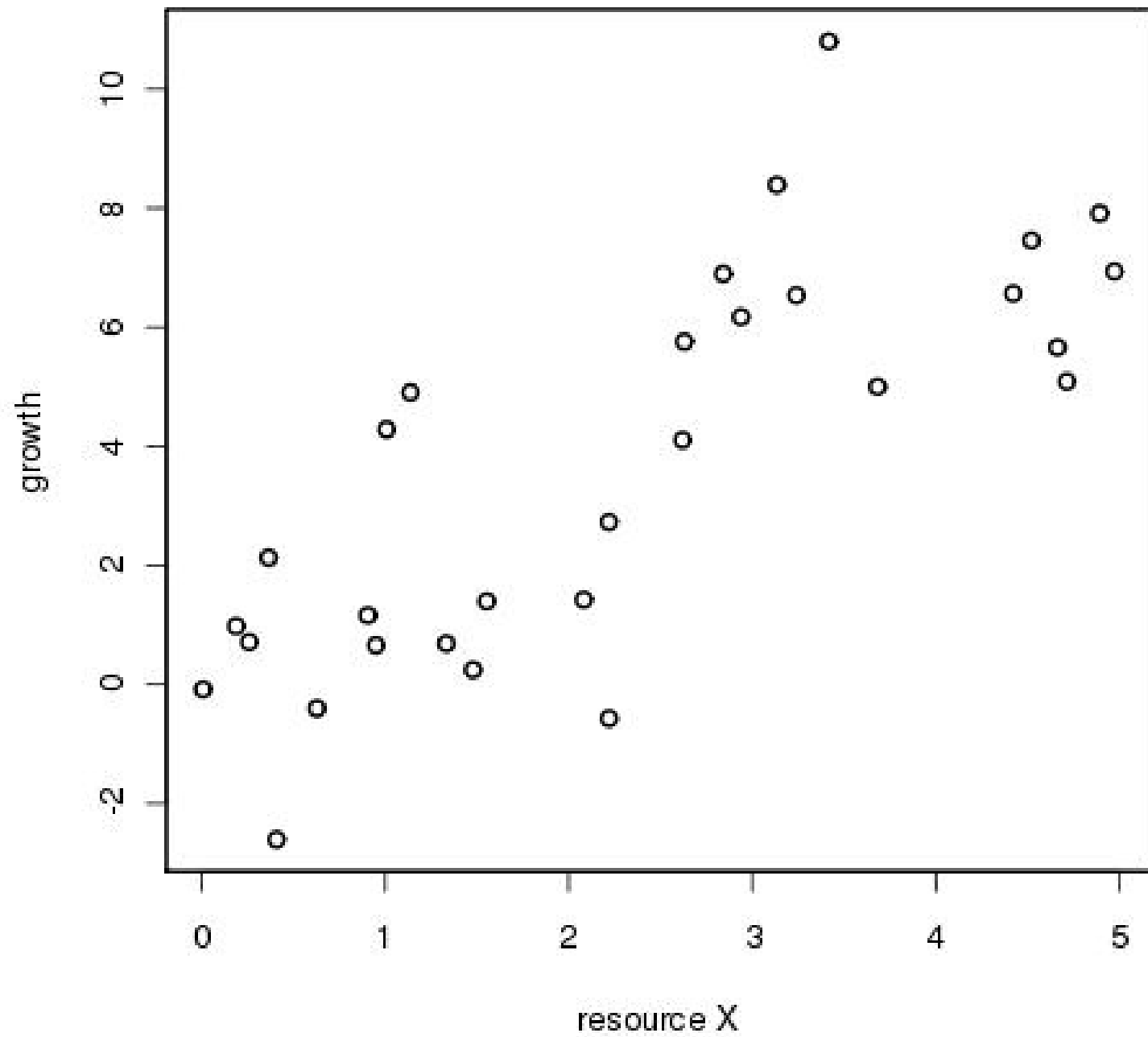


Repeated Measures and Interventions

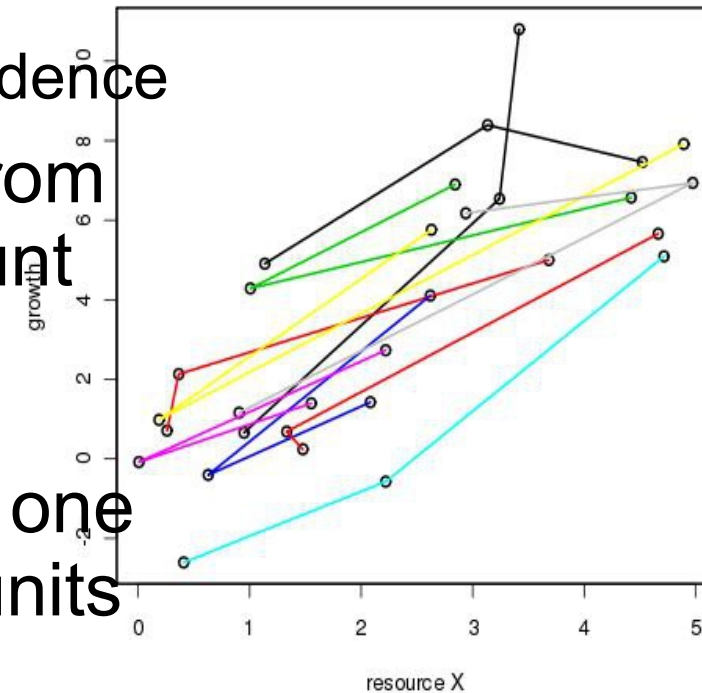
Longitudinal Data (aka Repeated Measures)

- The same observational unit (plot, individual, etc) is often measured repeatedly over time
- Usually have many such observation units
- Observations on the same unit over time are not independent



Alternatives for Repeated Measures

- Random effects
 - By time: Assumes all observational units move up or down in sync
