Model Selection

- Philosophy of science and multiple alternative models
- Trade-offs
- Likelihood-based metrics
  - Likelihood Ratio Test
  - AIC
- Bayesian metrics
  - DIC
  - Predictive Loss

“The” Scientific Method?

- Popper
  - Falsification of hypotheses
    - Hypotheses can not be proved, only disproved
- Stats: “Null hypothesis testing” (Fisher)
  - Single hypothesis is disproved by confrontation with the data
  - Likelihood the data would have been observed if the null hypothesis was true
  - If this probability (p-value) is small enough we reject the null

Alternative Philosophies of Science

- Kuhn – Scientific Paradigms
  - Dominant paradigm used until there is so much contradictory information that it is “overthrown”
  - Requires an alternate paradigm that is “better”
- Polanyi – Republic of science
  - Multiple views of the world by different scientists
  - Confrontation between views and data judged by plausibility, value, and interest
- Lakatos – Scientific research program
  - Confrontation of multiple hypothesis with data as arbitrator

Null models

- All these alternatives acknowledge
  - There may be multiple alternative models
  - Simple null models often scientifically trivial, uninteresting
  - Doesn't make sense to reject a model if there is not an alternative
- Likelihood and Bayesian stats both well suited to “judge” the contest between multiple competing hypotheses and data

Models vs Hypotheses

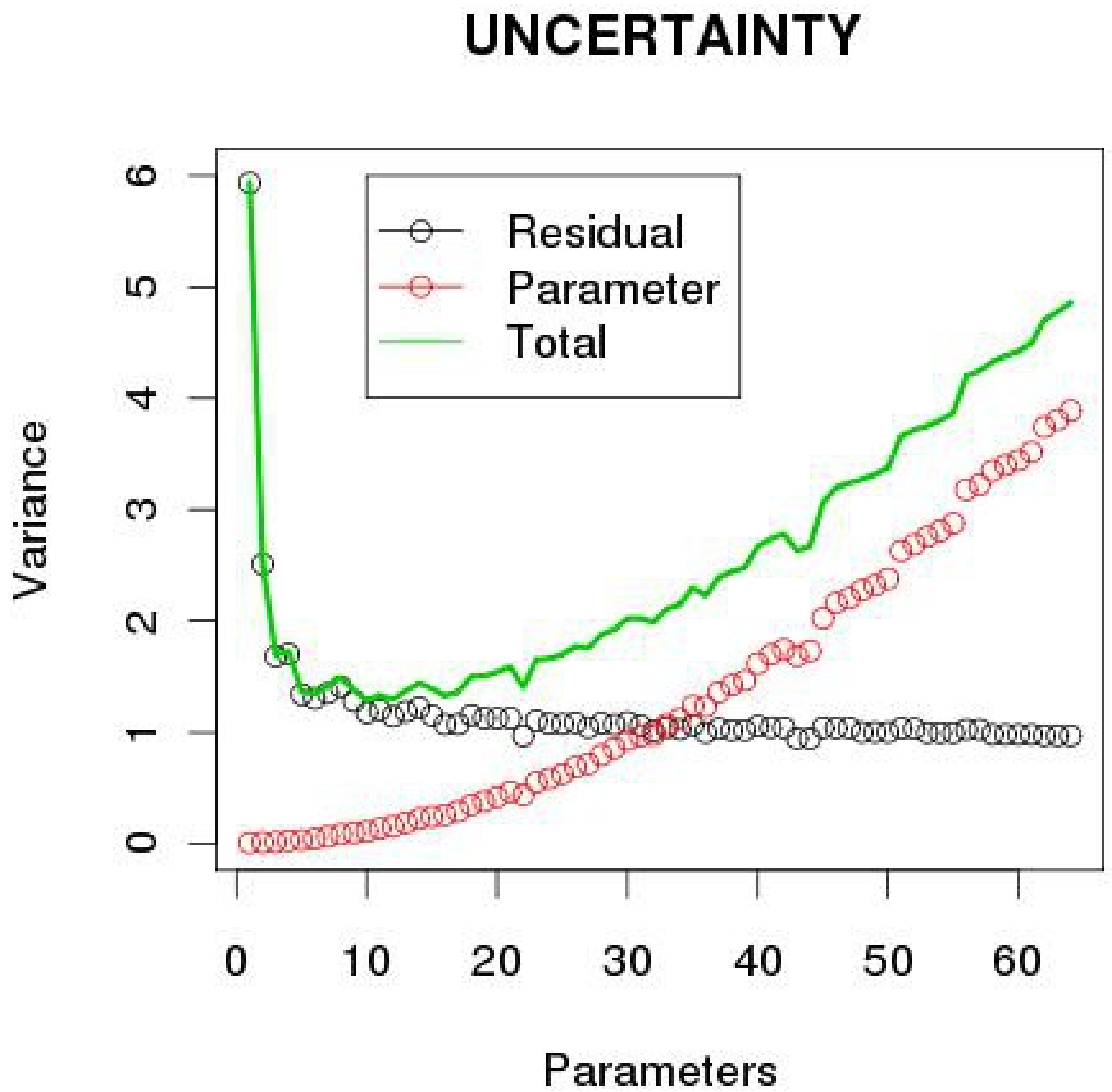
- Models usually more specific than hypothesis
- Hypoth: Birds forage more efficiently in flocks
- Models: <u>C</u>onsumption vs <u>S</u>ize
  - Consumption proportional $C = aS$
  - Consumption saturates $C = \frac{aS}{1 + bS}$
  - Increases then decreases $C = aS\,e^{-bS}$

