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On Maximum Lifetime Routing in Wireless Sensor Networks

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Abstract— **Lifetime maximization is an important optimization problem specific to Wireless Sensor Networks (WSNs) since they operate with limited energy resources which are therefore eventually depleted. This paper considers first the problem of routing in a WSN with the objective of lifetime maximization based on a simple model for battery dynamics. Specifically, we discuss the equivalence of two different formulations and solutions in the existing literature. We then revisit a related problem, the optimal allocation of a total energy amount over all nodes so as to maximize network lifetime. We prove that this is equivalent to a shortest path problem on a weighted graph and can therefore be efficiently solved. Finally, we present a more realistic model for battery dynamics, and numerically solve the lifetime maximization problem. The empirical results obtained indicate that, while a static routing policy is not expected to be optimal, such a policy is a good approximation of the optimal dynamic routing policy.**

I. Introduction

A Wireless Sensor Network (WSN) is a spatially distributed wireless network consisting of low-cost autonomous nodes which are mainly battery powered and have sensing and wireless communication capabilities [1]. Power consumption is a key issue in WSNs, since it directly impacts their lifetime in the likely absence of human intervention for most applications of interest. According to [2], the majority of power consumption is due to the radio component. Due to limited on-board power, nodes rely on short-range communication and form a multi-hop network to deliver information to a base station. Routing can be a challenging problem in WSNs. It aims to deliver data from the data sources (e.g., sensors) to a data sink (e.g., base station) in an energy-efficient and reliable way. A survey of state-of-the-art routing algorithms is provided in [3]. One of the non-standard metrics of interest is the network lifetime which we seek to maximize. First, this is specifically intended for battery-powered networks such as a WSN. Second, there has been no firm definition of the term "lifetime" for such a network. For example, while some researchers, e.g., [4] define the network lifetime as the time until the first node depletes its battery, it may just as well be defined as the time until the data source cannot reach the data sink [5].

The lifetime maximization problem in WSNs falls within the category of "energy-aware" routing problems. Early work on energy-aware routing problems, e.g. [6] and [7], focuses on finding routes that result in low cost or high residual battery energy. An explicit lifetime maximization problem is formulated in [4] in the form of a linear programming (LP) problem. Approximation heuristics that can be solved distributively in the network are also provided. Starting from a different perspective – a model for the battery dynamics, [8] formulates an optimal control problem for lifetime maximization with probabilistic routing.

Our contribution in this paper is to extend the results developed in [8] in three directions. First, we show that we can transform the set of NLP subproblems into the LP formulation in [4]. Second, we show that the initial energy allocation problem can be reformulated into a shortest path problem on a graph where the arc weights equal the link energy costs. This allows us to solve the initial energy allocation problem using existing efficient network flow algorithms, such as Dijkstra's algorithm [9]. Third, we look into the lifetime-maximization problem with more realistic battery models, where some interesting first results are obtained.

The paper is organized as follows. In Section II, we review the lifetime maximization problems in [4] and [8], as well as the initial energy allocation problem in [8]. In Section III we show that although different in their forms, the two lifetime maximization problems are equivalent. In Section IV we show that the initial energy allocation problem can be further reduced to a shortest path problem. In Section V we introduce new, more realistic battery models, as well as new approaches to solve the problem. Conclusions are given in Section VI.

II. Review of Previous results

A. Probabilistic routing model

Consider a simple WSN with single-class data, a single source and a single sink. Assume the WSN has $N+1$ nodes, numbered $0, ..., N$. Let node N be the data sink (base station), and let node 0 be the data source. We assume that nodes $1, ..., N-1$ are located in the area between the source node 0 and the data sink N, and are ordered according to the distance to the sink N. That is, denoting by d_{ij} the distance between node i and j, we have $d_{iN} > d_{jN}$ if $i < j$. We assume that any node i will forward data to node j only if $i < j$.

