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IFAC PapersOnLine 51-16 (2018) 223–228

Hybrid System Modeling of Multi-Agent Coverage Problems with Energy Depletion and Repletion ⋆

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Abstract: We present a hybrid system model describing the behavior of multiple agents cooperating to solve an optimal coverage problem under energy depletion and repletion constraints. The model captures the controlled switching of agents between coverage (when energy is depleted) and battery charging (when energy is replenished) modes. Our analysis contains three parts. The first part shows how the model guarantees the feasibility of the coverage problem by defining a guard function on each agent's battery level to prevent it from dying on its way to a charging station. The second part provides two scheduling algorithms to solve the contention problem of agents competing for the only charging station in the mission space. The third part shows the optimality of the motion plan adopted in the proposed model.

Keywords: Multi-agent coverage, optimal control, gradient methods, infinitesimal perturbation analysis (IPA), hybrid systems

1. INTRODUCTION

Systems consisting of cooperating mobile agents are often used to perform tasks such as coverage (Zhong and Cassandras (2011); Leonard and Olshevsky (2013)), surveillance (Tang and Ozguner (2005)), monitoring and sweeping (Smith et al. (2012)). A coverage task is one where agents are deployed so as to cooperatively maximize the coverage of a given mission space (Meguerdichian et al. (2001)), where "coverage" is measured in a variety of ways, e.g., through the joint detection probability of random events cooperatively detected by the agents. Widely used methods to solve the coverage problem include distributed gradient-based algorithms (Zhong and Cassandras (2011)) and Voronoi-partition-based algorithms (Cortes et al. (2004)). These approaches typically result in locally optimal solutions, hence possibly poor performance. To escape such local optima, a boosting function approach is proposed in Sun et al. (2014) where the performance is ensured to be improved. Recently, the coverage problem was also approached by exploring the submodularity property (Zhang et al. (2016)) of the objective function, and a greedy algorithm is used to guarantee a provable bound relative to the optimal performance (Sun et al. (2017)).

In most existing frameworks, agents are assumed to have unlimited on-board energy to perform the coverage task. However, in practice, battery-powered agents can only work for a limited time in the field. For example, most commercial drones powered by a single battery can fly for only about 15 minutes. Therefore, in this paper we take into account such energy constraints and add another dimension to the traditional coverage problem. The basic setup is similar to that in Zhong and Cassandras (2011). Agents interact with the mission space through their sensing capabilities which are normally dependent upon their physical distance from an event location. Outside its sensing range, an agent has no ability to detect events. Unlike other multi-agent energy-aware algorithms whose purpose is to reduce energy cost, we assume that a charging station is available for agents to visit according to some policy. The objective is to maximize an overall environment coverage measure by controlling the movement of all agents in a cooperative manner while guaranteeing that no agent runs out of energy while in the mission space.

We provide a solution to the above problem by modeling the behavior of an agent through three different modes: coverage (Mode 1), to-charging (Mode 2), and in-charging (Mode 3). We assume that an agent has no prior knowledge of the mission space except for the location of the charging station and the positions of agents within its communication range. While in Mode 1, each agent moves along the gradient direction of the objective function at the maximum velocity so as to cooperatively maximize the coverage measure. As an agent's energy is depleted, the agent switches to Mode 2 according to a guard function designed to guarantee that a minimum energy amount is preserved to reach the charging station from its current location while traveling at maximum speed. Note that an agent shares its position and battery state information with the charging station only when it is in the to-charging mode (Mode 2). Since the charging station is shared by all agents, there can only be at most a single agent at the station at any time. Therefore, two scheduling algorithms are proposed to resolve contention among low-energy agents: (*i*) First-Request-First-Serve (FRFS), and (*ii*) Shortest-Distance-First (SDF). These two scheduling algorithms

⋆ This work was supported in part by NSF under grants ECCS-1509084, IIP-1430145, and CNS-1645681, by AFOSR under grant FA9550-12-1-0113, by DOE under grant DOE-46100, and by Bosch and MathWorks.

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2018.08.038

are described in detail in Subsection 4.2. The charging station is perceived as a centralized controller executing a scheduling algorithm by dictating agent speeds so that a queue is formed by agents while in Mode 2. In Mode 3, an agent is located at the charging station and a model is developed for the battery charging dynamics using the dwell time of an agent at the station as a controllable parameter to be optimized.