- "All models are wrong but some are useful"

  -- George Box

Model selection

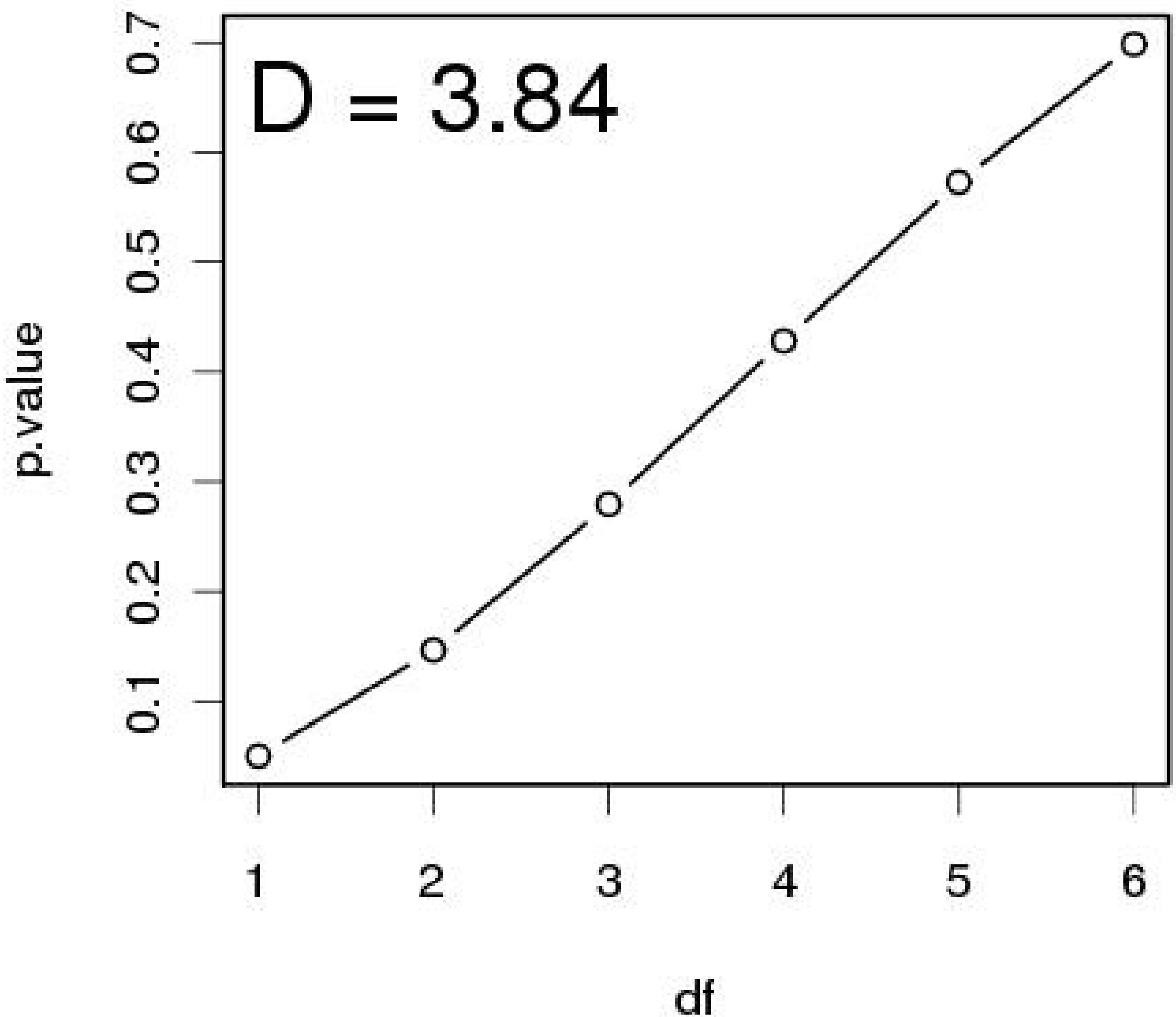
- Focus on choosing between multiple competing models rather than refuting a single null model
- How do we judge models?
  - Complexity
    - Number of parameters
  - Uncertainty
    - Model residuals
    - Parameter error (identifiability)
  - Data as ultimate arbiter
- "Make everything as simple as possible, but not simpler." - A. Einstein