In [4] and other routing algorithms, the routing control is not at a dynamic, per-packet level. Given the information generation rate, the routing algorithms determine the flow rate on each link so as to maximize the lifetime. On the per-packet level, one way to implement such flow rate is to use probabilistic routing. At each node i, $p_{ij}(t)$ is the probability of forwarding the information to node j, and $\sum_{i<j\leq N} p_{ij}(t) = 1$. Then, $\frac{1}{T}\int_0^T p_{ij}(t)\,dt$ equals the proportion of flow going through node i that enters link (i, j).

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As pointed out in [8], probabilistic routing has a security advantage over cost based fixed routing when facing attacks. For example, an intruder may falsify cost information such that it becomes a packet "sink-hole" enticing all neighboring nodes to direct their output to the intruder. The neighboring nodes will also broadcast the low cost, thus attracting more flow to the intruder. Probabilistic routing allows portions of the flow to go through different paths, thus diversifying the risk.

B. LP formulation

In [4], a linear programing (LP) formulation for lifetime maximization is proposed. The lifetime T of a WSN is defined as the first time a node runs out of battery. The assumption is that the energy in a battery depletes linearly with respect to the quantity of information forwarded, and does not depend on the chemical dynamics of the battery itself. The LP formulation in [4] can accommodate multiple classes and multiple data sources/sinks. We apply the LP formulation to the aforementioned network where there exists only a single source and single class.

Define q_{ij} as the amount of information (bits) transferred from node i to node j over the lifetime T. Denote by e_{ij}^{t} the energy (Joules) needed for node i to transmit 1 bit to node j, and e_{ji}^{r} is the energy needed for node i to receive 1 bit from node j. Both e_{ij}^{t} and e_{ji}^{r} are assumed known for each link (i,j). Generally, e_{ij}^{t} is an increasing function of the distance between node i and node j, and e_{ji}^{r} is usually constant (denoted by e^{r}). Denote by E_i the known initial amount of energy deposit at node i and R_0 the information generation rate (bits/seconds) at the source node. The LP problem is formulated as follows:

Problem 1 ([4]):

$$\max_{\{q_{ij},T\}} T$$

$$\text{s.t.} \sum_{j>i} e_{ij}^{t} q_{ij} + \sum_{j:i<j} e_{ji}^{r} q_{ji} \leq E_i, \quad 0 \leq i \leq N-1 \tag{1}$$

$$\sum_{j>i} q_{ij} - \sum_{j<i} q_{ji} = 0, \quad 1 \leq i \leq N-1 \tag{2}$$

$$\sum_{j>0} q_{0j} = TR_0 \tag{3}$$

$$q_{ij} \geq 0, \quad 0 \leq i < j \leq N$$

In Problem 1, $\{q_{ij}, T\}$ are control variables. (1) is the constraint of energy usage at each node. (2) is the flow conservation constraint at non-source nodes, and (3) specifies the flow generation at the source node. Problem 1 does not attempt to solve for the routing probabilities directly. Instead, it solves for the total quantity of information one node sends to another. From the routing probability point of view, any probabilistic routing vector $\{p_{ij}(t)\}$, constant or time-variant, can be optimal as long as the total quantity transferred from node i to node j is q_{ij}^{*}:

$$\int_0^T p_{ij}(t) \left(\sum_{k>i} q_{ik}^{*} \right) dt = q_{ij}^{*}$$

Therefore, the simplest way is to construct static routing probabilities $\{p_{ij}^{*}\}$ by normalizing $\{q_{ij}^{*}\}$:

$$p_{ij}^{*} = \frac{q_{ij}^{*}}{\sum_{k>i} q_{ik}^{*}}, \quad 0 \leq i \leq N-1$$

that is, p_{ij}^{*} is the proportion of data forwarded from i to j.

C. Optimal control formulation

In [8] the lifetime maximization problem is studied from a different perspective, that is, using the dynamics of the batteries in a WSN. This is a more general approach than Problem 1 because it accommodates the network dynamics in a more detailed way, modeled through differential equations. From this point of view, it is not obvious that a static routing probability policy suffices to be the optimal routing policy. In modeling the battery dynamics, [8] used a simple model, that is, the rate of energy consumption is proportional to the load at the node. In this model, the battery is treated like a simple linear energy storage device, omitting any internal chemical reaction. This model is essentially the same as the one assumed in [4].