The contributions of this paper are summarized as follows. First, a hybrid system model is developed so that the optimal coverage problem can be transformed into a parametric optimization problem which can be subsequently solved using Infinitesimal Perturbation Analysis (IPA) techniques (Cassandras et al.[2010]Cassandras (Wardi)). Second, we formulate the transition of an agent from the coverage mode (Mode 1) to the in-charging mode (Mode 3) as a shortest time optimization problem so as to ensure that agents minimize the time spent in the to-charging mode (Mode 2). Finally, two scheduling policies, FRFS and SDF, are proposed to allow agents to share the charging station effectively while also guaranteeing that no agent runs out of energy during the entire process.

2. PROBLEM FORMULATION

Consider a bounded mission space $\Omega \in \mathbb{R}^2$, which is modeled as a non-self-intersecting polygon. We deploy N agents in the mission space to detect possible events that may occur in it. By viewing the position of agent i in $\mathbb{R}^2$, its coordinates $s_i = [x_i, y_i]^T$ obey the following dynamics:

$$\dot{x}_i(t) = v_i(t)\cos w_i(t), \tag{1}$$

$$\dot{y}_i(t) = v_i(t)\sin w_i(t), \tag{2}$$

with $v_i(t)$ denoting the speed, and $w_i(t)$ the heading direction of agent i. We assume that $v_i(t) \in [0, v]$, and $w_i(t) \in [0, 2\pi)$, where v is the maximum speed of an agent. The mission space does not contain obstacles. If it does, the problem can be modified appropriately as done in Zhong and Cassandras (2011).

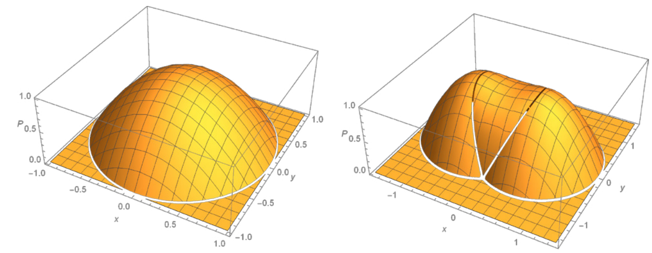
In contrast to traditional multi-agent coverage problems, agents are assumed to have a limited on-board energy supply, which is modeled by the state-of-charge $q_i(t)$ of its battery (i.e., the fraction of the battery available at time t). The dissipation of energy is proportional to a quadratic function of the velocity, yielding the following dynamics:

$$\dot{q}_i(t) = -\alpha v_i^2(t), \tag{3}$$

where α is a scaling constant to ensure that $0 \le q_i(t) \le 1$. When $q_i(t)$ is negative, this implies that agent i is "dead" in the mission space.

Remark 1. The energy depletion model (3) is a simplified version of $\dot{q}_i(t) = -v_i^2(t) - au_i^2(t)$ used in Setter and Egerstedt (2016), where $u_i(t)$ is the acceleration. There are also more sophisticated state-of-charge dynamics in the literature, e.g., Manwell and McGowan (1994), Moura et al. (2010).

To prevent agents from dying in the mission space, a charging station is available to all agents to replenish their energy supply during the mission time. Without loss of generality, we assume that the charging station is located at the origin with coordinates $(0, 0)$. At the charging station, the charging process has the following dynamics:



(a) A single agent at (0,0) (b) Two agents at $(\pm 0.5, 0)$

Fig. 1. Sensing probability of an area when agents present

$$\dot{q}_i(t) = \beta, \tag{4}$$

where $\beta > 0$ is the charging rate. We assume that only one agent can be served at the charging station at any time.

Our objective is to maximize the coverage of the mission space $\Omega \in \mathbb{R}^2$ over a time interval $[0, T]$, and at the same time keep all agents alive, that is, $q_i(t) > 0$ for all $t \in [0, T]$. The case $q_i(t) = 0$ can occur only at the charging station $(0, 0)$. Therefore, we consider the following optimization problem for each agent i:

