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Call Allocation in Cellular Communication Systems with Overlapping Coverage[1]

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Abstract

This paper addresses the problem of increasing the capacity of a Cellular Communication System. Rather than using the traditional channel allocation schemes, this paper tries to increase the system capacity by utilizing the unavoidable overlap between the coverage areas of adjacent base stations and allocate new calls to the "least sensitive" base station. Several simulation results are included that show the benefits of the proposed algorithms compared to the algorithms that already exist in the literature.

1 Introduction

In recent years mobile communications have experienced tremendous growth. In order to cope with the increased demand, the service area is divided into cells where frequency channels may be reused as long as there is sufficient distance separation to prevent interference [1]. In addition, frequency allocation algorithms may further increase the system capacity, at least for systems that use either frequency or time division multiple access schemes (FDMA or TDMA respectively). Several channel allocation algorithms exist that fall in three general categories. These are *Fixed Channel Allocation* (FCA), *Dynamic Channel Allocation* (DCA), and *Hybrid Channel Allocation* (HCA) schemes. For a comprehensive study of the channel allocation schemes the reader is referred to [2].

In all of the channel allocation schemes mentioned above, a mobile terminal is always connected to the base station with the highest signal-to-noise ratio, which in general is the base station that is physically located closer to the mobile user. In practice, in order to achieve a complete area coverage, it is inevitable that some mobile terminals depending on their actual location, may be able to establish a communication link with two or more base stations, i.e. they receive a signal with sufficiently high signal-to-noise ratio from multiple base stations. Since such terminals can be connected to two or more base stations, it may be possible to increase the network capacity by connecting new mobile terminals to the "least congested" base station. To this end, two algorithms have been developed, namely, Directed Retry (DR) and Directed Handoff (DH) [3, 4, 5]. DR directs a new call to the base station with the greatest number of available channels, while DH may redirect an existing call from one cell to a neighboring one to further increase the system capacity. The performance of DR is analyzed in [6, 7]. Furthermore, some fairness issues are address in [8].

Both, DR and DH perform a "call allocation" to base stations based on some state information, i.e., the number of available channels. In effect, the two algorithms try to balance the number of available channels over all base stations. However, using Lagrangean relaxation, it is well-known that the solution of a convex constrained optimization problem like this one is obtained by balancing the partial derivatives (i.e. sensitivities), of the objective function with respect to the control variables [9]. Note however, that for problems like the channel or call allocation, we cannot define derivatives because both channels and mobile users are discrete quantities. This has motivated Cassandras et. al. [10] to develop a discrete-variable equivalent algorithm, where, rather than using partial derivatives, they employ "finite differences". The same idea has motivated the development of the two algorithms that we develop in this paper, namely *Immediate Neighborhood* (IN) and *Extended Neighborhood* (EN) which constitutes the main contribution of this paper. These algorithms use *sensitivity* information (in the form of a finite difference) to determine the base station that a new call should be assigned to. These algorithms enhance the performance of DR and DH by decreasing the call loss probability while requiring fewer reconfigurations[1]. Furthermore, both of the proposed algorithms can be easily implemented in either a centralized or decentralized controller without imposing significant computational overhead. Moreover, simulation results demonstrate

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[1]Reconfigurations refer to "induced handoffs" which force existing users to switch to a different channel as to accommodate more users.

that using overlapping cell structures leads to increased system capacities and can outperform DCA schemes. Finally, one can expect such structures to perform even better in hierarchical models due to the overlapping between micro and macro cells.

2 Modeling Assumptions

In this section we describe the model of the cellular system that we consider. As in [7, 8], we assume a cellular network where base stations (B_i, $i = 1, 2, \cdots$) are organized in a hexagonal pattern as shown in Figure 1. Furthermore, we assume a flat terrain and uniform propagation therefore each cell (C_i, $i = 1, 2, \cdots$) is represented by a hexagon inscribed in a circle of radius r, where, without loss of generality we assume that $r = 1$. The coverage area of each base station is represented by a circle around B_i, where, in order to achieve full coverage of the service area its radius R must be greater than or equal to 1. Note that a model with overlapping cells is realistic since even under the idealized conditions described above, due to the geometry, it is necessary that at least 20% of the cell's area overlaps with neighboring cells.