Define $r_i(t)$ as the residual energy of node i at time t and $G_i(t)$ as the rate of inflow information at node i. Let v_{ij} be:

$$\begin{aligned} v_{ij} &= e_{ij}^{t} - e_{iN}^{t}, \quad i < j < N \\ v_{0N} &= e_{0N}^{t} \\ v_{iN} &= e_{iN}^{t} + e^{r} \end{aligned}$$

where e_{ij}^{t} and e^{r} are the communication parameters used in Problem 1. Let $p_{ij}(t)$ be the instantaneous routing probability at time t. The lifetime maximization problem is formulated as an optimal control problem as follows:

Problem 2 ([8]):

$$\min_{\{r_i(t),T,G_i(t),p_{ij}(t)\}} \int_0^T -1 dt$$

$$\text{s.t. } \dot{r}_i(t) = -G_i(t) \left(\sum_{i<j<N} p_{ij}(t) v_{ij} + v_{iN} \right), \tag{4}$$

$$0 \leq i \leq N-1$$

$$G_i(t) = \sum_{0 \leq j < i} p_{ji}(t) G_j(t), \tag{5}$$

$$1 \leq i \leq N-1$$

$$G_0(t) = R_0 \tag{6}$$

$$\sum_{i<j\leq N} p_{ij}(t) = 1, \quad 0 \leq i \leq N-1 \tag{7}$$

$$p_{ij}(t) \geq 0, \quad 0 \leq i < j \leq N$$

with boundary conditions specifying the initial energy and the terminal condition:

$$r_i(0) = E_i, \quad 0 \leq i \leq N-1$$

$$\min_i r_i(T) = 0, \quad 0 \leq i \leq N-1 \tag{8}$$

In Problem 2, the control variables are $\{r_i(t), T, G_i(t), p_{ij}(t)\}$. (4) captures the battery dynamics. (5) is the flow conservation constraint at non-source nodes, and (6) specifies the flow generation at the source node. (7) is the normalization constraint for routing probabilities. Problem 2 is a difficult optimal control problem especially due to the boundary condition (8) and the non-linear dynamics and constraints with respect to control variables. However, Theorem 1 in [8] proved that there exists a static optimal solution to Problem 2. That is, the optimal routing probabilities $\left\{p_{ij}^*(t)\right\}$ remain constant during the whole network lifetime. Therefore, $\{G_i(t), \dot{r}_i(t)\}$ are also constant. Define the static routing probabilities as $\{p_{ij}\}$ and the flow rate at node i as G_i. We can then drop $r_i(t)$. Instead, the lifetime of node j can be specified by:

$$t_j = E_j \left[G_j \left(\sum_{j<k<N} p_{jk} v_{jk} + v_{jN} \right) \right]^{-1}$$

So the only difficulty here is the non-differentiable boundary condition (8). To deal with this problem, in [8], all N possible cases are considered according to which node depletes its battery first. Hence, the lifetime maximization problem is alternatively formulated as a set of non-linear optimization problems:

Problem 3 ([8]): For each $0 \le i \le N-1$:

$$\min_{\{p_{jk}\}} G_i \left(\sum_{i<j<N} p_{ij} v_{ij} + v_{iN} \right) \tag{9}$$

$$\text{s.t. } t_j = E_j \left[G_j \left(\sum_{j<k<N} p_{jk} v_{jk} + v_{jN} \right) \right]^{-1}, \tag{10}$$

