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Optimal Control of Autonomous Vehicles for Non-Stop Signalized Intersection Crossing*

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Abstract— **This paper is devoted to the development of an optimal acceleration/speed profile for autonomous vehicles in free flow mode approaching a traffic light without stopping. The design objective is to achieve both short travel time and low energy consumption as well as avoid idling at a red light. This is achieved by taking full advantage of the traffic light information based on infrastructure-to-vehicle communication. The direct adjoining approach is used to solve both free and fixed terminal time optimal control problems subject to state constraints. We show that we can derive a real-time online analytical solution, distinguishing our method from most existing approaches based on numerical calculations. Extensive simulations are executed to compare the performance of autonomous vehicles under the proposed speed profile and human driving vehicles. The results show quantitatively the advantages of the proposed algorithm in terms of energy consumption and travel time.**

I. INTRODUCTION

Connected and automated vehicles (CAVs), commonly known as self-driving or autonomous vehicles, provide an intriguing opportunity for enabling users to better monitor transportation network conditions and to improve traffic flow. Their proliferation has rapidly grown, largely as a result of Vehicle-to-X (or V2X) technology [1] which refers to an intelligent transportation system where all vehicles and infrastructure components are interconnected with each other. Such connectivity provides precise knowledge of the traffic situation across the entire road network, which in turn helps optimize traffic flows, enhance safety, reduce congestion, and minimize emissions. Controlling a vehicle to improve energy consumption has been studied extensively, e.g., see [2], [3], [4], [5]. By utilizing road topography information, an energy-optimal control algorithm for heavy diesel trucks is developed in [4]. Based on Vehicle-to-Vehicle (V2V) communication, a minimum energy control strategy is investigated in car-following scenarios in [5]. Another important line of research focuses on coordinating vehicles at intersections to increase traffic flow while also reducing energy consumption. Depending on the control objectives, work in this area can be classified as dynamically controlling traffic lights [6] and as coordinating vehicles [7], [8], [9], [10]. More recently, an optimal control framework is proposed in [11] for CAVs to cross one or two adjacent intersections in an urban area. The state of art and current trends in the coordination of CAVs is provided in [12].

Our focus in this paper is on an optimal control approach for a single autonomous vehicle approaching an intersection in terms of energy consumption and taking advantage of traffic light information. The term "ECO-AND" (short for "Economical Arrival and Departure") is often used in the literature to refer to this problem [13]. Its solution is made possible by vehicle-to-infrastructure (V2I) communication, which enables a vehicle to automatically receive signals from upcoming traffic lights before they appear in its visual range. Clearly, an autonomous vehicle can take advantage of such information in order to go beyond current "stop-and-go" to achieve "stop-free" driving. Along these lines, the problem of avoiding red traffic lights is investigated in [14], [15], [16], [17], [18]. Avoiding red lights with probabilistic information at multiple intersections is considered in [16], where the time horizon is discretized and deterministic dynamic programming is utilized to numerically compute the optimal control input. The work in [17] devises an optimal speed profile given the feasible target time, which is within some green light interval. A velocity pruning algorithm is proposed in [18] to identify feasible green windows, and a velocity profile is optimized numerically in terms of energy consumption.

Here, we investigate the optimal control problem of autonomous vehicles approaching a traffic light where the objective function is a weighted sum of both travel time and energy consumption. The problem is challenging due to the following reasons. First, finding a feasible green light interval leads to a Mixed Integer Programming (MIP) problem formulation. In general, solving MIP problems requires a significant amount of computation, and the optimality of the solution is not guaranteed due to the non-convexity of the problem involved with integer variables. The second reason comes from state constraints related to speed limits. The inclusion of bounds on state variables poses a significant challenge for most optimization methods. To overcome the above difficulties, we devise a two-step method. Specifically, we first address the problem without the traffic light constraint, which means that the terminal time is free, and the mixed integer constraints are removed. If the terminal time obtained from the free terminal time optimal control problem is within some green light interval, then the problem is solved. However, if the terminal time falls within some red light interval, then the optimal terminal time could be either the end of the previous green light interval or the beginning of the next green light interval by using the monotonicity

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property of the objective function. Then, we transform the original problem into a fixed terminal time optimal control problem with feasible terminal times, and comparing the corresponding performance leads to the optimal solution of the original problem.