 - probably won't solve lack of independence
 - By unit: Assumes a unit is offset from “average” by some constant amount
- Autoregressive: AR(1)
 - Assumes each unit is similar from one time step to the next but not that units are synchronized
- With short t.s. almost impossible to distinguish AR vs individual effects



Example: Growth

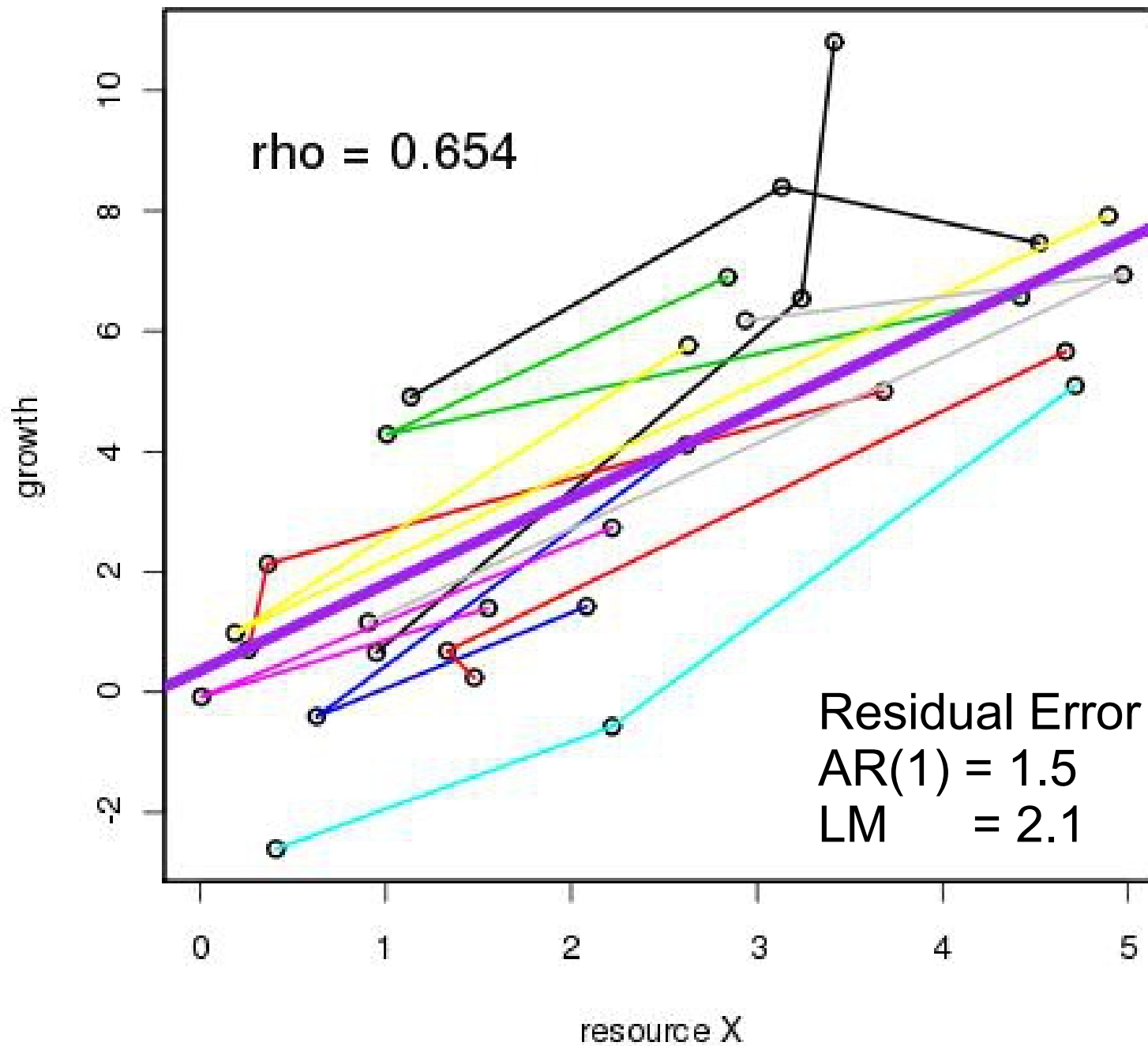
- Consider a population of individuals censused 3 times who's growth is a function of X

