually.

Thomas and Worrall (2018) study optimal relational contracts between two agents neither of whom can commit, both contribute effort to a common joint output, and cannot save or invest. They consider both the case where the agents have strictly concave utility, and where they have linear utility and subject to limited liability. In the former case, their results turn out similar to ours: over-investment never occurs; and convergence to the first-best is monotone if the first-best is sustainable.

Dasgupta and Roy Chowdhury (2022) provide an alternative explanation of progressive lending in a framework with a nonconvexity, where the borrower (with linear utility) has an opportunity to graduate to a higher occupation or productive activity which requires a minimum investment. The lender provides the borrower an opportunity both to save and borrow, enabling the latter to accumulate wealth and graduate at an endogenous finite date. Over time, savings banked with the lender increase, which the borrower forfeits in case of default. The endogenous growth of collateral permits the lender to extend larger loans which are repaid. In a context with a similar nonconvexity, Liu and Roth (2022) present a model of a debt poverty trap that arises when the lender is profit-maximizing and cannot commit to long term contracts. Mookherjee and Ray (2002) present a model where poverty traps can arise without any technological nonconvexity, with a profit-maximizing lender with limited commitment, and a borrower with concave utility subject to ex ante moral hazard. Comparing our results with these papers, we highlight the role of lender commitment in preventing debt traps.

A number of papers on dynamic lending in microfinance focus on unobserved borrower heterogeneity, and the possible role of progressive lending[7] in screening borrowers. In all of these models, defaults necessarily occur on the equilibrium path. This limits their relevance to the microfinance setting where repayment rates are near 100%. For instance, Ghosh and Ray (2016) show how progressive lending can help screen out bad borrowers who always default from good borrowers, by providing small initial loans followed by larger ones after the bad borrowers have been eliminated. Egli (2004) shows that progressive lending may fail to identify a “bad” type, since a bad borrower may camouflage herself as a “good” borrower (who always repays) in order to get a higher amount of loan later on which she defaults with certainty. Shapiro (2015) examines a framework with uncertainty over borrowers’ discount rates. He shows that even in the efficient equilibrium almost all the borrowers default. An earlier paper with the same feature is Aghion and Morduch (2000), where in a two period context the borrower repays the first period loan to get a higher amount of loan in the second period on which she subsequently defaults. In contrast to these papers, we provide a theory of progressive lending without any equilibrium default and without any unobserved heterogeneity.[8]

3. Model

Consider an agent with endowment w, in an infinite horizon discrete time framework. Her current payoff is given by $u(c)$ where c denotes consumption. $u(\cdot)$ is time-stationary, twice differentiable, strictly increasing and strictly concave. The agent’s objective is to maximize the present discounted value of her lifetime utility: $\sum_{t=0}^{\infty}\delta^t u(c_t)$; where $\delta \in (0,1)$ is the discount factor.

3.1. Autarky

The agent always has access to (i) a neoclassical production technology $g(\cdot)$ which is strictly increasing, strictly concave and satisfies the Inada conditions: $g'(0)=\infty$ and $g'(\infty)=0$ and (ii) a linear savings opportunity at a constant rate of return $r=\frac{1}{\delta}-1$. Together, these imply that the agent has access to a transformation possibility of resources from any date t to $t+1$ at a rate of return bounded below by $r=\frac{1}{\delta}-1$. This possibility is represented by the function $\phi(k)$ which provides total resources available at the next date if the agent invests a total of k, shown in Fig. 1.
Letting k_δ^A denote the solution to $\delta g'(k)=1$, in autarky the problem of the agent with endowment w is to

$$\underset{\{k_{\tau+1}\}_{\tau=0}^{\infty}}{\text{Maximize}}\; u\big(w-k_1\big)+\sum_{\tau=1}^{\infty}\delta^{\tau-t}u\big(\phi(k_\tau)-k_{\tau+1}\big)$$

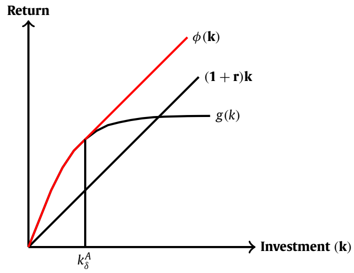
Subject to:

$$k_1 \le w \text{ and } \forall\tau \ge 1 \quad k_{\tau+1} \le \phi(k_\tau).$$

We denote the solution to this standard Ramsey optimal growth problem by $V^A(w)$. It is well known that V^A is differentiable, with $V^{A'}(w)=u'(w-k_A(w))$, where $k_A(w)$ is the optimal investment rule which is nondecreasing in w.

[7] In practice, various mechanisms operate simultaneously. Two empirical papers which distinguish the effects of progressive lending from other mechanisms such as group lending are Giné et al. (2010) and Fischer (2013).

[8] Observe for a time consistent borrower repayment schedule does not alter her decision to repay. This conforms with the finding of Field and Pande (2008) that repayment schedule does not affect the default rate. However, for a time inconsistent borrower, repayment rate improves with frequency of repayments (see Fischer and Ghatak (2010) for a formal theoretical model).



$$\phi(k) = \begin{cases} g(k) & if \quad k \leq k_\delta^A \\ g(k_\delta^A) + (1+r)(k - k_\delta^A) & \text{otherwise.} \end{cases}$$

Fig. 1. Autarky Technology.

3.2. The benevolent lender

We start by considering the case where contracts are designed to maximize the expected payoff of the borrower, subject to a breakeven constraint for the lender. Section 3.5 later shows how the main results extend to Pareto-efficient contracts, where the breakeven constraint for the lender is replaced by a minimum profit constraint.

The lender provides the agent with access to a more productive technology $z(k)$, as well as to loans. Subject to a breakeven constraint, the benevolent lender seeks to maximize the agent's welfare. The cost of capital of the lender is r, equal to the agent's rate of return on savings. The production function $z(\cdot)$ dominates $g(\cdot)$ in terms of both absolute and marginal returns to investment: $z(k) > g(k)$ and $z'(k) > g'(k)$ for all k. This can represent a higher TFP, or a higher price at which the output can be sold. Alternatively, it could represent a different production opportunity, which enables the borrower to raise returns by allocating investment between the two projects.[9] Yet another interpretation is that sanctions involve confiscation of a fraction $1 - \sigma$ of the borrowers capital stock. Default by a borrower with capital stock k would then result in operating in autarky thereafter with the same production function $z(\cdot)$ but with a starting capital stock of σk. Hence the outside option payoff is $V^A(g(k))$ where $g(k) = z(\sigma k)$, and $z'(k) > g'(k)$ obtains as long as $\sigma z'(\sigma k) < z'(k)$ for any k, a condition that is satisfied if $z(\cdot)$ has constant elasticity. We impose no restriction on the extent of sanctions, i.e., the difference between the two production functions.[10]

Let $y(k)$ denote the return on investment k which is optimally allocated between production and savings:

$$y(k) = \begin{cases} z(k) & if \quad k \leq k_\delta \\ z(k_\delta) + (1+r)(k - k_\delta) & \text{otherwise,} \end{cases}$$

where k_δ denote the first-best investment, which solves $\delta y'(k) = 1$.

3.2.1. Credit contracts

The borrower is subject to *ex post* moral hazard and cannot commit to repaying loans. The lender on the other hand can commit to a long-term contract providing access to the improved technology, stipulated investments and financial transfers to the agent, as a function of past history which includes past investments, loans and repayments (all of which are verifiable).

Standard arguments imply that attention can be restricted without loss of generality to contracts where all loans are repaid on the equilibrium path, and any default is followed by suspension of technology and loan access at all future dates. Hence we can focus attention on incentive compatible contracts. A contract is a sequence $p \equiv \{p_0, p_1, ...\}$ and $k \equiv \{k_0, k_1, ...\}$ of stipulated net transfers and investments at each date $t = 0, 1, ..$, conditional on absence of any past defaults. When $p_t > 0$, the borrower effectively obtains a loan, while $p_t < 0$ denotes payments made by the borrower.