II. PROBLEM FORMULATION

The dynamics of the vehicle are modeled by a double integrator

$$\dot{x}(t) = v(t), \tag{1}$$
$$\dot{v}(t) = u(t), \tag{2}$$

where $x(t)$, $v(t)$, and $u(t)$ are the position, velocity, and acceleration of the vehicle, respectively. At time t_0, the initial position and velocity are given as $x(t_0) = 0$ and $v(t_0) = v_0$ respectively. Let us use l to denote the distance to the traffic light, and t_p the intersection crossing time of the vehicle. The traffic light switches between green and red at an intersection are dictated by a rectangular pulse signal $f(t)$ with a period T:

$$f(t) = \begin{cases} 1 & \text{for } kT \le t \le kT + DT, \\ 0 & \text{for } kT + DT < t < (k+1)T, \end{cases}$$

where $f(t) = 1$ indicates that the traffic light is green, and $f(t) = 0$ indicates that the traffic light is red as shown in Fig. 1. The parameter $0 < D < 1$ is the fraction of the time period T during which the traffic light is green, and $k \in \mathbb{Z}_{\ge 0}$ is a non-negative integer.

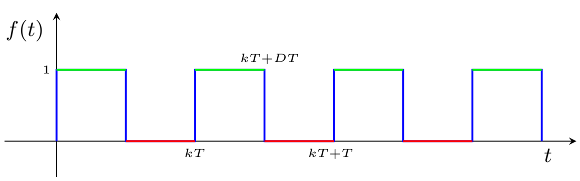

Fig. 1. Traffic light signal

Our objective is to make the vehicle cross an intersection without stopping with the aid of traffic light information as well as to minimize both travel time and energy consumption. Thus, we formulate the following problem:

Problem 1: ECO-AND Problem

$$\min_{u(t)} \quad \rho_t (t_p - t_0) + \rho_u \int_{t_0}^{t_p} u^2(t)\, dt \tag{3}$$

subject to

$$(1) \text{ and } (2), \tag{4}$$
$$x(t_p) = l, \tag{5}$$
$$v_{\min} \le v(t) \le v_{\max} \tag{6}$$
$$u_{\min} \le u(t) \le u_{\max} \tag{7}$$
$$kT \le t_p \le kT + DT, \tag{8}$$

for some $k \in \mathbb{Z}_{\ge 0}$. In (3), the term $J^t = t_p - t_0$ is the travel time while $J^u = \int_{t_0}^{t_p} u^2(t)\, dt$ captures the energy consumption; see [19].

In order to normalize these two terms for the purpose of a well-defined optimization problem, first note that the maximum possible value of J^t is $l/v_{\min}$. Depending on the relationship between $v_{\min}$, $v_{\max}$, $u_{\max}$ and l, there are two different cases for the maximum possible value of J^u. Following some calculations (details can be found in [20]), we can specify the two weighting parameters ρ_t and ρ_u as follows: $\rho_t = \rho \frac{v_{\min}}{l}$ and

$$\rho_u = \begin{cases} \frac{1-\rho}{(v_{\max}-v_{\min})u_{\max}} & \text{if } l \ge v_{\min}\frac{v_{\max}-v_{\min}}{u_{\max}} + \frac{1}{2}\frac{(v_{\max}-v_{\min})^2}{u_{\max}} \\ \frac{1-\rho}{\left(\sqrt{v_{\min}^2+2u_{\max}l}-v_{\min}\right)u_{\max}} & \text{otherwise} \end{cases}$$

capturing the normalized trade-off between the travel time and energy consumption by setting $0 \le \rho \le 1$. When $\rho = 0$, the problem reduces to minimizing the energy consumption only; when $\rho = 1$, we seek to minimize the travel time only.

