Optimal Routing of Energy-Aware Vehicles in Transportation Networks With Inhomogeneous Charging Nodes

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***Abstract*— We study the problem of routing for energy-aware battery-powered vehicles (BPVs) in networks with charging nodes. The objective is to minimize the total elapsed time, including travel and recharging time at charging stations, so that the vehicle reaches its destination without running out of energy. Relaxing the homogeneity of charging stations, and here, we investigate the routing problem for BPVs through a network of "inhomogeneous" charging nodes. We study two versions of the problem: the single-vehicle (user-centric) routing problem and the multiple-vehicle (system-centric) routing problem. For the former, we formulate a mixed-integer nonlinear programming (NLP)problem for obtaining an optimal path and charging policy simultaneously. We then reduce its computational complexity by decomposing it into two linear programming problems. For the latter, we use a similar approach by grouping vehicles into "subflows" and formulating the problem at a subflow-level with the inclusion of traffic congestion effects. We also propose an alternative NLP formulation obtaining near-optimal solutions with orders of magnitude reduction in the computation time. We have applied our optimal routing approach to a subnetwork of the eastern Massachusetts transportation network using actual traffic data provided by the Boston Region Metropolitan Planning Organization. Using these data, we estimate cost (congestion) functions and investigate the optimal solutions obtained under different charging station and energy-aware vehicle loads.**

***Index Terms*— Vehicle routing, optimization, transportation networks, electric vehicles, game theory.**

I. Introduction

A. Literature Review

IN VIEW of the energy recuperation ability in EVs, [4] addresses an energy-optimal routing problem which exploits this ability to extend their cruising range; in particular, a general shortest-path algorithm is extended and adapted to this problem. Considering both limited energy supply and energy recuperation ability, [5] studies the energy-efficient routing problem for EVs. Employing a generalization of Johnson's potential shifting technique in Dijkstra's algorithm, a computationally efficient route planning algorithm is proposed in this work, applicable to any road network graph whose edge costs represent energy consumption or energy recuperation. In [6] the problem of energy-optimal routing for EVs is studied in a graph-theoretic context, subject to specific characteristics such as energy recuperation, battery capacity limitations and dynamic energy cost, and a heuristic algorithm is proposed to determine an optimal path.

A multi-constrained route optimization problem for EVs is studied in [7] where, applying penalty function method, the problem is transformed into an unconstrained optimization problem and then a particle swarm optimization algorithm is proposed to find a suboptimal solution. In [8] several routing problems are considered, including the shortest path and the Traveling Salesman Problem (TSP), by incorporating all costs in terms of gas prices. The goal is to find the least expensive route for an origin-destination pair or the least expensive tour in the case of a TSP. For the shortest path problem, equivalent to our single vehicle routing problem, a Dynamic Programming (DP) algorithm is proposed to find a least cost path from an origin to a destination in a network with inhomogeneously priced refueling stations. The same problem is revisited in [9], where the recharging cost is assumed to be a nonlinear function of the battery charging level. Again, the goal is to find a minimum-cost path for an EV. Discretizing the state space, a DP-based algorithm is proposed to determine an optimal path. In [10], the Electric Vehicle Routing Problem with Time Windows and recharging stations (E-VRPTW) is introduced. In this case, the charging scheme simply forces vehicles to be always fully recharged. In [11], the problem of locating charging stations and also determining optimal routes for commercial EVs is formulated as an integer programming problem. Combinatorial optimization methods for different aspects of EV management, such as energy-efficient routing and facility location problems, are studied in [12]. In recent work, [13] investigates the user-optimal network flow equilibrium with different scenarios for flow dependency of energy consumption of Battery Electric Vehicles (BEVs).

In this paper, we study the EV routing problem in a network with charging stations and the goal is to determine an optimal route and charging policy which minimizes the total traveling time. We impose an energy constraint in order to prevent an EV from running out of energy during its journey. In our earlier work [1], we studied this problem with the assumption that the charging nodes are *homogeneous* in the sense that

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TABLE I
CLASSIFICATION OF CHARGING STATIONS [15]

Charge Method	Nominal Supply Voltage(volts)	Max. Current (Amps)	Miles per every hour charging
AC Level 1	120 VAC, 1-phase	12 A	< 5
AC Level 2	208-240 VAC, 1-phase	32 A	up to 62
DC Charging	300 - 460VDC	400 A Max.	up to 300

their charging capabilities are indistinguishable. In contrast, relaxing the homogeneity assumption, here we study the EV routing problem in a network containing *inhomogeneous* charging nodes, i.e., charging rates at different nodes are not identical. In fact, depending on an outlet's voltage and current, charging an EV battery could take anywhere from minutes to hours and the Society of Automotive Engineering (SAE) classifies charging stations into three categories [14]–[16] as shown in Tab. I. Thus, charging rates and times are highly dependent on the class of the charging station and they clearly affect the solution of our optimization problem.

As in [1], we view this as a network routing problem where vehicles control not only their routes (we will use "path" and "route" interchangeably) but also recharging amounts at various nodes in the network.

The contributions of this paper are as follows. First, we study the user-centric problem and propose a MINLP formulation to determine the optimal path and charging amounts simultaneously. Because of the inhomogeneity in charging nodes, the problem is more complicated than that in [1] and it can no longer be reduced to a simple Linear Program (LP). However, we can still prove certain optimality properties allowing us to reduce the dimensionality of the original problem. Further, by adopting a locally optimal charging policy, we derive an LP formulation through which near-optimal solutions are obtained. We note that the main difference between this single-vehicle problem and the one considered in [8] is that we aim to minimize the total elapsed time over a path, as opposed to a fueling cost, and must, therefore, include two parameters per network arc (we will use "arc" and "link" interchangeably): energy consumption and traveling time. Next, we study a multiple-vehicle energy-aware routing problem, where congestion effects are incorporated. Similar to [1], by grouping vehicles into "subflows" we are able to reduce the complexity of the original problem, although we can no longer obtain an LP formulation. Moreover, we provide an alternative flow-based formulation which reduces the computational complexity of the original MINLP problem by orders of magnitude with numerical results showing little loss in optimality. Finally, we consider actual traffic data provided by the Boston Region Metropolitan Planning Organization (MPO) for the Eastern Massachusetts transportation network and apply our algorithms to an interstate highway subnetwork. For the single vehicle routing problem, we observe that the optimal route may deviate from the shortest path for different combinations of charging stations. For the multiple vehicle routing problem, we solve the problem for different values of the number of subflows and notice that a near optimal solution can be obtained under a relatively small such number. We also investigate the effect of increasing the EV inflow rate on the optimal policy and observe that for small rates the shortest path remains the optimal solution; however, for higher rates, due to congestion effects, a fraction of vehicles deviates from the shortest path.

It is worth mentioning that we have investigated a Dynamic Programming (DP) approach for both single vehicle and multiple vehicle routing problems in [17]. For the former, the DP formulation results in optimal solutions with lower computational complexity compared to the MINLP formulation proposed here, however for the latter, the problem size significantly increases with the number of subflows and the DP algorithm is eventually outperformed by the MINLP approach proposed here, as the number of subflows increases.

The paper is organized as follows. In Section II, we address the single-vehicle routing problem in a network with inhomogeneous charging nodes and identify properties which lead to its simplification. In Section III, the multiple-vehicle routing problem is formulated, first as a MINLP and then as an alternative flow optimization problem. To investigate the routing and recharging solutions for an actual transportation network, we use real traffic data for the Eastern Massachusetts transportation network, and provide numerical results for a sub-network illustrating our approaches and providing insights on the relationship between recharging speed and optimal routes as well as the effect of inflows rates on the optimal solution. Conclusions and further research directions are outlined in Section IV.

II. SINGLE VEHICLE ROUTING

The single vehicle routing problem represents the "user-centric" point of view in which the objective is to find the optimal path and charging policy for a single EV minimizing its total traveling time. As in earlier work [1], we model the transportation network as a directed graph $G = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N} = \{1, \ldots, n\}$ is the set of nodes, each of them representing a charging node, and $\mathcal{A}$ is the set of links with $(i, j) \in \mathcal{A}$ denoting a link connection node i to j and $|\mathcal{A}| = m$. Node $i \in \mathcal{N}/\{n\}$ represents a charging station with g_i denoting the charging time per unit of energy at node i. In contrast to [1], here g_i, $1, \ldots, n$, are node-dependent parameters and not identical. Without loss of generality, let us assume all nodes have a charging capability (if node i does not have such capability, we can simply set $g_i = \infty$). We define $I(i)$ and $O(i)$ as the set of start nodes (respectively, end nodes) of arc that are incoming to (respectively, outgoing from) node i, that is, $I(i) = \{j \in \mathcal{N} | (j, i) \in \mathcal{A}\}$ and $O(i) = \{j \in \mathcal{N} | (i, j) \in \mathcal{A}\}$.