$$\begin{array}{ll} \max\limits_{w_i(t),\, v_i(t)} & \frac{1}{T}\int_0^T H(\mathbf{s}(t))\,dt \\ \text{s.t.} & q_i(t) \ge 0, \\ & q_i(t) > 0 \text{ when } s_i(t) \ne \mathbf{0}, \\ & (1), (2) \\ & 0 \le v_i(t) \le v, \\ & (4) \text{ if charging, } (3) \text{ otherwise} \\ & \text{if } s_i(t) = \mathbf{0}, \\ & \text{then } s_j(t) \ne \mathbf{0} \text{ for all } j \ne i \\ & i = 1, \dots, N, \end{array} \tag{5}$$

where $\mathbf{s} = [s_1^T, \dots, s_N^T]^T$ is a column vector that contains all agent positions, T is the time horizon, and $H(\mathbf{s}(t))$ is the coverage metric. We adopt the coverage objective function used in Zhong and Cassandras (2011) by first defining a reward function $R(x, y)$ with $(x, y) \in \Omega$ to capture the "value" of a point (x, y) in the mission space, and assume $\int\int_\Omega R(x, y)\,dxdy < \infty$. Thus, $R(x, y)$ may have larger values for points whose coverage may carry more significance. Clearly, if all points in Ω are treated indistinguishably, then $R(x, y) = 1$ for all $(x, y) \in \Omega$.

Each agent has an isotropic sensing system with range δ_i, that is, an agent is able to cover the area

$$\Omega_i(x_i, y_i) = \left\{(x, y) \,|\, (x - x_i)^2 + (y - y_i)^2 \le \delta_i^2\right\}.$$

The sensing probability of an agent at a point (x, y) within its sensing range $\Omega_i(x_i, y_i)$ is characterized by the sensing function $p_i(x, y, x_i, y_i) \in [0, 1]$ and depends on the distance between the agent location (x_i, y_i) and the point (x, y). In particular, it is monotonically decreasing in the distance between (x_i, y_i) and (x, y) and if a point (x, y) is out of the sensing range of agent i, that is, $(x, y) \notin \Omega_i(x_i, y_i)$, then $p_i(x, y, x_i, y_i) = 0$. For any given point (x, y) in the sensing range of multiple agents, assuming independence among agent sensing capabilities, the joint event detection probability is given by (Zhong and Cassandras (2011))

$$P(x, y, \mathbf{s}) = 1 - \prod_{i=1}^N [1 - p_i(x, y, x_i, y_i)]. \tag{6}$$

Finally, the coverage metric $H(\mathbf{s})$ is defined as

$$H(\mathbf{s}) = \int\int_{\Omega} R(x, y) P(x, y, \mathbf{s}) \, dxdy.$$

Other reasonable sensing quality metrics are also possible, as in Stipanovic et al. (2013) and Panagou et al. (2015). Note that $H(\mathbf{s})$ is a function mapping a vector $\mathbf{s} \in \mathbb{R}^{2N}$ into $\mathbb{R}$.

For simplicity, in what follows we assume that all points in the mission space are indistinguishable and set $R(x, y) = 1$. Even though the precise form of the function $p_i(x, y, x_i, y_i)$ does not affect our subsequent analysis, for ease of calculation in the sequel we take it to be

$$p_i(x, y, x_i, y_i) = 1 - \frac{(x - x_i)^2 + (y - y_i)^2}{\delta_i^2}, \tag{7}$$

for all $(x, y) \in \Omega_i$. Figure 1 depicts the sensing probability for two cases: a single agent (Fig. 1(a)) and two agents with overlapping sensing ranges (Fig. 1(b)), respectively. Here the sensing range of agents is set to $\delta_i = 1$.

Returning to problem (5), there are two challenges we face. First, recall that an agent has no prior knowledge of either the mission space or the battery levels of other agents; it only knows the location of the charging station and of its neighbors, where the neighborhood set of agent i is defined as $\mathcal{N}_i = \{j \,|\Omega_j \cap \Omega_i \neq \emptyset\}$. In addition, the charging station is only provided the location and battery state information of agents when they are in the to-charging mode. Under this information structure, it is clearly impossible to tackle the coverage problem in a centralized way. The second challenge stems from the fact that, unlike the traditional coverage problem in Zhong and Cassandras (2011) where the goal is to find the optimal equilibrium locations of agents, (5) is a *dynamic* multi-agent coverage problem: due to the energy dynamics and constraints in (5), such an equilibrium may never exist, as agents move back and forth between coverage and battery charging modes. Thus, in general, finding the optimal speed $v_i^*(t)$ and the optimal heading $w_i^*(t)$ in problem (5) for all $i = 1, \ldots, N$ and all t is a challenging task since its solution amounts to a notoriously hard two-point-boundary-value problem similar to other dynamic multi-agent optimization problems, e.g., see Lin and Cassandras (2015). In the following, we will show how to solve this problem by modeling the combined cooperative coverage-recharging processes as a hybrid system. Nevertheless, the proposed solution may not be an optimal solution to the problem (5) due to the aforementioned challenges.