In (6)-(7), the parameters $v_{\min} \ge 0$ and $v_{\max} > 0$ are the minimum and maximum allowable speeds for road vehicles, respectively, while the parameters $u_{\min}$ and $u_{\max}$ are the maximum allowable deceleration and acceleration, respectively. Note that when $u < 0$, the vehicle decelerates due to braking and when $u > 0$ the vehicle accelerates. Finally, the integer constraint (8) reflects the requirement that t_p belongs to an interval when the light is green (see Fig. 1).

III. MAIN RESULTS

Problem 1 is a Mixed Integer Programming (MIP) problem. Existing approaches to such problems turn out to be computationally very demanding for off-line computation, not to mention obtaining analytical solutions in a real-time on-line context. We propose a two-step approach, which allows us to efficiently obtain an analytical solution on line, under the assumption that the vehicle operates in free flow mode. The first step is to solve Problem 1 without the integer constraint (8). If the optimal arrival time t_p^* is within some green light interval, then the problem is solved. However, if

$$kT + DT < t_p^* < kT + T,$$

for some k, then we solve Problem 1 twice with the constraint (8) replaced by $t_p = kD + DT$ and $t_p = kT + T$, respectively. We compare the performance obtained with different terminal times, and the solution produced by the one with better performance naturally yields the optimal solution.

In the following, we first seek the optimal solution to Problem 1 without the constraint (8), which is termed "free terminal time optimal control problem".

A. Free Terminal Time Optimal Control Problem

The free terminal time optimal control problem is given below.

Problem 2: Free Terminal Time Optimal Control Problem

$$\min_{u(t)} \rho_t (t_p - t_0) + \rho_u \int_{t_0}^{t_p} u^2(t)\, dt \tag{9}$$

subject to

$$(1) \text{ and } (2), \tag{10}$$

$$x(t_p) = l, \tag{11}$$

$$v_{\min} \leq v(t) \leq v_{\max}, \tag{12}$$

$$u_{\min} \leq u(t) \leq u_{\max}, \tag{13}$$

where ρ_t and ρ_u are given in Section II.

From the objective function (9), it can be seen that a minimum energy consumption solution should avoid braking, that is, $u(t) \geq 0$ for $t \in [t_0, t_p]$. We will show this fact in the following lemma.

Lemma 1: The optimal solution $u^*(t)$ to Problem 2 satisfies $u^*(t) \geq 0$ for all $t \in \left[t_0, t_p^*\right]$.

Due to space constraints, the proof is omitted but may be found in [20].

In addition, it follows from this lemma that whenever $v(\tau) = v_{\max}$ (which may not be possible in some cases), we must have $u(t) = 0$ for all $t \in [\tau, t_p]$. Based on these observations, we can derive necessary conditions for the solution to Problem 2, which are summarized in the following theorem.

Theorem 1: Let $x^*(t)$, $v^*(t)$, $u^*(t)$, t_p^* be an optimal solution to Problem 2 and assume that $\rho_t \neq 0$ and $\rho_u \neq 0$. Then, the optimal control $u^*(t)$ satisfies

$$u^*(t) = \arg \min_{0 \leq u(t) \leq u_{\max}} \rho_u u^2 + \frac{\rho_t}{v^*\left(t_p^*\right)}(t - \tau) u, \tag{14}$$

where τ is the first time on the optimal path when $v(\tau) = v_{\max}$ if $\tau < t_p$; $\tau = t_p^*$ otherwise.

Due to space constraints, the proof is omitted but may be found in [20].

Recall that the theorem was proved under the assumption that $\rho_t \neq 0$ and $\rho_u \neq 0$. The special cases when either $\rho_t = 0$ or $\rho_u = 0$ are considered in the following two corollaries.

Corollary 1: Let $x^*(t)$, $v^*(t)$, $u^*(t)$, t_p^* be an optimal solution to Problem 2 when $\rho_t = 0$. Then, the optimal control $u^*(t)$ satisfies $u^*(t) = 0$ for all $t \in [t_0, t_p^*]$.