$$0 \le j \le N-1$$

$$G_j = \sum_{0 \le k<j} p_{kj} G_k, \quad 1 \le j \le N-1 \tag{11}$$

$$G_0 = R_0 \tag{12}$$

$$t_j \le t_i, \quad j \ne i \tag{13}$$

$$\sum_{k<j \le N} p_{kj} = 1, \quad 0 \le k \le N-1 \tag{14}$$

$$p_{kj} \ge 0, \quad 0 \le k < j \le N \tag{15}$$

The objective here is to maximize t_i, equivalent to minimizing the load on node i, as specified in (9). Problem 3 is a set of N non-linear programming (NLP) problems that are to be solved in parallel. The optimal lifetime is obtained through:

$$T^* = \max\{t_i^*\}$$
$$i^* = \arg\max\{t_i^*\}$$

and the optimal routing probability is obtained from the corresponding $\left\{p_{jk}^*\right\}_{i^*}$.

III. Reducing Problem 3 to Problem 1

One of the important contributions of [8] is proving the optimality of static routing probabilities, based on a simple model for the battery dynamics: the instantaneous energy depletion rate is linear with respect to the data flow rate. However, the simplified optimization problem is a NLP with N cases. In fact, we can show that this set of N NLP problems can be reduced to Problem 1.

Theorem 1: Problem 3 can be reduced to Problem 1 without loss of optimality.

Proof: First, we combine the N subproblems. Note that

$$t_i = E_i \left[G_i \left(\sum_{i<j<N} p_{ik} v_{ik} + v_{iN} \right) \right]^{-1},$$

where E_i is a given quantity. Because t_i is positive, we can rewrite the objective (9) of subproblem i as:

$$\max_{\{p_{jk}\}} t_i$$

Since the optimal lifetime T^* is given by $T^* = \max\{t_i^*\}$, without loss of optimality, we can add the following constraint into subproblem i:

$$T \le t_i$$

and rewrite again the objective to be

$$\max_{\{p_{jk}\}} T$$

Then, subproblem i's optimal solution must satisfy $T^* = t_i^*$ for some i trivially. Because in subproblem i, we have

$$t_j^* \le t_i^* = T^*$$

we can rewrite constraints (10), (13) as:

$$t_i = E_i \left[G_i \left(\sum_{i<k<N} p_{ik} v_{ik} + v_{iN} \right) \right]^{-1}$$

$$T \le E_j \left[G_j \left(\sum_{j<k<N} p_{jk} v_{jk} + v_{jN} \right) \right]^{-1}, \quad j \ne i$$

so as to eliminate all $\{t_j, j \ne i\}$. Combining the two constrains above and multiplying both sides by $G_j \left(\sum_{j<k<N} p_{jk} k_{jk} + k_{jN} \right)$, we simply have:

$$T G_j \left(\sum_{j<k<N} p_{jk} v_{jk} + v_{jN} \right) \le E_j$$

Hence, for each subproblem i, we end up with the same optimization problem because index i does not appear in the problem. Therefore the N identical subproblems collapse into one NLP problem. To show that it can be transformed into a LP problem, we multiply both sides of constraints (11) and (12) by T, and multiply both sides of constraints (14) and (15) by TG_k, respectively. Because G_k is the flow rate at node k, TG_k is the total number of bits transmitted at node k. Introduce new variables $\{q_{ij}, 0 \le k < j \le N\}$ such that $q_{ij} = TG_i p_{ij}$. Then, we can rewrite the optimization problem as:

$$\max_{\{p_{jk}\}} T$$

$$\sum_{j<k<N} q_{jk} e_{jk}^t + \sum_{0<k<j} e^r q_{kj} \le E_j, \quad 0 \le j \le N-1$$

$$\sum_{j<k<N} q_{jk} = \sum_{k<j} q_{kj}, \quad 1 \le j \le N-1$$

$$\sum_{0<k<N} q_{0k} = T R_0$$

$$q_{kj} \geq 0, \quad 0 \leq k < j \leq N$$

which is the same formulation as Problem 1. ■

Theorem 1 has reduced the original N non-linear sub-problems into one single linear program, which can be solved efficiently and even in distributed fashion [10].