Assuming nodes 1 and n as origin and destination respectively, the goal is that to determine an energy-feasible path and charging policy to optimize the traveling time from node 1 to node n for a single EV. For each link $(i, j) \in \mathcal{A}$, there exist two cost parameters: the required traveling time τ_{ij} and the energy consumption e_{ij}. Note that $\tau_{ij} > 0$ (if nodes i and j are not connected, then $\tau_{ij} = \infty$), whereas e_{ij} is allowed to be negative due to a BPV's potential energy recuperation effect [4]. Note that here we consider τ_{ij} and e_{ij} as fixed parameters where their values depend on the traffic conditions

TABLE II
TABLE OF NOTATION

$G = (\mathcal{N}, \mathcal{A})$	Directed graph with set of nodes, $\mathcal{N}$, and set of links $\mathcal{A}$
(i, j)	A link connecting nodes i and j
d_{ij}	Distance between nodes i and j
t_{ij}^0	Free-flow travel time of link (i, j)
f_{ij}	Flow on link (i, j)
C_{ij}	Effective capacity of link (i, j)
f_{ij}^{eq}	Equilibrium flow for NEVs
e_{ij}	Energy consumption when traveling through link (i, j) for single vehicle problem
$e_{ij}^k(\mathbf{x_{ij}})$	Energy consumption, as a function of flow, of all vehicles in subflow k while traveling through link (i, j)
τ_{ij}	Traveling time of link (i, j) for single vehicle problem
$\tau_{ij}(\mathbf{x_{ij}})$	Traveling time for each vehicle passing link (i, j) as a function of flow
x_{ij}	Binary variables for selecting link (i, j)
x_{ij}^k	Binary variables for selecting link (i, j) for subflow k
r_i	Energy recharging amount at node i
r_i^k	Energy recharging amount at node i for subflow k
g_i	Charging rate at node i
E_i	Vehicle's residual battery energy at node i for single vehicle problem
E_i^k	Residual energy of subflow k at node i
B	Battery capacity of a single EV
B^k	maximum charging amount kth subflow can get
N	Number of subflows
R	Total flow rate into the network at the origin node

at the time one solves the user-centric problem (as illustrated in Section II-C using actual traffic data). Clearly, this is not the case for the system-centric problem in which routing decisions affect congestion over a link and, consequently, the traveling time and energy consumption of the link. Defining the EV's battery capacity as B, we assume $0 < e_{ij} < B$ for all $(i, j) \in \mathcal{A}$, i.e. all links in the network are energy-feasible for the EV (this assumption is for simplicity and does not affect our analysis). Table II summarizes the main notation that we use throughout the paper.

Since the EV has limited battery energy it may not be able to reach the destination without recharging. Thus, recharging amounts at charging nodes $i \in \mathcal{N}$ are also decision variables. We define the decision variables as follows: m binary variables for selecting link (i, j) denoted by $x_{ij} \in \{0, 1\}$, $i, j \in \mathcal{N}$, and $n - 1$ non-negative real variables for the energy recharging amount at node $i \in \mathcal{N}/\{n\}$ denoted by r_i. Moreover, in order to model the energy dynamic of the EV while traveling through the network, we define $n - 1$ non-negative real variables as E_i denoting the vehicle's residual battery energy at node i. Then, for all E_j, $j \in O(i)$, we have:

$$E_j = \begin{cases} E_i + r_i - e_{ij} & \text{if } x_{ij} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

which can also be expressed as

$$E_j = \sum_{i \in I(j)} (E_i + r_i - e_{ij}) x_{ij}, \quad x_{ij} \in \{0, 1\}$$

We formulate the user-centric problem as follows:

$$\min_{x_{ij}, r_i,\ i,j \in \mathcal{N}} \ \sum_{i=1}^{n} \sum_{j=1}^{n} \tau_{ij} x_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} r_i g_i x_{ij} \tag{2}$$

$$s.t. \sum_{j \in O(i)} x_{ij} - \sum_{j \in I(i)} x_{ji} = b_i, \quad \text{for each } i \in \mathcal{N} \tag{3}$$

$$b_1 = 1,\ b_n = -1,\ b_i = 0, \ \text{ for } i \neq 1, n \tag{4}$$

$$E_j = \sum_{i \in I(j)} (E_i + r_i - e_{ij}) x_{ij}, \ \text{ for } j = 2, \ldots, n \tag{5}$$

$$0 \leq E_i \leq B, \quad E_1 \text{ given, for each } i \in \mathcal{N} \tag{6}$$

$$x_{ij} \in \{0, 1\}, \quad r_i \geq 0 \tag{7}$$

This is a Mixed Integer Non-Linear Programming (MINLP) problem with $m + 2(n - 1)$ variables and it will be referred to as **P1**. The objective of this problem, as observed in equation (2), is to determine a path from the origin (node 1) to the destination (node n), as well as a recharging amount at each node minimizing the total elapsed time. Constraints (3)-(4) stand for the flow conservation which enforces the condition that, starting from node i, only one path can be selected, i.e., $\sum_{j \in O(i)} x_{ij} \leq 1$. It is easy to check that this also implies $x_{ij} \leq 1$ for all i, j since $b_1 = 1$, $I(1) = \varnothing$. The vehicle's energy dynamics at each node are modeled using (5). Finally, (6) indicates that the vehicle cannot run out of energy before reaching a node or exceed a given capacity B. Note that the nonlinearity in the problem appears in the objective function as well as constraint (5). All other parameters are predetermined according to the network topology. A crucial difference between **P1** and the MINLP introduced in [1] is that here the charging rates g_i in (2) are node-dependent.

A. Properties

As discussed, **P1** is a MINLP problem which is computationally demanding. In the sequel, we reduce the computational complexity of **P1** by deriving some key properties of an optimal solution. Applying these properties we obtain a lower-dimensional problem with $m + (n - 1)$ variables. The main difficulty in this problem lies in the coupling of the decision variables x_{ij} and r_i in (5) and the following lemma will enable us to eliminate r_i from (2).

Lemma 1: Given (2)-(7), an optimal solution $\{x_{ij}, r_i, E_i\}$, $i, j \in \mathcal{N}$ satisfies:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(r_i x_{ij} - e_{ij} x_{ij}) g_i = \sum_{i=1}^{n}\sum_{j=1}^{n}(E_j - E_i) g_i x_{ij} \tag{8}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} E_j (g_i - g_j) x_{ij} - E_1 g_1 \tag{9}$$

Proof: Multiplying both sides of (1) by g_i gives:

$$E_j g_i = \begin{cases} (E_i + r_i - e_{ij}) g_i & \text{if } x_{ij} = 1, \\ 0 & \text{otherwise .} \end{cases}$$

which can be expressed as

$$\sum_{i \in I(j)} E_j g_i x_{ij} = \sum_{i \in I(j)} (E_i + r_i - e_{ij}) g_i x_{ij}$$

Summing both sides over $j = 2, \ldots, n$ and rearranging yields:

$$\sum_{j=2}^{n}\sum_{i \in I(j)} E_j g_i x_{ij} - \sum_{j=2}^{n}\sum_{i \in I(j)} E_i g_i x_{ij} = \sum_{j=2}^{n}\sum_{i \in I(j)} (r_i - e_{ij}) g_i x_{ij}$$

Based on (1), $E_i = 0$ for all nodes which are not in the selected path. Thus we can rewrite the equation above as

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(r_i x_{ij} - e_{ij} x_{ij}) g_i = \sum_{i=1}^{n}\sum_{j=1}^{n}(E_j - E_i) g_i x_{ij}$$

which establishes (8). Finally, (9) follows by observing that if P is an optimal path we can re-index nodes so that $P = \{1, \ldots, n\}$ with $g_n = 0$. Thus, we have $\sum_{i=1}^{n}\sum_{j=1}^{n} E_i g_i x_{ij} = E_1 g_1 + \ldots + E_{n-1} g_{n-1}$ which can also be written as $E_1 g_1 + \sum_{i=2}^{n}\sum_{j=2}^{n} E_j g_j x_{ij}$ where $x_{ij} = 0$ for all (i, j) not in the optimal path. Therefore,

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(E_j - E_i) g_i x_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} E_j (g_i - g_j) x_{ij} - E_1 g_1$$

which proves (9). ∎

Lemma 2: If $\sum_i r_i^* > 0$ in the optimal routing policy, then $E_n^* = 0$.

Proof: This is the same as the homogeneous charging node case; see [1, Lemma 2].

Using Lemma 1, we replace $\sum_{i=1}^{n}\sum_{j=1}^{n} r_i g_i x_{ij}$ in (2) and eliminate the presence of r_i, $i = 2, \ldots, n-1$, from the objective function and the constraints. Thus, **P1** is reduced to the following MINLP problem with only $m + (n-1)$ decision variables, referred to as **P2**:

$$\min_{\substack{x_{ij}, E_i,\\ i,j \in \mathcal{N}}} \sum_{i=1}^{n}\sum_{j=1}^{n}\big(\tau_{ij} x_{ij} + e_{ij} g_i x_{ij} + E_j (g_i - g_j) x_{ij}\big) - E_1 g_1 \tag{10}$$

$$s.t \sum_{j \in O(i)} x_{ij} - \sum_{j \in I(i)} x_{ji} = b_i \tag{11}$$

$$b_1 = 1,\ b_n = -1,\ b_i = 0 \ \text{ for } \ i \neq 1, n \tag{12}$$

$$0 \leqslant E_j - (E_i - e_{ij}) x_{ij} \leqslant B \quad \forall\, i, j \in N \tag{13}$$

$$0 \leqslant E_i \leqslant B,\ E_1 \text{ given}, \quad \forall\, i \in N \tag{14}$$

$$x_{ij} \in \{0, 1\} \tag{15}$$

Constraint (13) is derived from (5). Assuming $x_{ij} = 1$, i.e. arc (i, j) is part of the optimal path, we can recover $r_i = E_j - E_i + e_{ij}$ and constraint (13) is added to prevent any vehicle from exceeding its capacity B in an optimal path. Solving this problem gives both an optimal path and residual battery energy at each node.

Although **P2** has fewer decision variables, it is still a MINLP which is hard to solve for large networks. Specifically, the computation time is highly dependent on the number of nodes and arcs in the network. In what follows we introduce a *locally optimal* charging policy, leading to a simpler problem, by arguing as follows. Looking at (10), the term $\sum_{i=1}^{n}\sum_{j=1}^{n} E_j (g_i - g_j) x_{ij}$ is minimized by selecting each E_j depending on the sign of $(g_i - g_j)$:

Case 1: $g_i - g_j < 0$, i.e., node i has a faster charging rate than node j. Therefore, E_j should get its maximum possible value, which is $B - e_{ij}$. This implies that the vehicle must be maximally charged at node i.