Corollary 2: Let $x^*(t)$, $v^*(t)$, $u^*(t)$, t_p^* be an optimal solution to Problem 2 when $\rho_u = 0$. Then, the optimal control $u^*(t)$ satisfies

$$u^*(t) = \begin{cases} u_{\max} & \text{for } t \in [t_0, \tau), \\ 0 & \text{for } t \in \left[\tau, t_p^*\right], \end{cases} \tag{15}$$

where τ is the first time on the optimal path when $v^*(\tau) = v_{\max}$.

The proofs of the above two corollaries are straightforward by setting $\rho_t = 0$ and $\rho_u = 0$, respectively, in (14) in Theorem 1.

Based on the vehicle dynamics (1) and (2), the initial conditions $x(t_0) = 0$ and $v(t_0) = v_0$, and the terminal condition $x^*\left(t_p^*\right) = l$, the optimal control law (14) and the optimal time t_p^* can be uniquely determined. In the following, we will classify the results into different cases depending on the values of the model parameters. In order to do so, we define two functions:

$$f(v_0) = l - \frac{v_{\max}^2 - v_0^2}{2u_{\max}} - u_{\max} v_{\max}^2 \frac{\rho_u}{\rho_t} + \frac{1}{6} u_{\max}^3 v_{\max}^2 \frac{\rho_u^2}{\rho_t^2},$$

$$g(v_0) = l - 2v_0 \sqrt{(v_{\max} - v_0) v_{\max} \frac{\rho_u}{\rho_t}} - \frac{4}{3}(v_{\max} - v_0) \sqrt{(v_{\max} - v_0) v_{\max} \frac{\rho_u}{\rho_t}}.$$

Depending on the signs of these two functions, the optimal solution consisting of $u^*(t)$ and t_p^* can be classified as shown in Table I, where all detailed calculations are omitted due to space constraints but may be found in [20]. Referring to this table, the optimal control is parameterized by the following function

$$\Phi(t|a, b, c) = \begin{cases} u_{\max} & \text{when } t \leq a \\ c(t - b) & \text{when } a < t < b \\ 0 & \text{when } t \geq b \end{cases}$$

The dash in Φ means that the variable t cannot reach the upper bound and/or the lower bound, and therefore that case is inapplicable here. The parameters shown in Table I are defined as follows:

$$t_1 = t_0 + \frac{\left(1 - u_{\max}^2 \frac{\rho_u}{\rho_t}\right) v_{\max} - v_0}{u_{\max}}, \; t_3 = t_0 + \frac{v_1 - v_0}{u_{\max}},$$

$$t_2 = t_1 + 2u_{\max} v_{\max} \frac{\rho_u}{\rho_t}, t_4 = t_0 + 2\sqrt{(v_{\max} - v_0) v_{\max} \frac{\rho_u}{\rho_t}}$$

where

$$v_1 = \sqrt{\frac{2u_{\max} l + v_0^2}{1 + \frac{4u_{\max}^2}{1 - \frac{\rho_u}{\rho_t} u_{\max}^2} \frac{\rho_u}{\rho_t} + \frac{8}{3} \frac{u_{\max}^4}{\left(1 - \frac{\rho_u}{\rho_t} u_{\max}^2\right)^2} \frac{\rho_u^2}{\rho_t^2}}}$$

and v_2 is the solution of the following equation:

$$l = \frac{2}{3}(v_0 + 2v_2) \sqrt{(v_2 - v_0) v_2 \frac{\rho_u}{\rho_t}}.$$

The parameters $\delta_1, \delta_2, \delta_3, \delta_4$ specifying in Table I the optimal time t_p^* when the vehicle arrives at the traffic light in each of the four possible cases are given below:

$$\delta_1 = t_2 + \frac{f(v_0)}{v_{\max}}, \quad \delta_2 = t_3 + 2u_{\max} \frac{v_1}{1 - \frac{\rho_u}{\rho_t} u_{\max}^2} \frac{\rho_u}{\rho_t},$$

$$\delta_3 = t_4 + \frac{g(v_0)}{v_{\max}}, \quad \delta_4 = t_0 + 2\sqrt{(v_2 - v_0) v_2 \frac{\rho_u}{\rho_t}}.$$

