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Multi-Agent Coverage Control with Energy Depletion and Repletion*

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Abstract— **We develop a hybrid system model to describe the behavior of multiple agents cooperatively solving an optimal coverage problem under energy depletion and repletion constraints. The model captures the controlled switching of agents between coverage (when energy is depleted) and battery charging (when energy is replenished) modes. It guarantees the feasibility of the coverage problem by defining a guard function on each agent's battery level to prevent it from dying on its way to a charging station. The charging station plays the role of a centralized scheduler to solve the contention problem of agents competing for the only charging resource in the mission space. The optimal coverage problem is transformed into a parametric optimization problem to determine an optimal recharging policy. This problem is solved through the use of Infinitesimal Perturbation Analysis (IPA), with simulation results showing that a full recharging policy is optimal.**

I. INTRODUCTION

A coverage task is one where agents are deployed so as to cooperatively maximize the coverage of a given mission space [1], where "coverage" is measured in a variety of ways, e.g., through the joint detection probability of random events cooperatively detected by the agents. Widely used methods to solve the coverage problem include distributed gradient-based algorithms [2] and Voronoi-partition-based algorithms [3]. These approaches typically result in locally optimal solutions, hence possibly poor performance. To escape such local optima, a boosting function approach is proposed in [4] where the performance is ensured to be improved. Recently, the coverage problem was also approached by exploring the submodularity property [5] of the objective function, and a greedy algorithm is used to guarantee a provable bound relative to the optimal performance [6].

In most existing frameworks, agents are assumed to have unlimited on-board energy to perform the coverage task. However, in practice, battery-powered agents can only work for a limited time in the field [7]. Therefore, in this paper we take into account such energy constraints [8], [9], [10] and add another dimension to the traditional coverage problem. The basic setup is similar to that in [2]. Agents interact with the mission space through their sensing capabilities which are normally dependent upon their physical distance from an event location. Outside its sensing range, an agent has no ability to detect events. Unlike other multi-agent energy-aware algorithms whose purpose is to reduce energy cost [11], we assume that a charging station is available for agents to visit according to some policy. The objective is to maximize an overall environment coverage measure by controlling the movement of all agents in a cooperative manner while guaranteeing that no agent runs out of energy while in the mission space.

We provide a solution to the above problem by modeling the behavior of an agent through three different modes: coverage (Mode 1), to-charging (Mode 2), and in-charging (Mode 3). We assume that an agent has no prior knowledge of the mission space except for the location of the charging station and the positions of agents within its communication range. While in Mode 1, each agent moves along the gradient direction of the objective function at the maximum velocity so as to cooperatively maximize the coverage measure. As an agent's energy is depleted, the agent switches to Mode 2 according to a guard function designed to guarantee that a minimum energy amount is preserved to reach the charging station from its current location while traveling at maximum speed. Note that an agent shares its position and battery state information with the charging station only when it is in the to-charging mode (Mode 2). Since the charging station is shared by all agents, there can only be at most a single agent at the station at any time. Therefore, two scheduling algorithms are proposed to resolve contention among low-energy agents: (i) First-Request-First-Serve (FRFS), and (ii) Shortest-Distance-First (SDF). The charging station is perceived as a centralized controller executing a scheduling algorithm by dictating agents' speeds so that a queue is formed by agents while in Mode 2. In Mode 3, an agent is located at the charging station and a model is developed for the battery charging dynamics using the dwell time of an agent at the station as a controllable parameter to be optimized. The details for the modeling can be found in [12], and here we focus on using IPA in order to obtain the optimal dwell time of all agents at the charging station.

II. PROBLEM FORMULATION

Consider a bounded mission space $\Omega \in \mathbb{R}^2$, which is modeled as a non-self-intersecting polygon. We deploy N agents in the mission space to detect possible events that may occur in it. By viewing the position of agent i in $\mathbb{R}^2$, its coordinates $s_i = [x_i, y_i]^T$ obey the following dynamics:

$$\dot{x}_i(t) = v_i(t) \cos w_i(t), \quad (1)$$

$$\dot{y}_i(t) = v_i(t) \sin w_i(t), \quad (2)$$

with $v_i(t)$ denoting the speed, and $w_i(t)$ the heading direction of agent i. We assume that $v_i(t) \in [0, v]$, and $w_i(t) \in [0, 2\pi)$, where v is the maximum speed of an agent. The mission

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space does not contain obstacles. If it does, the problem can be modified appropriately as done in [2].

In contrast to traditional multi-agent coverage problems, agents are assumed to have a limited on-board energy supply, which is modeled by the state-of-charge $q_i(t)$ of its battery (i.e., the fraction of the battery available at time t). The dissipation of energy obeys the following dynamics:

$$\dot{q}_i(t) = -\alpha v_i^2(t) - \gamma, \tag{3}$$

where α is a scaling constant and γ is associated with the energy costs of sensing and computation. When $q_i(t)$ is negative, this implies that agent i is "dead" in the mission space.