$$\mu_{i,t} = \beta_0 + \beta_1 X_{i,t}$$

- If we assume a common variance and autocor.

$$\vec{g}_i \sim N_3(\vec{\mu}_i, \Sigma)$$

$$\Sigma = \frac{\sigma^2}{1 - \rho^2} \begin{vmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{vmatrix}$$



Bayesian Approach

```
model{
  beta ~ dmnorm(b0,Vb)
  sigma ~ dgamma(0.01,0.01)
  rho ~ dunif(-1,1)
  SIGMA <- inverse(1/sigma/(1-rho^2)*rho^H)

  for(i in 1:n){
    mu[i,] <- beta[1]+beta[2]*X[i,]
    g[i,] ~ dmnorm(mu[i,],SIGMA)
  }
}
```

State Space approach

- Bayesian version of AR(1) repeated measures did not separate process and measurement uncertainty
- Easy to extend the State Space model to the repeated measures context with a common rho

$$g_{i,t} \sim N(\beta_0 + \beta_1 X_{i,t} + \rho g_{i,t-1}, \sigma^2)$$

$$g_{i,t}^{(o)} \sim N(g_{i,t}, \tau^2)$$

Biggest difference is that the code will loop over both individuals and time

Generalization of Repeated Measures

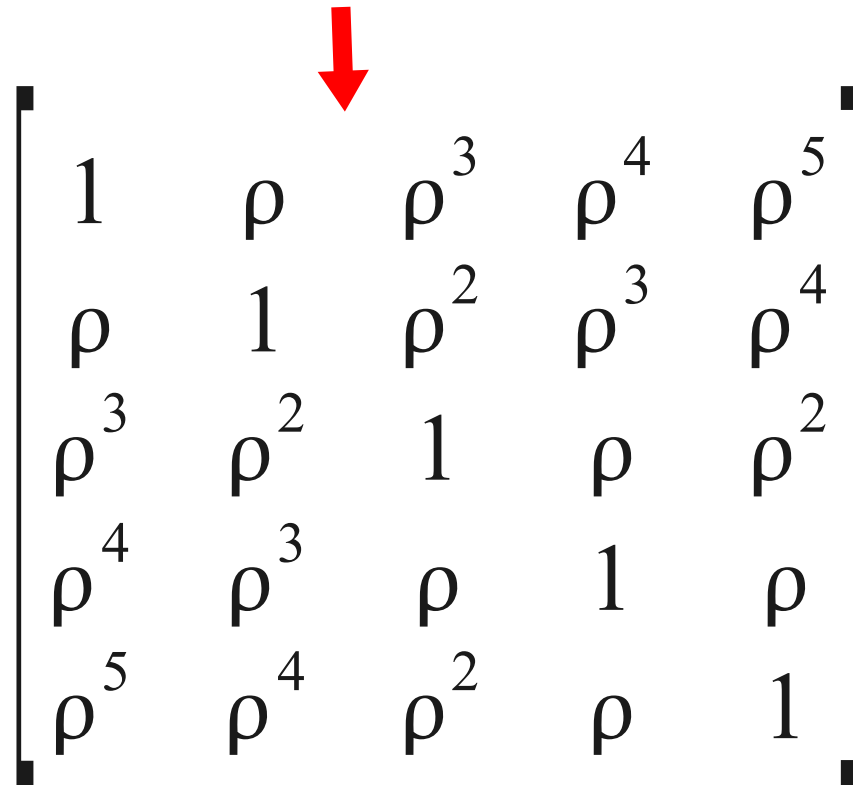
- Can accommodate mixed models / random effects (though some would be redundant)
- Can accommodate nonlinear models
- Covariance matrix ONLY works with NORMAL PDF
- State Space version can work with most any PDF because we separate the data model (not autocorrelated) from the process error (autocorrelated)