Remark 1: This remark pertains to the underlying criteria for the optimal solution classification in Table I. The first row determines whether or not the maximum acceleration $u_{\max}$ will be used for a given initial speed v_0. The optimality conditions tell us that the vehicle starts with the maximum acceleration when the initial speed is relatively low. The second row determines if the road length l is large enough for a vehicle to reach its maximum speed for a given initial speed v_0. In general, the optimal control contains three phases: full acceleration, linearly decreasing acceleration, and no acceleration. The first column specifies the case where all three phases are included with switches defined by t_1, t_2.

TABLE I

OPTIMAL SOLUTION CLASSIFICATION FOR PROBLEM 2

	$\frac{v_0}{v_{\max}} < 1 - u_{\max}^2 \frac{\rho_u}{\rho_t}$		$1 - u_{\max}^2 \frac{\rho_u}{\rho_t} \leq \frac{v_0}{v_{\max}}$	
	$f(v_0) \geq 0$	$f(v_0) < 0$	$g(v_0) \geq 0$	$g(v_0) < 0$
u^*	$\Phi\left(t \mid t_1, t_2, \frac{\rho_t}{2\rho_u v_{\max}}\right)$	$\Phi\left(t \mid t_3, -, \frac{\rho_t}{2\rho_u v_1}\right)$	$\Phi\left(t \mid -, t_4, \frac{\rho_t}{2\rho_u v_{\max}}\right)$	$\Phi\left(t \mid -, -, \frac{\rho_t}{2\rho_u v_2}\right)$
t_p^*	δ_1	δ_2	δ_3	δ_4

The second column corresponds to the case of low initial speeds and short-length roads. Under optimal control in this case, the vehicle starts with full acceleration, but the road length is so short that the maximum speed cannot be reached. Therefore, the optimal control contains only the first two phases. The third column corresponds to the case of large initial speeds and long-length roads. The vehicle starts with linearly decreasing acceleration, and then proceeds with no acceleration when the speed reaches the limit $v_{\max}$. Here, the optimal control contains only the last two phases. The last column corresponds to the case of large initial speeds and short-length roads. Therefore, the vehicle uses only linearly decreasing acceleration.

B. *Fixed Terminal Time Optimal Control Problem*

In this section, we consider the case where the optimal time t_p^* obtained in the free terminal time optimal control problem (Problem 2) is within some red light interval, that is,

$$kT + DT < t_p^* < kT + T.$$

In this case, the candidate optimal arrival time t_p^* in Problem 1 is either $kT + DT$ or $kT + T$. Therefore, we can compare the performance obtained under either one of these two terminal times, and select the one with better performance to determine the optimal arrival time for Problem 1. In both cases, the travel time is now fixed, hence the only objective is to minimize the energy consumption. Thus, we have the following problem formulation:

Problem 3: Fixed Terminal Time Optimal Control Problem

$$\min_{u(t)} \int_{t_0}^{t_p} u^2(t)\, dt \tag{16}$$

subject to

$$(1) \text{ and } (2) \tag{17}$$

$$x(t_p) = l \tag{18}$$

$$t_p = kT + DT \text{ or } kT + T \tag{19}$$

$$v_{\min} \leq v(t) \leq v_{\max} \tag{20}$$

$$u_{\min} \leq u(t) \leq u_{\max} \tag{21}$$

1) Arrival Time $t_p = kT + DT$: In this case, it is clear that the vehicle must use less time than the one specified by t_p^* in Problem 2 and more acceleration. Define a function

$$h(v_0) = \begin{cases} v_0 t_p + \frac{1}{2} u_{\max} t_p^2 - l & \text{for } t_p \leq \frac{v_{\max} - v_0}{u_{\max}} \\ v_{\max} t_p - \frac{1}{2} \frac{(v_{\max} - v_0)^2}{u_{\max}} - l & \text{for } t_p > \frac{v_{\max} - v_0}{u_{\max}} \end{cases}$$

Observe that the terminal time $t_p = kT + DT$ is possible if and only if $h(v_0) \geq 0$. The main result for this case is given in the following theorem.

