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The Penetration Rate Effect of Connected and Automated Vehicles in Mixed Traffic Routing

Arian Houshmand[1], Salomón Wollenstein-Betech[1], and Christos G. Cassandras[1]

***Abstract*— We study the problem of routing Connected and Automated Vehicles (CAVs) in the presence of mixed traffic (coexistence of regular vehicles and CAVs). In this setting, we assume that all CAVs belong to the same fleet, and can be routed using a centralized controller. The routing objective is to minimize a given overall fleet traveling cost (travel time or energy consumption). We assume that regular vehicles (non-CAVs) choose their routing decisions selfishly to minimize their traveling time. We propose an algorithm that deals with the routing interaction between CAVs and regular uncontrolled vehicles. We investigate the effect of assigning system-centric routes under different penetration rates (fractions) of CAVs. To validate our method, we apply the proposed routing algorithms to the Braess Network and to a sub-network of the Eastern Massachusetts (EMA) transportation network using actual traffic data provided by the Boston Region Metropolitan Planning Organization. The results suggest that collaborative routing decisions of CAVs improve not only the cost of CAVs, but also that of the non-CAVs. Furthermore, even a small CAV penetration rate can ease congestion for the entire network.**

I. INTRODUCTION

Every year Americans face more than 6.9 billion hours of delay in traffic which costs the US more than 160 billion dollars in urban congestion costs [1]. In addition, due to heavy traffic congestion, an annual amount of 3.1 billion gallons of fuel is being wasted in traffic [1].

The advent of Connected and Automated Vehicles (CAVs) has been facilitated by the emergence of vehicle automation technologies, as well as new forms of telecommunication technologies, such as Dedicated Short-Range Communication (DSRC) [2]. The latter has enabled Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication capabilities. Therefore, CAVs can help reduce traffic congestion and environmental impacts of our daily commute, as well as improve safety through collaborative decisions.

Many studies have been performed to investigate how CAVs can transform the future of cities [3]. For instance, we may be able to eliminate traffic lights and create unsignalized intersections to reduce congestion and energy consumption [4]. Another interesting area of focus is cooperative adaptive cruise control (CACC) [5], [6].

In this paper, we seek to find how optimizing routing decisions of CAVs affects the overall energy consumption costs and travel times of all vehicles. We investigate the interaction between CAVs and regular vehicles and their effects on total travel time and energy consumption in traffic networks. We assume that all CAVs belong to the same fleet, and the fleet operator is trying to minimize their costs (energy or time) by systematically routing the fleet given their origin-destination (O-D) demand.

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[1]The authors are with the Division of Systems Engineering, Boston University, Brookline, MA 02446 USA arianh@bu.edu; salomonw@bu.edu; cgc@bu.edu

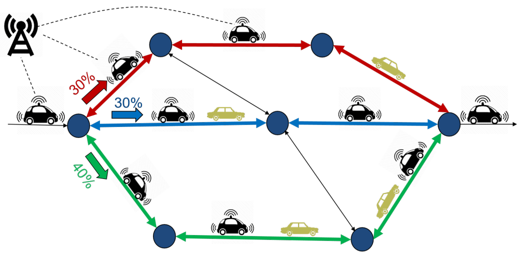


Fig. 1: The centralized controller is assigning routes to CAVs entering the network. Red, blue, and green links show three different routes for CAVs.

Similar to this work, Mehr et al. [7] studied how the presence of CAVs can affect mobility in traffic networks. They assumed CAVs can benefit from CACC by creating shorter headway which increases the road capacity. In this context, they adopted the mixed traffic road capacity model from [8]. Their results show that if all vehicles (CAVs and non-CAVs) make selfish routing decisions, the presence of CAVs might worsen traffic conditions. In contrast to [7], we adopt the viewpoint that there is a centralized controller capable of routing all CAVs given their origin and destination. Moreover, we do not assume the shorter headway for CAVs which was considered in [7]. We show that optimal routing of CAVs under these assumptions can not only benefit CAVs, but also help non-CAVs to save time and energy.