3. HYBRID SYSTEM MODEL

Our first step is to construct a hybrid system model to guarantee that the constraints in (5) are satisfied for all t. To ensure that the problem is well-posed, we assume that

$$\beta \geq N\alpha v^2. \tag{8}$$

This assumption is sufficient to guarantee the feasibility of the hybrid system model to be constructed. In particular, by treating the charging station as a server, the charging rate is β if it is occupied at all times, and referring to (3), the worst-case energy depletion rate over all agents is $N\alpha v^2$. Thus, the condition (8) is sufficient to prevent any agent from running out of energy (dying) anywhere in the mission space. However, this assumption is not necessary in the sense that the problem may be feasible even when (8) is not satisfied.

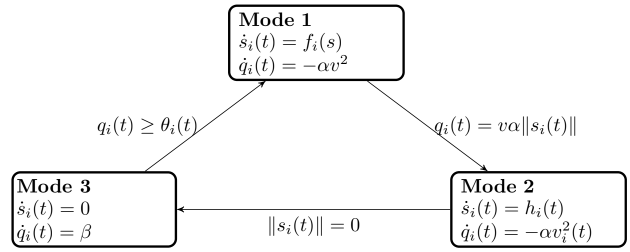


Fig. 2. A hybrid system model

For any agent, we define three different modes: coverage (Mode 1), to-charging (Mode 2) and in-charging (Mode 3). This hybrid system consists of a single cycle for each agent: Mode 1→Mode 2→Mode 3→Mode 1 as shown in Fig. 2 and detailed next. Due to the simplicity of the hybrid model, it is unnecessary to invoke a formal hybrid automaton modeling framework.

At Mode 1, $v_i(t) = v$, and

$$\cos w_i(t) = \frac{\frac{\partial H(t)}{\partial x_i(t)}}{\sqrt{\left(\frac{\partial H(t)}{\partial x_i(t)}\right)^2 + \left(\frac{\partial H(t)}{\partial y_i(t)}\right)^2}}, \tag{9}$$

$$\sin w_i(t) = \frac{\frac{\partial H(t)}{\partial y_i(t)}}{\sqrt{\left(\frac{\partial H(t)}{\partial x_i(t)}\right)^2 + \left(\frac{\partial H(t)}{\partial y_i(t)}\right)^2}}, \tag{10}$$

where the calculations of detailed expressions for $\frac{\partial H(t)}{\partial x_i(t)}$ and $\frac{\partial H(t)}{\partial y_i(t)}$ are given in Appendix A. To ease notation, we rewrite the dynamics in (1), (2) and (3) as

$$\dot{x}_i(t) = f_i^x(t), \quad \dot{y}_i(t) = f_i^y(t) \tag{11}$$
$$\dot{q}_i(t) = -\alpha v^2,$$

Here $f_i^x(t) = v \cos w_i(t)$ and $f_i^y(t) = v \sin w_i(t)$, where the expressions of $\cos w_i(t)$ and $\sin w_i(t)$ are given by (9), and (10), respectively. Moreover, $f_i(s)$ in Fig. 2 is given by $f_i(s) = [f_i^x, f_i^y]^T$. In other words, agent i travels at the maximum speed, and the heading direction follows the gradient direction of the coverage metric with respect to agent i's location. The state-of-charge of the battery monotonically decreases with rate αv^2 and when it drops to a certain value, the agent switches to Mode 2.