Case 2: $g_i - g_j \geqslant 0$, i.e., node j has a faster or same charging rate as node i. In this case, E_j should get its minimum value $E_j = 0$. This implies that the vehicle should get the minimum charge needed at node i in order to reach node j.

We define $\pi_{\mathbf{C}}$ to be the charging policy specified as above and note that it does not guarantee the global optimality of E_i thus selected in (10) which can easily be checked by a counterexample. However, it allows us to decompose the optimal routing problem from the optimal charging problem. If, in addition, we consider only solutions for which the vehicle is recharged at least once (otherwise, the vehicle is not energy-constrained and the problem is of limited interest), we can obtain the following result.

Theorem 1: If $\sum_i r_i^* > 0$ (i.e. the vehicle has to be recharged at least once), then under charging policy $\pi_{\mathbf{C}}$, the solution x_{ij}^*, $i, j \in \mathcal{N}$, of the original problem (2) can be determined by solving the LP problem:

$$\min_{x_{ij}, i, j \in \mathcal{N}} \sum_{i=1}^{n}\sum_{j=1}^{n}\big(\tau_{ij} + e_{ij} g_i + K(g_i - g_j)\big) x_{ij} \tag{16}$$

$$K = \begin{cases} B - e_{ij} & \text{if } g_i < g_j, \\ 0 & \text{otherwise .} \end{cases} \tag{17}$$

$$s.t. \sum_{j \in O(i)} x_{ij} - \sum_{j \in I(i)} x_{ji} = b_i \tag{18}$$

$$b_1 = 1, b_n = -1, b_i = 0 \quad for i \neq 1, n \tag{19}$$

$$0 \leqslant x_{ij} \leqslant 1 \tag{20}$$

Proof: Applying charging policy $\pi_{\mathbf{C}}$ in (10) we change the objective function to $\sum_{i=1}^{n}\sum_{j=1}^{n}\big(\tau_{ij} + e_{ij} g_i + K(g_i - g_j)\big) x_{ij} - E_1 g_1$ where K is as in (17). Therefore, x_{ij}^* can be determined by the following problem:

$$\min_{x_{ij}, i, j \in \mathcal{N}} \sum_{i=1}^{n}\sum_{j=1}^{n}\big(\tau_{ij} + e_{ij} g_i + K(g_i - g_j)\big) x_{ij} - E_1 g_1$$

$$K = \begin{cases} B - e_{ij} & \text{if } g_i < g_j, \\ 0 & \text{otherwise .} \end{cases}$$

$$s.t. \sum_{j \in O(i)} x_{ij} - \sum_{j \in I(i)} x_{ji} = b_i, \quad \text{for each } i \in \mathcal{N}$$

$$b_1 = 1,\ b_n = -1,\ b_i = 0, \ \text{for } i \neq 1, n$$

$$x_{ij} \in \{0, 1\}$$

which is a typical shortest path problem formulation. Moreover, according to the property of minimum cost flow problems [18], the above integer programming problem is equivalent to the LP with the integer restriction on x_{ij} relaxed. Finally, since E_1 and g_1 are given, the problem reduces to (16), which proves the theorem. ∎

Remark 1: If $g_i = g_j$ for all i, j in (16), the problem reduces to the homogeneous charging node case studied in [1] with the same optimal LP formulation as in Theorem 1. With $g_i \neq g_j$ however, the LP formulation cannot guarantee global optimality, although the routes obtained through Theorem 1 may indeed be optimal (see Section II.C), in which case the optimal charging amounts are obtained as described next.

B. Determination of Optimal Recharging Amounts r_i^*

Once we determine an optimal route P, it is relatively easy to find a feasible solution for r_i, $i \in P$, to satisfy the constraint (5) and minimize the total charging time on the selected path. It is obvious that the optimal charging amounts r_i^* are non-unique in general. Without loss of generality we re-index nodes so that we may write $P = \{1, \ldots, n\}$. Then, the problem resulting in an optimal charging policy is

$$\begin{aligned} \min_{r_i,\ i\in P} \quad & \sum_{i\in P} g_i r_i \\ s.t.\ \ & E_{i+1} = E_i + r_i - e_{i,i+1} \\ & 0 \le E_i \le B, \quad E_1 \text{ given} \\ & r_i \ge 0 \quad \text{for all } i \in \mathcal{N} \end{aligned} \tag{21}$$

This is an LP where E_i and r_i are decision variables. Unlike the homogeneous charging node problem in [1] where the objective function includes charging prices p_i associated with nodes, i.e., $\sum_{i\in P} p_i r_i$, this is not the case here, since there is a tradeoff between selecting faster-charging nodes and possible higher costs at such nodes. However, the advantage of the decoupling approach is that if an optimal path is determined, an additional cost minimization problem can be formulated to determine optimal charging times at nodes on this path.

C. Numerical Example for the Eastern Massachusetts Transportation Network

Using an actual traffic dataset provided by the Boston Region Metropolitan Planning Organization (MPO), our goal is to find the optimal path and charging policy for a single EV to travel from an origin to a destination in the Eastern Massachusetts transportation network shown in Fig. 1. The dataset includes traffic data in the form of spatial average speeds for major roadways and arterial streets in Eastern Massachusetts for every minute of year 2012 as described in [19]. In this paper, we use the same network - a representative interstate highway sub-network - presented in [19] for the numerical examples as shown in Fig. 2(a). This network is composed of 8 nodes and 24 links including 701 road segments. The topology of this sub-network is also shown in Fig. 2(b).

As discussed in the beginning of this section, we assume a fixed traveling time, τ_{ij}, and energy consumption, e_{ij}, for each link in the network depending on given traffic conditions at the time the single-vehicle routing problem is solved. The value of these parameters depend on given traffic conditions at the time the single-vehicle routing problem is solved. To calculate the traveling time, first we convert the speed data to flow data on each link using Greenshield's model [20] (details are provided in [19]). Then, the average traveling time on each link $(i, j) \in \mathcal{A}$ is estimated by incorporating the calculated flow and link capacity in a delay function with the general form of $t_a = t_{ij}^0 h(f_{ij}/C_{ij})$, where t_{ij}^0, f_{ij}, and C_{ij} denote free flow traveling time, link flow, and link capacity respectively. For example, adopting the corresponding data-driven delay function estimated in [19], Tab. III shows the average traveling time of each link during the AM period (7-9 AM) of a working day in April 2012.

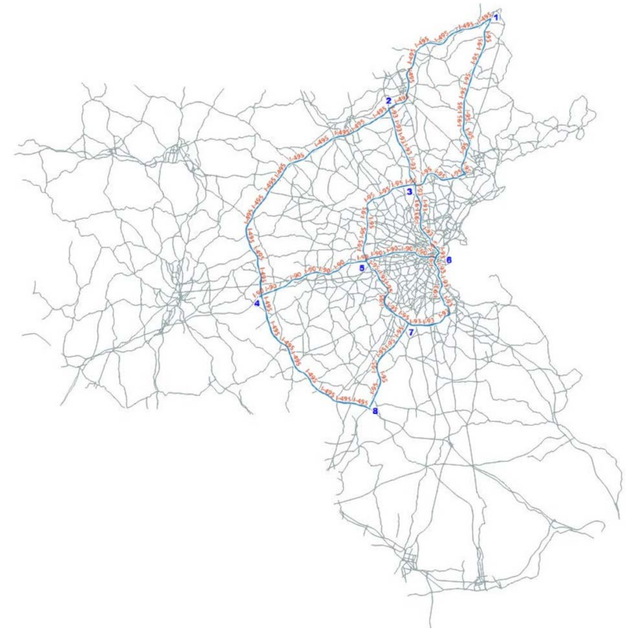
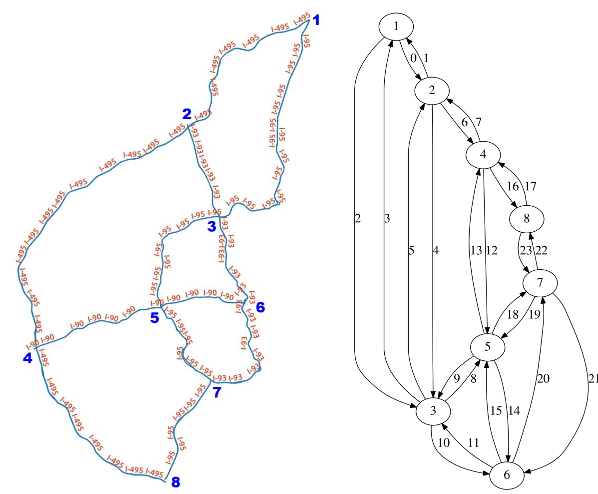


Fig. 1. Road map of eastern massachusetts.



(a) (b)

Fig. 2. (a) An interstate highway sub-network of Eastern Massachusetts (the blue numbers indicate node indices); (b) The topology of the sub-network.

