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Chapter 5

Multi-photon quantum interferometry

by

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Contents

§ 1. Introduction: practical optical tests of Bell inequalities

The often deeply counterintuitive predictions of quantum mechanics have been the focus of intensive discussions and debates among physicists since its introduction. In the early 1930s, the first explicit quantum theory of measurement was presented in the work of Landau and Peierls [1931], von Neumann [1932], Fock [1932] and Pauli [1933]. Since these early days, the *entangled states* (quantum states of multiple particles that cannot be represented as products of independent single-particle states: $\Psi \neq \psi_1 \otimes \psi_2 \otimes \cdots \otimes \psi_n$, where ψ_i are subsystem states, for n particles) were prepared in order to better differentiate quantum behavior from classical behavior. As Schrödinger [1935], who defined entanglement, put it, entanglement is "the characteristic trait of quantum mechanics".

After the theorem of von Neumann concerning hidden-variables theories, of which quantum mechanics might be the statistical counterpart, and the argument of Einstein, Podolsky and Rosen (EPR) [1935], that quantum mechanics is an incomplete theory of physical objects, practical tests of nonclassical behavior were carried out by Wu and Shaknov [1950] with states of spin, as discussed by David Bohm [1951] and analyzed by Bohm and Aharonov [1957] in the 1950s [1]. Next the way was prepared for the systematic study of quantum-scale behavior that could not be explained by the class of local, deterministic hidden variables theories. In 1964, John Bell derived a general inequality that introduced a clear empirical borderline between local, classically explicable behavior and less intuitive forms of behavior (such as nonlocality, contextuality and stochasticity), which he called "nonlocality" (Bell [1964]) [2]. The atomic cascade decay was at the time the best optical source of entangled states. An experiment using such a source was made soon after by Kocher and Commins [1967], to distinguish between quantum mechanics and local hidden-variables theories. Clauser, Horne,

[1] In this experiment, polarization measurements were made of two high-energy photons produced by spin-zero positronium annihilation (Wu and Shaknov [1950]).

[2] The premise resulting from Bell's heuristics motivated by his "locality" thesis was later shown (Jarrett [1984]) to be decomposable into the conjunction of two independent conditions, known as parameter and outcome independence (Shimony [1990]).

Shimony and Holt (CHSH) then modified Bell's treatment to fit any experimental arrangement similar to that of a two-spin atomic system, deriving the inequality

$$|E(\theta_1, \theta_2) + E(\theta_1, \theta_2') + E(\theta_1', \theta_2) - E(\theta_1', \theta_2')| \leqslant 2,$$

where the E's are the expectation values of the products of measurement outcomes for measurement parameter values $\theta_1, \theta_1', \theta_2$ and θ_2' (Clauser, Horne, Shimony and Holt [1969]). This result set the stage for more definitive experimental tests of quantum theory that involved a new generation of quantum optical sources of entangled photon pairs, i.e., of two-photons.

Radiative atomic cascade two-photon sources were used by groups at Berkeley, Harvard and Texas A&M in the early 1970s (see Clauser and Shimony [1978]). However, one particular problem with Bell inequality tests arose during this period: only single-channel polarizers consisting of a stack of glass plates at the Brewster angle were available, giving access to only positive measurement outcomes (see Aspect [1999]). In 1976, John Clauser performed one such experiment with results that suggested that such Bell-type inequalities might in fact hold, so that a local hidden-variable theory might be valid at the atomic scale. However, advances in laser physics and optics allowed for a new generation of elegant experiments by the group of Aspect at Orsay in the early 1980s, based on the use of photon pairs produced by nonlinear laser excitations of an atomic radiative cascade and the use of two-channel polarizers. These experiments paved the way for future quantum-interferometric experiments involving entangled photons, finally producing an unambiguous violation of a Bell-type inequality by tens of standard deviations and strong agreement with quantum-mechanical predictions (Aspect, Grangier and Roger [1981]).

Though producing photon pairs using atomic cascade decays had allowed for the initial demonstration of the significance of entangled states, the method was obviously far from optimal considering the sophisticated tests already proposed (Fry and Thompson [1976]). Meanwhile, a new, more powerful source of entangled photons, the optical parametric oscillator (OPO, fig. 1), was being developed independent of tests of basic principles of quantum mechanics. OPOs were operational in major nonlinear optics research groups around the world almost immediately following the development of the laser (for more details, see Bloembergen [1982]). It was seven years after the first OPOs were introduced that an international collection of experimental groups [Byer at Stanford University (Harris, Oshman and Byer [1967]), Magde and Mahr at Cornell University (Magde and Mahr [1967]), and Klyshko et al. at Moscow State University (Klyshko [1967])] independently discovered the spontaneous emission of polarized photons in an optical parametric amplifier.

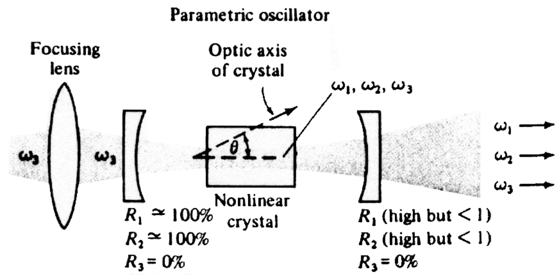

Fig. 1. Optical parametric oscillator (OPO).

This very weak OPO spontaneous noise occupied a very broad spectral range, from near the blue pump frequency through to the infrared absorption band. A corresponding spatial distribution of different frequencies followed the well-known and simple phase-matching conditions of nonlinear optical systems (see § 2). This effect had several names at the time: "parametric fluorescence", "parametric luminescence", "spontaneous parametric scattering", and "splitting". The existence of such a process follows from the quantum consideration of a parametric amplifier developed by Louisell, Yariv and Siegman [1961]. The Hamiltonian for the down-conversion process is given by

$$H_{\mathrm{I}} = \tfrac{1}{2} \int \mathrm{d}v\, \boldsymbol{P} \bullet \boldsymbol{E}_p(r,t) = \tfrac{1}{2} \int \mathrm{d}v\, \chi^{(2)}_{12p} \boldsymbol{E}_1(r,t)\, \boldsymbol{E}_2(r,t)\, \boldsymbol{E}_p, \tag{1.1}$$

where $\boldsymbol{P}$ is the nonlinear polarization induced in the medium by the pump field $\boldsymbol{E}$. The polarization is defined in terms of the second-order dielectric susceptibility of the medium $\chi^{(2)}_{12p}$, coupling the pump field to the two output fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$. The field annihilation operators for photons at two output frequencies ω_1 and ω_2 can be written as