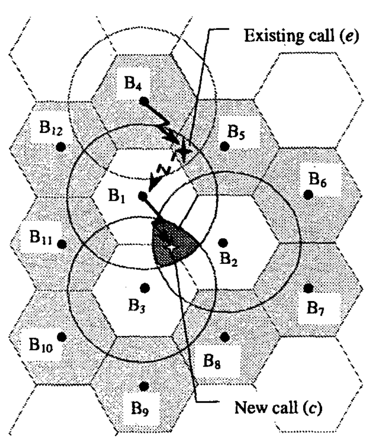

Figure 1: Overlapping Cell Structure

Next, we define several sets that will be useful when describing the algorithms presented in the sequel.

$A(B_i)$ is the set that consists of B_i and all base stations that are *adjacent* to it.

$IN(c)$ *Immediate Neighborhood* is the set of all base stations with which mobile terminal c can establish a communication link.

$EN(c)$ *Extended Neighborhood* is the set of all base stations that are adjacent to the base stations that are in $IN(c)$, i.e., $EN(c) = \bigcup_{j \in IN(c)} A(j)$.

$M(B_i)$ is the set of all mobile terminals that are connected to base station B_i, and

$H(B_i)$ is the set of all mobile terminals that are not connected to base station B_i but which can establish a satisfactory communication link with B_i, i.e., these are calls connected to a base station in $A(B_i) \setminus B_i$ and are located in the area that overlaps with the coverage area of B_i.

For example, in Fig. 1, $A(B_1) = \{B_1, \cdots, B_5, B_{11}, B_{12}\}$, $IN(c) = \{B_1, B_2, B3\}$, $EN(c) = \{B_1, \cdots, B_{12}\}$, $c \in M(B_1)$, $c \in H(B_2)$, and $c \in H(B_3)$.

To complete the notational definitions we assume that base station B_i is assigned a fixed number of channels K_i. Furthermore, we use m_i to denote the number of channels that are currently available in B_i (i.e., $m_i = K_i - |M(B_i)|$). Finally, by $B^*(c)$ we denote the base station that is located closest to mobile c or has the highest signal-to-noise ratio.

In addition, the results of this paper follow the basic assumptions made in [3], that is:

1. Call arrivals are described by a Poisson process[2] with rate λ while the call duration is exponentially distributed with mean $1/\mu$.
2. Blocked calls are cleared and do not return.
3. Fading and co-channel interference are not explicitly accounted for. Propagation and interference considerations are simply represented by the constraint that, if a channel is used in a given cell, it cannot be reused in a ring of R cells around that cell, $R = 1, 2, \cdots$.
4. Certain mobile terminals may be connected to multiple base stations, i.e. mobiles that are located in the intersection of the coverage areas of two or more base stations as shown in Figure 1.
5. Finally, for the simulation results presented in Section 4, mobile terminals are assumed stationary, i.e. in this model we do not account for mobile users that roam from one cell to another.

3 Proposed Algorithms

As mentioned in the introduction, DR and DH perform the call allocation to the base stations based on the current state information, i.e., the number of available channels in each cell, whereas, it would be desirable to use sensitivity information to perform the optimization. To derive such information, we need an objective function and given that we are interested in minimizing the number of lost calls, it is natural to consider the steady state loss probability as given by the Erlang B formula.

$$\Pr[\text{blocked call at } B_i] = \frac{\rho_i^{K_i}}{K_i!} \left(\sum_{j=0}^{K_i} \frac{\rho_i^j}{j!} \right)^{-1} \tag{1}$$

where K_i is the number of channels assigned to B_i and $\rho_i = \lambda_i / \mu_i$ is the traffic intensity in cell C_i.