UNCERTAINTY
Residual
Parameter
Total
Variance
Parameters
0
1
2
3
4
5
6
0
10
20
30
40
50
60

Likelihood Ratio Test

- $LR = L(x|\theta_{\circ}) / L(x|\theta_{\bullet})$
- $D = -2\ln L(x|\theta_{\circ}) - -2\ln L(x|\theta_{\bullet})$
- The test statistic D is known to be distributed with a χ^2 distribution
- Degrees of freedom = Difference in # of param.
  - Overall, L increases (-lnL declines) with # of param.
  - Penalizes model with more parameters
- p-val = 1-pchisq(D,df)


D = 3.84
p.value
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
2
3
4
5
6
df

LRT pro/con

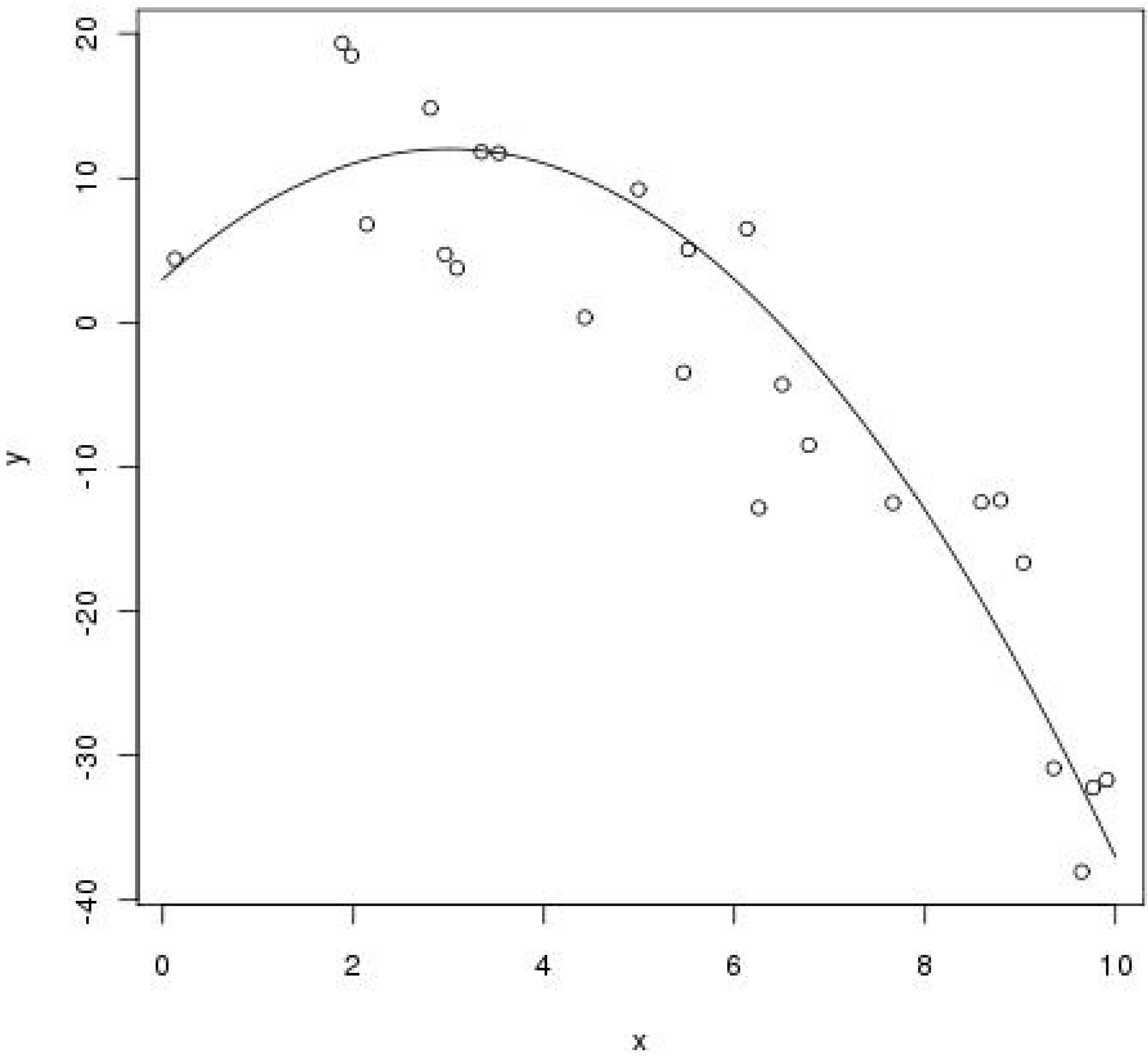
- Only applies to *nested* models
- Asymptotically, slightly biased toward more complex models
- Provides a p-value

- Additional reminders:
  - **ALL** model selection criteria require application to the same data with same sample size
  - e.g. If adding covariate Z requires rows to be dropped because of missing values, have to drop from the model w/o Z as well