Remark 1: In [13], the agent motion is modeled by a double integrator and the energy dynamics are modeled as $\dot{q}_i(t) = -v_i^2(t) - au_i^2(t)$, where $v_i(t)$ is the velocity and $u_i(t)$ is the acceleration. We assume that an agent's speed can be controlled directly, therefore, we do not include the acceleration in (3). The communication cost which depends on the distances from neighbor agents will be considered in future work based on: $\dot{q}_i(t) = -\alpha v_i^2(t) - \eta \sum_{j \in \mathcal{N}_i(t)} \left\| s_i(t) - s_j(t) \right\|^2$, where α and η are two scalars, $\mathcal{N}_i$ is the set of neighbors of agent i defined as $\mathcal{N}_i = \left\{ j \middle| \Omega_j \cap \Omega_i \neq \emptyset \right\}$, and Ω_i is the sensing range of agent i to be defined later.

To prevent agents from dying in the mission space, a charging station is available to all agents to replenish their energy supply during the mission time. Without loss of generality, we assume that the charging station is located at the origin with coordinates $(0,0)$. At the charging station, the charging process has the following dynamics:

$$\dot{q}_i(t) = \beta, \tag{4}$$

where $\beta > 0$ is the charging rate. We assume that only one agent can be served at the charging station at any time.

Our objective is to maximize the coverage of the mission space $\Omega \in \mathbb{R}^2$ over a time interval $[0,T]$ with T being a finite time horizon, and at the same time keep all agents alive, that is, $q_i(t) > 0$ for all $t \in [0,T]$. The case $q_i(t) = 0$ can occur only at the charging station $(0,0)$. Therefore, we consider the following optimization problem for each agent i:

$$\begin{aligned} \max_{w_i(t),\, v_i(t)} \quad & \tfrac{1}{T}\int_0^T H(\mathbf{s}(t))\,dt \\ \text{s.t.} \quad & q_i(t) \geq 0, \\ & q_i(t) > 0 \text{ when } s_i(t) \neq \mathbf{0}, \\ & (1),\ (2) \\ & 0 \leq v_i(t) \leq v, \\ & (4) \text{ if charging, } (3) \text{ otherwise} \\ & \text{if } s_i(t) = \mathbf{0}, \\ & \text{then } s_j(t) \neq \mathbf{0} \text{ for all } j \neq i \\ & i = 1,\ldots,N, \end{aligned} \tag{5}$$

where $\mathbf{s} = [s_1^T, \ldots, s_N^T]^T$ is a column vector that contains all agent positions, and

$$H(\mathbf{s}) = \int\int_{\Omega} R(x,y)\, P(x,y,\mathbf{s})\, dxdy$$

is the coverage metric. For simplicity, in what follows we assume that all points in the mission space are indistinguishable and set $R(x,y) = 1$. Here

$$P(x,y,\mathbf{s}) = 1 - \prod_{i=1}^{N} \left[1 - p_i(x,y,x_i,y_i)\right]. \tag{6}$$

Even though the precise form of the function $p_i(x,y,x_i,y_i)$ does not affect our subsequent analysis, for ease of calculation in the sequel we take it to be

$$p_i(x,y,x_i,y_i) = 1 - \frac{(x-x_i)^2 + (y-y_i)^2}{\delta_i^2}, \tag{7}$$

for all $(x,y) \in \Omega_i$. The physical meanings of $P(x,y,\mathbf{s})$ and $R(x,y)$ are discussed in [2] and [12].

Remark 2: We emphasize that the particular forms of $R(x,y)$ and $p_i(x,y,x_i,y_i)$ in (7) are only adopted for ease of calculation. It is worth noting that the optimal coverage theory applies to any reasonable $R(x,y)$ and $p_i(x,y,x_i,y_i)$, such as

$$p_i(x,y,x_i,y_i) = \alpha_i \exp\left[-\beta_i \sqrt{(x-x_i)^2 + (y-y_i)^2}\right]$$

used in [2], where $0 < \alpha_i \leq 1$ and $\beta_i > 0$ are sensing parameters.

Returning to problem (5), there are two challenges we face. First, recall that an agent has no prior knowledge of either the mission space or the battery levels of other agents; it only knows the location of the charging station and of its neighbors. In addition, the charging station is only provided with the location and battery state information of agents when they are in the to-charging mode. Under this information structure, it is clearly impossible to tackle the coverage problem in a centralized way. The second challenge stems from the fact that, unlike the traditional coverage problem in [2] where the goal is to find the optimal equilibrium locations of agents, (5) is a *dynamic* multi-agent coverage problem: due to the energy dynamics and constraints in (5), such an equilibrium may never exist, as agents move back and forth between coverage and battery charging modes. Thus, in general, finding the optimal speed $v_i^*(t)$ and the optimal heading $w_i^*(t)$ in problem (5) for all $i = 1,\ldots,N$ and all t is a challenging task since its solution amounts to a notoriously hard two-point-boundary-value problem similar to other dynamic multi-agent optimization problems, e.g., see [14]. In the following, we will show how to solve this problem by modeling the combined cooperative coverage-recharging processes as a hybrid system to obtain a sub-optimal solution.