The contributions of this paper are summarized as follows. We first review a system-centric (socially optimal) routing algorithm that minimizes the total travel time assuming 100% CAVs in the system. We then propose algorithms which assign system-centric time-optimal or energy-optimal routes (eco-routes) to CAVs in the presence of mixed traffic (both CAVs and non-CAVs in the system). Additionally, using the notion of *Wardrop equilibrium* [9], we model the user-centric (selfish) routing decisions of non-CAVs.

Our results indicate that optimal routing of CAVs can benefit both CAVs and non-CAVs in energy savings and travel times. In addition, we study the performance of the routing algorithms for various CAV penetration rates. We provide evidence that even under small CAV penetration rates, CAVs and non-CAVs benefit.

The remainder of this paper is organized as follows. In Section II we propose an algorithm which assigns system-centric time-optimal routes to CAVs in the presence of mixed traffic. In Section III, we formulate the system-centric eco-routing problem for CAVs to minimize their overall energy consumption costs in mixed traffic. In Section IV, we review the Traffic Assignment Problem and propose a framework to model non-CAV flow. In Section V, we use both a simple example and actual historical data to validate the performance of our routing algorithms. Finally, conclusions and further research directions are outlined in Section VI.

II. TIME-OPTIMAL ROUTING

The objective of the Time-Optimal routing problem is to minimize the overall travel time of CAVs. To achieve this goal, we assume (1) the central controller for CAVs has full information on the Origin-Destination (O-D) demand of both CAVs and non-CAVs and (2) non-CAVs route themselves selfishly (i.e., use the route that minimizes their individual travel time). In Section II-A, we calculate the system-centric (social) solution for the 100% CAV penetration rate. Subsequently, in Section II-B, we generalize the routing model to find optimal routes for CAVs in mixed traffic scenarios.

A. System-Centric Time-Optimal Routing

First we assume an all-CAV network, and we can route them using a centralized controller. The system-centric objective is to minimize total traveling time of CAVs in the network. In particular, we seek to find the route occupancy matrix (probabilities) for allocating vehicles to routes.

1) Problem Formulation: As in [10], we model the traffic network as a directed graph $G=(\mathscr{V},\mathscr{A},\mathscr{W})$ where $\mathscr{V}$ is the set of nodes, $\mathscr{A}$ is the set of links, and $\mathscr{W}=\{\mathbf{w}_i:\mathbf{w}_i=(w_{si},w_{ti}),i\in[\![\mathscr{W}]\!]\}$ is the set of all O-D pairs. We assume that all O-D pairs start and end at one of the network's nodes. Let the node-link incidence matrix for the strongly-connected and directed graph G be denoted by $\mathbf{N}\in\{0,1,-1\}^{|\mathscr{V}|\times|\mathscr{A}|}$. Let us define $d^w\geq 0$ as the flow demand from w_s to w_t for any O-D pair $\mathbf{w}=(w_s,w_t)$. Moreover, the route choice probability matrix is defined as $\mathbf{P}=[p_{ir}]$, where p_{ir} is the probability of taking route r while traveling through O-D pair i. Let $\mathbf{g}=(g_i;i\in[\![\mathscr{W}]\!])$ be the O-D demand vector.

Let us define the power-set of routes $\mathbf{R}=\{\mathscr{R}_i;i\in[\![\mathscr{W}]\!]\}$, where $\mathscr{R}_i$ is the set of routes for each O-D pair i. Finally, the link-route incidence matrix is denoted by $\mathbf{A}=\{\alpha^i_{a,r};i\in[\![\mathscr{W}]\!],r\in\mathscr{R}_i,a\in\mathscr{A}\}$ in which:

$$\alpha^i_{a,r}=\begin{cases}1; & \text{if route } r\in\mathscr{R}_i \text{ uses link a}\\ 0; & \text{otherwise.}\end{cases}$$

Additionally, let A_i be the sub-matrix of $\mathbf{A}$ which includes the columns of $\mathbf{A}$ where $r\in\mathscr{R}_i$. The total flow is denoted by $\mathbf{x}=\{x_a;a\in\mathscr{A}\}$ where x_a is the flow on each link $a\in\mathscr{A}$.