Note however that for the "call allocation" problem we are interested in, the control parameter is the number of

[2]A suggestion for relaxing this assumption is also included in Section 3.4.

available channels (m) at the time of a new call arrival, which does not appear in (1). Rather, the number of available channels corresponds to the initial conditions of the differential equation that has a solution given by (1). Since, the Erlang B formula is a steady state measure, it does not depend on the initial conditions hence, the number of available channels does not appear in (1) and thus any sensitivity function with respect to the initial conditions will always be equal to zero. This suggests that in order to use Lagrangean relaxation type techniques to solve this problem, it may be necessary to devise a "surrogate" performance measure. A natural choice would be to substitute the steady state measure with a *transient* one. So, rather than directly trying to minimize the steady state loss probability, one can try to minimize the loss probability over an interval of length τ time units that starts at the time of a new call arrival. In this case, one would expect that minimizing the surrogate function would also minimize the steady state loss probability. At this point it is worth pointing out that the selection of the "surrogate" function is not unique and that it is possible that other functions might further improve the performance of the proposed algorithms. For example, one may define a family of functions based on measurement data that might anticipate the change in traffic intensity over the period of a day.

To derive a transient objective function recall that any base station can be modeled by an $M/M/m/m$ queueing system (see [11]). Such a system generates a birth-death Markov chain with a probability mass function $\boldsymbol{\pi}_i(t)$ which is the solution of the differential equation

$$\frac{d\boldsymbol{\pi}_i(t)}{dt} = \boldsymbol{\pi}_i(t)\mathbf{Q}_i \qquad (2)$$

where $\boldsymbol{\pi}_i(t) = [\pi_0^i(t), \cdots, \pi_{K_i}^i(t)]$ and $\pi_j^i(t)$ is the probability that at time t there will be j, $j = 0, \cdots, K_i$, active calls connected to base station B_i. Furthermore, $\mathbf{Q}_i$ is the standard *transition rate matrix* for an $M/M/m/m$ queueing system. Finally, in order to specify a solution of (2), one needs an initial condition $\boldsymbol{\pi}_i(0)$ which for our purposes it will be the number of available channels m_i at the time of a new arrival. In other words, $\boldsymbol{\pi}_i(0) = \mathbf{e}_{K_i - m_i}$ where $\mathbf{e}_j$ is a $(K_i + 1)$-dimensional vector with all of its elements equal to zero except the jth one which is equal to 1.

What we are after is the probability that a call is lost within the next τ time units, i.e., $\pi_{K_i}^i(\tau)$, which for small τ is clearly going to be a function of the initial conditions, i.e. the number of free channels m_i at the time of the new call arrival, $t = 0$. Hence, for every cell C_i we define the following "surrogate" objective function:

$$L_i(m_i) = \pi_{K_i}^i(\tau), \quad \text{s.t.} \quad \boldsymbol{\pi}_i(0) = \mathbf{e}_{K_i - m_i} \qquad (3)$$

Based on this objective function, we then define the following finite difference

$$\Delta L_i(m_i) = L_i(m_i - 1) - L_i(m_i), \; m_i = 0, \cdots, K_i \quad (4)$$

with a boundary condition $L_i(-1) = \infty$. Next, we describe our optimization algorithms (IN) and (EN).

3.1 Immediate Neighborhood (IN)

The IN algorithm is very similar to DR. Their only difference is that IN assigns the new call to the "least sensitive" base station with respect to the number of available channels, while DR assigns the new call to the base station with the largest number of available channels. Note that if traffic is uniformly distributed over the entire coverage area then the two algorithms behave exactly the same, since the least sensitive base station will always be the one with the most available channels. This is also observed in some of the simulation results that we present in the next section. More specifically, the algorithm works as shown in Table 1.

Table 1: Immediate Neighborhood (IN) Algorithm

	When a new call c arrives,
1.	Let $i^* = \arg\min_{i \in IN(c)}\{\Delta L_i(m_i)\}$
2.	Assign c to base station B_{i^*}

3.2 Extended Neighborhood (EN)

The Extended Neighborhood algorithm rather than looking for the least sensitive base station among the cells in the immediate neighborhood ($IN(c)$), it searches the entire extended neighborhood ($EN(c)$). If the least sensitive base station (say i^*) is within $IN(c)$, then it assigns the call to i^* as in IN. If the least sensitive base station is not in $IN(c)$, then this scheme looks for an existing call that is currently connected to a base station in $IN(c)$, and is located in the intersection of the least sensitive base station with any of the cells in $IN(c)$. For example, in Figure 1, call e is connected to B_1 and is located in the intersection of the coverage area of B_4 (the least sensitive base station among $EN(c)$). Then, the algorithm induces a handoff, connecting e to B_4 while assigning c to B_1. Note, that if there is no call in the intersection of the least sensitive base station with any of the cells in $IN(c)$, then the scheme looks for the next best option. EN is shown in Table 2.