III. HYBRID SYSTEM MODEL

Our first step is to construct a hybrid system model to guarantee that the constraints in (5) are satisfied for all t. To ensure that the problem is well-posed, we assume that

$$\beta \geq N(\alpha v^2 + \gamma). \tag{8}$$

This assumption is sufficient to guarantee the feasibility of the hybrid system model to be constructed. In particular, by treating the charging station as a server, the charging rate is β

if it is occupied at all times, and referring to (3), the worst-case energy depletion rate over all agents is $N(\alpha v^2+\gamma)$. Thus, the condition (8) is sufficient to prevent any agent from running out of energy (dying) anywhere in the mission space. However, this assumption is not necessary in the sense that the problem may be feasible even when (8) is not satisfied.

For any agent, we define three different modes: coverage (Mode 1), to-charging (Mode 2) and in-charging (Mode 3). This hybrid system consists of a single cycle for each agent: Mode 1→Mode 2→Mode 3→Mode 1 and detailed next.

At Mode 1, $v_i(t)=v$ (the maximum speed is optimal for each agent in Mode 1 which can be easily shown by applying the Pontryagin's maximum principle to the optimal coverage problem without energy constraints), and

$$\cos w_i(t)=\frac{\frac{\partial H(t)}{\partial x_i(t)}}{\sqrt{\left(\frac{\partial H(t)}{\partial x_i(t)}\right)^2+\left(\frac{\partial H(t)}{\partial y_i(t)}\right)^2}}, \tag{9}$$

$$\sin w_i(t)=\frac{\frac{\partial H(t)}{\partial y_i(t)}}{\sqrt{\left(\frac{\partial H(t)}{\partial x_i(t)}\right)^2+\left(\frac{\partial H(t)}{\partial y_i(t)}\right)^2}}, \tag{10}$$

where the calculations of detailed expressions for $\frac{\partial H(t)}{\partial x_i(t)}$ and $\frac{\partial H(t)}{\partial y_i(t)}$ are given in [12]. We rewrite the dynamics in (3) as

$$\dot{q}_i(t)=-\alpha v^2-\gamma. \tag{11}$$

In other words, agent i travels at the maximum speed, and the heading direction follows the gradient direction of the coverage metric with respect to agent i's location. The state-of-charge of the battery monotonically decreases with rate $\alpha v^2+\gamma$ and when it drops to a certain value, the agent switches to Mode 2.

A transition from Mode 1 to Mode 2 occurs when the guard function $g_i(s_i,q_i)=q_i^2(t)-v^2\alpha^2\|s_i(t)\|^2$ is zero, where $\|s_i(t)\|=\sqrt{x_i^2(t)+y_i^2(t)}$. At Mode 2, the speed $v_i(t)$ is determined by the scheduling algorithm used to assign an agent to the charging station and the heading direction is constant and determined by the location of agent i at the time of switching from Mode 1 to Mode 2, say τ_2. The details of the FRFS and SDF scheduling algorithms are discussed in [12], [15]. Then, the motion dynamics and the state-of-charge dynamics are:

$$\dot{x}_i(t)=-v_i(t)\frac{x_i(\tau_2)}{\|s_i(\tau_2)\|},\quad \dot{y}_i(t)=-v_i(t)\frac{y_i(\tau_2)}{\|s_i(\tau_2)\|}, \tag{12}$$

$$\dot{q}_i(t)=-\alpha v_i^2(t)-\gamma. \tag{13}$$

The speed $v_i(t)$ in Mode 2 is piecewise constant or constant depending on which scheduling algorithm is used to resolve conflicts when multiple agents request to use the charging station at the same time (note that we assume no energy loss at points where the speed may experience a jump).

A transition from Mode 2 to Mode 3 occurs when agent i arrives at the charging station and the guard function $g_i(s_i)=\|s_i(t)\|^2$ is zero. At Mode 3, an agent remains at rest at the charging station, therefore, it satisfies the dynamics

$$\dot{x}_i(t)=0, \dot{y}_i(t)=0. \tag{14}$$

While the agent is in charging mode, the state-of-charge dynamics are given by $\dot{q}_i(t)=\beta$, where $\beta\geq N(\alpha v^2+\gamma)$ is the charging rate.