Considering $a\in\mathscr{A},i\in[\![\mathscr{W}]\!],r\in\mathscr{R}_i$ we can formulate the system-centric time-optimal problem as follows:

$$\min_{\mathbf{P}}\sum_{a\in\mathscr{A}}t_a(x_a)x_a \tag{1a}$$

$$\mathbf{x}=\mathbf{A}\mathbf{P}^T\mathbf{g} \tag{1b}$$

$$t_a(x_a)=t^0_a\sum_{i=1}^{n}\beta_i(\frac{x_a}{m_a})^{(i-1)} \tag{1c}$$

$$\textstyle\sum_{r\in\mathscr{R}_i}p_{ir}=1;\quad\forall i\in[\![\mathscr{W}]\!] \tag{1d}$$

$$p_{ir}\in[0,1];\quad\forall i\in[\![\mathscr{W}]\!],\forall r\in\mathscr{R}_i \tag{1e}$$

where $t_a(x_a)$ is the traveling time of link a as a function of its corresponding traffic flow x_a, which can be modeled as an increasing polynomial function using (1c). t^0_a, and m_a are the free flow travel time and flow capacity of link $a\in\mathscr{A}$ respectively. Moreover, $\beta=(\beta_i,i=1,2,...,n)$ is the vector of coefficient factors for calculating traveling time in (1c). A common value is $\beta=\{1,0,0,0,0.15\}$ which is the US Bureau of Public Roads (BPR) travel time function [11], [12]. The constraint (1d) enforces the requirement that the sum of all the fractions of vehicles traveling through an O-D pair is 1. The decision variable is the routing-probability matrix $\mathbf{P}=[p_{ir}]$ that for each O-D pair $i\in[\![\mathscr{W}_i]\!]$ assigns fractions of vehicles to existing routes $r\in\mathscr{R}_i$ between any given O-D pair. The inputs to the problem are the link-route incidence matrix ($\mathbf{A}$), and the O-D demand vector $\mathbf{g}$. The solution of Problem 1 is often referred to as the system-centric or social optimal solution in the transportation literature.

B. System-Centric Time-Optimal Routing in the Presence of Mixed Traffic

In this section, we address the system-centric time-optimal routing of CAVs in the presence of mixed traffic (CAVs and non-CAVs). In this case, only a portion of vehicles are CAVs and can be controlled through a centralized controller. As a result, instead of finding a routing scheme that minimizes total costs for all vehicles in the system, we focus on minimizing travel time for the CAV portion of traffic. We consider all CAVs as belonging to the same fleet and that we are trying to minimize total traveling time of the fleet. In order to solve this problem we make four assumptions: (1) The non-CAV traffic flow equilibrium is inferred from data (more details in Sec. IV). (2) There exists a centralized controller which can route the CAV portion of traffic. (3) Up to m number of routes are chosen for every O-D pair. (4) Travel time functions $t(\cdot)$ are strongly monotone and continuously differentiable.

1) Problem Formulation: Let us define the CAV penetration rate γ, as the portion of traffic that consists of CAVs (fraction of CAVs in the system). As in the system-centric case, we define $\mathbf{P}_c=[p^c_{ir}]$ to be the route choice probability matrix for the CAV portion of traffic. Moreover, $\mathbf{g}_c=\{g^c_i;i\in[\![\mathscr{W}]\!]\}$ is the O-D demand vector for CAVs. As mentioned before, we assume that the non-CAV traffic flow equilibrium is inferred from data, and is known. Let us define $\mathbf{x}^c=\{x^c_a;a\in\mathscr{A}\}$ and $\mathbf{x}^{nc}=\{x^{nc}_a;a\in\mathscr{A}\}$ as the flow of CAVs and non-CAVs in the system respectively, where x^c_a

and x_a^{nc} are the CAV and non-CAV flow on each link $a \in \mathscr{A}$. As a result, using the same notation as in Section II-A.1, the optimization problem can be written as:

$$\min_{\mathbf{P}_c} \sum_{a \in \mathscr{A}} t_a(x_a) x_a^c \tag{2a}$$
$$\mathbf{x} = \mathbf{x}^c + \mathbf{x}^{nc} \tag{2b}$$
$$\mathbf{x}^c = \mathbf{A}\mathbf{P}_c^T \mathbf{g}^c \tag{2c}$$
$$t_a(x_a) = t_a^0 \sum_{i=1}^{n} \beta_i (\frac{x_a}{m_a})^{(i-1)} \tag{2d}$$
$$\textstyle\sum_{r \in \mathscr{R}_i} p_{ir}^c = 1; \quad \forall i \in [\![\mathscr{W}]\!] \tag{2e}$$
$$p_{ir}^c \in [0,1]; \quad ; \forall i \in [\![\mathscr{W}]\!], \forall r \in \mathscr{R}_i \tag{2f}$$