The EN algorithm is also similar to the DH algorithm. Their differences lie in step 2 where EN is trying to minimize the sensitivities with respect to the number of available channels while DH tries to find the base station with the most available channels. Another difference is that EN is trying to find the least sensitive base station among all base stations in $EN(c)$ while DH is trying to find the base station with the most available channels in $A(B^*(c)) \subseteq EN(c)$.

Table 2: Extended Neighborhood (EN) Algorithm

When a new call (c) arrives,	
1.	Define $Q = EN(c)$
2.	Let $i^* = \arg\min_{i\in Q}\{\Delta L_i(m_i)\}$
3.	If $B_{i^*} \in IN(c)$, assign c to B_{i^*}. END.
4.	If there exists $e \in H(i^*) \cap \left(\bigcup_{j\in IN(c)} M(j)\right)$ go to 4.1, else go to 4.2.
4.1.	Handoff call e to B_{i^*} and assign c to the base station that e belonged to. END.
4.2	$Q = Q - \{B_{i^*}\}$. If $\lvert Q\rvert = 0$ call is blocked, otherwise, go to 2.

3.3 Decentralized Controller

The proposed schemes can be implemented with both centralized and decentralized controllers, which is particularly important as cells grow smaller to accommodate more customers. For such systems, the increased number of resource reconfigurations, either due to frequent roaming or induced handoffs, may put a considerable computational burden on a central controller therefore it would be preferable to use algorithms that can be implemented in a decentralized fashion to reduce the complexity of the controller.

Both IN and EN algorithms can be easily implemented with decentralized controllers that require minimal processing capabilities. For the IN algorithm, when a new call c is initiated, all base stations in $IN(c)$ send their current $\Delta L_i(m_i)$, $i \in IN(c)$, to the mobile terminal which in turn can determine $i^* = \arg\min_{i\in IN(c)}\{\Delta L_i(m_i)\}$ and therefore establish a link with base station B_{i^*}.

The same procedure can be utilized for the EN algorithm as well, however, in this case the base stations are required to have some information about the finite differences of their neighboring base stations. So, as in IN, when a new call c is initiated, every base station $i \in IN(c)$ sends ΔL_i^*, to the mobile terminal which in turn can determine $i^* = \arg\min_{i\in IN(c)}\{\Delta L_i^*\}$ and therefore establish a link with base station B_{i^*}. The difference lies in the values of ΔL_i^* that base station i sends to the mobile. In EN, every base station i maintains a table with the finite differences for all base stations in $A(B_i)$, $\mathbf{\Delta} = [\Delta L_i(m_i); \widetilde{\Delta L}_j(m_j), j \in A(B_i)\backslash B_i]$, where $\widetilde{\Delta L}_j(m_j)$ denotes the finite difference that the adjacent base station j has send to i. So, when a new call is initiated, rather than sending its own finite difference $\Delta L_i(m_i)$, base station i sends $\Delta L_i^* = \min\{\mathbf{\Delta}\}$. In this case, once a mobile terminal has decided on the i^*, it connects to it and the base station initiates the induced handoff if necessary.

The next question is how table $\mathbf{\Delta}$ is maintained. Every time base station i experiences a change in the number of available channels, i.e. when a call is either initiated or terminated, base station i sends its new finite difference to all of its neighbors $j \in A(B_i)$, according to the following rule:

$$\widetilde{\Delta L}_i(m_i) = \begin{cases} \Delta L_i(m_i) & \text{if } M(B_i)\bigcap H(B_j) \neq \emptyset \\ \infty & \text{otherwise} \end{cases} \quad (5)$$

Note that the transmitted $\widetilde{\Delta L}_i(m_i)$ of (5) automatically provides information on whether there exist an active call in the overlapping area of two base stations.