$$\begin{aligned} a_1(t) &= \mathrm{e}^{-\mathrm{i}\omega_1 t}(a_{10} \cosh gt + \mathrm{i}\mathrm{e}^{-\mathrm{i}\phi} a^{\dagger}_{20} \sinh gt), \\ a_2(t) &= \mathrm{e}^{-\mathrm{i}\omega_2 t}(a_{20} \cosh gt + \mathrm{i}\mathrm{e}^{-\mathrm{i}\phi} a^{\dagger}_{10} \sinh gt), \end{aligned} \tag{1.2}$$

where g is a parametric amplification coefficient proportional to the second-order susceptibility, the crystal length and the pump field amplitude, a_{i0} and $a^{\dagger}_{i0}$ are the initial operator values, and ϕ is determined by the pump wave phase. Accordingly, the average number of photons per mode in the output fields $n_1(t)$ and $n_2(t)$ is

$$\begin{aligned} n_1(t) &= \left\langle a^{\dagger}_1(t)\, a_1(t) \right\rangle = n_{10} \cosh^2 gt + (1 + n_{20}) \sinh^2 gt, \\ n_2(t) &= \left\langle a^{\dagger}_2(t)\, a_2(t) \right\rangle = n_{20} \cosh^2 gt + (1 + n_{10}) \sinh^2 gt, \end{aligned} \tag{1.3}$$

where n_{10} and n_{20} are the inputs into the $n_1(t)$ and $n_2(t)$ fields, respectively. This describes a two-component gain feature having the odd property that the "1" in the second terms means that there is nonzero output, even when both input fields are zero. This "extra" one photon per mode – due to vacuum fluctuations – can be viewed as stimulating spontaneous down-conversion.

The practical theory describing the generation of such radiation was analyzed in detail in 1967–1968 by Mollow and Glauber [1967], Giallorenzi and Tang [1968, 1969] and Klyshko [1967].

The next step in the theory was to analyze the statistics of photons appearing in such spontaneous conversion of one photon into a pair. This was done by Zel'dovich and Klyshko [1969], and Mollow in 1969 (and later treated in detail by Mollow [1973] and Kleinman [1968]), demonstrating the existence of very strong correlations between these photons in space, time and frequency. Burnham and Weinberg [1970] first demonstrated the unique and explicitly nonclassical features of states of two-photons generated in the spontaneous regime from the parametric amplifier. Quantum correlations involving two-photons were exploited again 10 years later in experimental work by Malygin, Penin and Sergienko [1981a,b]. Because of a very active research program at the University of Rochester led by Mandel, and the work of Alley at the University of Maryland, the use of highly correlated pairs of photons for the explicit demonstration of Bell inequality violations has become popular and convenient since the mid-1980s. The contemporary name for the process of generating these states, "spontaneous parametric down-conversion" (SPDC), has become widely accepted in the research community, and new, high-intensity sources of SPDC have been developed (see § 3).

A number of excellent reviews on the topic of two-photon quantum interference exist. Among the most comprehensive recent reviews covering the topic of entangled-photon interference are *Quantum Optics and the Fundamentals of Physics* by Perina, Hradil and Jurco [1994], *Optical Coherence and Quantum Optics* by Mandel and Wolf [1995], Hariharan and Sanders [1996], *Quantum Optics* by Scully and Zubairy [1997], and *The Physics of Quantum Information* by Bouwmeester, Ekert and Zeilinger [2000]. Though quantum optics has always kept the attention of the physics community, these reviews have mainly covered the subjects of quantum coherence, squeezed states, quantum non-demolition measurement and, most recently, quantum information. The main goal of this review is to exhibit several different contemporary trends in the development of entangled-photon interferometry using SPDC. We shall concentrate mainly on developments in the area of experimental two-photon interferometry, which has received a significant boost recently due to the importance of the properties of quantum entanglement in such exciting, but still relatively young areas as quantum teleportation, quantum cryptography and quantum computing (see also § 3 of ch. 1 in this volume).

§ 2. Two-photon interferometry with type-I phase-matched SPDC

Spontaneous parametric down-conversion (SPDC) of one photon into a pair is said to be of one two types, type I or type II, depending on whether the two photons of the down-conversion pair have the same polarization or orthogonal polarizations. The two photons of a pair can also leave the down-converting medium either in the same direction or in different directions, the collinear and noncollinear cases respectively. A medium is required for down-conversion, as conservation laws exclude the decay of one photon into a pair in vacuum. The medium is usually some sort of birefringent crystal, such as potassium dihydrogen phosphate (KDP), having a $\chi^{(2)}$ optical nonlinearity.

Upon striking such a nonlinear crystal there is a small probability (on the order of 10^{-7}) that an incident pump photon will be down-converted into a two-photon (see fig. 2). If down-conversion occurs, these conserved quantities are carried into that of the resulting photon pair under the constraints of their respective conservation laws, with the result that the phases of the corresponding wavefunctions match, in accordance with the relations

$$\omega_1 + \omega_2 = \omega_p, \qquad \boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{k}_{\mathrm{p}}, \tag{2.1}$$

known as the "phase-matching" conditions, where the $\boldsymbol{k}_i$ and ω_i are momenta and frequencies for the three waves involved. The individual photons (here labeled $i = 1, 2$) are often arbitrarily called "signal" and "idler", for historical reasons. When the two photons of a pair have different momenta or energies, entanglement will arise in SPDC, provided that the alternatives are in principle experimentally indistinguishable.

The two-photon state produced in type-I down-conversion can be written

$$|\,\psi\rangle = \int_0^{\omega_{\mathrm{p}}} \mathrm{d}\omega_1\, \phi(\omega_1, \omega_0 - \omega_1)\, |\omega_1\rangle\, |\omega_0 - \omega_1\rangle\,, \tag{2.2}$$

where $\phi(\omega_1, \omega_0 - \omega_1)$ is the frequency density and the two photons leave the nonlinear medium with the same polarization, orthogonal to the polarization of the pump beam photons. Down-conversion photons are thus produced in two

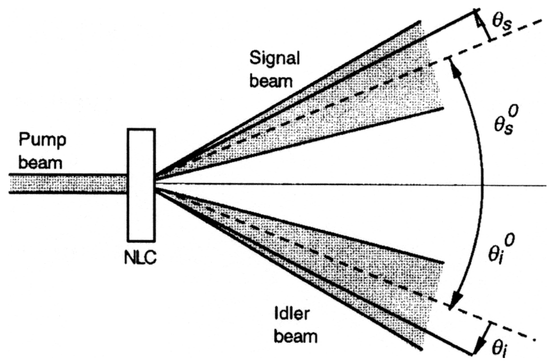

Fig. 2. Spontaneous parametric down-conversion (Saleh [1998]).

thick spectral cones, one for each photon, within which two-photons appear each as a pair of photons on opposite sides of the pump-beam direction (see fig. 2).