Theorem 2: Let $x^*(t)$, $v^*(t)$, $u^*(t)$ be an optimal solution to Problem 3 with $t_p = kT + DT$. Then, the optimal control $u^*(t)$ satisfies

$$u^*(t) = \arg \min_{0 \leq u(t) \leq u_{\max}} u^2 + \frac{u^*(t_0)^2 (t - \tau) u}{v_0 - v^*(t_p) + (\tau - t_0) u^*(t_0)}$$

where τ is the first time on the optimal path when $v(\tau) = v_{\max}$ if $\tau < t_p$; $\tau = t_p^*$ otherwise.

Due to space constraints, the proof is omitted but may be found in [20].

Given the terminal time $kT + DT$ and the road length l, the value of v_0 can be classified into one of the five cases as shown in Table II. Note that if Case i is infeasible for some v_0 and the given parameters, we can treat J_i^u as infinity. The performance associated with each case in Table II as well as the detailed calculations are omitted due to space constraints but may be found in [20]. After obtaining the performance for each case with $t_p = kT + DT$, we select the one with the smallest energy consumption, that is,

TABLE II

OPTIMAL SOLUTION CLASSIFICATION FOR PROBLEM 3 WITH $t_p = kT + DT$

	Optimal Control	Performance
Case I	$u^*(t) = u_{\max}$ until $v(t) = v_{\max}$ or t_p	J_1^u
Case II	$u^*(t_0) = u_{\max}$ and $v^*(t_p) = v_{\max}$	J_2^u
Case III	$u^*(t_0) = u_{\max}$ and $v^*(t_p) < v_{\max}$	J_3^u
Case IV	$u_0^* < u_{\max}$ and $v^*(t_p) = v_{\max}$	J_4^u
Case V	$u_0^* < u_{\max}$ and $v^*(t_p) < v_{\max}$	J_5^u

$$J_u^{kT+DT} = \min\{J_1^u, \ldots J_5^u\},$$

with the corresponding optimal acceleration profile.

2) Arrival Time $t_p = kT + T$: In this case, the vehicle must use less acceleration than in the free terminal time case. Depending on the initial speed v_0, there are three cases to consider. First, if $l = v_0 (kT + T - t_0)$, then the vehicle can cruise through the intersection with the constant speed v_0 without any acceleration (Case VI in Table III). The energy consumption in this case is $J_6^u = 0$. If, on the other hand, $l > v_0 (kT + T - t_0)$, then the problem can be solved using the result of the case $t_p = kT + DT$ analyzed above. Finally, if $l < v_0 (kT + T - t_0)$, then the vehicle must decelerate to reach the traffic light while in its green state. Therefore, the control input is only subject to the constraint $u_{\min} \leq u(t) \leq 0$. The main result in this case is given in the following theorem.

Theorem 3: Let $x^*(t)$, $v^*(t)$, $u^*(t)$ be an optimal solution to Problem 3 with $t_p = KT + T$. Then, the optimal solution $u^*(t)$ satisfies

$$u^*(t) = \arg \min_{u_{\min} \le u(t) \le 0} u^2 + \frac{u^*(t_0)^2 (\tau - t) u}{v^*(t_p) - v_0 - (\tau - t_0) u^*(t_0)},$$

where τ is the first time on the optimal path when $v(\tau) = v_{\max}$ if $\tau < t_p$; $\tau = t_p^*$ otherwise.
Due to space constraints, the proof is omitted but may be found in [20].