Constraint (2b) states that the total flow in the network is the summation of CAV flow ($\mathbf{x}^c$) and non-CAV flow ($\mathbf{x}^{nc}$). Since we are minimizing the travel time for the CAV share of traffic, in (2a) the traveling time of each link is multiplied only by the flow of CAVs (x_a^c). The inputs to the optimization problem are the link-route incidence matrix $\mathbf{A}$, O-D demand vector $\mathbf{g}^c$, and non-CAV flow $\mathbf{x}^{nc}$.

By solving Problem 2, we find optimal flows of CAVs over each O-D pair (route-probability matrix $\mathbf{P}_c$). In other words, when a CAV enters the network at an origin O given its destination D, the algorithm gives it the desired socially optimal route to follow in terms of a sequence of links.

As stated in Section 2.4 of [13] the system-centric problem can be reformulated as a user-centric problem by slightly changing the travel cost function. Therefore, the results on the existence and uniqueness of the solution for the user-centric problem (Section IV-A) extend to the system-centric case. As a requirement for such a result we need positive and strictly increasing travel time functions on $\mathbf{P}$ which is achieved by having increasing polynomial functions.

III. System-Centric Eco-Routing in the Presence of Mixed Traffic

Eco-routing refers to the procedure of finding the optimal route for a vehicle to travel between two points which utilizes the least amount of energy costs. This problem shares similar properties to (2), with the difference that we minimize energy instead of time. In this section, we first review an energy model for conventional vehicles and then formulate the system-centric eco-routing problem for CAVs.

A. Energy Consumption Modeling

Energy consumption of vehicles depends on many different factors including velocity and acceleration [14] of the vehicle, as well as the power-train's architecture. Since in eco-routing we are making high-level decisions that can affect the energy consumption, a low-fidelity model can be sufficient for our needs. Moreover, when solving the eco-routing problem, we are dealing with a large number of decision variables. Having a model with a simple mathematical function would allow us to speed up the calculation for practical purposes. Hence, we are looking for an energy model which can estimate the energy consumption as a function of the average speed of a vehicle. We adopt the empirical energy model for conventional vehicles proposed by Boriboonsomsin et al. [15]. This model is a polynomial function of of the average speeds of links. According to this empirical model), the average fuel consumption in grams per mile for every link $a \in \mathscr{A}$ can be calculated as follows:

TABLE I: Energy cost coefficients [15]

θ_0	θ_1	θ_2	θ_3	θ_4	θ_5
6.80	-1.4e-1	3.92e-3	-5.20e-5	2.57e-7	1.37e-1

$$ln(e_a) = \sum_{i=0}^{4} \theta_i (v_a)^i + \theta_5 R_a \tag{3}$$

in which e_a is the average energy consumption on link a in g/mi, v_a is the average speed of the link in mph, R_a is the road grade (in percentage), and $\theta = (\theta_i, i = 0,1,...,5)$ is the vector of coefficients for calculating the energy cost. Typical values of θ are given in Table I. Average fuel consumption per mile using (3) and θ values in Table I is shown in Fig. 2. In [16] we also reviewed a charge depleting(CD)/charge sustaining(CS) energy model [17] which can be used for PHEVs, HEVs, and EVs.