In a pioneering experiment in the mid-1980s, Hong, Ou and Mandel [1987] created noncollinear, type-I phase-matched SPDC photon pairs in KDP crystal using an ultraviolet continuous-wave (cw) laser pump beam (see fig. 3). These photon pairs were directed to a movable beamsplitter by two mirrors, so that the two resulting spatially superposed beams impinged on two photodetectors D_1 and D_2. Filters placed in the apparatus determined the frequency spread of the down-converted photons. This experiment empirically demonstrated the strong temporal correlation of the two-photons. The correlation function for two-photons is $g(t) = G(t)/G(0)$. In the experiment $G(t) = \int \mathrm{d}t\, \phi[(\omega_0/2) + \omega_1, (\omega_0/2) - \omega_1]$, where the down-converted light was frequency-degenerate, so that ϕ peaked at $\omega_1 = \frac{1}{2}\omega_0 = \omega_2$, with $\omega_0 = 351.1$ nm; g was nearly Gaussian in ω with a bandwidth $\Delta\omega$.

The probability of joint detection of the two photons of the pair at D_1 and D_2, at times t and $t+\tau$ respectively, in such an experiment is given by

$$\begin{aligned} \mathrm{P}_{12}(t) &= K \left\langle E_1^{(-)}(t)\, E_2^{(-)}(t+\tau)\, E_2^{(+)}(t+\tau)\, E_1^{(+)}(t) \right\rangle \\ &= K|G(0)|^2 \left\{ T^2|g(\tau)|^2 + R^2|g(2\Delta\tau - \tau)|^2 - RT[g^*(\tau)\, g(2\Delta\tau - \tau) + \text{c.c.}] \right\}, \end{aligned} \tag{2.3}$$

where the E_i are the electric fields at detectors D_i, and K is a constant characterizing the detectors. In the Hong–Ou–Mandel (HOM) experiment, the coincidence rate for photon joint detection at D_1 and D_2 was studied as the beamsplitter (BS) was translated vertically from its central location by small distances $c\,\delta\tau$, giving rise to optical path differences for the two outgoing beams. With $R/T = 0.95$, the corresponding joint count rate N_c exhibited a sharp dip,

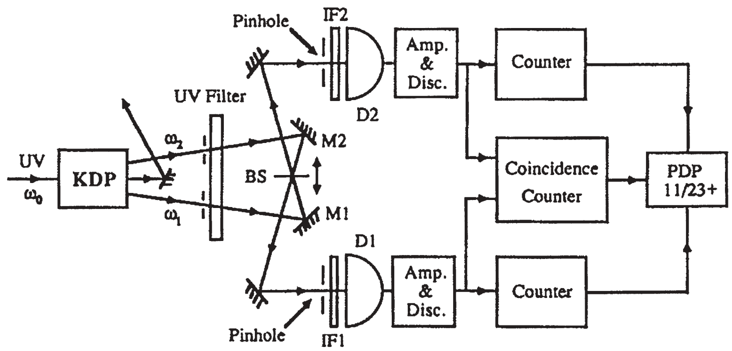

Fig. 3. Hong–Ou–Mandel interferometer (Hong, Ou and Mandel [1987]).

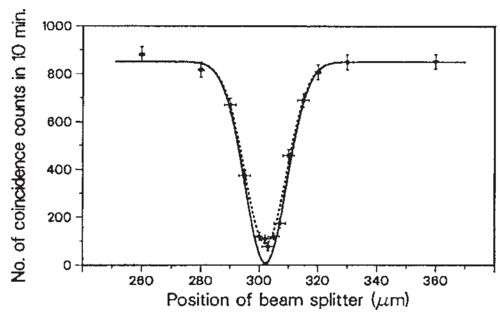

Fig. 4. Hong–Ou–Mandel dip (Hong, Ou and Mandel [1987]).

near the time difference $\delta\tau$, having a width determined by the length of the wavepacket (or, equivalently, the coherence time) of the two-photons.

This nonclassical coincidence dip was seen to fall to a few percent from the maximum value (see fig. 4), whereas classical optics predicts a visibility that cannot exceed 0.5 (Mandel [1983]) and Bell-type inequality violations can be obtained once coincidence visibilities exceed 71% (see, for example Tittel, Brendel, Gisin and Zbinden [1999]). Such a dip – hereafter referred to as the "Hong–Ou–Mandel dip" – also provided for an empirical measure of the time intervals between the two photon arrivals with sub-picosecond precision. Unlike methods requiring the observation of second-order (i.e., single-photon) interference, this technique does not require keeping path differences stable to within a fraction of a wavelength.

In 1988, a similar arrangement and light frequency was used by Ou and Mandel [1988a,b] to demonstrate the violation of Bell's inequality by six standard deviations, in addition to disagreement with classical optical

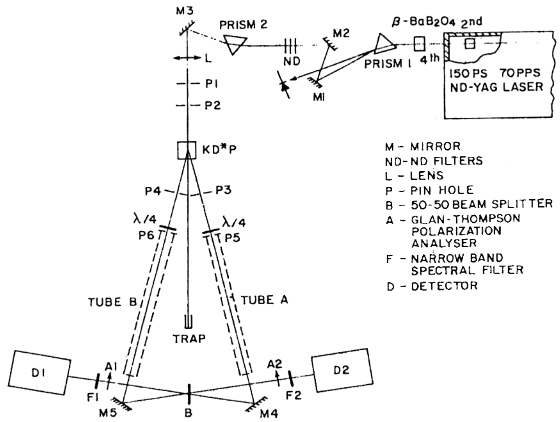

Fig. 5. Shih–Alley experiment (Shih and Alley [1988]).

predictions. In that experiment, the idler photon was rotated by 90 degrees in one beam before reaching the beamsplitter, and polarizers were placed before D_1 and D_2 at angles θ_1 and θ_2, respectively, to obtain count rates corresponding to the joint probabilities of the left-hand side of Bell's inequality. Taking into account alignment imperfections, the observed joint probabilities were found in this experiment to be in agreement with the quantum mechanical predictions and in violation of the CHSH inequality. According to quantum theory, the choices $\theta_1 = \pi/8$, $\theta_2 = \pi/4$, $\theta_1{}' = 3\pi/8$, $\theta_2{}' = 0$, for example, yield $S = \frac{1}{4}K(\sqrt{2} - 1) > 0$, where $S \leqslant 0$ is the Clauser–Horne variant of the inequality (Clauser and Horne [1974]). The corresponding two-photon interference visibility was empirically found to be $V = 0.76$.