TABLE III

TRAVELING TIME [hr] AND DISTANCE [mile] OF EACH LINK OF SUB-NETWORK OF FIG. 2(A)

τ_{12}	τ_{21}	τ_{13}	τ_{31}	τ_{23}	τ_{32}	τ_{24}	τ_{42}	τ_{35}	τ_{53}	τ_{36}	τ_{63}
0.31	0.33	0.46	0.46	0.14	0.18	0.54	0.55	0.25	0.28	0.21	0.23
τ_{45}	τ_{54}	τ_{56}	τ_{65}	τ_{48}	τ_{84}	τ_{57}	τ_{75}	τ_{67}	τ_{76}	τ_{78}	τ_{87}
0.29	0.25	0.18	0.18	0.34	0.35	0.20	0.19	0.27	0.28	0.2	0.2
d_{12}	d_{21}	d_{13}	d_{31}	d_{23}	d_{32}	d_{24}	d_{42}	d_{35}	d_{53}	d_{36}	d_{63}
21.49	22.83	32.03	32.7	10.08	11.98	37.63	38.67	16.21	16.8	12.88	12.94
d_{45}	d_{54}	d_{56}	d_{65}	d_{48}	d_{84}	d_{57}	d_{75}	d_{67}	d_{76}	d_{78}	d_{87}
6.94	17.16	10.77	10.68	24.43	24.37	12.51	12.33	16.51	16.3	13.92	14.19

TABLE IV

OPTIMAL PATHS OBTAINED BY SOLVING PROBLEMS **P-1** AND LP FOR DIFFERENT CHARGING NODE CONFIGURATIONS

$G = [g_1,\ g_2,\ ,\ldots, g_7]$	Problem	Path
$[g^1\ g^1\ g^1\ g^1\ g^1\ g^1\ g^1]$	**P1**:	1→3→5→7→8
	LP:	1→3→5→7→8
$[g^2\ g^1\ g^2\ g^2\ g^1\ g^2\ g^1]$	**P1**:	1→3→6→7→8
	LP:	1→3→6→7→8
$[g^1\ g^2\ g^1\ g^2\ g^1\ g^1\ g^1]$	**P1**:	1→2→4→8
	LP:	1→2→4→8

For simplicity we assume e_{ij} is proportional to the distance of each link (see Tab. III), i.e. $e_{ij} = \alpha d_{ij}$ where α is the energy consumption rate. In our numerical example we set $\alpha = 0.3$ [kWh/mile] as per the EPA fuel economy of a Nissan Leaf 2016 [21]. We then solve the problem for different configurations of charging stations in the network. The optimal paths obtained by solving the MINLP formulation as well as the decomposed LP are shown in Tab. IV where $G = [g_1,\ g_2,\ ,\ldots, g_{n-1}]$. In our numerical example we assume charging stations from 2 classes: level 1 with charging rate 41.67 [min/kWh] or $g^1 = 41.67/60$[hr/kWh], and level 2 with charging rate 10 [min/kWh] or $g^2 = 1/6$ [hr/kWh]. It is observed that both formulations (original MINLP and approximate LP) result in the *same* optimal path while their computational complexities are drastically different (from around 250 sec for **P1** to less than 1 sec for the LP formulation). Once the optimal path is determined, we can easily solve (21) to determine optimal charging amounts as well.

Next, we investigate the traffic congestion effect on the total traveling time. To do so, we solve the single vehicle routing problem for all days in April during AM periods. We assume $g_{1,\ldots,7} = 1/6$ and $E_1 = 0$. Fig. 3 shows the total elapsed time for a single EV to travel from node 1 to node 8 in different days in the month. For all days in April the optimal path is the same: (1→3→5→7→8). However, due to the traffic condition of different days, the traveling time may not be the same (recall that the energy consumption on each link is a linear function of its distance, the total charging time is independent of traffic conditions). Also, it is observed that the traveling time has a periodic pattern and its minimum value occurs at weekends and holidays as expected.

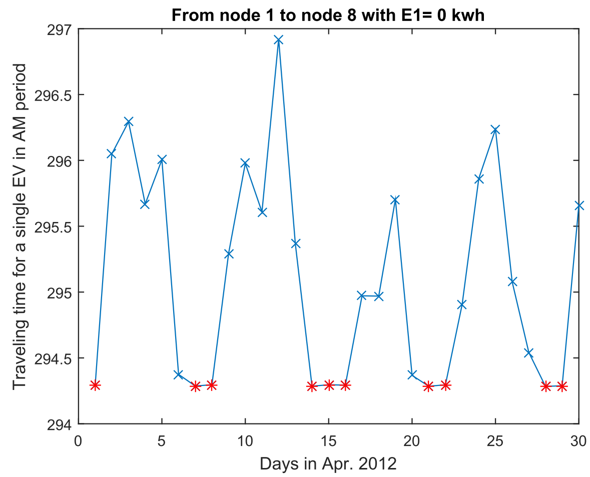


Fig. 3. Total elapsed time for a single EV traveling from node 1 to node 8 in April 2012 during AM period (Weekends / Holidays shown with red stars).

III. MULTIPLE VEHICLE ROUTING

Next, we investigate the system-centric problem, referred to as the multiple-vehicle routing problem, in a network with inhomogeneous charging nodes. As opposed to the user-centric policy, here we determine the routing and charging policies so as to optimize a *system-wide objective*. Thus, as discussed in Section II, the first technical difficulty here is the need to incorporate the effect of traffic congestion on both traveling time and energy consumption; therefore, the variables τ_{ij} and e_{ij} no longer have fixed values. A second difficulty is the implementation of an optimal routing policy, which requires signaling mechanisms and possibly incentive structures to enforce desired routes assigned to vehicles. This raises a number of additional research issues which are beyond the scope of this paper and likely to be addressed by the advent of Connected Automated Vehicles (CAVs).

As in [1], we formulate the problem by grouping subsets of vehicles into N "subflows". In the sequel, it will be observed that the problem complexity highly depends on N which should, therefore, be selected to render the problem manageable (the effect of N is discussed in Section III-D).

We define R as the EV flow rate entering the network at node 1. We divide it into N *subflows* such that each subflow includes the same type of vehicles (e.g., large vehicles vs smaller ones or vehicles with the same initial energy). Formulating the problem at the "subflow-level" (rather than individual EVs), we assume all vehicles in the same subflow follow the same routing and recharging decisions. Note that asymptotically, as $N \to \infty$, we can recover routing at the individual vehicle level. Clearly, it is not realistic to consider all vehicles in the system as EVs. In [22] we have addressed the routing problem for vehicle flows including both Electric Vehicles (EVs) and Non-Electric Vehicles (NEVs) for a network with homogeneous charging nodes and shown that a similar framework and analysis as in [1] are applicable. Here, we focus on routing of EVs while the NEV flows are not part of our optimization process. Instead, we treat them as uncontrollable interfering traffic and assume that their flow rates are known. It should be noted that extending this framework to include both EV and NEV flows (as done for the network with homogeneous nodes in [22]) is straightforward.

Our goal is to minimize the total elapsed time (latency) of the EVs traveling from origin to destination by determining optimal routes and energy recharging amounts for each vehicle subflow. The decision variables are as follows: Nm binary variables for selecting links for each subflow denoted by $x_{ij}^k \in \{0, 1\}$ for all links $(i, j) \in \mathcal{A}$ and subflows $k = 1, \ldots, N$, and $N(n-1)$ non-negative real variables for charging amounts at each node for all subflows represented by r_i^k, $i = 1, \ldots, n-1$ and $k = 1, \ldots, N$. Given traffic congestion effects, the time and energy consumption on each arc depends on the values of x_{ij}^k and the fraction of the total flow rate R associated with each subflow k; the simplest such flow allocation (which we will adopt) is one where each subflow is assigned R/N. Let $\mathbf{x_{ij}} = (x_{ij}^1, \cdots, x_{ij}^N)^T$ and $\mathbf{r_i} = (r_i^1, \cdots, r_i^N)^T$. Then, we denote the traveling time (delay) a vehicle will experience through link (i, j) by some nonlinear function $\tau_{ij}(\mathbf{x_{ij}})$. The corresponding energy consumption of the kth vehicle subflow through link (i, j) is a nonlinear function denoted by $e_{ij}^k(\mathbf{x_{ij}})$. As already mentioned, $\tau_{ij}(\mathbf{x_{ij}})$ and $e_{ij}^k(\mathbf{x_{ij}})$ can also incorporate the influence of uncontrollable (NEV) vehicle flows, which can be treated as parameters in these functions as further discussed in Section III-C. Similar to the user-centric case, we define $N(n-1)$ non-negative variables denoted by E_i^k for the residual energy of subflow k at node i, so that E_i^k represents the aggregated residual energy of all vehicles in the subflow. If the subflow does not go through node i, then $E_i^k = 0$. The problem is formulated as the following MINLP with $N(m + 2(n-1))$ variables:

$$\min_{\mathbf{x_{ij}}, \mathbf{r_i}, \; i,j \in \mathcal{N}} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} \left(\tau_{ij}(\mathbf{x_{ij}}) x_{ij}^k \frac{R}{N} + r_i^k g_i x_{ij}^k \right) \tag{22}$$

$$s.t. \text{ for each } k \in \{1, \ldots, N\}:$$

$$\sum_{j \in O(i)} x_{ij}^k - \sum_{j \in I(i)} x_{ji}^k = b_i, \quad \text{for each } i \in \mathcal{N} \tag{23}$$

$$b_1 = 1, \; b_n = -1, \; b_i = 0, \; \text{ for } i \neq 1, n \tag{24}$$

$$E_j^k = \sum_{i \in I(j)} (E_i^k + r_i^k - e_{ij}^k(\mathbf{x_{ij}})) x_{ij}^k, \quad j = 2, \ldots, n \tag{25}$$

$$E_1^k \text{ is given}, \quad E_i^k \geq 0, \quad \text{for each } i \in \mathcal{N} \tag{26}$$

$$x_{ij}^k \in \{0, 1\}, \quad r_i^k \geq 0 \tag{27}$$

We will refer to this problem as **P3**. The difference from the MINLP formulated in [1] is that we consider different charging rates g_i in the objective function. In the sequel, we discuss some properties of the optimal solution allowing us to reduce the complexity of this MINLP problem as we did for the user-centric case.

A. Properties

It can be seen that for each subflow k, the constraints (23)-(27) are similar to those in the user-centric case, though the term $\tau_{ij}(\mathbf{x_{ij}})$ in the objective function is no longer linear in general. Consequently, we can derive similar useful properties in the form of the following lemmas (proofs are very similar to those of the user-centric case and are omitted).