Missing Data Covariance Matrix

- Drops missing ROWS and COLUMNS from matrix

Missing 3rd observation so drop both the third row and third column from the full matrix

[X1 X2 X4 X5 X6]


$$\begin{bmatrix} 1 & \rho & \rho^3 & \rho^4 & \rho^5 \\ \rho & 1 & \rho^2 & \rho^3 & \rho^4 \\ \rho^3 & \rho^2 & 1 & \rho & \rho^2 \\ \rho^4 & \rho^3 & \rho & 1 & \rho \\ \rho^5 & \rho^4 & \rho^2 & \rho & 1 \end{bmatrix}$$

Hierarchical AR models

- Thus far have focused on autocorrelation in the STATE variables as part of the process model
- In hierarchical models, can also model autocorrelation in the hierarchical parameters
 - Time-varying model parameters

$$\vec{\alpha} \sim N(\mathbf{0}, \tau^2)$$



$$\vec{\alpha} \sim N(\mathbf{0}, \mathbf{T})$$

Fire in the Florida Everglades

Provides a means for addressing non-stationary processes

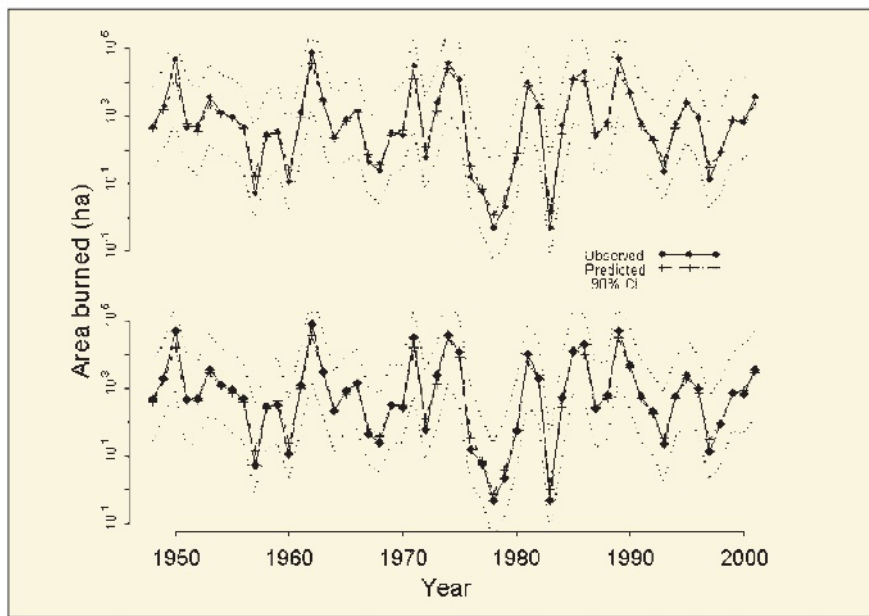


Figure 2. Predicted (median) and observed area burned in the April–May wildfire season in Everglades National Park, 1948–2001. (top) One-year predictions. (bottom) Three-month predictions that included mean November–January SOI as a covariate.