That same year, Shih and Alley [1988] used a similar experimental arrangement, but replaced the cw pump laser with a pulsed laser operating at 266 nm (fig. 5), to demonstrate a three-standard-deviation Bell-type inequality violation. In particular, it was found that $\delta = \left|\left[R_c(\frac{1}{8}\pi) - R_c(\frac{3}{8}\pi)\right]/R_0\right| = 0.34 \pm 0.03 > \frac{1}{4}$, where $\delta \leqslant \frac{1}{4}$ is the Freedman–Clauser variant of the inequality (Freedman and Clauser [1972]). Furthermore, the results were in good agreement with the quantum-mechanical prediction of $\delta = \frac{1}{4}\sqrt{2} \cong 0.35$.

In a variation on the same experimental arrangement, Rarity and Tapster obtained a coincidence dip by translating right-angle prisms, instead of fixed mirrors, placed in the beam paths before the beamsplitter. They next explored the frequency non-degenerate case of SPDC to obtain an interferogram exhibiting additional oscillations (Rarity and Tapster [1990a]). The time resolution was improved to approximately 40 fs and the observed visibility reached $V = 0.84$.

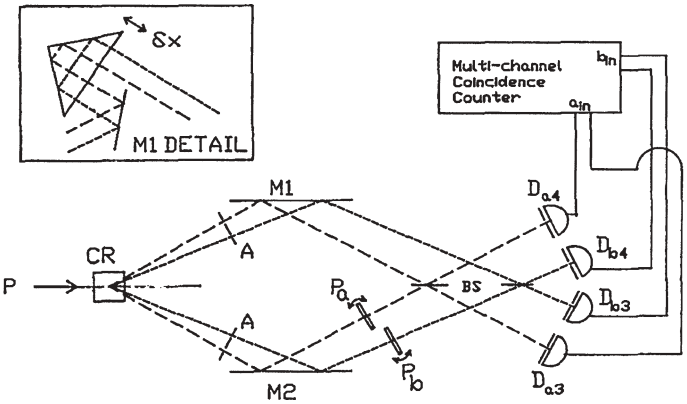

Fig. 6. Rarity–Tapster experiment (Rarity and Tapster [1990b]).

During the same period, after proposals by Horne, Pykacz, Shimony, Zeilinger and Zukowski (Horne and Zeilinger [1985], Zukowski and Pykacz [1988], Horne, Shimony and Zeilinger [1989]), Rarity and Tapster [1990b] used a modified arrangement involving two beamsplitters and two balanced Mach–Zehnder interferometers to test Bell's inequality. In this case, the variable of the state entanglement was momentum-direction and phase-shifting elements were placed in space-like separated locations (see fig. 6). The measured value for the left-hand side of the CHSH inequality using this arrangement was found to reach $S = 2.21$ at an interference visibility of $V = 0.78$, amounting to an inequality violation by 10 standard deviations. SPDC had also previously been used for similar experiments by Ou and Mandel [1988b] using polarization variables.

A different interferometric arrangement having two spatially separated, *unbalanced* Mach–Zehnder interferometers, each involving a phase shift ϕ_i ($i = 1, 2$) between the long and short beam paths, was also proposed by Franson [1989], in order to test a Bell-type inequality for position and energy without the involvement of polarization variables or polarizers. This latter sort of experiment was carried out by Franson [1991] (see fig. 7). The interferometer was pumped by a cw laser that produced energy-degenerate two-photons by SPDC. Brendel, Mohler and Martienssen [1992], Kwiat, Steinberg and Chiao [1993] and Shih, Sergienko and Rubin [1993] carried out similar experiments, though with somewhat different arrangements.

The initial two-photon state for such experiments can be written

$$|\psi\rangle = \tfrac{1}{2}\left(|S\rangle_1 |S\rangle_2 - e^{i(\phi_1+\phi_2)} |L\rangle_1 |L\rangle_2 + e^{i\phi_2} |S\rangle_1 |L\rangle_2 + e^{i\phi_1} |L\rangle_1 |S\rangle_2\right), \tag{2.4}$$

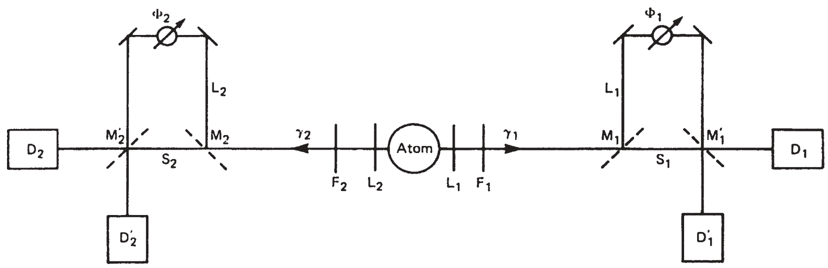

Fig. 7. Franson interferometer (Franson [1991]).

where S and L refer to the temporal position corresponding to short and long optical path lengths, respectively. By using a sufficiently large path-length difference between long and short options, the last two terms may be neglected – an entangled two-photon state results.

In the above experiments, the difference of optical paths in the two interferometers, ΔL, satisfies the requirement $cT_{\mathrm{coh}} \ll \Delta L$, where T_{coh} is the coherence time of the down-converted photons, so as to exclude single-photon (i.e., second-order) interference and to allow only two-photon (i.e., fourth-order) interference to occur. Since a cw laser was used as a pump, the photon emission times were unknowable (i.e., experimentally indistinguishable in principle), allowing no way of determining which of the two photons detected takes the short path and which takes the long path, while preserving the energy correlations between photons. The resulting coincidence counting rate was $R_{\mathrm{c}} = \frac{1}{4}\cos^2\left(\phi_1' + \phi_2'\right)$. This ostensibly allowed the demonstration of nonlocal effects due to quantum theory and the testing of local realistic theories because of their in-principle capability of providing 100% two-photon interference visibility while no single-photon interference would arise when the individual phases ϕ_i' ($\phi_1' = \phi_1/2$, $\phi_2' = \phi_2/2 + \phi_0$) were varied. However, it should be noted that it recently has been shown that such experiments cannot provide tests of local realism, since there exists a local hidden-variable model that reproduces quantum predictions for joint measurements using this apparatus (Larsson, Aerts and Zukowski [1998]).