A transition from Mode 1 to Mode 2 occurs when the guard function

$$g_i(s_i, q_i) = q_i(t) - v\alpha \|s_i(t)\| \tag{12}$$

is zero, where $\|s_i(t)\| = \sqrt{x_i^2(t) + y_i^2(t)}$. At Mode 2, the speed $v_i(t)$ is determined by the scheduling algorithm used to assign an agent to the charging station and the heading direction is constant and determined by the location of agent i at the time of switching from Mode 1 to Mode 2, say τ_2. Then, the motion dynamics and the state-of-charge dynamics are:

$$\dot{x}_i(t) = -v_i(t) \frac{x_i(\tau_2)}{\|s_i(\tau_2)\|}, \quad \dot{y}_i(t) = -v_i(t) \frac{y_i(\tau_2)}{\|s_i(\tau_2)\|} \tag{13}$$

$$\dot{q}_i(t) = -\alpha v_i^2(t). \tag{14}$$

The speed $v_i(t)$ in Mode 2 is piecewise constant or constant depending on which scheduling algorithm is used to resolve conflicts when multiple agents request to use the charging station at the same time, as discussed in Section 4.2 (note that we assume no energy loss at points where the speed may experience a jump). The function $h_i(t)$ in Fig. 2 is a column vector containing the right-hand side of (13).

A transition from Mode 2 to Mode 3 occurs when the guard function $g_i(s_i) = \|s_i(t)\|$ is zero, that is, agent i arrives at the charging station. At Mode 3, an agent remains at rest at the charging station, therefore, it satisfies the dynamics $\dot{x}_i(t) = 0$ and $\dot{y}_i(t) = 0$. While the agent is in charging mode, the state-of-charge dynamics are given by $\dot{q}_i(t) = \beta$, where $\beta \geq N\alpha v^2$ is the charging rate.

Finally, a transition from Mode 3 to Mode 1 occurs when the guard function $g_i(q_i) = \theta_i - q_i(t)$ is zero, where $\theta_i \in (0, 1]$ is a controllable threshold parameter indicating the desired state-of-charge at which the agent may stop its recharging process.

4. MAIN RESULTS

4.1 Feasibility

In the following, we will show that the constraints in (5) are satisfied for all $t \geq 0$, that is to say, the hybrid system model constructed guarantees the feasibility of the problem in (5).

Lemma 1. For agents in Mode 2 satisfying (13) and (14) with $v_i(t) = v$, the travel time and energy cost from $(x_i(\tau_2), y_i(\tau_2))$ to $(0, 0)$ are $\|s_i(\tau_2)\|/v$ and $\alpha v\|s_i(\tau_2)\|$, respectively. Moreover, the speed v is proportional to the energy cost and it is inversely proportional to the travel time.

Proof. Let τ_2 denote the time when agent i switches from Mode 1 to Mode 2. At the initial time τ_2, agent i at the location $(x_i(\tau_2), y_i(\tau_2))$ heads to the charging station with a constant speed v. Therefore, the arrival time τ_3 at the charging station is $\tau_3 = \tau_2 + \|s_i(\tau_2)\|/v$. According to (14), the energy cost of traveling with speed v is

$$q_i(\tau_2) - q_i(\tau_3) = \int_{\tau_3}^{\tau_2} \dot{q}_i(t)\,dt = \alpha v\|s_i(\tau_2)\|.$$

The proof is completed by observing the above expressions of travel time and energy cost in terms of v. □

Remark 2. In the above proof, we assume that $v_i(t)$ is a constant for the whole interval $[\tau_2, \tau_3]$. It is easy to show that the above result still holds when $v_i(t)$ is piecewise constant during the interval $[\tau_2, \tau_3]$.

Now we are ready to show the main result in this subsection.

Theorem 1. The hybrid system model guarantees the feasibility of the optimization problem (5).

Proof. Suppose that the problem (5) is feasible initially. First, we know the problem (5) is always feasible in Mode 1 since if $q_i(t) \leq v\alpha\|s_i(t)\|$ for $(x_i(t), y_i(t)) \neq \mathbf{0}$, the mode switches to Mode 2. From Lemma 1, we know that the energy that an agent spends on the trip to the charging station is proportional to its speed. The energy when an agent switches to Mode 2 is adequate for it to reach the charging station at the maximum speed. Lemma 1 also tells us that an agent will spend less energy if it heads to the charging station using a piecewise constant speed less than the maximum speed. Therefore, $q_i(t) > 0$ in Mode 2. It is trivial to verify that the constraints are satisfied at Mode 3. Therefore, for all three modes, all agents have non-negative battery levels. □

4.2 Schedulability

Since the charging station can only serve one agent at a time, a scheduling algorithm is needed to resolve conflicts among agents competing over access to it. Here, we consider two scheduling policies: First-Request-First-Serve (FRFS) and Shortest-Distance-First (SDF).