Nested Models

- The more complex model collapses to the simpler model when one or more of the parameters is FIXED
- Examples:
  - Weibull vs Exponential (Lab 3)
    (fix $c=1$)
  - Pine cone: combined vs AMB/ELEV (Lab 4)
  - Regression: Inclusion of additional covariates
    (fix slope = 0)

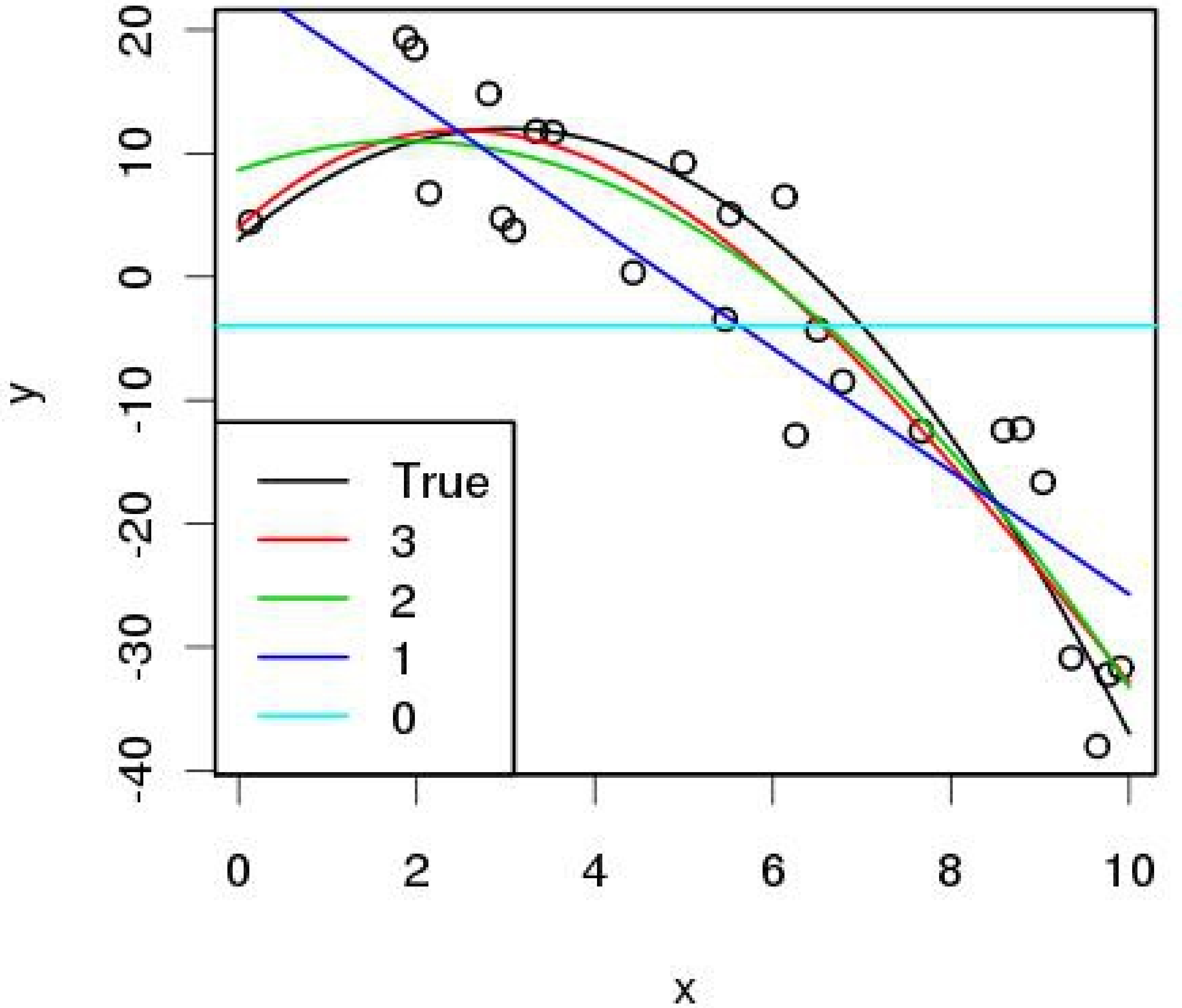
Example: Polynomial



Example: Polynomial

- Candidate models:
  - $Y = b0$
  - $Y = b0 + b1 \cdot x$
  - $Y = b0 + b1 \cdot x + b2 \cdot x^2$
  - $Y = b0 + b1 \cdot x + b2 \cdot x^2 + b3 \cdot x^3$
- Comparisons
  - 0 vs 1
  - 1 vs 2
  - 2 vs 3