3.4 Extensions for more general distributions

The selection of the "surrogate" objective function relies on the assumption that call arrivals at any cell are Poisson while call duration is exponential. The question that arises is what happens when these conditions are violated? As a first approximation, one may choose to ignore the effects of a non-Poisson arrival process, in other words use the finite difference as defined in equation (4). In fact this is what we did in the simulation results presented in the next section. Note that the induced handoffs introduced by the DH and EN algorithms change the statistics of the arrival process making it non-Poisson yet EN still works well. Another approach would be to use ideas from perturbation analysis (PA) to estimate the transient loss probability online; PA is the study of the sample paths generated by customer arrivals and departures and is well suited for transient measures. PA aims at predicting the effect of a specified change on the performance (or cost) of a discrete-event system, such as the cellular telephone network, by processing information from an observed *single* sample path. For details on PA theory, the readers are referred to [11] and references therein.

In order to evaluate a suitable "surrogate" measure online, at the arrival of a new call, one can start a timer and count the number of calls that will be lost within the next τ time units given that at the time of the call arrival, there are m_i free channels. This counter is denoted by $N_{m_i}^i$. Dividing by the number of arrivals observed when there are m_i free channels ($A_{m_i}^i$) will give us an estimate of the transient loss probability for the initial conditions of m_i free channels. Recording this ratio $\hat{L}_i(m_i) = N_{m_i}^i / A_{m_i}^i$ for all possible $m_i = 0, \cdots, K_i$ will allow us to get an estimate of the required finite differences $\widehat{\Delta L}_i(m_i) = \hat{L}(m_i - 1) - \hat{L}(m_i)$.

The above estimator of the transient loss probability has two drawbacks. First, it is a ratio of two random numbers $N_{m_i}^i$ and $A_{m_i}^i$, and as a result it may be biased and will generally have high variance. Second, if at steady state some of the states are not visited very often, then the number of samples collected for the corresponding m_is will be small, again resulting in an estimator with high variance. To solve these problems for every base station B_i, $i = 1, 2, \cdots$, we propose a simple algorithm shown in Table 3 which is motivated from the ideas of *Concurrent Estimation* [12] that constitutes a PA technique. Specifically, the algorithm provides the answer to a series of "what-if" questions, namely, what-if at the time of the call arrival instead of m_i, there were q free channels, for all $q = 0, \cdots, m_i - 1, m_i + 1, \cdots, K_i$.

Table 3: On-line estimation of $L_i(m_i)$

1.	Initialize $N_0^i = N_1^i = \cdots = N_{K_i}^i = 0,\ A^i = 0$
2.	At the time of a new call arrival (say n)
2.1	$A^i := A^i + 1$
2.2	Start a new timer $t_n^i = 0$, and define $x_n^i = 0$.
3.	While $t_n^i < \tau$
3.1	If a new arrival occurs $x_n^i := x_n^i + 1$ and update $N_j^i := N_j^i + 1$ for all $j > K_i - x_n$
3.2	If a call is terminated $x_n^i := x_n^i - 1$
4.	$\hat{L}_i(j) := \frac{N_j^i}{A^i}$, for $j = 0, \cdots, K_i$

In this algorithm, at the time of the arrival of the nth call at B_i, we start a timer t_n^i and a counter x_n^i. The timer t_n^i will measure an interval of length τ time units. The counter x_n^i corresponds to the number of channels demanded over a time interval τ and can take any integer value greater than or equal to $-K_i$ (Note that x_n^i takes negative values when the number of calls terminated is greater than the new call arrivals). Now, suppose that at the beginning of the interval, there were j free channels, then if a new call arrives when $x_n^i + j \geq K_i$ it will be blocked. Allowing j to take *all* possible values $j = 0, \cdots, K_i$ provides us the answer to the what-if questions posed earlier. Furthermore, the denominator of the estimator of Step 4 is no longer a random variable since we can choose to update its value periodically, say every A arrivals. Such estimator will be unbiased and will generally have smaller variance.

4 Simulation Results

In this section we present simulation results for the call loss probability of the IN and EN algorithms and compare them with the corresponding results of DR and DH. In addition, we use the analytical results derived in [13, 14] to compare the performance of these algorithms with the performance of DCA algorithms. Specifically, we reproduce the lower bounds on two DCA algorithms: (a) The "Timid DCA" scheme which allows a new mobile terminal (MT) to establish a connection via any channel that is not used in any of the cells that are located in the R consecutive rings surrounding the closest base station. (b) The "Aggressive DCA" scheme which allows a new MT to get any channel, even if it is used in an adjacent cell, and force existing calls to search for new interference free channel in their area[3]. These bounds are the result of an *ad-hoc* Erlang-B model which uses the Erlang B formula (1) and substitutes the traffic intensity $\rho \to N\rho$ and the total number of available channels $K \to \delta K$, where N is the reuse factor[4] [1], and δ is the normalized channel utilization[5].