TABLE III
OPTIMAL SOLUTION CLASSIFICATION FOR PROBLEM 3 WITH $t_p = kT + T$

	Optimal Control	Performance
Case VI	$u^*(t) = 0$ and $v^*(t) = v_0$	J_6^u
Case VII	$u^*(t_0) = u_{\min}$ and $v^*(t_p) = v_{\min}$	J_7^u
Case VIII	$u^*(t_0) = u_{\min}$ and $v^*(t_p) > v_{\min}$	J_8^u
Case IX	$u^*(t_0) < u_{\min}$ and $v^*(t_p) = v_{\min}$	J_9^u
Case X	$u^*(t_0) < u_{\min}$ and $v^*(t_p) < v_{\min}$	J_{10}^u

The classification of all possible solutions with $t_p = kT + T$ is shown in Table III. The performance associated with each case in this table as well as the detailed calculations are omitted due to space constraints but may be found in [20]. After obtaining the energy consumption from J_6^u through J_{10}^u, we can select $J_u^{kT+T} = \min\{J_6^u, \ldots, J_{10}^u\}$, where J_i^u can be treated as infinity if Case i is infeasible. Finally, we can compare the two values of the performance obtained, that is,

$$\begin{aligned} J^{kT+DT} &= \rho_t (kT + DT) + \rho_u J_u^{kT+DT} \\ J^{kT+T} &= \rho_t (kT + T) + \rho_u J_u^{kT+T} \end{aligned}$$

and determine the optimal performance to be the one with a smaller value.

Remark 2: This remark pertains to some practical issues of the ECO-AND solution. First, we assume that the vehicle goes straight through the intersection (no turns). When vehicles turn at the intersection, the arrival speed should be constrained for purposes of safety and ride comfort. Second, when multiple lanes are available, we assume that the vehicle can safely change to a lane without vehicles ahead of it that might affect its free flow mode operation. Finally, when $t_p^* = kT$, the vehicle is commanded to approach a traffic light at the exact time the traffic light changes from red to green. Since this may provide discomfort to the driver and/or passengers, a safety buffer δ may be added to the start of green lights, i.e., $t_p^* = kT + \delta$.

IV. NUMERICAL EXAMPLES

We have simulated the system defined by the vehicle dynamics (1) and (2) and associated constraints and optimal control problem parameters with values given as follows. The minimum and maximum speeds are 2.78 m/s and 22.22 m/s. The maximum acceleration and deceleration are set to 2.5 m/s^2 and -2.9 m/s^2, respectively.

In the following, the weights in (3) are set using $\rho = 0.9549$, that is, $\rho_t = 0.0133$, and $\rho_u = 9.2798 \times 10^{-4}$. In this case, the values $1 - u_{\max}^2 \frac{\rho_u}{\rho_t} = 0.5630$, and $\frac{v_m + v_M}{2v_M} = 0.5626$, are almost the same. Thus, if we randomly generate

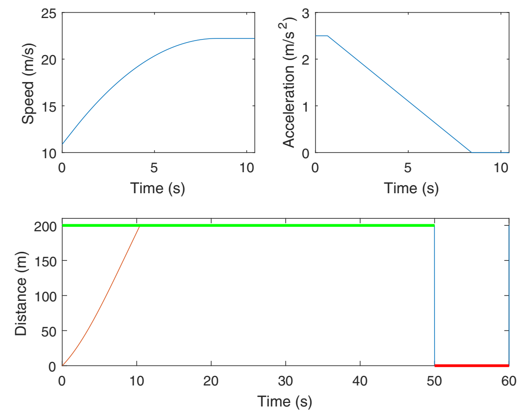

Fig. 2. Case I in Table I with $v_0 = 10.8869$.