B. Eco-routing Problem Formulation for Conventional Vehicles

In order to formulate the eco-routing problem for conventional vehicles, we use energy model (3). This problem is almost the same as (2), with the only difference that $t_a(x_a)$ should be replaced with $e_a(x_a)$, which is the average fuel consumption per mile for traveling link $a \in \mathscr{A}$. Considering this, we formulate the eco-routing problem of CAVs for conventional vehicles as follows:

$$\min_{\mathbf{P}_c} \sum_{a \in \mathscr{A}} c_{gas} l_a e_a(v_a(x_a)) x_a^c \tag{4a}$$
$$\mathbf{x} = \mathbf{x}^c + \mathbf{x}^{nc} \tag{4b}$$
$$\mathbf{x}^c = \mathbf{A}\mathbf{P}_c^T \mathbf{g}^c \tag{4c}$$
$$t_a(x_a) = t_a^0 \sum_{i=0}^{n} \beta_i (\frac{x_a}{m_a})^i \tag{4d}$$
$$v_a(x_a) = \frac{l_a}{t_a(x_a)} \tag{4e}$$
$$ln(e_a) = \sum_{i=0}^{4} \theta_i (v_a)^i + \theta_5 R_a \tag{4f}$$
$$\textstyle\sum_{r \in \mathscr{R}_i} p_{ir}^c = 1; \quad \forall i \in [\![\mathscr{W}]\!] \tag{4g}$$
$$p_{ir}^c \in [0,1]; \quad \forall i \in [\![\mathscr{W}]\!], \forall r \in \mathscr{R}_i \tag{4h}$$

where c_{gas} is the cost of gas (\$/gal), and l_a is the length of link $a \in \mathscr{A}$. Moreover, e_a is the average energy consumption per link's length $\forall a \in \mathscr{A}$, and $\theta = (\theta_i, i = 0,1,...,5)$ is the energy cost coefficient (Table I).

IV. Non-CAV Flow Modeling

One of our assumptions is that the non-CAV flow is an input to our models, and can be inferred from actual traffic data. However, since we currently do not have CAVs in cities, we model the non-CAV flow by considering how non-CAVs

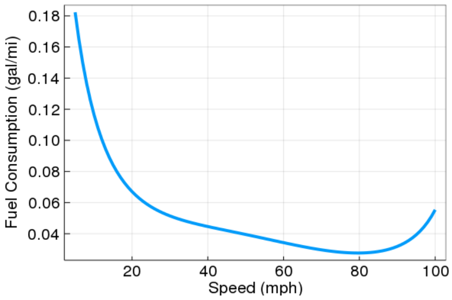


Fig. 2: Average fuel consumption of conventional vehicles using Boriboonsomsin model ($R_a = 0$)

react to the optimal decisions made by CAVs. To achieve this task, we assume non-CAVs act selfishly by minimizing their travel time. This modeling framework has been extensively studied and is often referred to as the *Traffic Assignment Problem* (TAP) [13]. As a result, we propose an iterative method for finding non-CAV flow considering the routing decisions of CAVs. The basis of this methodology is that whenever CAVs change their routing decisions, non-CAVs adjust theirs and vice versa. For this particular problem, we consider an iterative procedure to find an equilibrium for mixed traffic flow of CAVs and non-CAVs.

In order to obtain the non-CAV flow for a given CAV penetration rate γ, we first consider only non-CAVs in the network and the O-D demand of non-CAVs is given by:

$$\mathbf{g}^{nc} = (1-\gamma)\mathbf{g} \tag{5}$$

Even though we choose a uniform demand distribution for non-CAVs, without loss of generality, we can use any other given demand for both CAVs and non-CAVs.

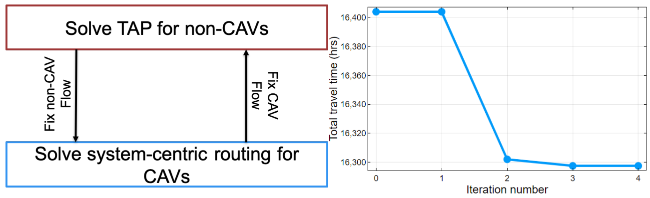
Considering a non-CAV demand $\mathbf{g}^{nc}$, we solve the selfish (user-centric) routing problem which minimizes their travel time. In this respect, we use the *Method of Successive Averages* (MSA) [18]. After finding $\mathbf{x}^{nc}$ using the MSA, we solve the time optimal (2) or energy optimal (4) routing problem for the CAV portion of traffic considering its demand to be:

$$\mathbf{g}^{c} = \gamma\mathbf{g} \tag{6}$$

Since non-CAVs were unaware of CAVs in the system while solving the TAP, we re-solve the problem considering CAV flow on each link. Hence, we re-iterate by considering the CAV solution $\mathbf{x}^c$. Furthermore, the TAP is solved again for non-CAVs. Re-iteration of this process continues until convergence (Figs. 3).