The optimal contracting problem is

$$V(w) \equiv \max_{\langle \{p_t\}_{t=0}^\infty, \{k_t\}_{t=0}^\infty \rangle} [u(w + p_0 - k_0) + \sum_{t=1}^\infty \delta^t u(y(k_{t-1}) + p_t - k_t)] \tag{1}$$

subject to: Lender's Break-even Constraint

[9] If the production function in the new opportunity is represented by a strictly concave twice-differentiable function $f(k)$ satisfying Inada conditions, the agent will optimally allocate a total investment of k into $x(k)$ in the new technology and $k - x(k)$ in the old, such that $f'(x(k)) = g'(k - x(k))$. This results in total production $z(k) = f(x(k)) + g(k - x(k))$ which dominates the old one in the sense described.

[10] If sanctions are zero, then of course the Bulow-Rogoff result holds and no lending is possible. However our results concerning long run outcomes would continue to apply, as the borrower will converge to a first-best outcome under autarky.

$$\text{LBC}: \quad p_0 + \sum_{t=1}^{\infty} \delta^t p_t \le 0$$

and Incentive Compatibility Constraints

$$\text{IC}_t: \quad V_t \equiv \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(y(k_{\tau-1}) + p_\tau - k_\tau) \ge V^A(y(k_{t-1})) \qquad \forall t.$$

We do not include non-negativity constraints on the borrower's consumption owing to Inada conditions. The borrower has the option to default at any date t; hence the incentive compatibility constraint requires that continuation payoff does not fall below the autarkic payoff corresponding to a wealth equal to the current output (and the borrower does not make any further payments due to the lender, nor abide by the mandated investments). The following preliminary observations will be subsequently useful.

Lemma 1.

(a) The optimal contract can be implemented by a sequence of one period loan contracts $l_t, t = 1, 2, ..$ charging the interest rate r, which are always repaid on the equilibrium path, where

$$l_0 = p_0, l_t = p_t - (1+r)l_{t-1}, \forall t \tag{2}$$

$$\textit{and} \qquad \lim_{T\to\infty} \delta^T l_T \le 0 \tag{3}$$

(b) The value function $V(\cdot)$ is strictly increasing and V strictly dominates the value function in autarky: $V(w) > V^A(w)$ for all w.

Part (a) is obvious, where the payment sequence p_t is equivalently represented by the one-period loan sequence l_t, defined by (2), so $p_t = \sum_{k=0}^{t} \delta^k l_{t-k}$. We can interpret the transfer p_t at any date as the composition of a fresh loan l_t that is offered, after the agent has repaid the previous loan (i.e., paid the lender $(1+r)l_{t-1}$). Observe that $\sum_{t=0}^{T} \delta^t p_t = \delta^T l_T$. Hence, the break-even constraint (LBC) reduces to (3). In part (b) the borrower's payoff $V(w)$ is strictly increasing in her initial wealth, since this permits the borrower to consume more at t = 0 without disturbing any of the incentive constraints (which pertain to $t \ge 1$). Moreover, this payoff must strictly exceed her outside option $V^A(w)$ corresponding to an initial endowment of w, because it is always feasible for a lender to break even by providing the borrower with access to the technology $y(\cdot)$ unaccompanied by any financial transfers.

3.3. First-best contracts

Consider the optimal contract when all the incentive constraints are dropped. It involves full consumption smoothing (via choice of transfers p_t) and efficient investment $k_t = k_\delta$. The constant consumption $c^*(w)$ is obtained by the requirement that the resulting present value of consumption $\frac{c^*(w)}{1-\delta}$ equals the present value of endowment/output minus investment: $w - k_\delta + \dfrac{\delta}{1-\delta}[y(k_\delta) - k_\delta]$, so

$$c^*(w) = (1-\delta)w + \delta y(k_\delta) - k_\delta. \tag{4}$$

The borrower then attains welfare $V^*(w) = \dfrac{u(c^*(w))}{1-\delta}$.

As the first best contract is stationary, all the incentive constraints collapse to a single constraint $c^*(w) \ge (1-\delta)y(k_\delta) + \delta g(k_\delta^A) - k_\delta^A$.[11] The first-best is incentive compatible if and only if the borrower's wealth exceeds the following threshold:

$$w^* \equiv y(k_\delta) - \frac{(\delta y(k_\delta) - k_\delta) - (\delta g(k_\delta^A) - k_\delta^A)}{1-\delta} \tag{5}$$

Since the $y(\cdot)$ technology dominates $g(\cdot)$, it follows that the threshold w^* is smaller than the first-best output $y(k_\delta)$. An agent with wealth in the interval $[w^*, y(k_\delta))$ obtains a loan at the first date of $\delta[y(k_\delta) - w]$ and then repays $(1-\delta)[y(k_\delta) - w]$ at every subsequent date. These repayments are motivated by the lender threatening a defaulter with loss of access to the more productive technology, which allows the borrower to earn additional surplus $(\delta y(k_\delta) - k_\delta) - (\delta g(k_\delta^A) - k_\delta^A)$ at every date in the future. Observe that $\delta^T l_T = \delta^{T+1}[y(k_\delta) - w]$ which converges to 0 as $T \to \infty$, so the 'no Ponzi scheme' condition (3) is satisfied. This is sustained by a stationary sequence of one-period loans of size $\delta[y(k_\delta) - w]$ at every date. If w falls below w^* the loan is too large, resulting in a repayment obligation that would motivate the borrower to default.

[11] If the borrower were to default, she would enter autarky with an initial wealth of $y(k_\delta)$. Since this exceeds $g(k_\delta^A)$ the autarkic steady state output, the borrower would smooth consumption perfectly, and attain a per period consumption of $c^A \equiv (1-\delta)y(k_\delta) + \delta g(k_\delta^A) - k_\delta^A$.

For borrowers starting with wealth above $y(k_\delta)$, there is no need to borrow to achieve the first-best allocation: they invest k_δ in production, and supplement this by saving $\delta[w - y(k_\delta)]$ at every date.

Note that the incentive problem arises only for intermediate ranges of the discount factor. If δ approaches 1, the first best can be sustained for any initial w, as the threshold w^* goes to minus infinity. While if δ approaches zero, the threshold approaches zero (as in this case, the efficient investment approaches zero), and the demand for loans vanishes.

It follows the first-best contract is incentive compatible if and only if the borrower is wealthy enough to start with: $w \geq w^*$.

3.4. Second-best contracts for poor borrowers

Now we focus on poor agents, who start with a wealth below this threshold $w < w^*$ and characterize the features of the optimal contract. We show that the optimal strategy can be implemented by *progressive lending*, where loan amounts are strictly increasing in net wealth. Investment and consumption grow over time, though allocations are distorted at every date. However, these distortions eventually vanish.

Before proving these results formally, we provide a sketch of the underlying argument.

(i) First, recall from Lemma 1 that the lender loses nothing by restricting attention to a sequence of one-period loans and mandated investments offered conditional on repayment of past loans and achievement of stipulated investment targets. Moreover, we show that the contract at any date can be conditioned on a single state variable, a measure of net wealth of the borrower equal to value of current output, less debt repayments due. This recursive representation simplifies the analysis by enabling us to represent the incentive constraints in a tractable manner.[12]

(ii) Given this recursive representation, at any date with a given current wealth, the problem can be reduced to selecting a *target wealth* for the next period. This represents the fundamental trade-off the agent faces between current consumption and future wealth.