Finally, a transition from Mode 3 to Mode 1 occurs when the guard function $g_i(q_i)=\theta_i-q_i(t)$ is zero, where $\theta_i\in(0,1]$ is a controllable threshold parameter indicating the desired state-of-charge at which the agent may stop its recharging process.

Remark 3: The energy consumption of sensing and computation is relatively small compared with motion and communication. In the rest of the paper, we assume that $\gamma=0$ and agents still perform the coverage task while they are in Modes 2 and 3 without costing extra energy. Our approach also is able to deal with the case that an agent turns off its sensing and communication functionalities while it is in either Mode 2 or Mode 3. In this case, the coverage problem applies to fewer agents. Therefore, the energy costs in Mode 2 are only related to the agent's speed. It is worth noting that the hybrid model does not rely on a detailed energy depletion model of the state-of-charge in Mode 1.

The results on feasibility and rationality of the proposed hybrid model can be found in [12]. Here we only show the results on schedulability and optimality of the parameter θ.

Remark 4: This remark pertains to the centralization/decentralization of the proposed method. In Mode 1, we solve the basic coverage maximization problem by using gradient-based methods which are distributed as shown in [2] even though it seems that the heading angle depends on all agents through $H(\mathbf{s})$ in (9) and (10). This follows from the fact that a truncated sensing model limits the gradient calculation of the objective function by agent i to its own sensing range and its neighbors. In Mode 2, an agent locally determines its trajectory, but its speed depends on the scheduling algorithm which requires some degree of centralized coordination over all agents which are in Mode 2. In Mode 3, the recharging process is decentralized once the optimal recharging level is found by using the IPA methodology as shown in the next section.

IV. Main Results

We now address the question of selecting an optimal charging level, denoted by $\theta=[\theta_1,\ldots,\theta_N]$, when an agent is in the charging mode. This problem boils down to optimizing the parameter θ so that the objective function in (5) is maximized. By writing explicitly the dependence on θ, the optimization problem becomes

$$J(\theta)=\max_{\theta}\frac{1}{T}\int_0^T H(s(\theta,t))\,dt.$$

Even though θ is only used in Mode 3, its optimal value affects the entire hybrid system model. By controlling θ, we directly control the switching times of agents from Mode 3 to Mode 1, and indirectly control the switching times of agents from Mode 1 to Mode 2. The switching times of agents from Mode 2 to Mode 3 are controlled by the proposed scheduling algorithms. Also note that the parameter θ is constant. We can obtain optimal charging thresholds through off-line analysis and implement the coverage task on line by all agents in distributed fashion. We use IPA [16] to determine the optimal θ.

Before proceeding, we briefly review the IPA framework for general stochastic hybrid systems as presented in [16], which plays an instrumental role in obtaining the optimal dwell time of all agents at the charging station.

Let $\{\tau_k(\theta)\}$, $k = 1,\ldots,K$, denote the occurrence times of all events in the state trajectory of a hybrid system with dynamics $\dot{x} = f_k(x,\theta,t)$ over an interval $[\tau_k(\theta), \tau_{k+1}(\theta))$, where $\theta \in \Theta$ is some parameter vector and Θ is a given compact, convex set. For convenience, we set $\tau_0 = 0$ and $\tau_{K+1} = T$. We use the Jacobian matrix notation: $x'(t) \equiv \frac{\partial x(\theta,t)}{\partial \theta}$ and $\tau_k' \equiv \frac{\partial \tau_k(\theta)}{\partial \theta}$, for all state and event time derivatives. It is shown in [16] that

$$\frac{d}{dt}x'(t) = \frac{\partial f_k(t)}{\partial x}x'(t) + \frac{\partial f_k(t)}{\partial \theta}, \tag{15}$$

for $t \in [\tau_k, \tau_{k+1})$ with boundary condition:

$$x'(\tau_k^+) = x'(\tau_k^-) + [f_{k-1}(\tau_k^-) - f_k(\tau_k^+)]\tau_k', \tag{16}$$

for $k = 0,...,K$. In order to complete the evaluation of $x'(\tau_k^+)$ in (16), we need to determine τ_k'. We classify events into two categories. An event is exogenous if it causes a discrete state transition at time τ_k independent of the controllable vector θ and, therefore, satisfies $\tau_k' = 0$. Otherwise, the event is *endogenous* and there exists a continuously differentiable function $g_k : \mathbb{R}^n \times \Theta \to \mathbb{R}$ such that $\tau_k = \min\{t > \tau_{k-1} : g_k(x(\theta,t),\theta) = 0\}$ and

$$\tau_k' = -[\frac{\partial g_k}{\partial x} f_k(\tau_k^-)]^{-1}(\frac{\partial g_k}{\partial \theta} + \frac{\partial g_k}{\partial x}x'(\tau_k^-)) \tag{17}$$

as long as $\frac{\partial g_k}{\partial x} f_k(\tau_k^-) \neq 0$ (details may be found in [16]).