ARMA(2,2) with temporally varying parameters

Beckage and Platt 2003

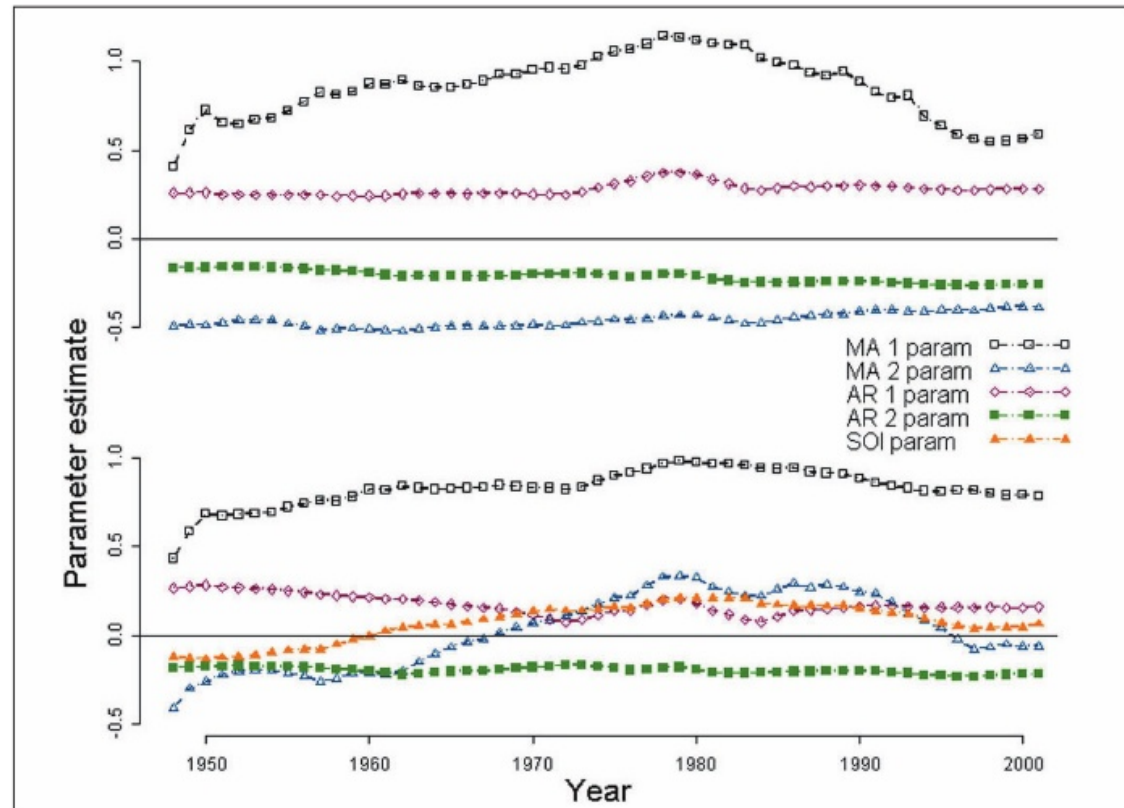


Figure 3. Evolution of model parameters (median values) through time. (top) One-year predictions. (bottom) Three-month predictions that included mean November–January SOI as a covariate.

Intervention Analysis

- Treatment effects in TIME
 - Pretreatment data establishes unit differences
- Hypotheses are usually that one or more parameters changed with treatment
- Alternate model (NULL) is no change in parameters with treatment
- Time can be modeled as
 - Covariate (explicit)
 - Time varying treatment (implicit)
 - Autocorrelation