(iii) We break up this problem into two stages. At the first stage, fix a target wealth, and select loan and investment amounts to maximize current consumption, subject to the target wealth constraint, and next period's incentive compatibility constraint (ensuring that the borrower is willing to repay the loan amount in the next period). The solution to this problem defines the current cost (in terms of current consumption sacrifice) to attain a given target wealth at the next date. Then at the second stage, given current wealth, the target wealth for the next date is chosen optimally, trading off the benefit of a higher wealth later against the incremental sacrifice of current consumption.

(iv) Owing to concavity of utility in current consumption, wealthier agents face a lower marginal cost of raising the wealth target for the subsequent period. Therefore, the target wealth is non-decreasing in current wealth. This implies that the sequence of wealths is either monotonically increasing or decreasing over time, and therefore must converge.

(v) Since the borrower's net wealth converges, so must her consumption — implying that for large enough t, the agent's consumption must be smoothed nearly perfectly. Hence the investment distortion must also vanish asymptotically, since the agent can always self-finance some extra investment. A first-best allocation must therefore be attained in the limit. And the agent's limiting wealth must be at least w^*, the first-best threshold.

(vi) However a first-best allocation cannot be achieved in finite time, because this would result in a consumption distortion without a co-existing investment distortion.[13] Hence the agent's wealth must be strictly less than w^* at all dates. This is only possible if wealth is rising and converging to w^*.

(vii) The argument for optimality of progressive lending (i.e., loan sizes are increasing in net wealth) is somewhat more involved, and is based on showing that consumption grows faster on the equilibrium path than in the counter-factual event of default. Intuitively, this is because the technology available to the agent on the equilibrium path has a higher rate of return than the autarkic technology.[14]

Now we provide the results and proofs formally. For that let $c_t = y(k_t) + p_t - k_t$ denote the agent's consumption at date $t \geq 0$.

Lemma 2. *$c_t \geq c_{t-1}$ for all t.*

[12] Specifically, the value of continuing the relationship the next period onwards should not fall below the outside option corresponding to the Ramsey autarkic value starting with an endowment equal to the current output, but with the inferior autarkic technology.

[13] Specifically, if the first-best could be attained at some finite date T, consumption would be smooth after T, and strictly higher than at $T-1$. Moreover output at T would be first-best, which requires first-best investment at $T-1$. In other words, the distribution of consumption between T and $T-1$ would be distorted, while investment is not. This cannot be optimal: the agent could reduce investment slightly at $T-1$ so as to reduce the consumption distortion, while the reduction in output at T would have a zero first order welfare effect.

[14] Since the incentive constraint binds, the present value of consumption is the same on and off the equilibrium path. Therefore current consumption must be lower on the equilibrium path. This implies a higher marginal welfare impact of increasing wealth on the equilibrium path, i.e. wealthier borrowers can credibly commit to repaying larger loans.

Proof. Suppose otherwise, and $c_t < c_{t-1}$ for some t. Lower p_{t-1} slightly, and raise p_t correspondingly to keep $p_{t-1} + \delta p_t$ unchanged. This smooths consumption, raising V_l for every $l \leq t$, while leaving it unchanged for every $l > t$. Hence LBC and all incentive constraints are preserved, while raising borrower welfare. ■

To make further progress we use Lemma 1 to obtain a recursive formulation of the problem in terms of one-period loans. We study the 'relaxed' problem where the asymptotic breakeven constraint (3) is ignored. This relaxed problem can be stated as

$$\underset{\langle\{l_t\}_{t=0}^{\infty},\{k_t\}_{t=0}^{\infty}}{\text{Maximize}} \Big[u(w + l_0 - k_0) + \sum_{t=1}^{\infty} \delta^t u\big(y(k_{t-1}) - \frac{l_{t-1}}{\delta} + l_t - k_t\big)\Big]$$

$$\text{subject to} \qquad \text{IC: } V_t \equiv \sum_{\tau=t}^{\infty} \delta^{\tau-t} u\big(y(k_{\tau-1}) - \frac{l_{\tau-1}}{\delta} + l_\tau - k_\tau\big) \geq V^A\big(y(k_t)\big) \qquad \forall t \geq 1. \tag{6}$$

We will show later that the solution to this relaxed problem will end up automatically satisfying the breakeven constraint (3). Hence the optimal contract can be characterized by the solution to the relaxed problem. What makes this problem tractable is that the relaxed problem admits the following convenient recursive representation.

Observe that starting from any date t, the effect of past history is summarized in the single state variable $w_t \equiv y(k_{t-1}) - \frac{l_{t-1}}{\delta}$, the borrower's net wealth which is the value of current output less inherited debt. So the contracting problem can be restated as follows.

Lemma 3. *The maximum attainable welfare $V(w)$ for a borrower with initial wealth w must satisfy*

$$V(w) = \max_{l,k}[u(w + l - k) + \delta V(y(k) - \frac{l}{\delta})] \quad \textit{subject to:} \quad V(y(k) - \frac{l}{\delta}) \geq V^A\big(y(k)\big) \tag{7}$$

This problem can be broken into two stages. At the first stage, given any 'target' wealth $\omega = y(k) - \frac{l}{\delta}$ for the next date, select (l, k) to minimize the net investment cost, i.e., the sacrifice of current consumption $k - l$, subject to the incentive constraint $V(\omega) \geq V^A(y(k))$. Let the resulting minimized cost be denoted by $C(\omega)$. Formally,

$$C(\omega) = \min_{l,k}(k - l) \quad \text{subject to:} \quad y(k) - \frac{l}{\delta} = \omega \quad \text{and} \quad V(\omega) \geq V^A(y(k)). \tag{8}$$

Then at the second stage, select the optimal target wealth $\omega = W(w)$ for the next date, given current wealth w. We summarize this as follows.

Lemma 4. *The maximum attainable welfare $V(w)$ for a borrower with initial wealth w must satisfy*

$$V(w) \equiv u(w - C(W(w)) + \delta V(W(w)) = \max_{\omega}[u(w - C(\omega)) + \delta V(\omega)] \tag{9}$$

Let us start with the first stage cost minimization problem. Given target wealth ω and capital choice k, the associated current loan must be $\delta y(k) - \delta\omega$. Hence we can simplify (8) and reduce it to choice of investment alone as follows:

$$C(\omega) = \delta\omega + \min_{k}[(k - \delta y(k)) \quad \text{subject to:} \quad V(\omega) \geq V^A(y(k))] \tag{10}$$

So when $\omega \geq w^*$, the borrower invests k_δ from the very first period and $C(\omega)$ reduces to $\delta\omega - [\delta y(k_\delta) - k_\delta]$. While if $\omega < w^*$, the incentive constraint binds and in particular

$$V(\omega) = V^A\big(y(k)\big) \tag{11}$$

so the resulting investment is $\kappa(\omega) = y^{-1}\big(V^{A^{-1}}(V(\omega))\big)$. Since $V(\omega) < V(w^*) = V^A\big(y(k_\delta)\big)$, and V^A, V and y are increasing, there will be underinvestment ($\kappa(\omega) < k_\delta$) when $\omega < w^*$. We summarize this in the following lemma.

Lemma 5. *For target wealths ω smaller than w^*, investment $\kappa(\omega)$ is smaller than the efficient level k_δ, and equal to the efficient level otherwise.*

So given target wealth ω, optimal investment is uniquely determined:

$$\kappa(\omega) \equiv \begin{cases} k_\delta & \text{if } \omega \geq w^* \\ y^{-1}\big(V^{A^{-1}}(V(\omega))\big) & \text{otherwise} \end{cases}$$

and the cost function is

$$C(\omega) = \delta\omega + \kappa(\omega) - \delta y(\kappa(\omega)) \tag{12}$$

which is continuous and strictly increasing. Clearly the marginal cost of target wealth is δ for wealthy borrowers ($w > w^*$) and larger than δ for poor borrowers.