A. Traffic Assignment Problem (TAP) and Wardrop equilibrium

The objective of the Traffic Assignment Problem is to find link flows in a transportation network given the O-D demands and cost functions. A standard solution to this problem is to find travel flows that minimize their travel times. Such a solution individually optimizes every vehicle's travel time based on network conditions. This leads to a *Nash Equilibrium* that in transportation networks is known as the



(a) (b)

Fig. 3: (a) Procedure for solving the system-centric routing problem; (b) Convergence plot for iterating through TAP and social problem

Wardrop Equilibrium [9]. The resulting flows $\mathbf{x}^*$ (equilibrium flows) require that for every O-D pair $\mathbf{w}$, and any route r connecting (w_s, w_t), the associated travel time is not greater than the traveling time from any other route. Formally

$$t_a(x_a^*) \leq t_a(x_{a'}^*) \quad \forall a, a' \in \mathscr{A} \tag{7}$$

or equivalently

$$t_a(x_r^*) \leq t_r(x_{r'}^*) \quad \forall r, r' \in \mathscr{R}_i, \quad \forall i \in [\![\mathscr{W}]\!] \tag{8}$$

To obtain such flows, we can solve

$$\min_{\mathbf{x}\in\mathscr{F}} \quad \Phi(\mathbf{x}) = \sum_{a\in\mathscr{A}} \int_0^{x_a} t_a(s)ds \tag{9}$$

where $\mathscr{F}$ is the set of feasible flow vectors defined by

$$\mathscr{F} = \left\{\mathbf{x} : \exists \mathbf{x}^{\mathbf{w}} \in \mathbb{R}_+^{|\mathscr{A}|} \text{ s.t. } \mathbf{x} = \sum_{\mathbf{w}\in\mathscr{W}} \mathbf{x}^{\mathbf{w}}, \mathbf{N}\mathbf{x}^{\mathbf{w}} = \mathbf{d}^{\mathbf{w}}, \forall \mathbf{w} \in \mathscr{W}\right\}, \tag{10}$$

and where $\mathbf{x}^{\mathbf{w}}$ is the flow vector attributed to O-D pair $\mathbf{w}$. Recall that $t_a(\cdot)$ in (2d) is continuous. Since $\mathscr{F}$ is a compact set, the Weierstrass Theorem [9] implies that there exists a solution to this minimization problem. Moreover, since cost functions are non-decreasing (by assumption), then $\Phi(\cdot)$ is convex and therefore a unique solution exists [9].

Now, let us write the TAP in terms of non-CAV flows and take into account the presence of the CAV flow in the network.

$$\min_{\mathbf{x}^{nc}} \sum_{a\in\mathscr{A}} \int_{x_a^c}^{x_a^c + x_a^{nc}} t_a(s)ds \tag{11a}$$

$$s.t \quad \mathbf{x}^{nc} = \sum_{\mathbf{w}\in\mathscr{W}} \mathbf{x}^{nc,w} \tag{11b}$$

$$\mathbf{N}\mathbf{x}^{nc,w} = \mathbf{d}^{nc,w}, \quad \forall \mathbf{w} \in \mathscr{W} \tag{11c}$$

$$\mathbf{x}^{nc,w} \geq \mathbf{0} \tag{11d}$$

V. Numerical Results

In order to validate the proposed routing algorithms we perform two case studies. First we analyze the widely explored Braess network (Fig. 4), and study the effect of the CAV penetration rate on the total time savings and energy savings in this network. As an alternative benchmark, we applied the algorithms to a sub-network of the Eastern Massachusetts interstate highways (Fig. 5b). For finding the energy optimal routes, we assume the road grade is zero ($R_a = 0$ in (3)), and we assume the cost of gas is 2.75 \$/gal. We solve the NLP problems using IPOPT [19] in Julia [20].