Denote the time-varying cost along a given trajectory as $L(x,\theta,t)$, so the cost in the k-th inter-event interval is $J_k(x,\theta) = \int_{\tau_k}^{\tau_{k+1}} L(x,\theta,t)dt$ and the total cost is $J(x,\theta) = \sum_{k=0}^{K} J_k(x,\theta)$. Differentiating and applying the Leibniz rule with the observation that all terms of the form $L(x(\tau_k),\theta,\tau_k)\tau_k'$ are mutually canceled with $\tau_0 = 0, \tau_{K+1} = T$ fixed, we obtain

$$\begin{aligned}\frac{\partial J(x,\theta)}{\partial \theta} &= \sum_{k=0}^{K} \frac{\partial}{\partial \theta}\int_{\tau_k}^{\tau_{k+1}} L(x,\theta,t)dt \\ &= \sum_{k=0}^{K}\int_{\tau_k}^{\tau_{k+1}} \frac{\partial L(x,\theta,t)}{\partial x}x'(t) + \frac{\partial L(x,\theta,t)}{\partial \theta}dt. \end{aligned} \tag{18}$$

Now let us return to our problem and define the following notations

$$\tau_k' = \frac{\partial \tau_k(\theta)}{\partial \theta}, \quad \mathbf{x}_i' = \frac{\partial x_i(\theta)}{\partial \theta}, \quad \mathbf{y}_i' = \frac{\partial y_i(\theta)}{\partial \theta}$$

which are row vectors, and

$$\begin{aligned}\frac{\partial \mathbf{x}(\theta,t)}{\partial \theta} &= \left[\mathbf{x}_1'(\theta,t)^T, \cdots, \mathbf{x}_N'(\theta,t)^T\right]^T, \\ \frac{\partial \mathbf{y}(\theta,t)}{\partial \theta} &= \left[\mathbf{y}_1'(\theta,t)^T, \cdots, \mathbf{y}_N'(\theta,t)^T\right]^T,\end{aligned}$$

are matrices.

Let us assume that all agents start with the battery level

$$q_i(0) > v\alpha \|s_i(0)\|,$$

for $i = 1,\ldots,N$, that is, all agents start with Mode 1.

For $t \in [\tau_1, \tau_2)$, applying (15) to (9) and (10) yields that

$$\frac{d}{dt}\mathbf{x}_i'(\theta,t) = v\frac{\partial \cos w_i(t)}{\partial \mathbf{x}(\theta,t)}\frac{\partial \mathbf{x}(\theta,t)}{\partial \theta} + v\frac{\partial \cos w_i(t)}{\partial \mathbf{y}(\theta,t)}\frac{\partial \mathbf{y}(\theta,t)}{\partial \theta}, \tag{19}$$

$$\frac{d}{dt}\mathbf{y}_i'(\theta,t) = v\frac{\partial \sin w_i(t)}{\partial \mathbf{x}(\theta,t)}\frac{\partial \mathbf{x}(\theta,t)}{\partial \theta} + v\frac{\partial \sin w_i(t)}{\partial \mathbf{y}(\theta,t)}\frac{\partial \mathbf{y}(\theta,t)}{\partial \theta}, \tag{20}$$

where the detailed calculations of $\frac{\partial \cos w_i(t)}{\partial \mathbf{x}(\theta,t)}$, $\frac{\partial \cos w_i(t)}{\partial \mathbf{y}(\theta,t)}$, $\frac{\partial \sin w_i(t)}{\partial \mathbf{x}(\theta,t)}$, and $\frac{\partial \sin w_i(t)}{\partial \mathbf{y}(\theta,t)}$ are given in [15]. Note that for agents $j \notin \mathscr{N}_i$,

$$\frac{\partial \cos w_i(t)}{\partial x_j(t)} = 0, \quad \frac{\partial \sin w_i(t)}{\partial x_j(t)} = 0.$$

For the state-of-charge, we have

$$\frac{d}{dt}\mathbf{q}_i'(\theta,t) = \mathbf{0},$$

by applying (15) to (11), which implies that $\mathbf{q}_i'(\theta,\tau_2^-) = \mathbf{q}_i'(\theta,\tau_1^+)$. By solving the differential equations (19) and (20), we can obtain $\mathbf{x}_i'(\theta,\tau_2^-)$ and $\mathbf{y}_i'(\theta,\tau_2^-)$.