IV. The Initial Energy Allocation Problem

A. Formulation

The initial energy allocation problem as formulated in [8] is simply Problem 3 with the $\{E_i\}$ set as control variables and added constraints:

$$\sum_{0 \leq i \leq N-1} E_i = \bar{E}$$
$$E_i \geq 0, \quad 0 \leq i \leq N-1$$

In [8], Theorem 2 proved that in the optimal solution, all nodes deplete their batteries at the same time. Hence, the initial energy allocation problem can be decomposed into two parts: (1) an NLP (called NLPMIN) that computes the optimal routing probabilities; (2) a simple calculation to allocate the battery energy proportional to the node energy load. The NLPMIN problem is as follows:

Problem 4 ([8]-NLPMIN):

$$\min_{p_{ij}} \sum_{0 \leq i \leq N-1} G_i \left(\sum_{i<j<N} p_{ij} v_{ij} + v_{i,N} \right)$$
$$\text{s.t.} \quad G_j = \sum_{0 \leq k < j} p_{kj} G_k, \quad 1 \leq j \leq N-1$$
$$G_0 = R_0$$
$$\sum_{i<j \leq N} p_{ij} = 1, \quad 0 \leq i \leq N-1$$
$$p_{ij} \geq 0, \quad 0 \leq i < j \leq N$$

After solving Problem 4, the optimal initial energy allocation can be obtained from:

$$E_i^* = \bar{E} \cdot \frac{G_i^* \left(\sum_{i<j<N} p_{ij}^* v_{ij} + v_{i,N} \right)}{\sum_{0 \leq i \leq N-1} G_i^* \left(\sum_{i<j<N} p_{ij}^* v_{ij} + v_{i,N} \right)}$$

that is, the initial energy allocation is proportional to the consumption rate. Hence all nodes will deplete energy together.

B. Shortest path problem transformation

We can show that the initial energy allocation problem is equivalent to a shortest path problem on a weighted graph. The WSN in our problem can be modeled as a directed graph with a source (node 0) and a destination (node N). An arc (i, j) is a unidirectional transmission link from node i to j. The weight w_{ij} of arc (i, j) is defined as:

$$w_{ij} = e_{ij}^t + e_{ji}^r$$

that is, the energy consumption to transmit 1 bit of information from node i to node j.

Information is generated from the source (node 0), and routed to the sink (node N) via paths on the graph. A path P is denoted by a sequence of nodes $\{0, n_1^P, n_2^P, ..., N\}$ where $n_i^P < n_j^P$ if $i < j$. The cost of the path P, denoted by C_P is the sum of the weight of the links it goes through:

$$C_P = \sum_i w_{n_i^P n_{i+1}^P}$$

Therefore, for each bit of information, the total energy cost to deliver it from node 0 to node N on path P is C_P.

Theorem 2: In the optimal solution to Problem 4, if there exist multiple paths from node 0 to node N where nodes in the path are allocated positive energy, then they all have the same cost.

Proof: We can prove the result by contradiction. Suppose in the optimal solution there exist two distinct paths P_1^* and P_2^* such that $C_{P_1^*} > C_{P_2^*}$. First, we can remove from the network the nodes not allocated any energy without loss of optimality, as they are not used. Let the amount of information passing through P_1^* and P_2^* be $q_{P_1^*}$ and $q_{P_2^*}$, respectively. Then, we have:

$$q_{P_1^*} C_{P_1^*} + q_{P_2^*} C_{P_2^*} = \bar{E}$$
$$q_{P_1^*} + q_{P_2^*} = R_0 T^*$$

where T^* is the optimal lifetime. Let $\varepsilon > 0$ be a small positive number. Because $C_{P_1^*} > C_{P_2^*}$, we have

$$\left(q_{P_1^*} - \varepsilon \right) C_{P_1^*} + \left(q_{P_2^*} + \varepsilon \right) C_{P_2^*} < \bar{E}$$

which implies that by time T^*, there is still unused energy left in some nodes so the original solution is not optimal. The theorem is thus proven. ■