A time–frequency Bell inequality test has also been proposed, though not realized (Davis [1989]). The idea is to measure the detection time of one of two-photons produced by down-conversion of a bandwidth-limited, pulsed pump beam, with respect to the center of each down-converted pump pulse and the spectral frequency of the other. The detection time of the first photon provides information regarding the arrival time of the second photon from the same two-photon. Thus the arrival time and the spectral frequency of the second photon become known precisely enough to violate the time–frequency bandwidth

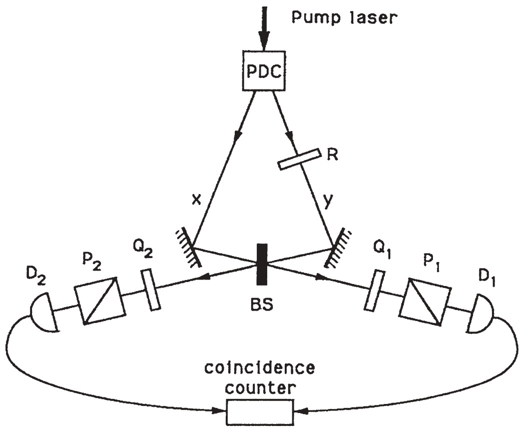

Fig. 8. Apparatus for testing local realism with a Bell inequality (Torgerson, Branning, Monken and Mandel [1995]).

product. Using time-dependent-physical-spectrum (TDPS) measurements, using a single-photon detector behind a Fabry–Perot etalon, the photon arrival time is to be measured to a precision limited only by the reciprocal of the etalon frequency resolution. The quality factor of the etalon then determines which combination of the time and frequency $\hat{\Gamma}_Q$, is observed. Positive or negative values are to be attributed based on whether or not the photon is transmitted through the etalon within a given time window. The necessary correlations between such measurements on space-like separated photons will arise due to their simultaneous production under phase-matching constraints. Considering three sets of coincidence count rates under the assumptions of counterfactual definiteness of photon properties and the Einstein locality condition, given the above direct correlations allows a Bell-type inequality to be derived for these variables:

$$\Pr[\hat{F}^{(1)} = +, \hat{T}^{(2)} = -] \leqslant \Pr[\hat{F}^{(1)} = +, \hat{\Gamma}^{(2)} = -] + \Pr[\hat{\Gamma}^{(1)} = +, \hat{T}^{(2)} = -], \tag{2.5}$$

where F and T refer to the special cases of pure frequency and pure time measurements, and the superscripts each refer to one of the two photons, arbitrarily labeled 1 and 2.

There followed a truly remarkable violation of local realism by roughly 40 standard deviations. This result was achieved in an experiment by Torgerson, Branning, Monken and Mandel [1995] (see fig. 8). Motivated by the ambiguous results of Bell-type inequality tests in which two photons pass through QWPs before reaching polarization analyzers, these workers obtained tremendous inequality violations that removed any lingering questions about nature's ability to violate such inequalities.

In late 1994, an argument was presented to the effect that many experiments involving SPDC cannot be used to properly test Bell-type inequalities because the states they utilized are in fact product states (De Caro and Garuccio [1994]). That is, only by post-selecting from the full ensemble of down-conversion events that one-half of events in which joint-detections occur, can Bell tests be simulated. In particular, such a method appears invalid because the intrinsic efficiency of detection required for loophole-free tests is 67% (Kwiat [1995], Kwiat, Eberhard, Steinberg and Chiao [1994], Eberhard [1993]). However, it was subsequently pointed out that even type-I phase-matched down-conversion sources can be configured so as to produce genuine entanglement without the need for post-selection (Kwiat [1995]). This concern can be completely avoided by using type-II phase-matched down-conversion sources that produce a state truly entangled in regard to polarization. Furthermore, the CHSH inequality may be slightly modified so as to allow the use of the full ensemble in a valid Bell-type inequality test.

§ 3. Two-photon interferometry with type-II SPDC

In the case of type-II spontaneous parametric down-conversion (SPDC), the two photons of each down-conversion pair have orthogonal rather than identical polarizations. This allows the entanglement of their states to involve polarization in addition to those other quantities potentially involved in the type-I case. This sort of entanglement, including multiple degrees of freedom, has been referred to as "hyper-entanglement" (Kwiat [1997]). In the type-II case, if the two photons of a pair leave the down-converting medium in different directions, *i.e.*, noncollinearly, their entanglement will involve both directions – as it is not possible to identify which photon went in each direction – and polarizations. Moreover, for a nearly monochromatic, continuous-wave laser pump any sort of down-conversion pair entanglement will involve energy, yielding hyper-entanglement with three relevant quantities.

Such states are generally given by

$$
\begin{aligned}
|\,\psi\rangle = \tfrac{1}{2}\int_0^{\omega_{\mathrm{p}}} \mathrm{d}\omega\,\phi(\omega,\omega_0-\omega)\,|\omega\rangle\,|\omega_0-\omega\rangle \\
\cdot\left(|\boldsymbol{k}_1\rangle\,|\boldsymbol{k}_1'\rangle + \exp[\mathrm{i}\phi]\,|\boldsymbol{k}_2\rangle\,|\boldsymbol{k}_2'\rangle\right)(|\mathrm{e}\rangle\,|\mathrm{o}\rangle + |\mathrm{o}\rangle\,|\mathrm{e}\rangle)\,,
\end{aligned}
\tag{3.1}
$$

where the orthogonal polarizations of the down-conversion photons are labeled "e" and "o", according to their orientation relative to the polarizations associated with the extraordinary and ordinary axes of the nonlinear crystal used for down-conversion. Unlike the case of type-I phase-matched down-conversion, the two

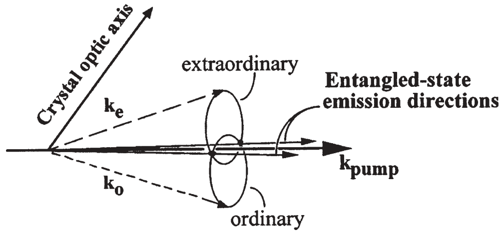

Fig. 9. SPDC under type-II phase-matching conditions (Kwiat [1997]).

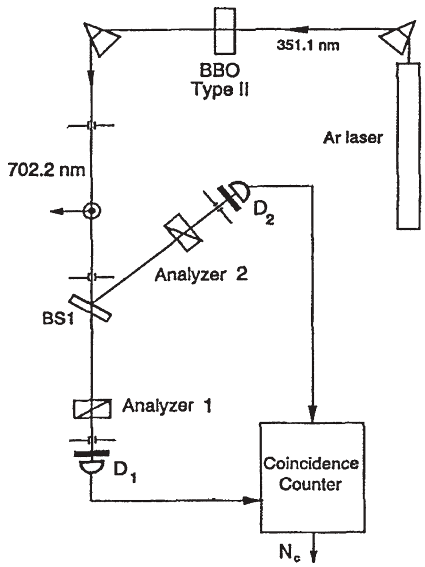

Fig. 10. Bell inequality tests using type-II phase-matched two-photons (Kiess, Shih, Sergienko and Alley [1993]).

down-conversion light cones are *not* concentric about the direction of the pump beam (see fig. 9, and contrast with fig. 2).