The second-stage problem involves choosing the target wealth ω. Observe first that the set of attainable target wealths is bounded above by $\frac{w}{\delta}$, since the marginal cost of target wealth is bounded below by δ, and current consumption must be non-negative owing to the Inada conditions. It is also bounded below, e.g., by the incentive constraint which requires $V(\omega) \geq V^A(y(0))$. Hence there always exists an optimal target wealth.[15] The optimal target wealth however may be non-unique. In what follows we consider any function $W(w)$ which is a measurable selection from the optimal target wealth correspondence.

Since V is strictly increasing, a higher target wealth is always valuable. The borrower must trade off a higher target wealth against the current cost. The concavity of u implies wealthier borrowers incur a lower marginal cost (in terms of sacrifice of utility from current consumption) of achieving higher future wealth. Hence those currently wealthier will remain wealthier in future.

Lemma 6. *$W(w)$ is nondecreasing in w.*

We are now ready to present our first main result, showing the absence of any poverty trap: the wealth of every poor borrower will converge to the first-best threshold w^*.[16]

Proposition 1. *If $w < w^*$, the sequence of net wealths w_t is strictly increasing, strictly smaller than w^* at every t, and converges to the first best threshold w^* as $T \to \infty$. The corresponding investment sequence k_t is nondecreasing and converges to k_δ, and consumption c_t is nondecreasing and converging to $c^*(w^*)$.*

Our next main result is that the optimal strategy involves progressive lending: when $w < w^*$ optimum loan size increases over time.

Proposition 2. *Starting with any $w < w^*$, the borrower obtains a loan $l(w)$ which is strictly positive and strictly increasing in w.*

Let $c(w), k(w), l(w)$ denote the optimal choices of current consumption, capital and loan for a borrower with current wealth w. The reasoning is based on observing that the optimal loan size is characterized by the binding incentive constraint: $V(y(k(w)) - \frac{l(w)}{\delta}) = V_A(y(k(w))$ for all $w < w^*$. The loan size $l(w)$ is rising in w if a marginal rise in w relaxes the incentive constraint, i.e., the rise in the no-default payoff ($V'(W(w)) = u'(c(W(w)))$ exceeds that of the outside option ($V'_A(y(k(w)) = u'(c_A(y(k(w)))))$), where $c_A(w')$ denotes the optimal consumption of a borrower in autarky when starting with wealth w'. In other words, consumption on the equilibrium path needs to be smaller than consumption on the corresponding autarkic outside option which generates equal welfare. This condition holds because the borrower has access to a more productive technology on the equilibrium path, implying a faster rate of consumption growth. Since the present value of consumption is the same on and off the equilibrium path, current consumption on the equilibrium path must be lower.

3.5. The profit-oriented lender: Pareto efficient contracts

We now consider a profit-oriented lender. Let π be an arbitrary profit target for the lender, and e denote the initial endowment of the borrower. We shall establish a relationship between e and π with w – the notation we used to denote the endowment of a borrower getting loans from a benevolent lender. A Pareto efficient contract then solves the following problem.

$$\max_{\langle\{p_t\}_{t=0}^{\infty},\{k_t\}_{t=0}^{\infty}\rangle} V_0 \equiv [u(e + p_0 - k_0) + \sum_{t=1}^{\infty} \delta^t u(y(k_{t-1}) + p_t - k_t)] \tag{13}$$

$$\text{s.t.} \quad \text{Lender's Profit Constraint} \quad \text{LPC}: \quad p_0 + \sum_{t=1}^{\infty} \delta^t p_t \leq -\pi$$

[15] This relies on the upper semi-continuity of the value function, which follows from a standard recursive argument.

[16] Observe also that the no Ponzi scheme condition (3) holds for the optimal contract for any poor borrower starting below w^*, because the wealth and behavior of such borrowers eventually are arbitrarily close to those of someone who starts with w^*. Hence as $T \to \infty$, $\delta^T l_T$ converges to $l(w^*) lim_{T\to\infty}\delta^T = 0$, and LBC is automatically satisfied at the solution to the relaxed problem in which it was dropped.

$$\text{Incentive Compatibility Constraints} \quad \text{IC}_t: \quad V_t \equiv \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(y(k_{\tau-1}) + p_\tau - k_\tau) \geq V^A(y(k_{t-1})), \quad \forall t \geq 1.$$

$$\text{Borrower Participation Constraint} \quad \text{BPC}: \quad V_0 \geq V^A(e).$$

As the lender is profit-oriented, we need to include two extra constraints – (i) obviously, the lender's profit constraint which ensures that the lender's profit is no less than π, and also (ii) the borrower's participation constraint – the optimal contract must provide the borrower payoff as high as her autarkic payoff starting with an endowment e.

Part (a) of the next lemma shows that the borrower's participation constraint (BPC) generates an upper bound on the lender's profit. Part (b) establishes a relationship between this Pareto efficient contract (13) and a version of problem (1).

Lemma 7.

(a) There exists an upper bound $\bar{\pi}(e) \in (0, \infty)$ to the profit that can be earned by the lender in any feasible contract.
(b) Given any feasible profit target $\pi \leq \bar{\pi}(e)$, a Pareto efficient contract solves the benevolent lender problem (1) with borrower initial endowment $w \equiv e - \pi$.

Part (a) states there is a *positive* upper bound $\bar{\pi}(e)$ to the profit that the lender can earn by contracting with a borrower with initial endowment $e > 0$: any higher profit would violate the borrower's participation constraint. Any lower profit than $\bar{\pi}(e)$ allows the existence of a feasible contract. The upper bound is positive since there is always a feasible contract if $\pi = 0$: access to the more profitable technology allows the borrower to attain a higher payoff compared to autarky in the absence of any transfers (i.e., if $p_t = 0$ for all t). Hence the borrower would be willing to pay a positive fee for such access, even if it is not bundled with any loans. Bundling access with loans will further increase the scope for the lender to earn profits. Part (a) therefore implies we can focus attention on the case where $\pi \leq \bar{\pi}(e)$. The borrower participation constraint can then be dropped.

Part (b) establishes that, given any $\pi \leq \bar{\pi}(e)$, it is sufficient to focus on a version of the benevolent lender problem (1). The only modification we need to make is the borrowers initial endowment from e to $w = e - \pi$, i.e., via a lump-sum transfer of π from the borrower to lender.[17] Observe that due to part (b), Propositions 1 and 2 extend to the framework with a profit-oriented lender.

We conclude this section by considering the effect of lowering π, the profit target of the lender. This corresponds either to a higher bargaining power of the borrower, or an increase in 'aid' disbursed by a non-profit lender. The latter interpretation corresponds to an external aid donor providing aid to facilitate lending by a non-profit MFI that seeks to design contracts to maximize the present value utility of the representative borrower, subject to a break-even constraint. If the external aid donor provides aid $a > 0$ per borrower, the MFI's profit target is lowered from 0 to $-a$.

Proposition 3. *Suppose the borrower has initial endowment $e < w^* + \pi$, and the profit target of the lender is lowered from π to $\pi - a$ with $a > 0$. This raises (weakly) the net wealth, borrowing and investment at every date, while long run wealth, borrowing, investment and consumption are unaffected.*

Hence the effects of aid are entirely 'front-loaded', just as in a neoclassical growth model. Proposition 3 follows straightforwardly from our preceding results. Given the initial endowment e of the borrower, this raises her initial net wealth from $e - \pi$ to $e - \pi + a$. Owing to the monotonicity of the $W(w), k(w), l(w)$ functions, this results in a (weak) increase in the net wealth, borrowing and investment of the borrower at the next date, and thereafter at every subsequent date. However in the long run the borrower's wealth must converge to the same limit w^*, and so must all the other outcomes.