At τ_2, the guard condition

$$\begin{aligned}&g_i(x_i(\theta,\tau_2), y_i(\theta,\tau_2), q_i(\theta,\tau_2)) \\ =&q_i^2(\theta,\tau_2) - v^2\alpha^2\|s_i(\theta,\tau_2)\|^2 = 0.\end{aligned}$$

This is an endogenous event. By applying (17) to the above guard function and the dynamics in (9), (10) and (11), we have

$$\tau_2' = \frac{q_i(\tau_2)\mathbf{q}_i'(\tau_2^-) - v^2\alpha^2\left[x_i(\tau_2)\mathbf{x}_i'(\tau_2^-) + y_i(\tau_2)\mathbf{y}_i'(\tau_2^-)\right]}{\alpha v^2 q_i(\tau_2) + v^3\alpha^2\left[x_i(\tau_2)\cos w_i(\tau_2^-) + y_i(\tau_2)\sin w_i(\tau_2^-)\right]}$$

with the boundary conditions

$$\begin{aligned}\mathbf{q}_i'(\tau_2^+) &= \mathbf{q}_i'(\tau_2^-) + \alpha\left[v_i^2(\tau_2^+) - v^2\right]\tau_2' \\ \mathbf{x}_i'(\tau_2^+) &= \mathbf{x}_i'(\tau_2^-) + \left[v\cos w_i(\tau_2^-) - v_i(\tau_2^+)\cos w_i(\tau_2^+)\right]\tau_2', \\ \mathbf{y}_i'(\tau_2^+) &= \mathbf{y}_i'(\tau_2^-) + \left[v\sin w_i(\tau_2^-) - v_i(\tau_2^+)\sin w_i(\tau_2^+)\right]\tau_2',\end{aligned}$$

which are obtained by applying (16) to the dynamics in (12) and (13).

Remark 5: Irrespective of the scheduling algorithm, if agent i is the first to request charging in the current queue, then $v_i(\tau_2^+) = v$, and $\mathbf{q}_i'(\theta,\tau_2^+) = \mathbf{q}_i'(\theta,\tau_2^-) = \mathbf{q}_i'(\theta,\tau_1^+)$.

In Mode 2, the right-hand sides of (12) and (13) are constant or piecewise constant depending on the scheduling algorithm. Therefore, we have

$$\frac{d}{dt}\mathbf{x}_i'(\theta,t) = 0, \quad \frac{d}{dt}\mathbf{y}_i'(\theta,t) = 0, \quad \frac{d}{dt}\mathbf{q}_i'(\theta,t) = 0,$$

according to (15). It is easy to see that

$$\mathbf{x}_i'(\theta,\tau_3^-) = \mathbf{x}_i'(\theta,\tau_2^+), \quad \mathbf{y}_i'(\theta,\tau_3^-) = \mathbf{y}_i'(\theta,\tau_2^+), \tag{21}$$

$$\mathbf{q}_i'(\theta,\tau_3^-) = \mathbf{q}_i'(\theta,\tau_2^+). \tag{22}$$

In the SDF scheduling algorithm, the velocity of agent i may be adjusted due to the competition to the charging station. This is the case when agent j, which is closer to the charing station than agent i, requests for the charging service. Such events are independent of θ and are, therefore, treated as exogenous events. In Mode 2, the relationships (21) and (22) hold independent of the scheduling methods, and the number of exogenous events.

At time τ_3, the guard function $g_i(x_i(\theta,\tau_3), y_i(\theta,\tau_3)) = \|s_i(\tau_3)\|^2 = 0$. This is an endogenous event. According to (17), we can calculate

$$\tau_3' = -\frac{x_i(\theta,\tau_3)\mathbf{x}_i'(\theta,\tau_3^-) + y_i(\theta,\tau_3)\mathbf{y}_i'(\theta,\tau_3^-)}{x_i(\theta,\tau_3)v_i(\tau_3^-)\cos w_i(\tau_3^-) + y_i(\theta,\tau_3)v_i(\tau_3^-)\sin w_i(\tau_3^-)}$$

based on the dynamics (12) and (13) and the boundary conditions are

$$\begin{aligned}
\mathbf{x}_i'(\theta,\tau_3^+) &= \mathbf{x}_i'(\theta,\tau_3^-) + v_i(\tau_3^-)\cos w_i(\tau_3^-)\tau_3' \\
\mathbf{y}_i'(\theta,\tau_3^+) &= \mathbf{y}_i'(\theta,\tau_3^-) + v_i(\tau_3^-)\sin w_i(\tau_3^-)\tau_3' \\
\mathbf{q}_i'(\tau_3^+) &= \mathbf{q}_i'(\theta,\tau_3^-) - \left[\alpha v_i^2(\tau_3^-) + \beta\right]\tau_3'
\end{aligned}$$

by applying (16) to the dynamics in (14), and (4).