Lemma 3: An optimal solution $\{\mathbf{x_{ij}}, \mathbf{r_i}\}$, $i, j \in \mathcal{N}$ satisfies:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} r_i^k g_i x_{ij}^k - \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} e_{ij}^k(\mathbf{x_{ij}}) g_i x_{ij}^k \tag{28}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} (E_j^k - E_i^k) g_i x_{ij}^k$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} E_j^k (g_i - g_j) x_{ij}^k - \sum_{k=1}^{N} E_1^k g_1 \tag{29}$$

Lemma 4: If $\sum_i r_i^{k*} > 0$ in the optimal routing policy, then $E_n^{k*} = 0$ for $k = 1, \ldots, N$.

Using Lemma 3, we replace $\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^N r_i^k g_i x_{ij}^k$ in (22) through (29) and r_i^k, $i = 1, \ldots, n-1$, $k = 1, \ldots, N$, is eliminated from the objective function (22). The term $\sum_{k=1}^N E_1^k g_1$ is also removed because it has a fixed value. Thus, a new MINLP formulation with $N(m + (n-1))$ variables is obtained to determine x_{ij}^{k*} and E_i^{k*} for all $i, j \in \mathcal{N}$ and $k = 1, \ldots, N$ as follows:

$$\min_{\substack{x_{ij}^k, E_i^k \\ i,j \in \mathcal{N}}} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} \big[\tau_{ij}(\mathbf{x_{ij}}) x_{ij}^k \frac{R}{N} + (e_{ij}^k(\mathbf{x_{ij}}) g_i + E_j^k (g_i - g_j)) x_{ij}^k \big] \tag{30}$$

$$s.t. \text{ for each } k \in \{1, \ldots, N\}:$$

$$\sum_{j \in O(i)} x_{ij}^k - \sum_{j \in I(i)} x_{ji}^k = b_i \tag{31}$$

$$b_1 = 1, \; b_n = -1, \; b_i = 0 \quad \text{for } i \neq 1, n$$

$$0 \leq E_j^k - (E_i^k - e_{ij}^k(\mathbf{x_{ij}})) x_{ij}^k \leq B^k \quad \forall (i, j) \in \mathcal{A} \tag{32}$$

$$E_1^k \text{ is given}, \quad E_i^k \geq 0, \quad \text{for each } i \in N \tag{33}$$

$$x_{ij}^k \in \{0, 1\} \tag{34}$$

We call this problem **P4**. Note that inequality (32) is derived from (25). Assuming $x_{ij}^k = 1$, i.e., arc (i, j) is part of the optimal path for the kth subflow, $r_i^k = e_{ij}^k(\mathbf{x_{ij}}) + E_j^k - E_i^k$. Thus, (32) ensures the optimal solution E_i^{k*} results in a feasible charging amount for the kth subflow, $0 \leq r_i^k \leq B^k$ where B^k is the maximum charging amount kth subflow can get. This value should be predetermined for each subflow based on the vehicle types and the fraction of total inflow in it. Similar to **P2** in the single-vehicle case, once we determine E_i^{k*} we can simply calculate optimal charging amounts using (25). Although **P4** has fewer decision variables than **P3**, its complexity still highly depends on the network size and number of subflows. Similar to the charging policy $\pi_{\mathbf{C}}$ used in Theorem 1, we introduce a charging policy by arguing as follows. Looking at (30), the term $\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^N E_j^k (g_i - g_j) x_{ij}^k$ is minimized by selecting each E_j^k depending on the sign of $(g_i - g_j)$:

Case 1: $g_i < g_j$, i.e., node i has faster charging rate than node j. Therefore, E_j^k should get its maximum value, i.e., the kth subflow should get its maximum charge at node i.

Case 2: $g_i \geqslant g_j$, i.e., the charging rate of node j is greater than or equal to node i. Therefore, E_j^k should get its minimum value of 0. This implies that the kth subflow should get the minimum charge needed at node i in order to reach node j.

Applying this policy in (30) and changing the objective function accordingly we introduce problem **P5** as follows:

$$\min_{x_{ij}^k} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{N} \big[\tau_{ij}(\mathbf{x_{ij}}) x_{ij}^k \frac{R}{N} + + (e_{ij}^k(\mathbf{x_{ij}}) g_i + K (g_i - g_j)) x_{ij}^k \big] \tag{35}$$

$$K = \begin{cases} B^k - e_{ij}^k(\mathbf{x_{ij}}) & \text{if } g_i < g_j, \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

$$s.t. \text{ for each } k \in \{1, \ldots, N\}:$$

$$\sum_{j\in O(i)} x_{ij}^k - \sum_{j\in I(i)} x_{ji}^k = b_i \tag{37}$$
$$b_1 = 1, b_n = -1, b_i = 0 \quad \text{for } i \neq 1, n$$
$$x_{ij}^k \in \{0, 1\} \tag{38}$$

Unlike the user-centric case, the objective function is no longer necessarily linear in x_{ij}^k, therefore, (35) cannot be further simplified into an LP problem as in Theorem 1. Nonetheless, we are still able to decompose the original problem into two smaller problems: a MINLP to determine routing variables x_{ij}^k and a NLP to find recharging amounts r_i^k over the optimal routes. Similar to the single-vehicle case, once the optimal routes for all subflows, P^k, $k = \{1, \ldots N\}$, are determined, we can obtain r_i^k by formulating a corresponding NLP which minimizes $\sum_{k=1}^N \sum_{i\in P^k} r_i^k g_i$ while satisfying the energy constraints (25)-(26). The computational effort required to solve this problem with Nm decision variables, depends on the dimensionality of the network and the number of subflows.

Next, we present an alternative formulation of (22)-(27) leading to a computationally simpler solution approach.

Remark 2: If $g_i = g_j$ for all i, j in (35), the problem reduces to the homogeneous charging node case with the exact same MINLP formulation as in [1] for obtaining an optimal path. However, **P5** cannot guarantee an optimal solution because of the locally optimal charging policy $\pi_{\mathbf{C}}$ which may not be feasible in a globally optimal solution (x_{ij}^{k*}, E_i^{k*}).

B. Flow Control Formulation

We begin by relaxing the binary variables in (27) by letting $0 \le x_{ij}^k \le 1$. Thus, we switch our attention from determining a single path for any subflow k to several possible paths by treating x_{ij}^k as the normalized vehicle flow on arc (i, j) for the kth subflow. This is in line with many network routing algorithms in which fractions x_{ij} of entities are routed from a node i to a neighboring node j using appropriate schemes ensuring that, in the long term, the fraction of entities routed on (i, j) is indeed x_{ij}. Following this relaxation, the objective function in (22) is changed to:

$$\min_{\mathbf{x_{ij}},\mathbf{r_i},\ i,j\in\mathcal{N}} \sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^N \tau_{ij}(\mathbf{x_{ij}})x_{ij}^k\frac{R}{N} + \sum_{i=1}^n\sum_{k=1}^N r_i^k g_i$$

Moreover, the energy constraint (25) needs to be adjusted accordingly. Let E_{ij}^k represent the fraction of residual energy of subflow k associated with the x_{ij}^k portion of the vehicle flow exiting node i. Therefore, the constraint (26) becomes $E_{ij}^k \ge 0$. We can now capture the relationship between the energy associated with subflow k and the vehicle flow as follows:

$$\left[\sum_{h\in I(i)} (E_{hi}^k - e_{hi}^k(\mathbf{x_{ij}})) + r_i^k\right]\cdot\frac{x_{ij}^k}{\sum_{h\in I(i)} x_{hi}^k} = E_{ij}^k \tag{39}$$
$$\frac{E_{ij}^k}{\sum_{j\in O(i)} E_{ij}^k} = \frac{x_{ij}^k}{\sum_{j\in O(i)} x_{ij}^k} \tag{40}$$

In (39), the energy values of different vehicle flows entering node i are aggregated and the energy corresponding to each portion exiting this node, E_{ij}^k, $j \in O(i)$, is proportional to the corresponding fraction of vehicle flows, as expressed in (40). Clearly, this aggregation of energy leads to an approximation, since one specific vehicle flow may need to be recharged in order to reach the next node in its path, whereas another might have enough energy without being recharged. This approximation foregoes controlling recharging amounts at the individual vehicle level and leads to approximate solutions of the original problem (22)-(27). Several numerically based comparisons are provided in the next section showing little or no loss of optimality relative to the solution of (22). Adopting this formulation with $x_{ij}^k \in [0, 1]$ instead of $x_{ij}^k \in \{0, 1\}$, we obtain the following simpler nonlinear programming problem (NLP):

$$\min_{\mathbf{x_{ij}},\mathbf{r_i},\ i,j\in\mathcal{N}} \sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^N \tau_{ij}(\mathbf{x_{ij}})x_{ij}^k\frac{R}{N} + \sum_{i=1}^n\sum_{k=1}^N r_i^k g_i \tag{41}$$

$s.t.$ for each $k \in \{1, \ldots, N\}$:

$$\sum_{j\in O(i)} x_{ij}^k - \sum_{j\in I(i)} x_{ji}^k = b_i, \quad \text{for each } i \in \mathcal{N} \tag{42}$$
$$b_1 = 1,\ b_n = -1,\ b_i = 0, \text{ for } i \neq 1, n$$
$$\left[\sum_{h\in I(i)} (E_{hi}^k - e_{hi}^k(\mathbf{x_{ij}})) + r_i^k\right]\cdot\frac{x_{ij}^k}{\sum_{h\in I(i)} x_{hi}^k} = E_{ij}^k \tag{43}$$
$$\frac{E_{ij}^k}{\sum_{j\in O(i)} E_{ij}^k} = \frac{x_{ij}^k}{\sum_{j\in O(i)} x_{ij}^k} \tag{44}$$
$$E_{ij}^k \ge 0, \tag{45}$$
$$0 \le x_{ij}^k \le 1, \quad r_i^k \ge 0 \tag{46}$$

As in our previous analysis, we are able to eliminate $\mathbf{r_i}$ from the objective function in (41) as follows.