4. Extension to uncertain productivity shocks

In autarky, the output of the borrower is now $g(k; s)$, where s is an i.i.d. shock with a CDF $J(.)$ over a finite support $[\underline{s}, \bar{s}]$, and g is twice differentiable, satisfying $g_k > 0 > g_{kk}, g_s > 0, g_{ks} \geq 0$ besides Inada conditions. This includes both the case of additive ($g(k; s) = \tilde{g}(k) + s$) and multiplicative shocks ($g(k; s) = s\tilde{g}(k)$). In the presence of the lender, the borrower has a production function $y(k; s)$ with higher output and marginal product of capital than the autarkic technology at any $(k; s)$ and satisfying all other analogous properties of $g(\cdot)$ mentioned above.

We assume that at any date t, the shock is observed *before* investment decisions are made. This is analogous to the model of Albuquerque and Hopenhayn (2004). If the shock is observed after investments are made, it can be verified that all the results continue to apply if the shocks are additive.[18]

[17] Observe that $\bar{\pi}(e) = \bar{\pi}(0) + e$, so there is a feasible contract and $V(w)$ is defined for all $w \geq -\bar{\pi}(0)$, i.e., including some negative values of borrower net wealth.

[18] The main complication arises from the need to keep track of a single dimensional measure of wealth the value of which can be targeted while making current investment decisions. If the current shock is known, the borrower and lender can predict the former's output that will result next period. If the

4.1. Autarky

The autarkic value function is for a state with current wealth e which is the value of output realized from investment at the previous date, and shock s applying to production at the current date:

$$V_A(e,s) = \max_{0\le k\le w}[u(e-k) + \delta E_{s'}\{V_A(g(k;s),s')\}] \tag{14}$$

Standard arguments imply that optimal investment $k_A(e,s)$ is non-decreasing in both arguments. This implies that wealth follows a monotone Markov process (Hopenhayn and Prescott (1992)): the distribution of wealth at the next date conditional on current wealth w first order stochastically dominates that conditional on a lower current wealth $w' < w$. Note that unlike Hopenhayn and Prescott (1992) we have not imposed any exogenous upper bound on capital stocks, so the state space is not compact and their results concerning invariant or limiting distributions do not necessarily apply.

4.2. Access to lender

A lender provides access to the superior technology $y(k;s)$, and a lending contract featuring a sequence of state-contingent investments $k_t(h_t)$ and transfers $p_t(h_t)$ where $h_t \equiv (s_0,\dots,s_t)$, provided the borrower has not defaulted at any previous date. As in the deterministic case, the analysis of Pareto efficient contracts with a minimum profit constraint π for the lender, reduces to the analysis of benevolent lender contracts with a borrower initial wealth of $w = e - \pi$.

Such a contract generates welfare for a borrower starting with wealth w and initial shock s_0:

$$u(w-k_0+p_0) + E_{s_1,s_2,\dots}\Big[\sum_{t=1}^{\infty}\delta^t u(y(k_{t-1},s_{t-1}) - k_t + p_t)\Big] \tag{15}$$

The lender's breakeven constraint is

$$E[\sum_{t=0}^{\infty}\delta^t p_t] \le 0 \tag{16}$$

while the incentive constraint requires at any date t and following any history h_t:

$$E\Big[\sum_{j=0}^{\infty}\delta^j u(y(k_{t-1+j}), s_{t+j}) - k_{t+j} + p_{t+j})|h_t\Big] \ge E_{s'}[V_A(y(k_{t-1}), s_t), s')] \tag{17}$$

A contract is feasible if it satisfies (16) and (17). Let welfare $V(w,s_0)$ denote the maximum value of (15) subject to the two feasibility constraints.

Analogous to the deterministic case, we obtain:

Lemma 8. *Any feasible contract can be implemented by a sequence of one period state-contingent loans satisfying $l_0 = p_0$, $l_t(h_t) = p_t(h_t) - \frac{l_{t-1}(h_{t-1})}{\delta}$, which (i) are always repaid on the equilibrium path, and (ii) satisfy the break-even condition:*

$$E[\lim_{T\to\infty}\delta^T l_T] \le 0 \tag{18}$$

In other words, the lender provides a fresh one period loan at each date-history pair, which provides the required net transfer after allowing for repayment of the previous loan. The break-even condition (18) reduces to a 'no-Ponzi' scheme requirement which holds in expectation. To ensure this, we shall assume there is a finite lower bound $\underline{w}$ on the borrower's net wealth imposed by law or MFI policy. This amounts to a limit on loan size that depends on anticipated current output:

$$l_t \le \delta[y(k_t,s_t) - \underline{w}] \tag{19}$$

As in the deterministic case, we shall ignore constraint (18) and then show that the solution to the 'relaxed' problem (which incorporates constraint (19) instead) satisfies it automatically.

We then obtain a recursive representation of the optimal contracting problem:

$$V(w,s) = \max_{k,l}[u(w-k+l) + \delta E_{s'}[V(y(k,s) - \frac{l}{\delta}, s')] \tag{20}$$

shock is unknown, the borrower's output cannot be predicted with certainty. However, in this case if the shock is additive, the mean output can be predicted which serves to pin down the distribution of output as well as wealth next period by their corresponding certainty equivalents (CE). That is, with production function $\tilde{y}(k)+s$, the wealth next period from choice of (k,l) today will be $\tilde{w}+s$ where $\tilde{w} = \tilde{y}(k) - \frac{l}{\delta}$. Hence future CE wealth $(\tilde{w}+Es)$ is determined by current decisions, and a higher CE wealth results in a wealth distribution that is 'larger' in the sense of first order stochastic dominance. Hence we can extend the theory to this case where we replace target wealths by target CE wealths.

subject to the incentive constraint

$$V(y(k,s) - \frac{l}{\delta}, s') \geq V_A(y(k,s), s'), \forall s' \in [\underline{s}, \bar{s}] \tag{21}$$

and borrowing constraint

$$l \leq \delta[y(k,s) - \underline{w}] \tag{22}$$

This is similar to the case of certainty, except that there is a separate incentive constraint for each possible realization of the shock at the next date, while the objective function involves only the corresponding expected value of the continuation utility.

The measure of net wealth is now $w \equiv y(k,s) - \frac{l}{\delta}$, which is bounded below by $\underline{w}$ and unbounded above. And we can continue to break down the recursive contracting problem into two steps. First, given current shock s and a target net wealth $W \geq \underline{w}$ in the next period, minimize the cost in terms of foregone current consumption:

$$C(W,s) \equiv \min(k - l) \quad \text{subject to: (21)} \quad \text{and} \quad y(k,s) - \frac{l}{\delta} = W \tag{23}$$

Then at the second step, select the target wealth $W(w,s)$ for next period:

$$V(w,s) = \max_{W \geq \underline{w}}[u(w - C(W,s)) + \delta E_{s'} V(W,s')] \tag{24}$$

As in the deterministic case, optimal target wealths are well-defined, and we consider any policy function which is a measurable selection from the optimal policy correspondence. Let an optimal investment and financing policy be denoted by $k(w,s), l(w,s)$ respectively.

Our main result for the case of uncertainty is the following.

Proposition 4.

(a) Wealth next period $w' = W(w;s)$ is non-decreasing in current wealth w and productivity shock s.
(b) $k(w,s) \leq k_\delta(s)$ for all w, where $\delta y'(k_\delta(s);s) = 1$.
(c) Along any history, the no-Ponzi scheme condition $\lim_{T\to\infty} \delta^T l_T \leq 0$ holds, and the lender breaks even.
(d) $k(w;s)$ is non-decreasing in w.
(e) Conditional on $w_t \geq w_{t-1}$ and a sequence of nondecreasing productivity shocks:

$$s_{t+j} \geq s_{t+j-1}, \forall j = 0, 1, 2.. \tag{25}$$

net wealth w_{t+j} and investment k_{t+j} are nondecreasing in j.
(f) For any s, there exists wealth threshold $w^*(s)$ such that $k(w,s) = k_\delta(s)$ and $l(w,s) \leq 0$ for all $w \geq w^*(s)$.