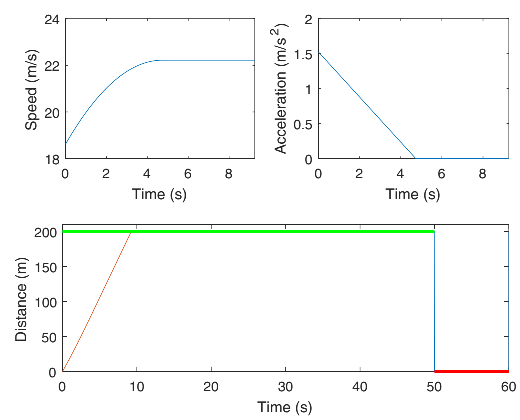

Fig. 3. Case III in Table I with $v_0 = 18.6182$.

the initial speed v_0 from a uniform distribution on the interval $[v_{\min}, v_{\max}]$, different initial speeds fall roughly equally into the two different cases in the first row in Table I. The total cycle time for the traffic light is 60 s with different patterns. We first test the optimal controller on a road of length 200 m. Figure 2 depicts the case when the initial speed is relatively low. The vehicle starts with full acceleration and, when the speed limit is reached, it switches to no acceleration. The vehicle arrives at the traffic light within the first green light cycle. When the initial speed is relatively large, the vehicle should not start with full acceleration. This is the case shown in Fig. 3.

In the above two figures, the traffic light starts at a green state. The following figure shows a case when the traffic light starts at a red state. It can be inferred from the first two plots that the arrival time obtained from the free terminal time optimal control problem should be within the red light interval. Figure 4 shows a case when the initial speed is low. The optimal arrival time obtained from the free terminal time optimal control is 12.1860 seconds. However, the traffic light in the first 40 seconds is red. The optimal time for the vehicle to arrive at the intersection is 40 seconds. The vehicle has adequate time to accelerate, therefore, it does not start with

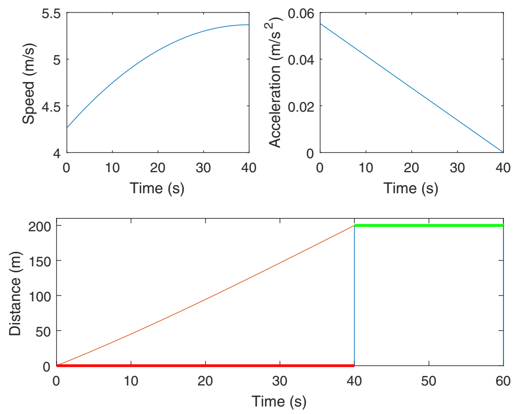

Fig. 4. Case V in Table II with $v_0 = 4.2634$.

full acceleration, and it is unnecessary to accelerate to the maximum speed.

In order to compare the performance between (i) autonomous vehicles under the optimal control developed and (ii) a human driver, we arbitrarily define the following rules as the behavior of an aggressive human driver:

- Full acceleration when the traffic light is green;
- No acceleration/deceleration when the traffic light is red.

We calculate the performance defined in (3) of both autonomous vehicles and human drivers for the different scenarios encountered from Fig. 2 to Fig. 4, and summarize the results in Table IV. The improvement is more than 10% for the case in Fig. 4. The improvement is calculated as the performance difference between the human driver and autonomous vehicle divided by the performance of the human driver. It is particularly challenging for a human driver to make a decision when he/she faces a steady red traffic light. Also note that the weighting parameter ρ is chosen to be in favor of travel time rather than energy efficiency. Therefore, the performance improvement would be larger when we decrease the weighting parameter ρ, which provides a trade-off between energy consumption and travel time.

TABLE IV
PERFORMANCE COMPARISON BETWEEN HUMAN DRIVER (HD) AND AUTONOMOUS VEHICLE (AV)

	HD	AV	Improvement
Fig. 2	0.1611	0.1574	2.3%
Fig. 3	0.1294	0.1263	2.4%
Fig. 4	0.5965	0.5310	10.98%

V. CONCLUSIONS

This paper provided the optimal acceleration/deceleration profile for autonomous vehicles approaching an intersection based on the traffic light information, which could be obtained from an intelligent infrastructure via V2I communication. The solution for the above problem had the key feature of avoiding idling at a red light. Comparing with similar problems solved by numerical calculations, we provided a real-time analytical solution. The proposed algorithm offered better performance in terms of travel time and energy consumption when compared with vehicles with aggressive human driver behavior, which has been verified through extensive simulations.

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