- 0 vs 1
  p=7.6e-10
- 1 vs 2
  p=0.00019
- 2 vs 3
  p=0.9238

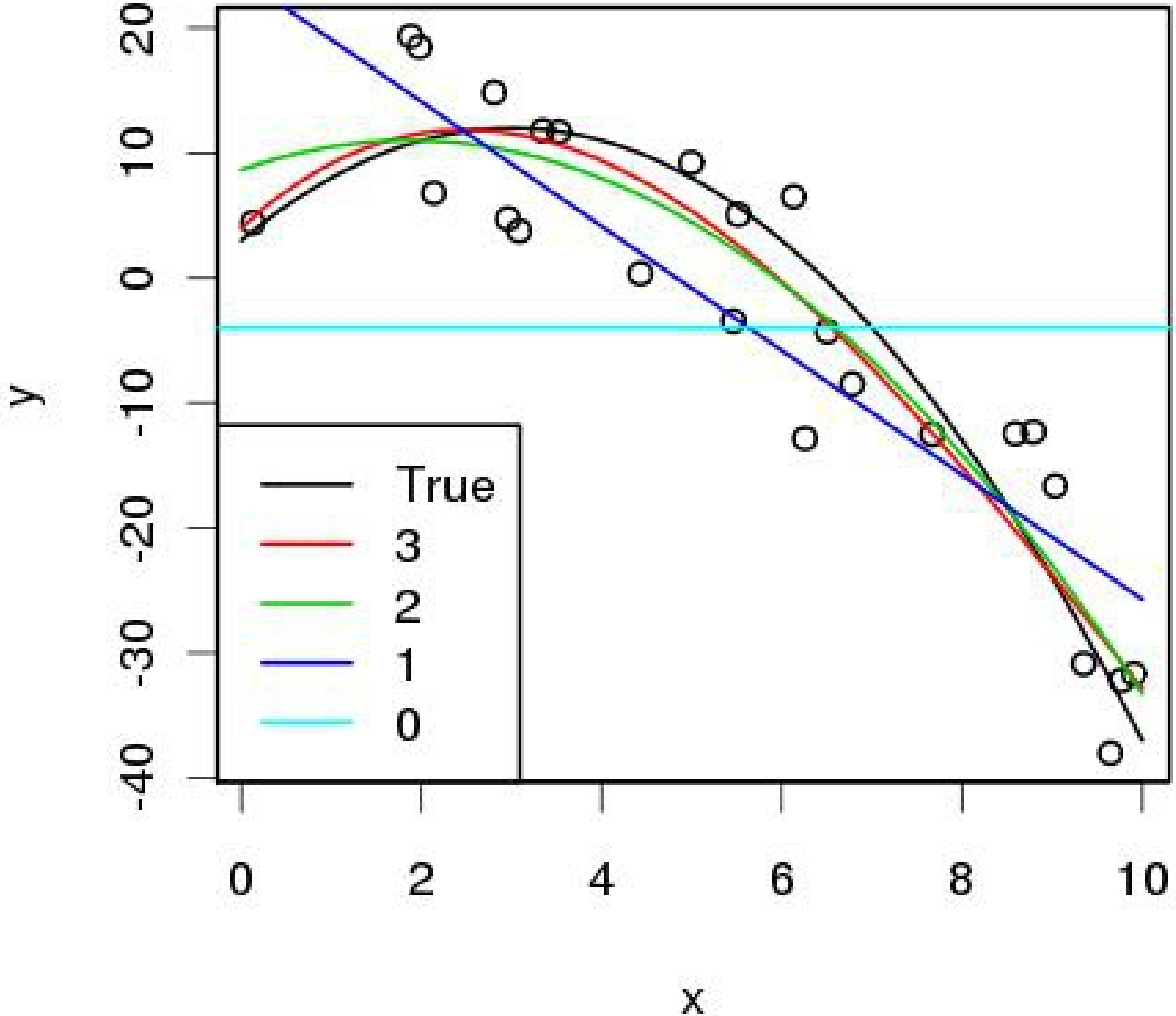


Akaike Information Criterion

$$AIC = -2\,lnL + 2\mathrm{p}$$

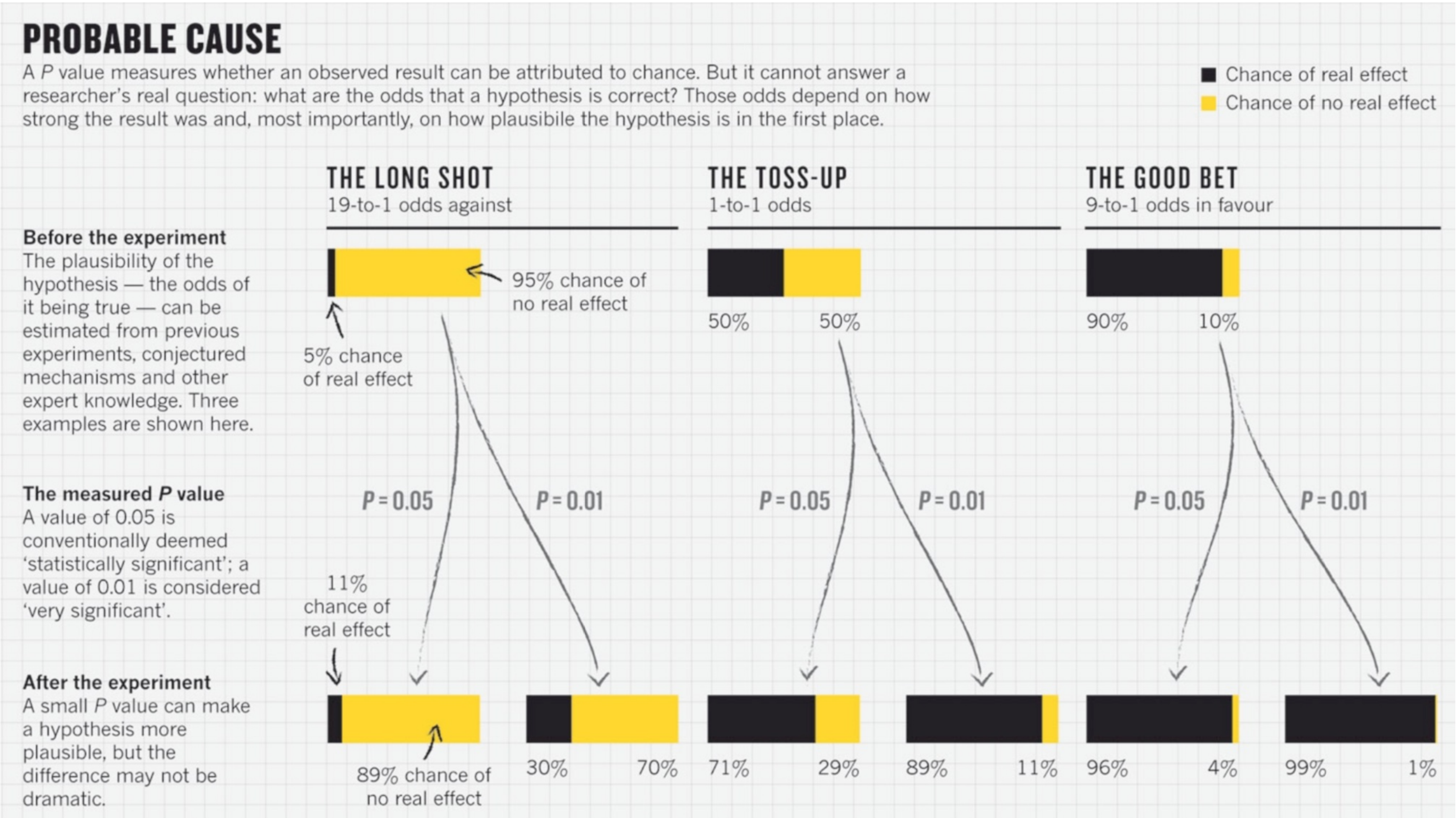
- p = number of parameters in the model
- Based on information theory
- Lowest value “wins”
- Often expressed relative to best model, ΔAIC
- No p-value
- “Rules of thumb”
  - 0-2 = similar 2-5 = weak support >5 = strong