The new ingredient in the type-II case (eq. 3.1), compared with the type-I case (eq. 2.2), is the involvement of polarization in the entanglement. Entangled states of this kind were used by Kiess, Shih, Sergienko and Alley [1993] to find CHSH inequality violation by 22 standard deviations. In that experiment, a 351.1 nm cw laser pump was used to produce two-photons in BBO crystal at 702.2 nm. These collinear-photon pairs were deflected by a nonpolarizing beamsplitter to two Glan–Thompson polarization analyzers followed by photodetectors, and the resulting coincidence detections were studied (see fig. 10).

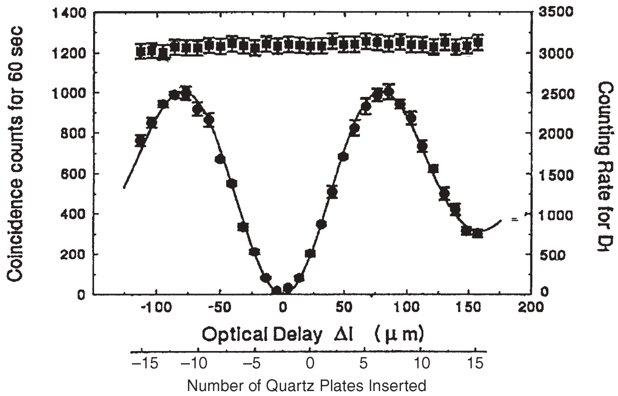

Fig. 11. Polarization two-photon coincidences varying optical delay (Rubin, Klyshko, Shih and Sergienko [1994]).

Shortly thereafter, a comprehensive theoretical treatment of these type-II phase-matched two-photons was given by Rubin, Klyshko, Shih and Sergienko [1994]. A review of several experiments done at the University of Maryland–Baltimore County verifying this treatment was presented therein. Quantum beating between polarizations was also observed as absolute polarizations were varied while relative polarization was kept orthogonal (see fig. 11).

A similar experimental arrangement was then used to demonstrate the violation of two Bell-type inequalities, one for polarization and one for space–time, in a single experimental arrangement (Pittman, Shih, Sergienko and Rubin [1995]). In order to test the latter, EPR states were produced by probability-amplitude cancellation. The experimental arrangement was similar to that of fig. 10, but included also a large quartz polarization delay line and a number of thinner reorientable birefringent quartz plates placed before the predetector polarization analyzers. Two optical paths to each detector were thus created, so that a two-photon state of the form

$$\Psi = A(X_1, X_2) - A(Y_1, Y_2)$$

was created, where 1 and 2 label the fast-axis path and the slow-axis path respectively, analogously to the short and long paths of the Franson interferometer, and X and Y indicate two orthogonal linear polarizations.

Notably different from the Franson interferometer, however, is that the entangled state here arises from probability-amplitude cancellation rather than from the use of a short coincidence counting time window. In the position test, by activating two spacelike separated Pockels cells, a coincidence counting

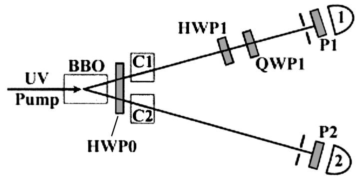

Fig. 12. High-intensity two-photon source (Kwiat, Mattle, Weinfurter, Zeilinger, Sergienko and Shih [1995]).

rate $R_c = R_0\,[1 - \cos(\omega_1\Delta_1 - \omega_2\Delta_2)]$ was found, where the Δ_i are the total optical delay between the optical paths of the two detectors, and ω_1 and ω_2 are the signal and idler frequencies. An inequality violation of more than 14 standard deviations was achieved. Similarly, a test in polarization was made by rotating polarization analyzers behind each Pockels cell with coincidence counting rate $R_c(\phi)$, where ϕ is the difference in polarization analyzer angles at counters 1 and 2, such that $\delta = \left|\left[R_c(\frac{1}{8}\pi) - R_c(\frac{3}{8}\pi)\right]/R_0\right| = 0.309 \pm 0.009 > \frac{1}{4}$. A violation of the constraints of local hidden variables theory by more than six standard deviations was observed.

In 1995, a new high-intensity, type-II phase-matched SPDC two-photon source was developed in order to take full advantage of two-photon entanglement involving polarization. Two-photons were produced noncollinearly and directly, i.e., without the use of extra beamsplitters or mirrors previously required to emulate entanglement post-selectively (see fig. 12) (Kwiat, Mattle, Weinfurter, Zeilinger, Sergienko and Shih [1995]).

This source allowed the observation of CHSH inequality violations by more than 100 standard deviations in less than 5 minutes. Furthermore, all four polarization Bell-states

$$\left|\Psi^{\pm}\right\rangle = \tfrac{1}{2}\left(|\mathrm{H},\mathrm{V}\rangle \pm |\mathrm{V},\mathrm{H}\rangle\right), \qquad \left|\Phi^{\pm}\right\rangle = \tfrac{1}{2}\left(|\mathrm{H},\mathrm{H}\rangle \pm |\mathrm{V},\mathrm{V}\rangle\right), \tag{3.2}$$

were readily produced. The use of a half-wave plate (HWP) allowed for polarization flipping between ordinary and extraordinary, that is H and V, states. It thus allowed for the exchange of states $|\Psi^-\rangle$ and $|\Phi^-\rangle$, and states $|\Psi^+\rangle$ and $|\Phi^+\rangle$. Similarly, a birefringent phase-shifter allowed for a sign change between two-photon joint amplitudes, so that an exchange between two-photon states $|\Psi^+\rangle$ and $|\Psi^-\rangle$, and between $|\Phi^+\rangle$ and $|\Phi^-\rangle$, was also accomplished. Bell-type inequalities were tested using all four Bell states, with significant violations in each case.

In addition to the problem of creating high-intensity sources two-photons with entanglement involving polarization, there have been other difficulties associated with entangled optical states. First, long crystals capable of producing entangled states with two polarizations give rise to nontrivial walk-offs. This problem can arise in the form of spatial walk-off: a photon of one polarization moves more quickly through the crystal than the other (yielding longitudinal walk-off) and, though they will leave the crystal collinearly, they can move in different directions while within the crystal (transverse walk-off). For sufficiently short crystals, one can completely compensate for the walk-off, as interference occurs pairwise between processes where the photon pair is created at equal distances but on opposite sides of the crystal central axis. This is accomplished by the introduction in each of the two photon paths of a similar crystal half as long (or in one path and of identical length) after polarization rotation of the photons. This makes the polarization that was previously fast the slow polarization, and vice versa (Rubin, Klyshko, Shih and Sergienko [1994], Kwiat, Mattle, Weinfurter, Zeilinger, Sergienko and Shih [1995]). Similarly, optimal transverse walk-off compensation is accomplished. However, for a sufficiently long crystal, the o and e rays may separate by more than the coherence length of the pump photons, making complete compensation impossible.