Lemma 5: For each subflow $k = 1, \ldots, N$,

$$\begin{aligned}\sum_{i=1}^n r_i^k g_i &= \sum_{i=1}^n\sum_{j=1}^n e_{ij}^k(\mathbf{x_{ij}})g_i + \sum_{i=1}^n\sum_{j\in O(i)} E_{ij}^k g_i - \sum_{i=1}^n\sum_{h\in I(i)} E_{hi}^k g_i \\ &= \sum_{i=1}^n\sum_{j=1}^n e_{ij}^k(\mathbf{x_{ij}})g_i + \sum_{i=1}^n\sum_{j\in O(i)} E_{ij}^k(g_i - g_j)\end{aligned}$$

Proof: Multiplying (43) by g_i and summing over all $i = 1, \ldots, n$, then using (42) and (44) proves the lemma. ∎

Using Lemma 5 we change the objective function (41) to:

$$\sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^N \left(\tau_{ij}(\mathbf{x_{ij}})x_{ij}^k\frac{R}{N} + e_{ij}^k(\mathbf{x_{ij}})g_i\right) + \sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^N E_{ij}^k(g_i - g_j) \tag{47}$$

Once again, we adopt a charging policy $\pi_{\mathbf{C}}$ as follows:

Case 1: If $g_i < g_j$, then E_{ij}^k gets its maximum value $(B^k - e_{ij}^k(\mathbf{x_{ij}}))x_{ij}^k$.

Case 2: If $g_i \ge g_j$, then E_{ij}^k gets its minimum value 0.

Applying this policy in (47) we can transform the objective function (41) to (48) and determine near-optimal routes x_{ij}^{k*}

by solving the following NLP:

$$\min_{\substack{\mathbf{x_{ij}}\\ i,j\in\mathcal{N}}} \sum_{k=1}^{N}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\tau_{ij}(\mathbf{x_{ij}})x_{ij}^{k}\frac{R}{N}+e_{ij}^{k}(\mathbf{x_{ij}})g_i+K(g_i-g_j)\right]$$

$$K=\begin{cases}(B^k-e_{ij}^{k}(\mathbf{x_{ij}}))x_{ij}^{k} & \text{if } g_i<g_j,\\ 0 & \text{otherwise}\end{cases}$$

$$s.t. \text{ for each } k\in\{1,\dots,N\}:$$

$$\sum_{j\in O(i)}x_{ij}^{k}-\sum_{j\in I(i)}x_{ji}^{k}=b_i,\quad \text{for each } i\in\mathcal{N}$$

$$b_1=1,\ b_n=-1,\ b_i=0,\ \text{for } i\neq 1,n$$

$$0\le x_{ij}^{k}\le 1 \tag{48}$$

Once again, there is no guarantee of global optimality due to applying charging policy $\pi_{\mathbf{C}}$. The values of r_i^k, $i=1,\dots,n$, $k=1,\dots,N$, can then be determined so as to satisfy the energy constraints (43)-(45), and minimizing $\sum_{k=1}^{N}\sum_{i\in P^k}^{n}r_i^k g_i$. Note that in the above formulation, the nonlinearity appears in the objective function due to the traffic congestion effect on traveling time and energy consumption. Thus, if $\tau_{ij}(\mathbf{x_{ij}})x_{ij}^k$ and $e_{ij}^k(\mathbf{x_{ij}})$ are convex functions, the NLP is a convex optimization problem and its solution can be found generally fast (Note that this is not in general the global optimum of the main problem). Finally, if $g_i=g_j$ for all i,j in (48), the problem reduces to the homogeneous charging node case with the same exact NLP flow control formulation as in [1].

C. Objective Function Selection

We now explain how to estimate the delay function, $\tau_{ij}(\mathbf{x_{ij}})$, in either (22) or (48) using the same actual traffic dataset from the Eastern Massachusetts transportation network as in Section II-C and based on the analysis given in [19]. We assume that the delay functions have the following form [23]:

$$\tau_{ij}\left(f_{ij}\right)=t_{ij}^{0}h\left(\frac{f_{ij}}{C_{ij}}\right), \tag{49}$$

where t_{ij}^0 is the free-flow travel time of link $(i,j)\in\mathcal{A}$, $h(\cdot)$ is strictly increasing and continuously differentiable on $\mathbb{R}_+$, and f_{ij} and C_{ij} denote the flow and the effective capacity of link $(i,j)\in\mathcal{A}$ respectively. The goal is to estimate $h(\cdot)$ functions based on actual traffic data.

As already mentioned, the dataset at our disposal, provided by the Boston Region Metropolitan Planning Organization (MPO), includes spatial average speeds and the flow capacity for each road segment of major roadways and arterial streets of Eastern Massachusetts (see Fig. 1). We assume that the observed traffic data correspond to user (Wardrop) equilibria. Applying Greenshield's traffic flow model [20], we first convert the spatial average speed data into equilibrium flows for each road segment. Then, by adopting the estimated traffic flows we obtain Origin-Destination (O-D) demand matrices. Finally, we formulate appropriate inverse problems [23] to recover the per-road cost (congestion) functions determining user route selection for each month and time-of-day period

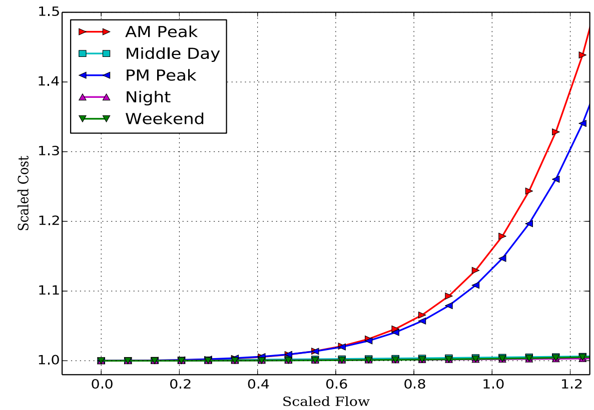


Fig. 4. Comparison of the estimated cost functions corresponding to different time periods.

(details are provided in [19]). Applying polynomial kernels in the corresponding Quadratic Programming (QP) problem [19], we estimate cost functions $h(\cdot)$ as polynomial functions. We estimate the cost functions for different scenarios: AM (7 am – 9 am), MD (11 am – 1 pm), PM (5 pm – 7 pm), and NT (9pm – 11 pm) for each day of January, April, July, and October, all in 2012.

The estimated $h(\cdot)$ functions corresponding to five different time periods for month April are shown in Fig. 4. We observe that the costs for the AM/PM peaks are much more sensitive to traffic flows than for the other three time periods (MD, NT, and weekend). This can be explained by taking into account the traffic condition during a day: from a congested road network in the AM/PM period to an uncongested road network during MD, NT, or weekend periods.

For the rest of the paper, we consider the estimated equilibrium flow on each link as the uncontrolled NEV flow. Then, our goal is to determine system-optimal routes and charging policies for the EV flow entering the network. Let us assume that EVs enter the network at a rate of R veh./hr. We then evenly divide the EV inflow into N subflows and the total flow entering link (i,j) becomes:

$$f_{ij}=\sum_{k}x_{ij}^{k}\frac{R}{N}+f_{ij}^{eq} \tag{50}$$

where the first term represents the assignment of EV subflows to link (i,j) and the second term is the equilibrium flow for NEVs inferred from the average speed data. Therefore, the time a vehicle spends on link (i,j) becomes

$$\tau_{ij}(\mathbf{x_{ij}})=t_{ij}^{0}h\left(\frac{\sum_k(x_{ij}^{k}\frac{R}{N})+f_{ij}^{eq}}{C_{ij}}\right) \tag{51}$$

As for $e_{ij}^k(\mathbf{x_{ij}})$, we assume that the energy consumption rates of subflows on link (i,j) are all identical, proportional to the distance between nodes i and j, giving

$$e_{ij}^{k}(\mathbf{x_{ij}})=\alpha d_{ij}\frac{R}{N}$$

TABLE V
UNCONTROLLED NEV FLOW ON EACH LINK DURING AM PERIOD [NO. veh/hr]

f_{12}^{eq}	f_{21}^{eq}	f_{13}^{eq}	f_{31}^{eq}	f_{23}^{eq}	f_{32}^{eq}	f_{24}^{eq}	f_{42}^{eq}	f_{35}^{eq}	f_{53}^{eq}	f_{36}^{eq}	f_{63}^{eq}
1287	1271	1725	1740	1713	3757	2231	172	3352	4870	3678	4218
f_{45}^{eq}	f_{54}^{eq}	f_{56}^{eq}	f_{65}^{eq}	f_{48}^{eq}	f_{84}^{eq}	f_{57}^{eq}	f_{75}^{eq}	f_{67}^{eq}	f_{76}^{eq}	f_{78}^{eq}	f_{87}^{eq}
4171	875	2464	2039	863	2099	4154	2802	3661	3776	1796	560

Therefore, the objective function in problems (35)-(38) and (48) in this case becomes

$$\min_{\substack{x_{ij}^k \\ i,j\in\mathcal{N}}} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{N}\Big[t_{ij}^0 h\left(\frac{\sum_k (x_{ij}^k \frac{R}{N}) + f_{ij}^{eq}}{C_{ij}}\right) x_{ij}^k \frac{R}{N} + \alpha d_{ij} g_i x_{ij}^k \frac{R}{N} + K(g_i - g_j)\Big] \quad (52)$$

D. Numerical Examples for the Eastern Massachusetts Transportation Network

We consider the same sub-network shown in Fig. 2. Our goal is to determine system-optimal routes and charging policies for the flow of EVs traveling from node 1 to node 8 while the effect of NEV flows on the traffic congestion should be included in the cost function. As discussed in Section III-C, we use real traffic data to calculate the uncontrolled NEV flow on each link. To do so, we use the average speed data on each road segment and infer the average flow data on that using the Greenshield's traffic flow model. Finally, the calculated flow for all segments composing a link are aggregated in order to calculate the uncontrolled NEV flow on each link [19]. Tab. V shows the calculated average flow on each link of the sub-network on April 3 during AM period and we consider them as the user equilibrium flow, f_{ij}^{eq}, in (50). We then use the data-derived estimated cost function in our formulations. As stated earlier, the cost function is in polynomial form since we apply polynomial kernels in the corresponding QP problem. For the April-Workday-AM period, the estimated $h(\cdot)$ function in (51) has the following form (red curve in Fig. 4):

$$h(x) = 0.11x^8 - 0.4705x^7 + 0.946x^6 - 0.9076x^5 + 0.6238x^4 - 0.1973x^3 + 0.057x^2 - 0.0032x + 1 \quad (53)$$