First Request First Serve Suppose that when agent i sends a charging request at τ_r^i, the charging station is not reserved. Then, agent i will use the maximum speed v to reach the charging station. If another agent j sends a charging request at $\tau_r^j > \tau_r^i$, the arrival time of agent j will be scheduled at $\max\{\tau_f^i, \tau_a^j\}$, where τ_f^i is the time when agent i finishes charging, and τ_a^j is the arrival time if agent j heads to the charging station at the maximum speed. There are two different cases: $\tau_f^i < \tau_a^j$ and $\tau_f^i \geq \tau_a^j$. For the former case, there are no conflicts between agents i and j. This is because when agent j arrives at the charging station using the maximum speed, agent i has already left the charging station. For the latter case, the speed of agent j will be set to

$$v_j(t) = \frac{\|s_j(\tau_r^j)\|}{\tau_f^i - \tau_r^j} \leq v,$$

for $\tau_r^j \leq t < \tau_f^i$. Therefore, agent j will arrive at the charging station right after agent i finishes charging. It is straightforward to extend the case of two agents to the case of multiple competing agents.

Shortest Distance First Suppose that agent i sends a charging request at τ_r^i. While agent i is on its way to the charging station, suppose that agent j, which is closer to the charging station at time τ_r^j, also sends a charging request. Therefore, if both agents travel at the maximum speed, agent j will arrive at the charging station before agent i. In this case, the speed of agents j is set as

$$v_j(t) = v,$$

and its arrival time is τ_a^j. The arrival time of agent i will be scheduled at $\max\{\tau_f^j, \tau_a^i\}$, where τ_f^j is the leaving time of agent j from the charging station and τ_a^i is the intended arrival time of agent i to the charging station. Similarly, there are two different cases: $\tau_f^j < \tau_a^i$, and $\tau_f^j \geq \tau_a^i$. For the former case, there are no conflicts between agents j and i. For the latter case, the speed of agent i is set as

$$v_i(t) = \begin{cases} v & \text{for } t \in [\tau_r^i, \tau_r^j), \\ \dfrac{\|s_i(\tau_r^j)\|}{\tau_f^j - \tau_r^j} & \text{for } t \in [\tau_r^j, \tau_f^j). \end{cases}$$

In this case, agent i is scheduled to arrive at the charging station right after agent j finishes charging. It is not difficult to extend this reasoning to the case of multiple agents: the one closer to the charging station always receives the highest priority to be served first.

4.3 Optimality

Mode 1 Since the feasibility of the problem in (5) has already been guaranteed by the hybrid system model constructed, the constraints on $q_i(t), i = 1, \ldots, N$ are no longer needed. In addition, since an agent does not know the state-of-charge of all other agents or the positions of non-neighbor agents, it does not know when it will switch to Mode 2. Thus, an agent in Mode 1 seeks to maximize the objective function $H(\mathbf{s}(t))$ at the fastest possible rate. Given the trajectories of other agents, the optimization problem for agent i in Mode 1 becomes

$$\begin{array}{ll} \max\limits_{w_i(t),\, v_i(t)} & H(\mathbf{s}(t)) \\ \text{s.t.} & 0 \le v_i(t) \le v \\ & (1) \text{ and } (2) \end{array} \tag{15}$$

Here, a continuous-time gradient method is used to seek the optimal solution of the optimization problem (15). The normalized gradient direction is shown in (9) and (10). We can show that when $v(t) = v$, the gradient method achieves the fastest rate of convergence. In particular, calculating the time derivative of $H(\mathbf{s}(t))$, we obtain

$$\begin{aligned} \dot{H}(\mathbf{s}(t)) &= \sum_{i=1}^{N} \left[\frac{\partial H}{\partial x_i} \dot{x}_i(t) + \frac{\partial H}{\partial y_i} \dot{y}_i(t) \right] \\ &= \sum_{i=1}^{N} v_i(t) \sqrt{\left(\frac{\partial H}{\partial x_i}\right)^2 + \left(\frac{\partial H}{\partial y_i}\right)^2}. \end{aligned}$$

Given the positions of other agents, we can achieve the largest increasing rate of $H(s(t))$ due to agent i by choosing $v_i(t) = v$.