Part (a) implies the evolution of wealth follows a monotone Markov process. (b) ensures there is never any over-investment, while (c) implies the break-even condition for the lender along every history. Part (d) says that investment is non-decreasing in wealth. These results imply part (e): conditional on a nondecreasing sequence of productivity shocks (combined with wealth rising initially), the wealth and investment of the borrower rises monotonically over time. This shows that the main result of Albuquerque and Hopenhayn (2004) regarding the effect of rising 'age' of the relationship (conditional on productivity shock) continues to hold in this setting. Once wealth rises sufficiently, (f) states that investment levels become first-best. As wealth rises further, the borrower becomes a lender rather than borrower and attains first-best investment. Of course, welfare is not first-best, owing to lack of insurance and resulting consumption distortions. Wealthy borrowers may suffer a string of negative productivity shocks and subsequent declines in wealth and forced to borrow again.

We have not provided any results concerning invariant or limiting wealth distributions. Such results could be obtained upon imposing exogenous upper bounds on lending and capital investment, as in Hopenhayn and Prescott (1992), which ensure a compact state space. Such bounds would be arbitrary and ad hoc, so we avoid imposing them. Whether such results can be obtained despite the absence of such bounds, remains an interesting open question.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Proof of Lemma 6. If this is false, there exist $w_1 < w_2$ with $\omega_1 \equiv W(w_1) > W(w_2) \equiv \omega_2$. Then $V(\omega_1) > V(\omega_2)$ and

$$u(w_2 - C(\omega_2)) - u(w_2 - C(\omega_1)) \geq \delta[V(\omega_1) - V(\omega_2)] > 0 \tag{26}$$

which implies $C(\omega_1) > C(\omega_2)$. On the other hand,

$$\delta[V(\omega_1) - V(\omega_2)] \geq u(w_1 - C(\omega_2)) - u(w_1 - C(\omega_1)) \tag{27}$$

which implies

$$u(w_2 - C(\omega_2)) - u(w_2 - C(\omega_1)) \geq u(w_1 - C(\omega_2)) - u(w_1 - C(\omega_1)) \tag{28}$$

This contradicts the concavity of u. ∎

One useful consequence of Lemma 6 is the following lemma which we will use in further analyses.

Lemma 9. *The value function $V(w)$ is differentiable almost everywhere, with derivative equal to $u'(c(w))$ wherever it exists, where $k(w), l(w)$ denote the investment and loan policy, and $c(w) \equiv w - k(w) + l(w)$ denotes the borrower's consumption policy. More generally, at every w, the right hand derivative of V is bounded below by $u'(c(w))$.*

Proof. Consider any w and a slightly higher wealth $w + \epsilon > w$. Since the incentive constraint in (7) does not depend on w, the policies $(k(w), l(w))$ and $(k(w+\epsilon), l(w+\epsilon))$ are feasible for both borrowers with starting wealth w and $w + \epsilon$. Therefore:

$$V(w+\epsilon) \equiv u(w+\epsilon - C(W(w+\epsilon))) + \delta V(W(w+\epsilon)) \geq u(w+\epsilon - C(W(w))) + \delta V(W(w))$$
$$V(w) \equiv u(w - C(W(w))) + \delta V(W(w)) \geq u(w - C(W(w+\epsilon))) + \delta V(W(w+\epsilon))$$

which implies

$$\begin{aligned} \frac{u(w+\epsilon - C(W(w+\epsilon))) - u(w - C(W(w+\epsilon)))}{\epsilon} &\geq \frac{V(w+\epsilon) - V(w)}{\epsilon} \\ &\geq \frac{u(w+\epsilon - C(W(w))) - u(w - C(W(w)))}{\epsilon} \end{aligned} \tag{29}$$

Take limits as $\epsilon \to 0+$. Since Lemma 6 implies $C(W(w))$ is nondecreasing in w, it is continuous almost everywhere. At any continuity point of $C(W(w))$, it follows that the right-hand derivative of V exists and equals $u'(w - C(W(w)))$. A parallel argument for the case of $\epsilon < 0$ with direction of inequalities reversed in (29) holds, implying the left-hand derivative of V also exists and equals $u'(w - C(W(w)))$. Finally observe that for any $\epsilon > 0$, it is always feasible to let the borrower consume the incremental wealth immediately, so right hand derivative of V is everywhere bounded below by $u'(c(w))$. ∎

Proof of Proposition 1. Consider any $w < w^*$. If $W(w) \leq w$, Lemma 6 implies that starting from w the sequence of net wealth is monotonically nonincreasing. Conversely, if $W(w) > w$, the sequence is monotonically nondecreasing. Hence either way, the sequence of net wealths must converge. This implies that the sequence of consumption and investments must also converge.

Next we show that the limiting wealth w_∞ cannot be smaller than w^*. Suppose otherwise. Then we claim there is a variation on the contract which is feasible and raises the borrowers welfare. Since $w_\infty < w^*$, for all large t we have $w_t < w^*$, and there is underinvestment in the limit ($k_\delta > k_\infty$). So $[\delta y'(k_t) - 1]$ is positive and bounded away from zero for all large t.

For all large t, the incentive constraint binds, hence $V(y(k_t) - \frac{l_t}{\delta}) = V^A(y(k_t))$. Consider an increase in k_t by $\epsilon > 0$, and let l_t change by $\Delta_t \epsilon$ where

$$\Delta_t = \delta y'(k_t)[1 - \frac{V^{A'}(y(k_t))}{u'(c_{t+1})}] \tag{30}$$

For ϵ sufficiently small, the IC is preserved because:

$$\begin{aligned} &\frac{\partial V(y(k_t+\epsilon) - \frac{l_t}{\delta} - \frac{\Delta_t}{\delta}\epsilon)}{\partial \epsilon}|_{\epsilon \to 0+} \\ &\geq u'(c_{t+1})[y'(k_t) - \frac{\Delta_t}{\delta}] \\ &= V^{A'}(y(k_t))y'(k_t) \end{aligned} \tag{31}$$

where the last equality follows from construction of Δ_t (equation (30)), and the preceding inequality follows from Lemma 9.

The resulting borrower's welfare at t is

$$V_t(\epsilon) \equiv u(w_t + l_t + \Delta_t\epsilon - k_t - \epsilon) + \delta V(y(k_t + \epsilon) - \frac{l_t + \Delta_t\epsilon}{\delta}) \tag{32}$$

implying that at $\epsilon = 0$, the rate of rise of V_t is at least:

$$\begin{aligned} & -u'(c_t)[1 - \Delta_t] + u'(c_{t+1})[\delta y'(k_t) - \Delta_t] \\ = {} & u'(c_t)[\delta y'(k_t) - 1] + [u'(c_{t+1}) - u'(c_t)][\delta y'(k_t) - \Delta_t] \end{aligned} \tag{33}$$

For t sufficiently large (33) is positive, because $[\delta y'(k_t) - 1]$ is positive and bounded away from zero, $[u'(c_{t+1}) - u'(c_t)]$ converges to zero, and $[\delta y'(k_t) - \Delta_t)] = \delta y'(k_t)\frac{V^{A'}(y(k_t))}{u'(c_{t+1})}$ is positive and converges to a finite number as $t \to \infty$.

Hence $w_\infty \geq w^*$. Since $W(w^*) = w^*$, Lemma 6 implies that $W(w) \leq w^*$ for any $w < w^*$. Hence $w_\infty \leq w^*$, and it follows that $w_\infty = w^*$.