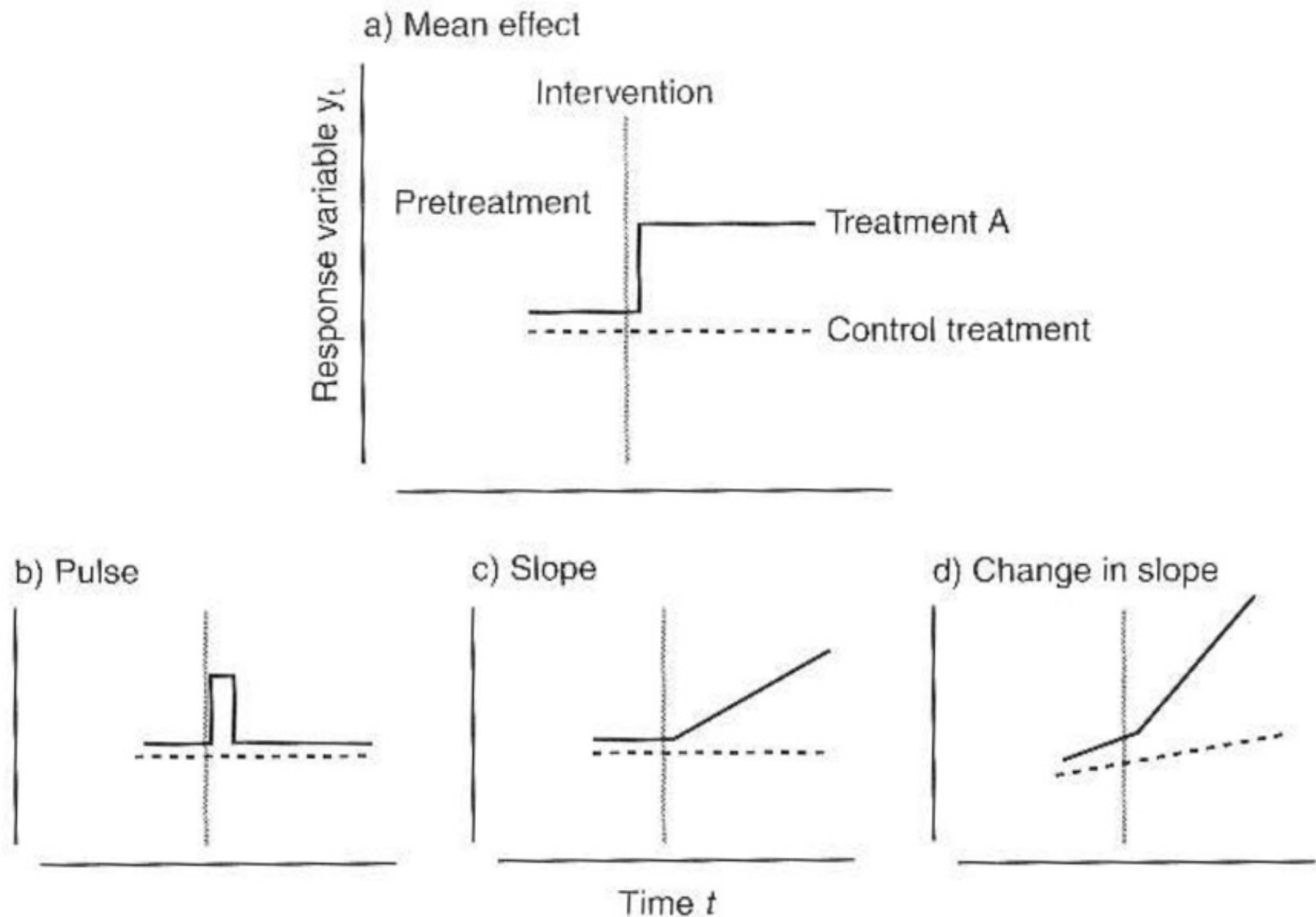


FIGURE 9.21. Four examples of ways in which an intervention might occur. The trend in the solid line “Treatment A” might not match that of the response to the treatment. For example, a step change in a treatment x (a) might elicit a trajectory of change in a response variable y .

Change point / threshold detection models

- Like Intervention models but the point in time where the change in parameters occurs is itself an unknown that needs to be estimated
- Challenging for likelihood models because the likelihood has a discontinuity

```
beta1 ~ dnorm(b0, Vb) ## prior betas
```

```
beta2 ~ dnorm(b0, Vb)
```

```
prec ~ dgamma(s1, s2) ## prior precision
```

```
K ~ dcat(pi) ##discrete prior for breakpoint
```

```
for (i in 1:n){
```

```
  mu1[i] <- beta1[1] + beta1[2]*time[i] ##prebreak
```

```
  mu2[i] <- beta2[1] + beta2[2]*time[i] ##postbreak
```

```
  mu[i] <- ifelse(i>K, mu2[i], mu1[i]) ##process model
```

```
  y[i] ~ dnorm(mu[i], prec) ## data model
```

```
}
```