- ΔAIC
- 0 47.77
- 1 11.91
- 2 0.00
- 3 1.99



P-value

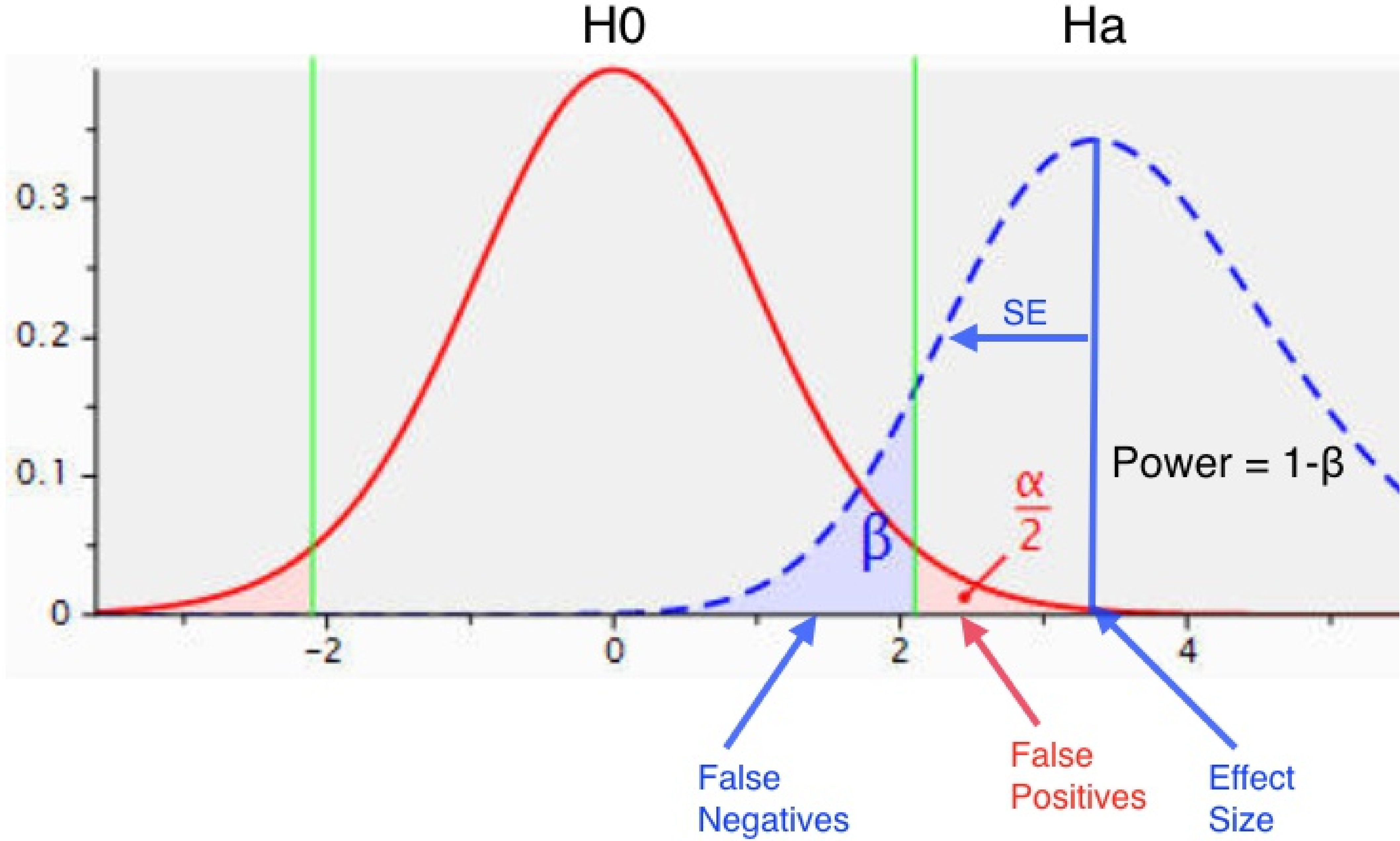
- Probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true.
- **Not** the probability that the null hypothesis is true
  - P-value can be close to zero when the posterior probability of the null is close to 1
- **Not** the probability of falsely rejecting the null hypothesis



https://www.nature.com/news/scientific-method-statistical-errors-1.14700

Example: Southern Brown Frog

- Researcher surveys a pond for the frog
- From prior experience 80% detection | present
- No frogs observed
- If null hypothesis is frogs are absent
  - P = 1.0 -- Fail to reject
  - Further surveys that fail to find the frog, p=1.0
- If null hypothesis is frogs are present
  - P = 0.2 – Fail to reject



Power = f(effect size, SE)