For the results presented next, we assume that there is a total of 70 channels. Any channel assigned to base station B_i cannot be reused in any base station in the ring of radius $R = 2$ cells around B_i. In this case, the reuse factor $N = 7$ and hence each base station is assigned 10 channels. In addition, we assume that the service area consists of 64 cells each of which is represented by a hexagon inscribed in a circle of radius $r = 1$, arranged in an 8×8 grid. Note that in order to achieve full area coverage, the coverage radius of each base station must be at least equal to 1. Finally, in order to reduce the effect of cells being located at the edges we have used the model in [3]: a cell on an edge is assumed to be adjacent to the cells on the opposite edge.

First we compare the call loss probabilities for several channel/mobile allocation schemes. Figure 2 compares the call loss probability when the traffic is uniform over the entire service area and the coverage radius of each base station is 1.14. In this case, about 50% of all MTs can hear a single base station, 43% can hear two base stations and 7% can hear three base stations. Note that DH and EN exhibit superior performance (lower call blocking probability) which is considerably better than the Timid DCA while for $\rho \geq 7.5$, EN outperforms even the aggressive DCA. Moreover, note the shape of the curves corresponding to the call loss probabilities of the DH and EN algorithms. These are considerably different from the smooth curve that one would expect from the Erlang-B formula. The reason is that the induced handoffs change the statistics of the arrival process which is no longer Poisson. Even though we ignore the change in the statistics and still use (4), the EN algorithm still performs very well.

A similar result is also presented in Figure 3 which compares the loss probabilities of the seven algorithms when the coverage radius is equal to 1.4, i.e. when 15% of the MTs can hear one base station, 33% can hear two base stations and 52% can hear three base stations. Note how the call loss probability has been dramatically reduced and note that both, EN and DH outperform even the aggressive DCA. However, increasing the coverage radius by that much will increase the co-channel interference between adjacent base stations and it is possible that such a configuration may not be feasible due to noise. On the other hand, we point

[3]Note that in [13] it is stated that no practical algorithm exists that can implement the aggressive DCA and conjecture that this bound may not be attainable.

[4]$N = i^2 + ij + j^2$ where i, j are integers. For R odd, $i = j = (R+1)/2$ and for R even $i = R/2$ and $R = j/2 + 1$.

[5]For a Timid DCA scheme $\delta(R = 1) = 0.693$, $\delta(R = 2) = 0.658$, $\delta(R = 3) = 0.627$ while for the aggressive DCA scheme $\delta = 1$ [13].

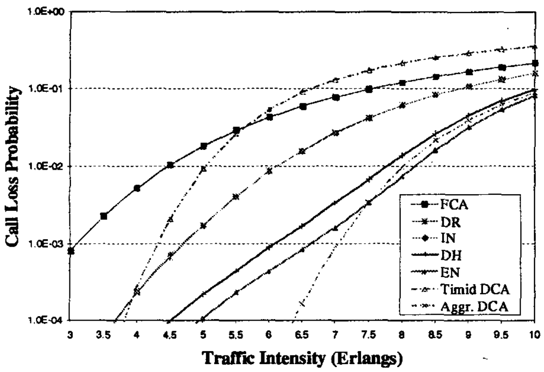

Figure 2: Call loss probabilities as a function of the traffic intensity ρ when the coverage radius is 1.14

out that such overlapping probabilities may be feasible for non-uniform traffic or in hierarchical cell structures. For example, when planning a system, the base stations may be placed is a way such that the overlapping areas are over high usage regions.