Applying this function in (51), we obtain the delay function for each link (i, j), τ_{ij}, based on actual traffic data. For the energy consumption function we set $\alpha = 0.3$ and distances between nodes are as shown in Tab. III. We assume the network has inhomogeneous charging nodes with a level 2 charging station at node 3 (charging rate of $g^2 = 1/6$ [hr/kWh]) and level 1 charging stations (charging rate of $g^1 = 41.67/60$ [hr/kWh]), for the rest, i.e., $G = [g^1\ g^1\ g^2\ g^1\ g^1\ g^1\ g^1]$. In our approach, we need to identify N subflows and we do so by evenly dividing the entire vehicle inflow into N subflows, each of which has R/N vehicles per unit time. In order to verify the accuracy of different formulations, we numerically solve the optimal and near-optimal problems **P3** and **P5**. Let us set $R = 1492$ [Veh./hr] as the flow of EVs traveling from node 1 to node 8. Tab. VI shows both optimal routes and suboptimal routes obtained by solving **P3** and **P5** respectively

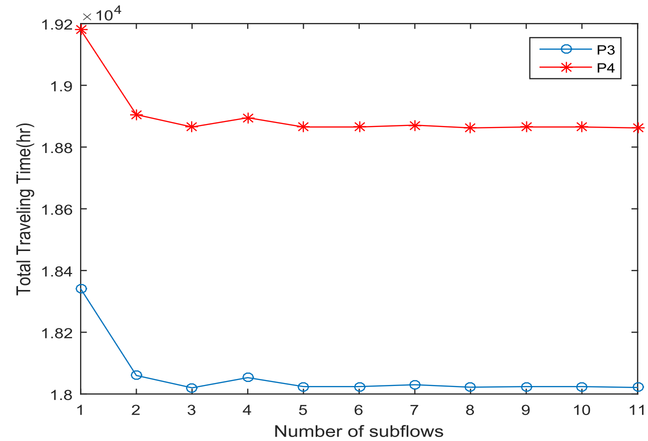


Fig. 5. Performance as a function of N (No. of subflows).

for different values of $N \in [1, \ldots, 11]$ and

$$G = [\frac{41.67}{60}\ \frac{41.67}{60}\ \frac{1}{6}\ \frac{41.67}{60}\ \frac{41.67}{60}\ \frac{41.67}{60}\ \frac{41.67}{60}].$$

We observe that vehicles are mainly distributed through two routes and the traffic congestion effect makes the flow distribution differ from the shortest path, $1 \to 3 \to 5 \to 7 \to 8$. The number of decision variables (hence, the solution search space) rapidly increases with the number of subflows. However, looking at Fig. 5 which gives the performance in terms of our objective functions in (22) and (35) as a function of the number of subflows, observe that the optimal objective value (**P3**) quickly converges around $N = 3$. Thus, even though the best solution is found when $N = 11$, a near-optimal solution can be determined under a small number of subflows. This suggests that one can rapidly approximate the asymptotic solution of the multiple-vehicle problem (dealing with individual vehicles routed so as to optimize a system-wide objective) based on a relatively small value of N.

Another observation is that although **P5** is a suboptimal formulation it results in the same paths as those obtained by solving **P3**. Next, we obtain a solution to the same problem using the NLP formulation (48) with $0 \le x_{ij}^k \le 1$. Since in this example all subflows are identical, we can further combine all x_{ij}^k over each (i, j), leading to the N-subflow relaxed problem:

$$\begin{aligned}
\min_{\substack{x_{ij} \\ i,j\in\mathcal{N}}} \ & \sum_{i=1}^{n}\sum_{j=1}^{n}\Big[t_{ij}^0 h\left(\frac{x_{ij}R + f_{ij}^{eq}}{C_{ij}}\right) x_{ij}R + \\
& \alpha d_{ij} g_i x_{ij} R + K(g_i - g_j)\Big] \\
& K = \begin{cases} (B - e d_{ij} R) x_{ij} & \text{if } g_i < g_j, \\ 0 & \text{otherwise} \end{cases} \\
s.t. \ & \sum_{j\in O(i)} x_{ij} - \sum_{j\in I(i)} x_{ji} = b_i, \quad \text{for each } i \in \mathcal{N} \\
& b_1 = 1,\ b_n = -1,\ b_i = 0,\ \text{for } i \ne 1, n \\
& 0 \le x_{ij} \le 1
\end{aligned} \quad (54)$$

TABLE VI
NUMERICAL RESULTS FOR SAMPLE PROBLEM

	P3	**P5**
N	1	1
obj	1.8341e4	1.9182e4
routes	$1\to3\to5\to7\to8$	$1\to3\to5\to7\to8$
N	2	2
obj	1.806e4	1.8905e4
routes	$1\to3\to5\to7\to8$ $1\to3\to6\to7\to8$	$1\to3\to5\to7\to8$ $1\to3\to6\to7\to8$
N	3	3
obj	1.8024e4	1.8865e4
routes	$1\to3\to5\to7\to8(\times2)$ $1\to3\to6\to7\to8$	$1\to3\to5\to7\to8(\times2)$ $1\to3\to6\to7\to8$
N	4	4
obj	1.8053e4	1.8895e4
routes	$1\to3\to5\to7\to8(\times3)$ $1\to3\to6\to7\to8$	$1\to3\to5\to7\to8(\times3)$ $1\to3\to6\to7\to8$
N	5	5
obj	1.8024e4	1.8865e4
routes	$1\to3\to5\to7\to8(\times3)$ $1\to3\to6\to7\to8(\times2)$	$1\to3\to5\to7\to8(\times3)$ $1\to3\to6\to7\to8(\times2)$
N	6	6
obj	1.8024e4	1.8865e4
routes	$1\to3\to5\to7\to8(\times4)$ $1\to3\to6\to7\to8(\times2)$	$1\to3\to5\to7\to8(\times4)$ $1\to3\to6\to7\to8(\times2)$
N	7	7
obj	1.803e4	1.8871e4
routes	$1\to3\to5\to7\to8(\times4)$ $1\to3\to6\to7\to8(\times3)$	$1\to3\to5\to7\to8(\times4)$ $1\to3\to6\to7\to8(\times3)$
N	8	8
obj	1.8022e4	1.8862e4
routes	$1\to3\to5\to7\to8(\times5)$ $1\to3\to6\to7\to8(\times3)$	$1\to3\to5\to7\to8(\times5)$ $1\to3\to6\to7\to8(\times3)$
N	9	9
obj	1.8024e4e4	1.8865e4
routes	$1\to3\to5\to7\to8(\times6)$ $1\to3\to6\to7\to8(\times3)$	$1\to3\to5\to7\to8(\times6)$ $1\to3\to6\to7\to8(\times3)$
N	10	10
obj	1.8024e4	1.8865e4
routes	$1\to3\to5\to7\to8(\times6)$ $1\to3\to6\to7\to8(\times4)$	$1\to3\to5\to7\to8(\times6)$ $1\to3\to6\to7\to8(\times4)$
N	11	11
obj	1.8021e4	1.8862e4
routes	$1\to3\to5\to7\to8(\times7)$ $1\to3\to6\to7\to8(\times4)$	$1\to3\to5\to7\to8(\times7)$ $1\to3\to6\to7\to8(\times4)$

This is a relatively easy to solve NLP problem. It is obvious from Fig. 4 that the $h(\cdot)$ function during AM period is a strictly convex function, thus the solution of this NLP is a unique global optimum. Using the same parameter settings as before, we obtain the objective value of $1.8862e45$ hrs and the optimal routes are:

$$63.24\% \text{ of vehicle flow: } (1\to3\to5\to7\to8)$$
$$36.76\% \text{ of vehicle flow: } (1\to3\to6\to7\to8)$$

Compared to the best solution ($N = 11$) in Fig. 5, the difference in objective values between the integer and flow-based solutions is less than 4.7%. This supports the effectiveness of a solution based on a limited number of subflows in the MINLP problem.

1) Effect of EV Inflow on Optimal Routes: Tab. VII shows the optimal paths obtained by solving both problem **P3** and the NLP relaxed problem (54) for different values of EV inflow, R.

TABLE VII
EFFECT OF FLOW RATE, R, ON OPTIMAL ROUTES

R=300 [Veh./hr]		
P3	$N = 1$	$N = 2$
routes	$1\to3\to5\to7\to8$	$1\to3\to5\to7\to8(\times2)$
NLP $\Rightarrow$ 100% of EV flow:$(1\to3\to5\to7\to8)$		
R=1492 [Veh./hr]		
P3	$N = 1$	$N = 11$
routes	$1\to3\to5\to7\to8$	$1\to3\to5\to7\to8(\times7)$ $1\to3\to6\to7\to8(\times4)$
NLP $\Rightarrow$ 63.24% of EV flow:$(1\to3\to5\to7\to8)$ 36.76% of EV flow:$(1\to3\to6\to7\to8)$		
R=2984 [Veh./hr]		
P3	$N = 1$	$N = 7$
routes	$1\to3\to5\to7\to8$	$1\to3\to5\to7\to8(\times3)$ $1\to3\to6\to7\to8(\times3)$ $1\to2\to3\to5\to7\to8$
42.21% of EV flow:$(1\to3\to5\to7\to8)$ **NLP** $\Rightarrow$ 41.32% of EV flow:$(1\to3\to6\to7\to8)$ 16.47% of EV flow:$(1\to2\to3\to5\to7\to8)$		

TABLE VIII
CPU TIME FOR SAMPLE PROBLEM

Fig.2	**P3**	**P5**	NLP approx.
N	8(near opt)	8(near opt)	-
CPU time(sec)	48179	10267	54

It is observed that the optimal routes will change with the inflow rate. For lower flows, e.g. $R = 300$, it is optimal that all EVs travel through the shortest path which means the corresponding change in $\frac{f_{ij}}{C_{ij}}$ in (49) is negligible. However, higher flows may cause congestion in some links, i.e., larger values of $\frac{f_{ij}}{C_{ij}}$ in (49), resulting in larger delays on those links τ_{ij}. Consequently,some EVs should deviate from the shortest path in the optimal routing.