Mode 2 Once an agent is in the to-charging mode, its trajectory no longer follows the gradient direction of the objective function. Therefore, we seek to minimize the time duration of an agent in Mode 2, that is, to solve the following minimum time optimization problem:

$$\begin{array}{ll} \min & J = \int_{\tau_2}^{\tau_3} 1 dt \\ \text{s.t.} & 0 \le v_i(t) \le v, \\ & (1) \text{ and } (2) \\ & s_i(\tau_3) = \mathbf{0} \end{array} \tag{16}$$

where τ_2 is the time when an agent switches to Mode 2, and τ_3 is the time when an agent switches to Mode 3.

Then we have the following theorem.

Theorem 2. The shortest time of an agent in Mode 2 is $J^* = \|s_i(\tau_2)\|/v$ and the corresponding energy cost is $v\alpha \|s_i(\tau_2)\|$.

Proof. The optimization problem in (16) is subject to (1) and (2), and the final state constraints $x_i(\tau_3) = 0$, $y_i(\tau_3) = 0$. The Hamiltonian is

$$\mathcal{H} = 1 + \lambda_i^x(t) v_i(t) \cos w_i(t) + \lambda_i^y(t) v_i(t) \sin w_i(t).$$

By the stationarity condition, we have $-\dot{\lambda}_i^x = -\dot{\lambda}_i^y = 0$. Therefore, we know λ_i^x and λ_i^y are constants. Again, by the stationarity condition, we have

$$0 = \frac{\partial \mathcal{H}}{\partial w_i} = -\lambda_i^x \sin w_i(t) + \lambda_i^y \cos w_i(t).$$

Then, w_i is a constant. By the final condition $x_i(\tau_3) = 0$, and $y_i(\tau_3) = 0$, we obtain

$$1 + \lambda_i^x v_i(\tau_3) \cos w_i + \lambda_i^y v_i(\tau_3) \sin w_i = 0.$$

Therefore, we have the condition $\lambda_i^x \cos w_i + \lambda_i^y \sin w_i < 0$ since $0 \le v_i(\tau_3) \le v$. According to Pontryagin's minimum principle, the optimal $v_i^*(t)$ must satisfy

$$\begin{aligned} &1 + v_i^*(t)(\lambda_i^x \cos w_i + \lambda_i^y \sin w_i) \\ &\qquad \le 1 + v_i(t)(\lambda_i^x \cos w_i + \lambda_i^y \sin w_i). \end{aligned}$$

It follows that $v_i^*(t) = v$ since $\lambda_i^x \cos w_i + \lambda_i^y \sin w_i < 0$. The state equations are given as follows:

$$\dot{x}_i(t) = \frac{\partial \mathcal{H}}{\partial \lambda_i^x} = v_i \cos w_i, \quad \dot{y}_i(t) = \frac{\partial \mathcal{H}}{\partial \lambda_i^y} = v_i \sin w_i. \tag{17}$$

Using the optimal control $v_i^* = v$, and taking into account the initial states, we can integrate the above state equations to get

$$\begin{aligned} x_i(t) &= x_i(\tau_2) + v(t - \tau_2) \cos w_i, \\ y_i(t) &= y_i(\tau_2) + v(t - \tau_2) \sin w_i. \end{aligned}$$

Thus, the final time τ_3 must satisfy

$$0 = x_i(\tau_2) + v(\tau_3 - \tau_2) \cos w_i = y_i(\tau_2) + v(\tau_3 - \tau_2) \sin w_i.$$

Here $\cos^2 w_i + \sin^2 w_i = 1$, which is

$$\frac{x_i^2(\tau_2)}{v^2(\tau_3 - \tau_2)^2} + \frac{y_i^2(\tau_2)}{v^2(\tau_3 - \tau_2)^2} = 1.$$

Therefore, the minimum-time is $J^* = \|s_i(\tau_2)\|/v$ and the optimal heading direction is determined by

$$\cos w_i = \frac{x_i(\tau_2)}{\|s_i(\tau_2)\|}, \quad \sin w_i = \frac{y_i(\tau_2)}{\|s_i(\tau_2)\|}.$$

By integrating both sides of (3), we have

$$q_i(\tau_3) - q_i(\tau_2) = -\alpha v \|s_i(\tau_2)\|,$$

and the proof is completed by multiplying both sides by -1. □

Remark 3. The guard function (12) guarantees that when an agent switches from Mode 1 to Mode 2, it has sufficient energy to travel to the charging station using the least time. The guard function (12) is optimal in the sense that an agent maximizes its time in Mode 1 and minimizes its time in Mode 2.