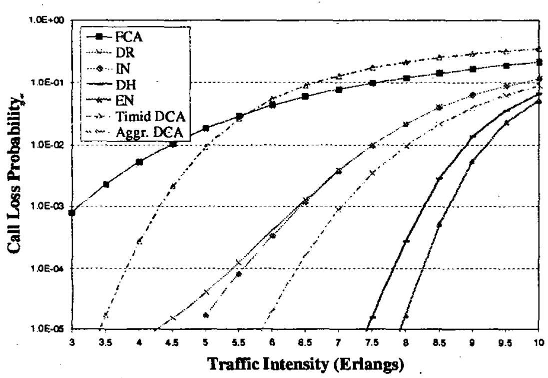

Figure 3: Call loss probabilities as a function of the traffic intensity ρ when the coverage radius is 1.4

One of the main advantages of EN is that it requires considerably less resource reconfigurations (call reallocations) than DH because it is utilizing sensitivity information and since it is using a larger neighborhood than DH. This is shown in Figure 4 which shows the number of induced handoffs per call for the two algorithms when the area coverage of each base station has radius $r = 1.14$ and $r = 1.4$. This figure shows that in addition to the lower call loss probability, EN requires considerably fewer reconfigurations than DH (less intracell induced handoffs) which is important because it implies significantly less work for the controller.

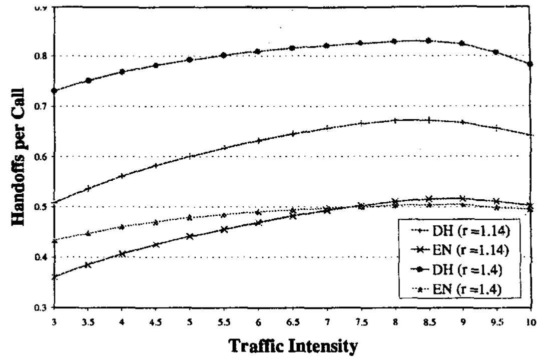

Figure 4: Average number of induced handoffs.

References

[1] V. H. MacDonald, "The cellular concept," *Bell System Tech. Journal*, vol. 58, pp. 15–41, Jan 1978.

[2] I. Katzela and M. Naghshineh, "Channel assignment schemes for cellular mobile telecommunication systems: A comprehensive survey," *IEEE Personal Communications*, vol. 3, pp. 10–31, Jun 1996.

[3] D. Everitt, "Traffic capacity of cellular mobile communication systems," *Computer Networks ISDN Systems*, vol. 20, pp. 447–54, Jan 1990.

[4] J. Karlsson and B. Eklundh, "A cellular mobile telephone system with load sharing-an enhancement of directed retry," *IEEE Transactions on Communications*, vol. 37, pp. 530–535, May 1989.

[5] B. Eklundh, "Channel utilization and blocking probability in a cellular mobile telephone system with directed retry," *IEEE Transactions on Communications*, vol. 34, pp. 329–337, Apr 1986.

[6] T.-P. Chu and S. Rappaport, "Microcellular communication systems," *IEEE Transactions on Vehicular Technology*, vol. 43, pp. 713–721, Aug 1994.

[7] T. Chu and S. Rappaport, "Overlapping coverage with reuse partitioning in cellular communication systems," *IEEE Trans. on Veh. Tech.*, pp. 41–54, 1997.

[8] X. Lagrange and B. Jabbari, "Fairness in wireless microcellular networks," *IEEE Transactions on Vehicular Technology*, vol. 47, pp. 472–479, May 1998.

[9] R. Galager, "A minimum delay routing algorithm using distributed computation," *IEEE Transactions on Communications*, vol. 25, pp. 73–85, 1977.

[10] C. G. Cassandras, L. Dai, and C. Panayiotou, "Ordinal optimization for a class of deterministic and stochastic discrete resource allocation problems," *IEEE Trans. on Automatic Control*, pp. 881–900, 1998.

[11] C. G. Cassandras, *Discrete Event Systems, Modeling and Performance Analysis.* IRWIN, 1993.

[12] C. G. Cassandras and C. Panayiotou, "Concurrent sample path analysis of discrete event systems," *J of Discrete Event Dynamic Systems*, pp. 171–195, 1999.

[13] L. Cimini, G. Foschini, C.-L. I, and Z. Miljanic, "Call blocking performance of distributed algorithms for dynamic channel allocation in microcells," *IEEE Trans. on Communications*, pp. 2600–7, Aug 1994.

[14] L. Cimini, G. Foschini, and L. Shepp, "Single-channel user-capacity calculations for self-organizing cellular systems," *IEEE Trans. on Communications*, vol. 42, pp. 3137–3143, Dec 1994.