Next, we show that first-best wealth w^* cannot be achieved at any finite date. Otherwise, there exists some date t with $w_t < w^*$ and $W(w_t) = w_{t+1} = w^*$. From Lemma 5 it follows that $k_(w_t) = k_\delta$, and hence $\delta y'(k_t) = 1$, while $c_{t+1} = c(w_{t+1}) = c^*(w^*)$. And c_t must be strictly lower than $c^*(w^*)$, otherwise the borrower achieves welfare at least $V(w^*)$ at a wealth $w_t < w^*$. So there is a consumption distortion resulting in $u'(c_{t+1}) - u'(c_t) < 0$. Now we can consider a sequence of reverse perturbations analogous to that constructed above, with $\epsilon < 0$ and converging to 0 from below. From (33) it is evident that this will raise welfare for ϵ close enough to zero.

Finally, the sequence of wealths must be strictly increasing at every date (otherwise $w_{t+1} = W(w_t) = w_t$, and wealth will remain at $w_t < w^*$ for ever). ■

Proof of Proposition 2. Whenever $w < w^*$ the IC binds, hence $V(W(w)) \equiv V(y(w) - \frac{l(w)}{\delta}) = V^A(y(w))$ implies $W(w) < y(w)$ since $V(w') > V^A(w')$ for all w'. Hence $l(w) \equiv \delta[y(w) - W(w)] > 0$.

Next, we show that $l(w)$ must be strictly increasing for all $w < w^*$.

Claim 1: To establish this, it suffices to show that $c(W(w)) < c^A(y(w))$ for all $w < w^*$, i.e., optimal consumption on the equilibrium path is smaller than the optimal consumption off the equilibrium path in the first period of any deviation. This is because the right hand derivative of V at $W(w)$ is bounded below by $u'(c(W(w)))$, while the slope of $V^A(y(w))$ equals $u'(c^A(y))$ (where $c(y), c^A(y)$ respectively denote the optimal consumptions at the first date along the equilibrium path and in autarky respectively, starting with wealth y). Hence $c(W(w)) < c^A(y(w))$ implies the slope of V at $W(w)$ is larger than $V^{A'}(y(w))$. So $W(w)$ must rise more slowly in w than $y(w)$, implying $l(w)$ is increasing.

Next, observe that if $k(w) \geq k_\delta^A$, off-equilibrium-path consumption is stationary. Also, consumption is growing on the equilibrium path (an argument similar to that used in Proposition 1 rules out the possibility that consumption is stationary on the equilibrium path). Then $c(W(w)) < c^A(y(w))$ follows from the IC: $V(W(w)) = V^A(y(w))$ (which implies the present value of consumption utility is the same on and off the equilibrium path). Applying Claim 1, the result is true at any w where $k(w) \geq k_\delta^A$.

So in what follows, we consider the case where $k(w) < k_\delta^A$ at some $w < w^*$. Suppose also that $c(W(w)) \geq c^A(y(w))$: we now show this leads to a contradiction. Consider an agent that starts with wealth w, and let $c_{t'}^{A,t}$ denote optimal consumption at date t' in autarky, resulting from a deviation at $t < t'$. Then, by hypothesis $c_1^{A,1} \leq c_1$.

Claim 2: If at any date t: $c_t^{A,t} \leq c_t$, the same must be true at $t+1$: $c_{t+1}^{A,t+1} \leq c_{t+1}$.

To establish Claim 2, suppose otherwise that at some date t: $c_t^{A,t} \leq c_t$ and $c_{t+1}^{A,t+1} > c_{t+1}$. Then

$$\Delta_t \equiv \delta y'(k_t)[1 - \frac{u'(c_{t+1}^{A,t+1})}{u'(c_{t+1})}] > 0$$

and so

$$u'(c_t)[1 - \Delta_t] < u'(c_t) \leq u'(c_t^{A,t}) = \delta\frac{\partial}{\partial k}[V^A(g(k_t^{A,t}))] \tag{34}$$

Since the IC at t binds, we have

$$u(c_t) + \delta V(y_{t+1} - \frac{l_t}{\delta}) = u(c_t^{A,t}) + \delta V^A(g(k_t^{A,t}))$$

so $c_t^{A,t} \leq c_t$ implies

$$V^A(g(k_t^{A,t})) \geq V(y_{t+1} - \frac{l_t}{\delta}) \geq V^A(y_{t+1})$$

where the last inequality again uses the IC at t. Hence $g(k_t^{A,t}) \geq y_{t+1}$. Since V^A is concave, this implies $\delta \frac{\partial}{\partial k}[V^A(g(k_t^{A,t}))] \leq \delta \frac{\partial}{\partial k}[V^A(y_{t+1})]$. Hence (34) implies

$$u'(c_t)[1-\Delta_t] < \delta \frac{\partial}{\partial k}[V^A(y_{t+1})]$$

Using an argument similar to that used in Proposition 1, it is feasible to increase investment slightly on the equilibrium path and raise the borrower's welfare, contradicting optimality of the original contract. Hence Claim 2 holds.

Finally, Claim 2 implies by induction that $c_{t'}^{A,t'} \leq c_{t'}$ for all $t' > t$. Since k_t converges to the first-best capital stock k_δ which strictly exceeds k_δ^A, there exists some date $T > t$ when $k_T = k(w_T) > k_\delta^A$. This implies $g(k_T^{A,T}) < y_{T+1}$. On the other hand, the fact that the IC binds at T and $c_T^{A,T} \leq c_T$ imply that $g(k_T^{A,T}) \geq y_{T+1}$, using a similar argument as in the previous paragraph. We therefore obtain a contradiction. This implies $c(W(w)) < c^A(y(w))$ for all $w < w^*$.

Applying Claim 1, the proof is complete. ■

Proof of Lemma 7. Denote the original problem (13) by $\mathcal{P}(e,\pi)$ and note that it can be rewritten as follows with a change of variable from transfers p_t to consumption $c_t \equiv y(k_{t-1}) + p_t - k_t, t \geq 1$ and $c_0 \equiv e + p_0 - k_0$:

$$\max_{\langle \{c_t\}_{t=0}^\infty, \{k_t\}_{t=0}^\infty \rangle} V_0 \equiv \sum_{t=0}^\infty \delta^t u(c_t) \tag{35}$$

subject to:

$$\text{LPC}: \quad \sum_{t=0}^\infty \delta^t c_t \leq e - \pi - \sum_{t=0}^\infty \delta^t k_t + \sum_{t=1}^\infty \delta^t y(k_{t-1})$$

$$\text{IC}_t: \quad V_t \equiv \sum_{\tau=t}^\infty \delta^{\tau-t} u(c_\tau) \geq V^A(y(k_{t-1})), \quad \forall t \geq 1.$$

$$\text{BPC}: \quad V_0 \geq V^A(e).$$

Similarly refer to the benevolent lender problem (1) by $\mathcal{P}^B(w)$, and using the same change of variables this can be written as:

$$V(w) \equiv \max_{\langle \{c_t\}_{t=0}^\infty, \{k_t\}_{t=0}^\infty \rangle} \sum_{t=0}^\infty \delta^t u(c_t) \tag{36}$$

subject to:

$$\text{LBC}: \quad \sum_{t=0}^\infty \delta^t c_t \leq w - \sum_{t=0}^\infty \delta^t k_t + \sum_{t=1}^\infty \delta^t y(k_{t-1})$$

$$\text{IC}_t: \quad V_t \equiv \sum_{\tau=t}^\infty \delta^{\tau-t} u(c_\tau) \geq V^A(y(k_{t-1})), \quad \forall t \geq 1. \tag{37}$$

We claim that the feasible set in $\mathcal{P}(e,\pi)$ is non-empty if and only if the feasible set in $\mathcal{P}^B(e-\pi)$ is non-empty and $V(e-\pi) \geq V^A(e)$. To show the 'if' part, observe that the solution to $\mathcal{P}^B(e-\pi)$ is feasible in $\mathcal{P}(e,\pi)$. For the converse, take any feasible contract $\{\hat{c}_t, \hat{k}_t\}$ in $\mathcal{P}(e,\pi)$. This is feasible in $\mathcal{P}^B(e-\pi)$. Hence the solution to the latter problem yields a payoff $V(e-\pi)$ of at least $\sum_{t=0}^\infty \delta^t u(c_t)$, which in turn is at least $V^A(e)$ owing to (BPC).