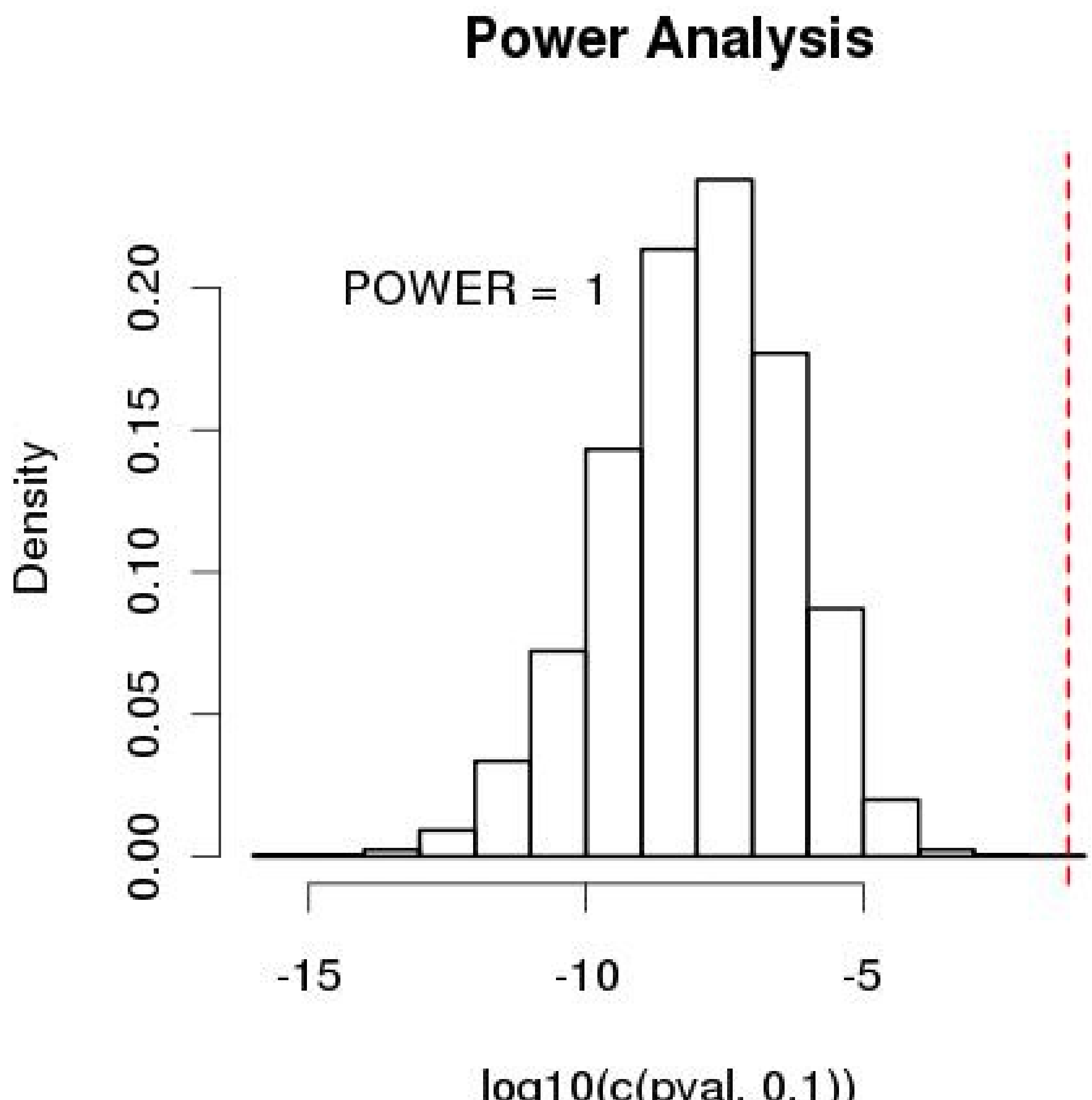
Power

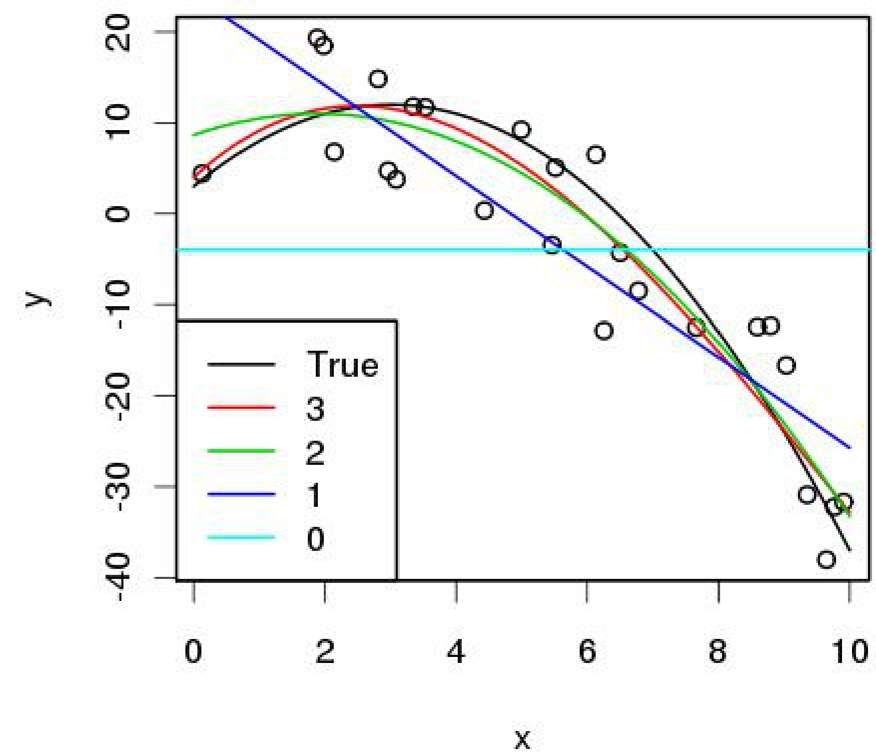
- Probability of correctly rejecting the null hypothesis
- Requires that some explicit alternative hypothesis is stated
  - Parameter values
  - Variance
  - Sample size
- Often calculated as a function of sample size
- For complex models, calculate through simulation

Generic Example

```
LnL.A = function(theta){
    -sum(dnorm(y,f(x,theta),sd)))
}
lnL.0 = function(mu){
    -sum(dnorm(y,mu,sd))
}
for(i in 1:nsim){
    Ey = f(x,theta)              ## process model
    y = rnorm(N,Ey,sd)      ## data model
    outA = optim(ic,lnL.A) ##fit of alternative
    out0 = optim(ic,lnL.0)  ##fit of null
    pval[i] = 1-pchisq(2*(outA$value-out0$value),df)
}
power = sum(pval < 0.05)/nsim
```

Example: Quadratic vs Linear LRT





- Results **specific** to parameter values and sample size chosen

Identifiability

- Data may not provide information on all parameters in a model
- Often requires restructuring model
- Not fixed by collecting more data
- Parameters often “trade-off when fitting”
- Simple examples
  - $N(\mu,\sigma^2+\tau^2)$
  - $N(a/b,\sigma^2)$
- Occur in both Likelihood and Bayes