We solve the eco-routing problem for conventional vehicles using the energy model discussed in Section III-A. As mentioned before, eco-routing results are extremely sensitive to the energy model. Given a more accurate energy model which is convex, smooth and differentiable we may get different results. Hence, the eco-routing results shown in this paper should only be considered as preliminary results which show the potential of saving energy using centralized routing of CAVs.

A. Braess Network Example

To demonstrate how optimal routing of CAVs under different penetration rates can affect both CAVs and non-CAVs, we first apply algorithm 2 to the well-known Braess network (Fig. 4). Note that in this case instead of using the BPR function (2d), we use the travel time functions shown on each link of the Braess network in Fig. 4. We consider a demand of 4000 veh/hr travels from node 1 to node 4, the lengths of links 1,2,3 and 5 equal to 30.5 miles and the length of link 4 equal to zero (by definition of Braess network). First we solve the time-optimal routing of CAVs under different penetration rates. Using the obtained flows, and energy model (3) we calculate energy costs for traveling through the network. Time-optimal results are shown in Figs. 6a, in which we compare traveling time of CAVs with non-CAVs under different penetration rates. The energy cost for traveling through the optimal routes are also shown in Fig. 6c. As shown in Fig. 6, introducing CAVs into the system not only improves the time saving of CAVs, but also helps non-CAVs to save time. This is because smart routing decisions of CAVs reduce the traffic intensity in the highly congested roads, which consequently helps non-CAVs to travel faster. As we inject CAVs into the system, we see that travel time (as well as energy cost) per vehicle of CAVs starts decreasing compared with the uncontrolled traffic. Moreover, the traveling time of commuting through the fastest route decreases as we inject more CAVs to the system. Typically, we expect a trade off between time saving and energy saving in routing problems [21], [22]. However, in Fig. 6 we see that time and energy follow the same trend. In other words, time savings result in energy savings. The reason for this behavior is the energy model used in the eco-routing problem 3. As we can see in Fig. 2, based on this model, the higher the speed, the better fuel efficiency. As a result, we get similar results for energy and time.

It is interesting to see that when a small percentage of CAVs are in the system, there is no improvement for anyone. This happens because CAVs are optimizing over their own small fraction of overall traffic and this fraction is not sufficient to change the network conditions. However, as we increase the penetration rate, CAVs create a more balanced flow distribution in the network from which both CAVs and non-CAVs can benefit. In the Braess network example, it can be seen that if all the cars in the system are replaced with

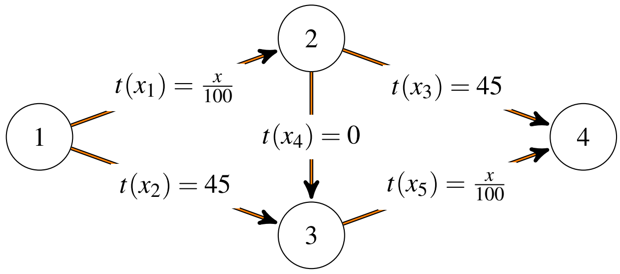
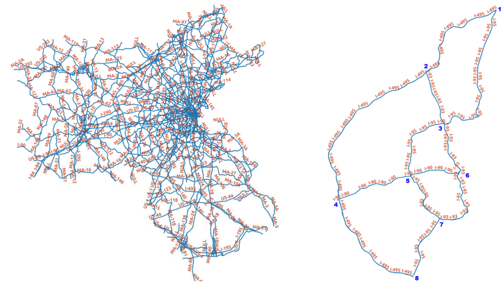


Fig. 4: Simple 5 link directed graph (Braess's network)



(a) (b)

Fig. 5: (a) All available road segments in the road map of Eastern Massachusetts [10] ; (b) Interstate highway sub-network of eastern Massachusetts

CAVs, we can save 18.9% in terms of travel time. This value is often referred to as the Price of Anarchy (PoA) [10].