Theorem 2 implies that to solve the initial energy allocation problem, we just need to find a shortest path from node 0 to node N on the network. This is because arc weights correspond to energy consumed. Thus, the shortest path on the graph weighted by the transmission energy costs guarantees the lowest cost to deliver every bit of data from node 0 to node N. Also, when there exist multiple paths with the same lowest cost, we only need to pick one of them. Then, we can allocate the energy to the nodes on the path. Let the shortest path be $\{0, n_1^*, n_2^*, ...n_l^*, N\}$, where l is the number of intermediate nodes. We know that the flow on this path is R_0. Therefore, the load (energy consumption rate) on each node is:

$$L_0 = R_0 e_{0 n_1^*}^t$$
$$L_{n_i^*} = R_0 \left(e_{n_i^* n_{i+1}^*}^t + e_{n_{i-1}^* n_i^*}^r \right), \quad 1 \leq i \leq l-1$$
$$L_{n_l^*} = R_0 \left(e_{n_l^* N}^t + e_{n_{l-1}^* n_l^*}^r \right)$$

So the optimal initial energy allocation is:

$$E_i^* = \begin{cases} 0 & i \notin \{0, n_1^*, n_2^*, ...n_l^*\} \\ \bar{E} \cdot \dfrac{L_i}{\sum_{j \in \{0, n_1^*, n_2^*, ...n_l^*\}} L_j} & i \in \{0, n_1^*, n_2^*, ...n_l^*\} \end{cases} \tag{16}$$

Shortest path problems can be solved very efficiently using existing algorithms, such as Dijkstra's algorithm. When a network has multiple sources $\{n_0^1, ..., n_0^C\}$, where C is the number of distinct sources, we can first compute the shortest path to sink for each source:

$$\{n_0^1, n_1^1, n_2^1, ..., n_l^1, N\}$$

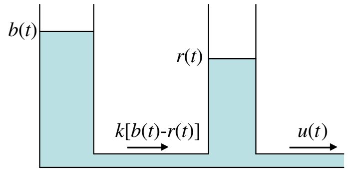


Fig. 1: The Kinetic Battery Model

$$\left\{n_0^2, n_1^2, n_2^2, ..., n_l^2, N\right\}$$
$$...$$
$$\left\{n_0^C, n_1^C, n_2^C, ..., n_l^C, N\right\}$$

Note that these paths may join each other at some point, therefore, the topology will be a tree rooted at the sink. Next, we compute the load $\left\{L_{n_i^c}\right\}$ at each node for each path, and superimpose them at nodes where multiple paths go through. Finally, we will obtain a set of nodes with load $\{L_i\}$. We can then use (16) to allocate the initial energy.

V. Lifetime Maximization with Battery Dynamics

The maximum lifetime analysis thus far is based on a simplistic model for the battery dynamics. However, in reality batteries do not satisfy such a simple linear model. For example, [11] shows that the lifetime of an alkaline battery decreases nonlinearly with respect to the work load. Hence, to perform a more accurate routing optimization, we need to incorporate more detailed battery dynamics into the system model.

A. *The Kinetic Battery Model*

Recent research on battery characteristics [11] points out that the simple linear discharge model is not a good approximation of battery capacity, due to the rate capacity effect and recovery effect. A Kinetic Battery Model (KBM) is proposed and shown in Figure 1. In Figure 1, a battery is modeled as two charge wells. One is the available-charge well (R-well) that directly connects to the output. The amount of energy in the R-well is denoted by $r(t)$. There is another well called the bound-charge well (B-well), which supplies electrons only to the R-well. The amount of energy in the B-well is denoted by $b(t)$, and electrons flow from the B-well to the R-well only when $r(t) < b(t)$, at a rate of $k[b(t) - r(t)]$ per unit time. $u(t)$ is the workload on the battery at time t. The battery depletes when $r(t)$ reaches zero. That is, it cannot provide available electrons. With the KBM, we can modify Problem 2 to accommodate the new dynamics:

$$\begin{aligned}\dot{r}_i(t) &= -G_i(t)\left(\sum_{i<j<N} p_{ij}(t)\, v_{ij} + v_{iN}\right) \\ &\quad + k\left[b_i(t) - r_i(t)\right], \quad 0 \le i \le N-1 \\ \dot{b}_i(t) &= -k\left[b_i(t) - r_i(t)\right]\end{aligned}$$

with boundary conditions:

$$\begin{aligned}r_i(0) &= E_i^r \\ b_i(0) &= E_i^b, \quad 0 \le i \le N-1 \\ \min_i r_i(T) &= 0, \quad 0 \le i \le N-1\end{aligned}$$