Next observe that $V(\cdot)$ is strictly increasing, since any increase in w can be accompanied by an equivalent increase in c_0 without violating either LBC or any IC_t.

Define $\bar{\pi}(e) \equiv \sup\{\pi | V(e-\pi) \geq V^A(e)\}$. It is evident that $\bar{\pi}(e) < \infty$, since $V^A(e) > 0$ for any $e > 0$. Also $\bar{\pi}(e) > 0$, since $V(e) > V^A(e)$, as the access to superior technology combined with financial autarky ensures a higher payoff to the borrower. Since $V(\cdot)$ is strictly increasing, it follows that a feasible contract in $\mathcal{P}(e,\pi)$ exists if and only if $\pi \leq \bar{\pi}(e)$. Finally, observe that given any $\pi \leq \bar{\pi}(e)$, BPC is redundant in $\mathcal{P}(e,\pi)$, and the two problems $\mathcal{P}(e,\pi)$, $\mathcal{P}^B(e-\pi)$ are the same as they involve the same objective function and constraint set. ■

Proof of Proposition 4.

(a) The same argument as in the deterministic case ensures $W(w;s)$ is non-decreasing in w. To show that it is non-decreasing in s, it suffices to show that C and marginal cost of target wealth $C_\omega(\omega,s)$ are both non-increasing in s.

Observe that

$$C(\omega,s) = \delta\omega - \delta Z(\omega,s) \tag{38}$$

where

$$Z(\omega, s) \equiv \max_{y \geq 0}[y - \frac{k(y,s)}{\delta}] \qquad \text{subject to:} \qquad V_A(y, s') \leq V(\omega, s'), \forall s' \tag{39}$$

and $k(Y, s)$ denotes the solution for k in $y(k, s) = Y$.

Since (using standard arguments) $V_A(y, s')$ is strictly increasing and continuous in y, we can define $Y(\omega)$ as the largest (or supremum) y satisfying the incentive constraint (IC) in (39). The IC can then be replaced by $y \leq Y(\omega)$. Standard arguments also imply $V(w, s)$ is increasing in w, hence $Y(\omega)$ is increasing. Therefore

$$Z(\omega, s) \equiv \max_{y \geq 0}[y - \frac{k(y,s)}{\delta}] \qquad \text{subject to:} \qquad y \leq Y(\omega) \tag{40}$$

As $k(y, s)$ is non-increasing in s, it follows that Z is non-decreasing in s, and C is non-increasing in s.

The unconstrained solution to (40) involves setting y equal to $y(\omega, s) = y_\delta(s) \equiv y(k_\delta(s), s))$, and the IC binds iff $y_\delta(s) > Y(\omega)$. Hence $y(\omega, s) = Y(\omega)$ if $y_\delta(s) > Y(\omega)$ and $y_\delta(s)$ otherwise. It follows that $C(W, s) = \delta W - y(k_\delta(s), s)) + \frac{1}{\delta} y(k_\delta(s), s))$ in the former case, and $\delta\omega - Y(\omega, s) + \frac{1}{\delta} k(Y(\omega, s), s)$ otherwise.

Hence the marginal cost of a higher wealth target $C_\omega(\omega, s)$ equals δ if ω is large enough that IC does not bind, so does not depend on s. When the IC does bind, the marginal cost is defined a.e. (whenever $Y'(\omega)$ exists), in which case it equals $\delta - Y'(\omega)[1 - \frac{1}{\delta} y_k(k(Y(\omega), s), s)]$. This is non-increasing in s because $y_k(k, s)$ is decreasing in k and increasing in s, $k(Y, s)$ is decreasing in s, and $Y'(\omega) > 0$.

(b) Suppose $\delta y'(k(w, s)) < 1$. Consider a small reduction in both capital and borrowing: $k = k(w, s) - \epsilon$ and $l = l(w, s) - \epsilon$, which then results in an increase in $y(k, s) - \frac{l}{\delta}$. The IC constraint (21) and borrowing constraint (22) continue to hold, continuation utility $E_{s'}[V(y(k, s) - \frac{l}{\delta}), s']$ rises and current consumption is unchanged. So the original contract could not have been optimal.

(c) From (b) and (22) we have $l_t(w, s) \leq \delta[y_\delta(s) - \underline{w}] \leq \delta \max_s[y_\delta(s) - \underline{w}]$, so the size of loans is uniformly bounded above, implying $\lim_{T\to\infty} \delta^T l_T \leq 0$ along any history.

(d) If this is false, there exists s and wealths $w > w'$ with $k(w, s) < k(w', s) \leq k_\delta(s)$. From (a), $y(k(w, s), s) - \frac{l(w,s)}{\delta} \geq y(k(w', s), s) - \frac{l(w',s)}{\delta}$, which implies

$$l(w', s) - l(w, s) \geq \delta[y(k(w', s), s) - y(k(w, s), s)] \tag{41}$$

Define $l'' \equiv [k(w', s) - k(w, s)] + l(w, s)$. Since $k(w, s) < k(w', s) \leq k_\delta(s)$, the Intermediate Value Theorem implies

$$\delta[y(k(w', s), s) - y(k(w, s), s)] > k(w', s) - k(w, s) = l'' - l(w, s). \tag{42}$$

(41) then implies that $l(w', s) > l''$. Therefore:

$$y(k(w', s), s) - \frac{l''}{\delta} > y(k(w', s), s) - \frac{l(w', s)}{\delta} \tag{43}$$

implying that the borrower with wealth w could feasibly borrow l'' and invest $k(w', s)$ while preserving the incentive and borrowing constraints. This would entail the same current consumption: $k(w, s) - l(w, s) = k(w', s) - l''$, and generate higher continuation utility because of (42), so we obtain a contradiction.

(e) Note to start with that $w_{t+1} = W(w_t, s_t) \geq w_t = W(w_{t-1}, s_{t-1})$ upon using (a), combined with $w_t \geq w_{t-1}$ and $s_t \geq s_{t-1}$. Now use the same argument inductively at all subsequent dates.

(f) If this is false, there is some s for which $k(w, s) < k_\delta(s)$ for all w. Observe first that this implies $l(w, s) \geq 0$ for all w, since otherwise $l(w, s) < 0$ for some w and the IC (21) does not bind for a borrower with wealth w. This borrower can increase k and l by the same small amount and raise continuation utility strictly, while keeping current consumption unchanged. Since $y'(k(w, s), s) > \frac{1}{\delta} > 1$, the borrowing constraint is also preserved. So the contract could not have been optimal.

Since $l(w, s) \geq 0$ for all w, (b) implies that net wealth next period $W(w, s) \equiv y(k(w, s), s)) - \frac{l(w,s)}{\delta} \leq y_\delta(s)$ for all w, i.e., is bounded above. Consumption in the next period is therefore bounded above because investment is nonnegative and borrowing is bounded above by $\max_{s'} y_\delta(s') - \underline{w}$.

On the other hand, current consumption for the borrower is bounded below by $w - k_\delta(s)$ since borrowing is non-negative for all w. Hence as $w \to \infty$, current consumption goes to ∞, while consumption in the next period is bounded above. Since the utility function satisfies Inada conditions, the marginal utility of current consumption must be lower than discounted marginal utility of consumption in the following period, for sufficiently large w. Such a borrower can reduce l slightly below $l(w, s)$ which preserves borrowing and incentive constraints, and raises welfare. So the contract cannot be optimal for large enough w. ■

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