2) CPU Time Comparison: Tab. VIII compares the computational effort in terms of CPU time for problems **P3**, **P5** and the flow control formulation to find optimal routes for the sample network shown in Fig. 2. Our results show that the flow control formulation results in a reduction of about 3 orders of magnitude in CPU time with almost the same solution as the optimal solution.

3) Selection of the Number of Subflows: Since the problem size increases with the number of subflows, N, a proper selection of this number is essential to render the problem computationally manageable and reflects a trade-off between proximity to optimality and computational effort needed to solve the problem. Our numerical results have shown that a small number of subflows are adequate to obtain convergence to near-optimal solutions. In [1] we have proposed a criterion and procedure for appropriate choice of the number of subflows for the network with homogeneous charging stations. In brief, the key idea is based on the fact that the decomposed MINLP problem (35)-(38) obtains the optimal solution for the homogeneous network ($g_i = g_j\ \forall i, j$), thus the corresponding relaxed NLP, i.e., problem (54) with $g_i = g_j$, gives a lower bound for the optimal objective value. We then defined a critical number of subflows, N^*, which guarantees near optimality and showed that by selecting N so that $N > N^*$, the average deviation between NLP solution and MINLP solution with N subflows never exceeds a predefined upper bound (for details refer to [1]).

For the network with inhomogeneous charging stations one should note that adopting a locally optimal charging policy, the decomposed MINLP (35)-(38) is suboptimal in general. Therefore, the corresponding relaxed NLP does not give a lower bound for the optimal objective value, though it does for the decomposed suboptimal MINLP. Nevertheless, since the routes obtained by solving the decomposed MINLP are near-optimal and the relaxed NLP gives a lower bound for its objective value, we may still use the same procedure as in [1] for selecting a "good" N. In our numerical results, it is observed that both **P3** and **P5** result in the same solutions for different values of N. Furthermore, for the for the value N with the lowest objective value, $N = 11$, the normalized flow on each path, that is $4/11 = 36.36\%$ for $1 \rightarrow 3 \rightarrow 6 \rightarrow 7 \rightarrow 8$ and $7/11 = 63.64\%$ for $1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 8$, has the least deviation from the solution of the NLP problem which is the main idea in the selection of N.

IV. Conclusion

We have studied the routing problem for energy-aware vehicles (typically, EVs) in road networks with inhomogeneous charging stations. We have considered two versions of the problem: user-centric (referred as single vehicle routing problem) vs. system-centric (referred as multiple-vehicle problem). For the former, we have shown how to reduce the complexity of this problem. For the latter, where traffic congestion effects are considered, we used a similar approach by aggregating vehicles into subflows and seeking optimal routing decisions for each such subflow. We also developed an alternative flow-based formulation which yields approximate solutions with a computational cost reduction of several orders of magnitude, so it can be used in problems of large dimensionality. We then applied real traffic data from the Eastern Massachusetts transportation network and investigated the user-optimal vs social-optimal routing policies for different scenarios.

So far, we have assumed all the charging stations have unlimited capacities and a vehicle begins the charging process as soon as it gets to a station. Our ongoing work focuses on considering queueing capacities for charging stations. For the system-centric routing problem, we are exploring extensions to stochastic vehicle flows where the objective is to minimize average vehicle travel times or to periodically re-solve the routing problem based on new traffic flow data. Moreover, in the system-centric case, a challenging problem is the implementation of an optimal charging and routing policy, i.e., how does an individual driver receive the explicit guidance from a central controller? Also, does the driver follow this policy? The first question may be addressed through Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication capabilities which are increasingly being made available to vehicles. The second question is more challenging since it deals with individual drivers' behavior who act "selfishly" in general. The emerging trend towards CAVs is likely to facilitate a centrally derived system-centric optimal routing policy which could be implemented through CAVs, a research topic of growing interest.

References

[1] S. Pourazarm, C. G. Cassandras, and T. Wang, "Optimal routing and charging of energy-limited vehicles in traffic networks," *Int. J. Robust Nonlinear Control*, vol. 26, no. 6, pp. 1325–1350, 2016.

[2] *Technology Roadmap: Electric and Plug-in Hybrid Electric Vehicles*, Int. Energy Agency, Paris, France, 2011.

[3] G. Laporte, "The vehicle routing problem: An overview of exact and approximate algorithms," *Eur. J. Oper. Res.*, vol. 59, no. 3, pp. 345–358, 1992.

[4] A. Artmeier, J. Haselmayr, M. Leucker, and M. Sachenbacher, "The optimal routing problem in the context of battery-powered electric vehicles," in *Proc. 2nd Int. Workshop Constraint Reason. Optim. Comput. Sustain. (CROCS CPAIOR)*, Bologna, Italy, May 2010, pp. 1–13.

[5] J. Eisner, S. Funke, and S. Storandt, "Optimal route planning for electric vehicles in large networks," in *Proc. 25th AAAI Conf. Artif. Intell.*, San Francisco, CA, USA, Aug. 2011, pp. 1108–1113.

[6] M. Sachenbacher, M. Leucker, A. Artmeier, and J. Haselmayr, "Efficient energy-optimal routing for electric vehicles," in *Proc. 25th AAAI Conf. Artif. Intell.*, 2011, pp. 1402–1407.

[7] U. F. Siddiqi, Y. Shiraishi, and S. M. Sait, "Multi-constrained route optimization for electric vehicles (EVs) using particle swarm optimization (PSO)," in *Proc. 11th Int. Conf. Intell. Syst. Design Appl. (ISDA)*, Cordoba, Spain, Nov. 2011, pp. 391–396.

[8] S. Khuller, A. Malekian, and J. Mestre, "To fill or not to fill: The gas station problem," *ACM Trans. Algorithms*, vol. 7, no. 3, Jul. 2011, Art. no. 36.

[9] T. M. Sweda and D. Klabjan, "Finding minimum-cost paths for electric vehicles," in *Proc. IEEE Int. Elect. Vehicle Conf. (IEVC)*, Mar. 2012, pp. 1–4.

[10] M. Schneider, A. Stenger, and D. Goeke, "The electric vehicle-routing problem with time windows and recharging stations," *Transp. Sci.*, vol. 48, no. 4, pp. 500–520, 2014.

[11] O. Worley, D. Klabjan, and T. M. Sweda, "Simultaneous vehicle routing and charging station siting for commercial electric vehicles," in *Proc. IEEE Int. Elect. Vehicle Conf. (IEVC)*, Mar. 2012, pp. 1–3.

[12] N. Touati-Moungla and V. Jost, "Combinatorial optimization for electric vehicles management," in *Proc. Int. Conf. Renew. Energies Power Quality*, Apr. 2010, pp. 942–947.

[13] F. He, Y. Yin, and S. Lawphongpanich, "Network equilibrium models with battery electric vehicles," *Transp. Res. B, Methodol.*, vol. 67, pp. 306–319, Sep. 2014.

[14] G. Joos, M. de Freige, and M. Dubois, "Design and simulation of a fast charging station for PHEV/EV batteries," in *Proc. IEEE Elect. Power Energy Conf. (EPEC)*, Aug. 2010, pp. 1–5.

[15] S. Bai, Y. Du, and S. Lukic, "Optimum design of an EV/PHEV charging station with DC bus and storage system," in *Proc. IEEE Energy Convers. Congr. Expo. (ECCE)*, Sep. 2010, pp. 1178–1184.

[16] *SAE Electric Vehicle and Plug in Hybrid Electric Vehicle Conductive Charge Coupler*, SAE Standard J1772, 2012.

[17] S. Pourazarm, C. G. Cassandras, and A. Malikopoulos, "Optimal routing of electric vehicles in networks with charging nodes: A dynamic programming approach," in *Proc. IEEE Int. Elect. Vehicle Conf. (IEVC)*, Dec. 2014, pp. 1–7.

[18] F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*, 8th ed. New York, NY, USA: McGraw-Hill, 2005.

[19] J. Zhang, S. Pourazarm, C. G. Cassandras, and I. C. Paschalidis, "The price of anarchy in transportation networks by estimating user cost functions from actual traffic data," in *Proc. 55th IEEE Conf. Decision Control*, Dec. 2016, pp. 789–794.

[20] B. D. Greenshields, J. R. Bibbins, W. S. Channing, and H. H. Miller, "A study of traffic capacity," in *Proc. Highway Res. Board*, vol. 14. 1935, pp. 448–477.

[21] *Fuel Economy Guide*, Office Energy Efficiency Renew. Energy, U.S. Dept. Energy, Washington, DC, USA, 2016.

[22] S. Pourazarm and C. G. Cassandras, "System-centric minimum-time paths for battery-powered vehicles in networks with charging nodes," in *Proc. 5th IFAC Conf. Anal. Design Hybrid Syst.*, Oct. 2015, pp. 111–116.

[23] D. Bertsimas, V. Gupta, and I. C. Paschalidis, "Data-driven estimation in equilibrium using inverse optimization," *Math. Programm.*, vol. 153, no. 2, pp. 595–633, 2014.

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