Due to the KBM, $\dot{r}_i(t)$ and $\dot{b}_i(t)$ are explicit functions of $r_i(t)$ and $b_i(t)$. Therefore, we cannot use Theorem 1 in [8], nor can we expect the optimal solution to be static. One way to solve optimal control problems numerically is to convert the continuous-time problem into a discrete-time one, and solve it using an LP formulation [12]. Time-optimal problems pose an additional layer of difficulty, since the number of discrete time slots is not determined so the number of variables is unknown. In [12], a workaround for minimum-time optimal control problems is proposed. Due to the non-standard terminal condition $\min_i r_i(T) = 0$ and the time-maximizing objective, we borrow the concept but tailor the workaround to solve our routing problem:

1) Choose an initial fixed terminal time T.
2) Solve a discrete time optimal control problem that maximizes residual energy with a fixed terminal time T. The optimal control problem can be formulated as an LP:

$$\begin{aligned}&\max_{\{r_i(t), b_i(t), q_{ij}(t)\}} \min_{0 \le i < N} r_i(T) \\ \text{s.t. } r_i(t+1) &= r_i(t) + \bar{k}\left[b_i(t) - r_i(t)\right] \\ &\quad - \left(\sum_{j>i} e_{ij}^t q_{ij}(t) + \sum_{j:i<j} e_{ji}^r q_{ji}(t)\right) \\ b_i(t+1) &= b_i(t) - \bar{k}\left[b_i(t) - r_i(t)\right] \\ \sum_{j>i} q_{ij}(t) &= \sum_{j<i} q_{ji}(t) \\ \sum_{j>0} q_{0j}(t) &= R_0 \\ r_i(0) &= E_i^r \\ b_i(0) &= E_i^b \\ q_{ij}(t) &\ge 0, \quad 0 \le i < j \le N, 0 \le t \le T-1 \\ r_i(t), b_i(t) &\ge 0, \quad 0 \le i < N\end{aligned}$$

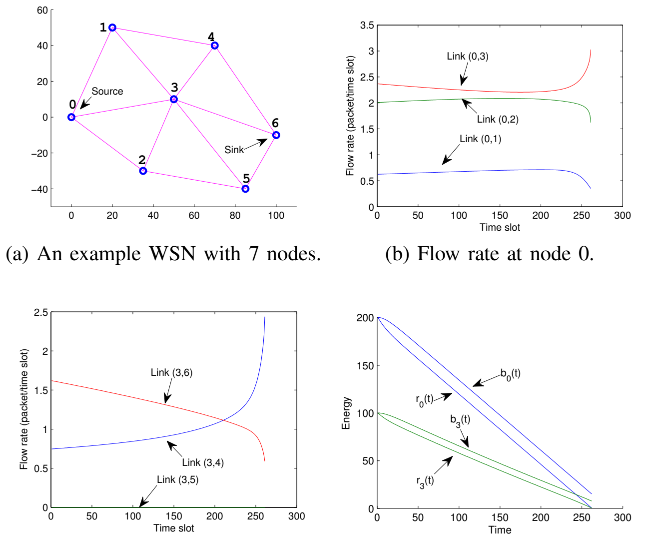
where $q_{ij}(t)$ is the amount of data transmitted from node i to j during time slot $[t, t+1)$.
3) If the LP in Step 2 is feasible, increase T. If the LP in Step 2 is infeasible, reduce T.
4) Stop when the LP is feasible for T but infeasible for $T+1$. We have obtained the maximum lifetime T.