Remark 6: When calculating τ_3', we find that both the numerator and denominator are zero due to $x_i(\theta,\tau_3) = y_i(\theta,\tau_3) = 0$. In this case, the value of τ_3' is calculated according to its limit in the direction $w_i(\tau_3^-)$. Let us put x_i and y_i in the polar coordinate, then $x_i = r\cos w_i(\tau_3^-)$ and $y_i = r\sin w_i(\tau_3^-)$. Replacing x_i and y_i in τ_3', it becomes

$$\begin{aligned}
\tau_3' &= -\lim_{r\to 0}\frac{r\cos w_i(\tau_3^-)\mathbf{x}_i'(\theta,\tau_3^-) + r\sin w_i(\tau_3^-)\mathbf{y}_i'(\theta,\tau_3^-)}{rv_i(\tau_3^-)\cos w_i^2(\tau_3^-) + rv_i(\tau_3^-)\sin w_i^2(\tau_3^-)} \\
&= -\frac{\cos w_i(\tau_3^-)\mathbf{x}_i'(\theta,\tau_3^-) + \sin w_i(\tau_3^-)\mathbf{y}_i'(\theta,\tau_3^-)}{v_i(\tau_3^-)}.
\end{aligned}$$

Note that in Mode 2, agents do not change their direction and $w_i(\tau_3^-) = w_i(\tau_2^+)$.

In Mode 3, during the cycle $[\tau_3, \tau_1)$, we can obtain

$$\frac{d}{dt}\mathbf{x}_i'(\theta,t) = 0, \quad \frac{d}{dt}\mathbf{y}_i'(\theta,t) = 0, \quad \frac{d}{dt}\mathbf{q}_i'(\theta,t) = 0$$

by applying (15) to the dynamic equations (14) and (4). Therefore, it is easy to calculate

$$\begin{aligned}
&\mathbf{x}_i'(\theta,\tau_1^-) = \mathbf{x}_i'(\theta,\tau_3^+), \quad \mathbf{y}_i'(\theta,\tau_1^-) = \mathbf{y}_i'(\theta,\tau_3^+) \\
&\mathbf{q}_i'(\theta,\tau_1^-) = \mathbf{q}_i'(\theta,\tau_3^+).
\end{aligned}$$

At time τ_1, the threshold

$$g_i(q_i(\theta,\tau_1)) = q_i(\theta,\tau_1) - \theta_i = 0.$$

This is an endogenous event. We can obtain

$$\tau_1' = \frac{1 - \mathbf{q}_i'(\theta,\tau_1^-)}{\beta},$$

and the boundary conditions

$$\begin{aligned}
\mathbf{x}_i'(\theta,\tau_1^+) &= \mathbf{x}_i'(\theta,\tau_1^-) - v\cos w_i(\tau_1^+)\tau_1' \\
\mathbf{y}_i'(\theta,\tau_1^+) &= \mathbf{y}_i'(\theta,\tau_1^-) - v\sin w_i(\tau_1^+)\tau_1' \\
\mathbf{q}_i'(\theta,\tau_1^+) &= \mathbf{q}_i'(\theta,\tau_1^-) + (\beta + \alpha v^2)\tau_1',
\end{aligned}$$

according to (17) and (16), respectively, based on the dynamics in (14), (4), (9), (10) and (11). Now the IPA derivative of $dJ/d\theta$ can be obtained by taking derivatives of $J(\theta)$ with respective to θ as shown in (18):

$$\frac{dJ}{d\theta} = \sum_{k=0}^{l}\frac{d}{d\theta}\int_{t_k}^{t_{k+1}} H_k(s,\theta,t)\,dt$$

and applying the Leibnitz rule we obtain, for every $k = 0,\ldots,l$,

$$\begin{aligned}
&\frac{d}{d\theta}\int_{t_k}^{t_{k+1}} H_k(s,\theta,t)\,dt \\
= \;& \int_{t_k}^{t_{k+1}}\left[\frac{\partial H_k(s,\theta,t)}{\partial \mathbf{x}}\mathbf{x}' + \frac{\partial H_k(s,\theta,t)}{\partial \mathbf{y}}\mathbf{y}'\right]dt \\
&+ H_k(s(t_{k+1}),\theta,t_{k+1})\mathbf{t}_{k+1}' - H_k(s(t_k),\theta,t_k)\mathbf{t}_k'
\end{aligned}$$

where t_k are event times of any agents, $t_0 = 0$ and $t_l = T$. The parameter θ is updated as

$$\theta_{n+1} = \theta_n + \lambda_n\frac{dJ(\theta_n)}{d\theta_n}, \tag{23}$$

where $\{\lambda_n\}$ is a step size sequence.

V. SIMULATION RESULTS

In this section, we illustrate the optimization process in (23), and compare the performance by using different scheduling algorithms (FRFS, and SDF).