After the Hong–Ou–Mandel (fig. 3) and Shih–Alley (fig. 5) experiments, it was often intuitively believed that the two-photon interference could be understood in terms of the simultaneous arrival – and hence possible interaction of the two photons of each pair at the common beamsplitter. This is incidental, however. The essential requirement is the equality of optical path length to within the coherence length of the photons, resulting in in-principle indistinguishability. Type-II phase-matched two-photons provided an opportunity to demonstrate this. Pittman, Strekalov, Migdall, Rubin, Sergienko and Shih used collinear type-II phase-matched SPDC in a similar arrangement to observe two-photon interference, where the two photons of each pair were made to reach the common beamsplitter at times greater than the coherence length of their 702.2 nm photons yet still yield two-photon interference (Pittman, Strekalov, Migdall, Rubin, Sergienko and Shih [1996]) (see fig. 13).

This provided a counterexample to the intuitive, local picture of some local influence at a common beamsplitter "telling them" which way to travel afterward. First, a phase shifter (τ_{ay}) was placed in the path of the signal photon. Then, since that alone could eliminate the indistinguishability of the two-photon alternatives necessary for coincidence interference, "postponed compensation" was used, the leading photon was delayed for $\tau_1 x = 2\tau_{ay}$ after the beamsplitter. Thus the arrival of the photons at the two detectors was accomplished in exactly the same order and time difference, successfully restoring indistinguishability of detection events, as can be clearly seen in a space–time portrayal of alternative events (see

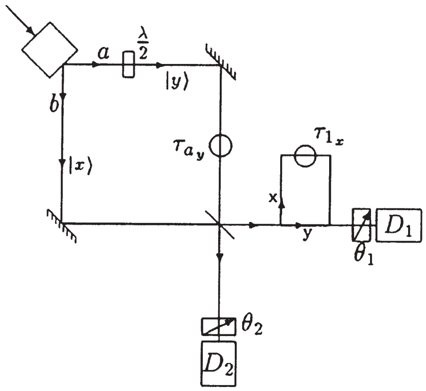

Fig. 13. Schematic of postponed compensation demonstration (Pittman, Strekalov, Migdall, Rubin, Sergienko and Shih [1996]).

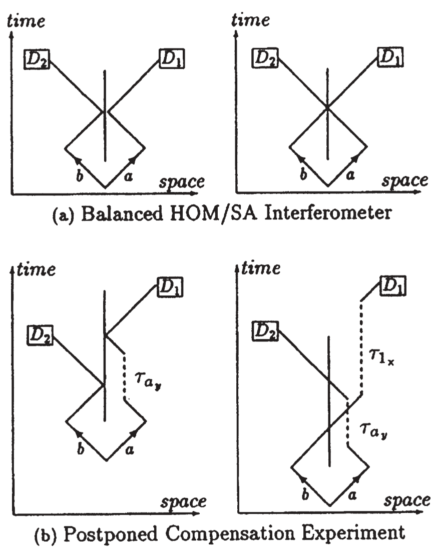

Fig. 14. Space–time diagram of restored indistinguishability (Pittman, Strekalov, Migdall, Rubin, Sergienko and Shih [1996]).

fig. 14). In fig. 13, the delay, state and path labels are identical, allowing for direct comparison with those of fig. 14, if one reorients the apparatus schematic so that the down-conversion crystal is placed at the bottom with its output directed upward. Such an apparatus later proved useful for high-precision polarization mode dispersion measurements (see below).

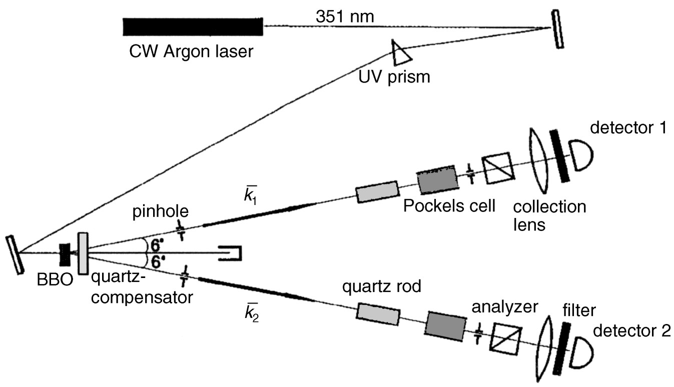

Fig. 15. Apparatus for postselection-free Bell-inequality test energy (Strekalov, Pittman, Sergienko, Shih and Kwiat [1996]).

In 1996, another step exhibiting the nonlocal character of two-photon quantum interference was taken when the new high-intensity type-II phase-matched SPDC source was used by some of the same investigators to make a post-selection-free test of a Bell inequality with entanglement involving energy (Strekalov, Pittman, Sergienko, Shih and Kwiat [1996]). Unlike the experiment of Franson (fig. 7), where a short-duration time window was used to post-select the coincidence alternatives of interest, this experimental arrangement avoided unwanted alternatives by design: the short–long and long–short alternatives were engineered out.

Noncollinear beams of 702.2 nm-photon pairs were created in a symmetrical configuration and passed through a quartz compensator, quartz compensator rods, Pockels cells and polarization analyzers (see fig. 15). The quantum state of two-photons that emerged from the birefringent rods along the two propagation directions was

$$\begin{aligned}|\Phi\rangle \propto &\left[|s\rangle_1\,|s\rangle_2\,e^{i(\alpha+\beta)} - |f\rangle_1\,|f\rangle_2\right]\sin(\phi_1+\phi_2)\\ &+\left[|s\rangle_1\,|f\rangle_2\,e^{i\alpha} - |f\rangle_1\,|s\rangle_2\,e^{i\beta}\right]\cos(\phi_1+\phi_2),\end{aligned} \tag{3.3}$$

where α and β are the phase shifts introduced by each of the two Pockels cells. The resulting interference visibility, 95%, was found to exceed the limit set by Bell locality by 17 standard deviations. It is noteworthy that the pair of quartz rods, rather than the usual pair of spatial paths as in the Franson apparatus, provide the interfering quantum alternatives in this experiment.

In 1998, Kwiat and Weinfurter showed how higher-dimensional Hilbert spaces could be used to distinguish Bell states (eq. 3.2), in what they called “embedded

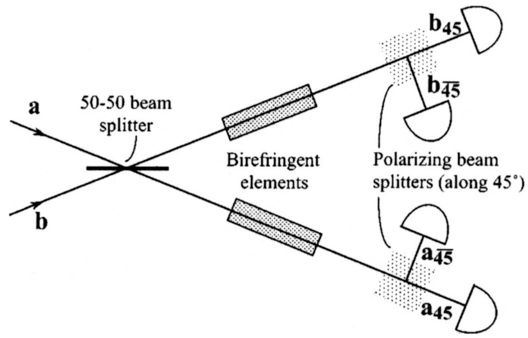

Fig. 16. Bell-state analyzer (Kwiat and Weinfurter [1998]).