B. *Numerical Example for Kinetic Battery Model*

We consider a simple 7-node network as an example as shown in Fig. 2a. Node 0 is the data source, and node 6 is the sink. We set the initial energy of node 0 to be: $E_0^b = 200, E_0^r = 200$. For the rest of the nodes $1 \le i \le 5$, $E_i^b = 100, E_i^r = 100$. The data generation rate at the source is $R_0 = 5$ (packets/time slot). The battery parameter $\bar{k} = 0.05$.

From the optimal control-LP workaround outlined in the previous section, we have obtained the maximum lifetime $T^* = 262$ (time slots). More interesting is to see how the routing under KBM differs from static routing with simple battery dynamics. Figure 2b shows the flow rate originating from node 0 to nodes 1, 2 and 3, and Fig. 2c shows the flow rate originating from node 3 to nodes 4, 5 and 6. We see that the flow rates all exhibit non-linear behavior.

We then examine how the energy is consumed at nodes. Figure 2d shows the depletion of energy at node 0 and



(a) An example WSN with 7 nodes. (b) Flow rate at node 0.

(c) Flow rate at node 3. (d) Battery residual energy at nodes 0 and 3.

Fig. 2: A numerical example with 7 nodes

node 3. Interestingly, we see that despite a small non-linear segment early on, the rest of the curves exhibit linearity and parallelism between $r_i(t)$ and $b_i(t)$. Referring to Fig. 1, we can see that the difference $b_i(t) - r_i(t)$ remains constant only when $\bar{k}[b_i(t) - r_i(t)]$ equals the load. This implies an approximately constant load might have been the case at these nodes, and suggests that the optimal (dynamic) solution may be *approximated* by a static solution. Therefore, we *hypothesize* that there may exist an optimal solution which has static flow rates (routing probabilities). To test this, we add the following constraint to force static flow rates:

$$q_{ij}(t+1) = q_{ij}(t), \quad 0 \le i < j \le N, 0 \le t \le T-1 \quad (17)$$

and re-solve the problem. Table I compares the solutions between the original dynamic optimization and the optimization with added constraints (17). First, we immediately see that both optimization problems provide the same lifetime! We compare the mean flow at links $(0, 1..3)$ and $(3, 4..6)$, where the mean is computed by:

$$\bar{q}_{ij} = \frac{1}{T} \sum_{t=0}^{T-1} \bar{q}_{ij}(t)$$

The comparison in Table I shows that the mean flow in both problems are very close, suggesting that the optimality of dynamic routing can at least be very well approximated by static routing whose link flow equals the mean flow in the dynamic routing. Further theoretical investigation is needed to explore the potential optimality of static routing policies, in effect replacing the dynamic solutions by equivalent fixed averages which are easier to obtain.

VI. Conclusions

The maximum lifetime routing problem was investigated in [4] and [8], the former based on an LP formulation and the latter on an optimal control formulation. We show that when the model for battery dynamics is a simple one, both

	Dynamic	Forced Static
T^*	262	262
$\bar{q}_{01}$	0.6666	0.6603
$\bar{q}_{02}$	2.0474	2.0412
$\bar{q}_{03}$	2.2860	2.2985
$\bar{q}_{34}$	0.9796	1.0098
$\bar{q}_{35}$	0.0000	0.0000
$\bar{q}_{36}$	1.3064	1.2887

TABLE I: Comparison of dynamic routing and static routing under Kinetic Battery Model

approaches are equivalent. In addition, we have shown that the initial energy allocation problem can be reformulated into a shortest path problem on a graph, allowing efficient solving of such problems.

When the battery model is no longer a simplistic one, we have to incorporate detailed dynamics into the formulation. The interesting empirical results we have obtained are that, while a static routing policy is not expected to be optimal, it turns out that such a policy can be a good approximation of the optimal dynamic routing policy. It is possible that average routing probabilities replacing dynamically varying ones are indeed optimal, a question which is the subject of ongoing research.

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