The mission space is a 60 by 50 rectangular area without obstacles. We consider a team of four agents with initial locations (2,2), (4,4), (6,6) and (8,8). The initial state-of-charge variables are randomly generated, which are 97%, 48%, 71%, and 46%, respectively. The maximum speed is $v = 5$, and the sensing range $\delta_i = 22$ for all $i = 1,\ldots,4$. The parameter $\alpha = 0.0001$, and $\beta = 4\alpha v^2 = 0.01$. Figures 1 and 2 show the evolution of θ under the FRFS and SDF scheduling algorithms, respectively, where the step size sequence $\{\lambda_n\}$ over iterations $n = 0,1,\ldots$, is chosen as $\{(\|\frac{dJ(\theta_n)}{d\theta_n}\|n^{\frac{3}{2}})^{-1}\}$. It can be seen from both figures that it is optimal to fully charge the battery for both scheduling algorithms. The simulation runs for $T = 5400$ by considering the optimal $\theta = 1$. The comparison of the coverage performance between different scheduling algorithms is depicted in Fig. 3. The coverage performance is $J(\theta) = 186407$ for FRFS, and $J(\theta) = 186095$ for SDF, respectively. The difference between the performance of the two scheduling algorithms is within 0.1%. Therefore, no general conclusions can be drawn on which scheduling algorithm is better, even though we might expect SDF to be preferable because it uses the distance information compared to FRFS.

A visual interactive simulation can be found at `http://www.bu.edu/codes/simulations/Coverage_ADHS`. Interested readers are encouraged to interact with the simulation by choosing different scheduling algorithms, as well as adjusting parameters such as the number of agents N, the sensing range δ_i, or the maximum speed v.

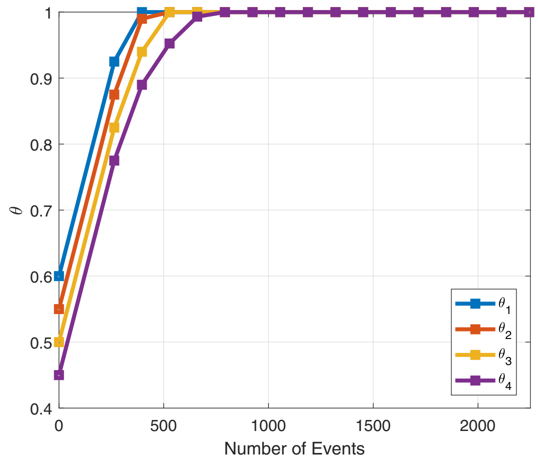

Fig. 1: The evolution of θ under the FRFS scheduling method

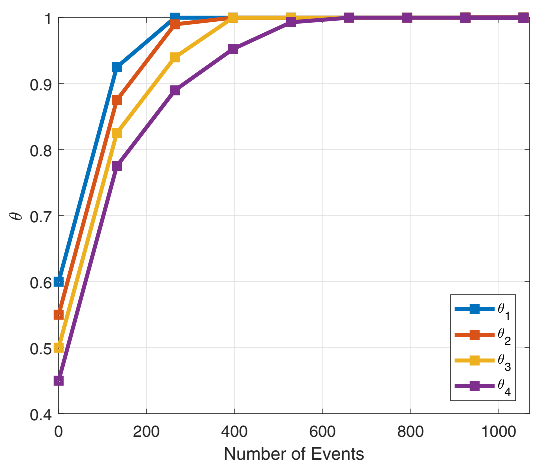

Fig. 2: The evolution of θ under the SDF scheduling method

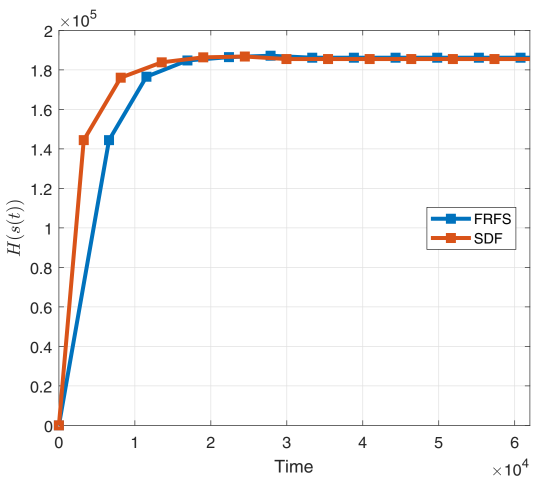

Fig. 3: The comparison of performance using different scheduling algorithms

VI. CONCLUSIONS

A hybrid system model is proposed to capture the behavior of multiple agents cooperatively solving an optimal coverage problem under energy depletion and repletion constraints. The proposed model links each agent's coverage, to-charging, and in-charging modes so as to form a cycle and the guard conditions are designed to maximize the coverage performance over a finite time horizon as well as to ensure that the agents never run out of energy. Full repletion is optimal to maximize the coverage objective function as shown by numerical calculations using IPA; and a theoretical proof is the subject of ongoing research. We are also working on the inclusion of energy expended for communication among agents (see Remark 1).

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