Mode 3 We now address the question of selecting an optimal charging level, denoted by $\theta = [\theta_1, \ldots, \theta_N]$, when an agent is in the charging mode. This problem boils down to optimizing the parameter θ so that the objective function in (5) is maximized. By writing explicitly the dependence on θ, the optimization problem becomes

$$J(\theta) = \max_{\theta} \frac{1}{T} \int_0^T H(s(\theta, t)) dt.$$

Even though θ is only used in Mode 3, its optimal value affects the entire hybrid system model. By controlling θ, we directly control the switching times of agents from Mode 3 to Mode 1, and indirectly control the switching times of agents from Mode 1 to Mode 2. The switching times of agents from Mode 2 to Mode 3 are controlled by the proposed scheduling algorithms. Also note that the parameter θ is constant. We can obtain optimal charging thresholds through off-line analysis and implement the coverage task on line by all agents in distributed fashion. To determine the optimal θ, we will use Infinitesimal Perturbation Analysis (IPA) techniques (Cassandras et al.[2010]Cassandras

(Wardi)), a task which is the subject of ongoing research to be reported in future work.

A visual interactive simulation can be found at `http://www.bu.edu/codes/simulations/Coverage_ADHS`. Interested readers are encouraged to interact with the simulation by choosing different scheduling algorithms, as well as adjusting parameters such as the number of agents N, the sensing range δ_i, or the maximum speed v.

5. CONCLUSION

A hybrid system model is proposed to capture the behavior of multiple agents cooperating to solve an optimal coverage problem under energy depletion and repletion constraints. The proposed model links each agent's coverage, to-charging, and in-charging modes so as to form a cycle and the guard conditions are designed to maximize the coverage performance over a finite time horizon as well as to ensure that the agents never run out of energy. The problem of controlling the amount of energy repletion that maximizes the coverage objective function is the subject of ongoing research, as is the inclusion of energy expended for communication among agents. In addition, it remains to investigate the performance of the two proposed scheduling algorithms and explore additional ones. We conjecture that SDF performs better than FRFS, but this remains to be verified. Finally, when obstacles are present in the mission space, finding optimal trajectories for agents in Mode 2 is a challenging task that we plan to address in future work.

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Appendix A. CALCULATION OF THE GRADIENT

To find the heading direction of agent i, we need to calculate the gradient of $H(\mathbf{s})$ at point (x_i, y_i), which is $\nabla H(s_i) = [\partial H/\partial x_i, \partial H/\partial y_i]^T$. According to Flanders (1973), we can calculate the gradient as

$$\frac{\partial H}{\partial x_i} = \iint_{\Omega} \frac{\partial P}{\partial x_i} + \int_{\partial\Omega} P\left(\frac{\partial x}{\partial x_i}dy - \frac{\partial y}{\partial x_i}dx\right),$$

where the integration in the second term is done in the counterclockwise direction over the boundary of Ω. Recalling the expressions of (6) and (7), we have

$$\int_{\partial\Omega} P\left(\frac{\partial x}{\partial x_i}dy - \frac{\partial y}{\partial x_i}dx\right) = 0.$$

This is because when $(x, y) \in \partial\Omega \cap \partial\Omega_i$, $P = 0$; when $(x, y) \in \partial\Omega \backslash \partial\Omega_i$, $\partial x/\partial x_i = \partial y/\partial x_i = 0$. Therefore, we can obtain

$$\frac{\partial H}{\partial x_i} = \iint_{\Omega_i} \frac{2(x - x_i)}{\delta_i^2} \prod_{j\in\mathcal{N}_i} (1 - p_j)\, dxdy. \tag{A.1}$$

Similarly, we have

$$\frac{\partial H}{\partial y_i} = \iint_{\Omega_i} \frac{2(y - y_i)}{\delta_i^2} \prod_{j\in\mathcal{N}_i} (1 - p_j)\, dxdy. \tag{A.2}$$

Remark 4. When the sensing range Ω_i of agent i is blocked by the boundary, the gradient can be derived similarly using a simple projection onto the feasible mission space. The detailed calculations for this case are thus not shown here.