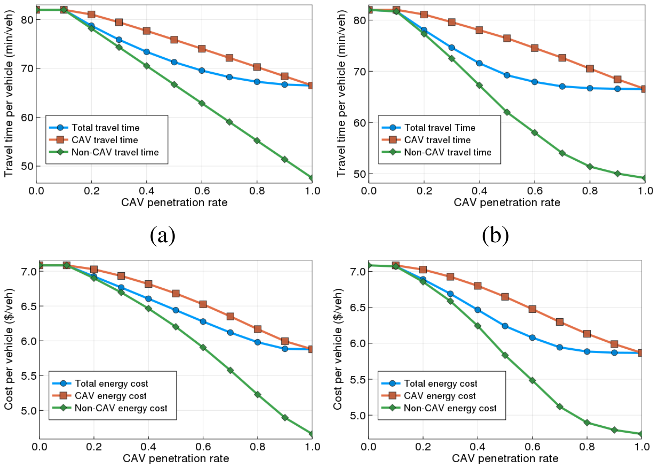
In addition to the time-optimal case, we also solve the eco-routing (energy-optimal) problem for CAVs using the Braess network. As we can see in Figs. 6b and 6d, energy-optimal results follow the same trend as time-optimal result. In other words, centralized eco-routing of CAVs can benefit both CAVs and non-CAVs. The maximum energy savings happens at the 100% CAV penetration rate (19.1%).

B. EMA Interstate Highway Network

To obtain more realistic results, we perform a data-driven case study using the actual traffic data from the Eastern Massachusetts (EMA) road network. These data were collected by *INRIX* and provided to us by the Boston Region Metropolitan Planning Organization. The sub-network including the interstate highways of EMA (Fig. 5b) is chosen for the case study. For this network, we use the O-D demand which has been estimated using an inverse optimization framework in [10]. In order to solve the problem we consider 56 O-D pairs, and allow up to 3 routes between each origin and destination (top 3 shortest routes). We then solve (2) and (4) to find the time optimal and energy optimal paths for CAVs respectively. Time optimal results are shown in Figs. 7a, and 7c, and the energy optimal results are shown in Figs. 7b and 7d. Note that the results for both the EMA and Braess networks are averaged over all the cars and O-D pairs.

VI. Conclusions and Future Work

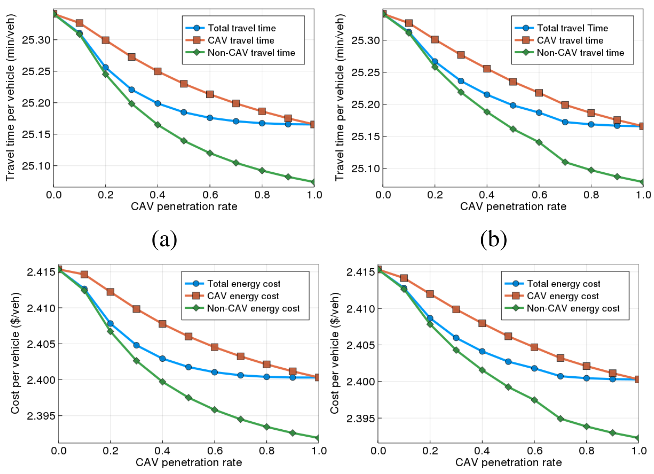
In this paper we proposed system-centric optimal routing algorithms for a fleet of CAVs in the presence of mixed



(a) (b)

(c) (d)

Fig. 6: Braess network routing results under different penetration rates by solving system-centric time-optimal problem (a), (c) and, system-centric energy-optimal problem (b), (d).



(a) (b)

(c) (d)

Fig. 7: EMA network routing results under different penetration rates by solving system-centric time-optimal problem (a), (c) and system-centric energy-optimal problem (b), (d).

traffic. We consider two objectives for routing: (1) minimizing travel time (2) minimizing energy consumption cost. Moreover, in order to model the routing behavior of regular vehicles, we assume that they make selfish decisions by minimizing their own travel time. Then, by iteratively solving the TAP, and finding optimal routes for CAVs we estimate the non-CAV flow in the network. Historical traffic data and a simple illustrative example were used to validate the models. The results indicate that optimal routing of CAVs can not only benefit CAVs, but the smart routing decisions of CAVs helps ease traffic congestion in the network which helps regular vehicles as well. Additionally, we empirically showed that even a small CAV penetration rate has significant impact on the overall traveling cost of the network.

In ongoing work, we are considering multiple fleets of CAVs, in which each fleet is trying to minimize its own cost.

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