Bell-state analysis" (Kwiat and Weinfurter [1998]). Previously, Vaidman and Luetkenhaus had shown that the Bell states cannot be fully distinguished using only linear optical elements, if multiple entanglements are not involved. The main idea of Kwiat and Weinfurter was to use entanglements including variables beyond those of the Bell states, for example energies in the case of a polarization-state analysis, and the additional interferometric measurements they allow, to fully distinguish the four Bell states. In the chosen example, first photon pairs in Bell states are sent to a 50–50 beamsplitter and the emerging beams sent to polarizing beamsplitters (see fig. 16).

The scheme works here because only state $|\Psi^-\rangle$ can give rise to one photon in each beamsplitter output beam, allowing the state to be readily identified by coincidence measurements in detector pairs **a** and **b**. Then, by looking at only one of the wings corresponding to one beam, one is able to distinguish between the remaining three states. Birefringent material, with axes oriented to correspond to the H–V basis states, was introduced for this purpose. That gives rise to temporal shifts between H and V polarization components, distinguishing $|\Psi^+\rangle$ from $|\Phi^\pm\rangle$. $|\Psi^+\rangle$ was distinguished by detecting two photons in one wing separated in time, making sure that this time difference is still less than the coherence time of the pump photons. Finally, in the $\pm 45^{\circ}$ basis, $|\Phi^\pm\rangle$ are distinguished from one another by the polarizing beamsplitter: for $|\Phi^+\rangle$ there will be a simultaneous arrival of two photons at one detector, while for $|\Phi^-\rangle$ there will be simultaneous detections at the two detectors of one wing.

Next, a further improvement in the brightness of down-conversion photon pairs whose entanglement involves momentum was made by Kwiat, Waks, White, Appelbaum and Eberhard [1999]. By using two nonlinear crystals (BBO) under type-I phase-matching conditions, these workers were able to achieve a brightness many orders of magnitude higher than previous sources. They

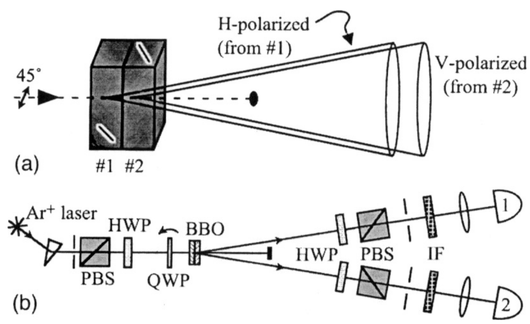

Fig. 17. Two-crystal ultra-bright two-photon source (Kwiat, Waks, White, Appelbaum and Eberhard [1999]).

accordingly observed a far higher violation of a CHSH Bell-type inequality, as much as 242 standard deviations within three minutes. Considerations of source symmetry suggest that an increase in intensity of 10 000 times was achieved in the entire output. Two relatively thin (0.59 mm) BBO crystals were identically cut and oriented so that their optic axes lay in perpendicular planes. They were then pumped by a 351.1 nm laser line polarized 45 degrees to both the e and o crystal axes (see fig. 17). The result was the production of two-photons in which all pairs of a given color were entangled, as the amplitudes for the orthogonal modes were non-zero in only one of the two and in alternate crystals. That is, two-photons will be described by the polarization state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle\,|\mathrm{H}\rangle + \mathrm{e}^{\mathrm{i}\phi}\,|\mathrm{V}\rangle\,|\mathrm{V}\rangle\right). \tag{3.4}$$

The phase ϕ is adjustable by changing crystal tilt, by phase shifting one of the output beams or adjusting the phase relation between horizontal and vertical polarization components of the pump state. A two-photon interference visibility of $V = 0.996$ was thus achieved.

In another significant step forward, Zeilinger et al. tested a Bell-type inequality under strict locality conditions, in order to close one loophole of previous Bell tests, since it is conceivable that polarizers might somehow communicate their settings to one another before two-photons reach them (Weihs, Jennewein, Simon, Weinfurter and Zeilinger [1998]). In this they surpassed the previous attempt of Aspect, Dalibard and Roger [1982], whose polarizer orientations had only been rapidly, periodically changed during the photons' flight from their source to polarizers separated by a distance of 12 m, which accordingly suffered from what has come to be known as the "periodicity loophole" (Shimony [1990], Weihs, Weinfurter and Zeilinger [1997]).

Zeilinger et al. separated their polarizers by 400 m, thereby allowing them a full 1.3 μs to make ultrafast physically random (as opposed to physically deterministic pseudorandom) polarizer orientations, as well as to independently register measurement results from each wing of the apparatus. At the end of each cycle of the experiment, the two sets of data were brought together to view the correlations. 97% coincidence interference visibility was obtained, and the CHSH inequality was violated by 30 standard deviations. There remains only the "detection loophole", to such tests after this work. This final loophole can be blocked when photodetection efficiencies can be improved beyond 0.841 (Shimony [1990]).

As a practical application of two-photon interferometry, two-photon interference patterns similar to the HOM dip (see fig. 4) have proven useful for measuring polarization mode dispersion (PMD), the difference in propagation rate between two polarization modes in a birefringent medium (see, for example, Dauler, Jaeger, Muller, Migdall and Sergienko [1999]). PMD is important, among other reasons, in understanding propagation of polarized light in optical fibers, such as has been proposed for the purposes of quantum cryptography (see below). The extreme constraint on the simultaneity of the creation of the two photons of a down-conversion pair allows for the high resolution achieved using such a method. The PMD can be determined with sub-femtosecond resolution by studying the effect of dispersive media on this interference feature. An important advantage of this technique, relative to some non-white light interferometric methods, is that it determines the optical delay absolutely, as opposed to simply measuring the delay modulo a wavelength. The PMD is directly determined from the temporal shift of the HOM-type interference feature produced by the insertion of a birefringent sample into the interferometer (see fig. 18).

Two ways of producing a coincidence event are arranged so that they cannot be distinguished (even in principle). A differential delay line is used to delay one polarization relative to the other. The coincidence rate from spatially separated detectors is recorded as this delay line is varied. When the two photons are separated at the beamsplitter by more than their coherence time the two coincidence events can be distinguished, so no interference is possible; the total coincidence rate is simply the sum of the two individual rates. When the two photons reach the beamsplitter to within their coherence time, however, destructive interference occurs, as the detector polarizers are oriented at 45 degrees and 135 degrees. The two types of coincidence, the first photon produced in the e polarization and the second in the o polarization, or vice versa, become indistinguishable. The temporal correlations are limited by the length

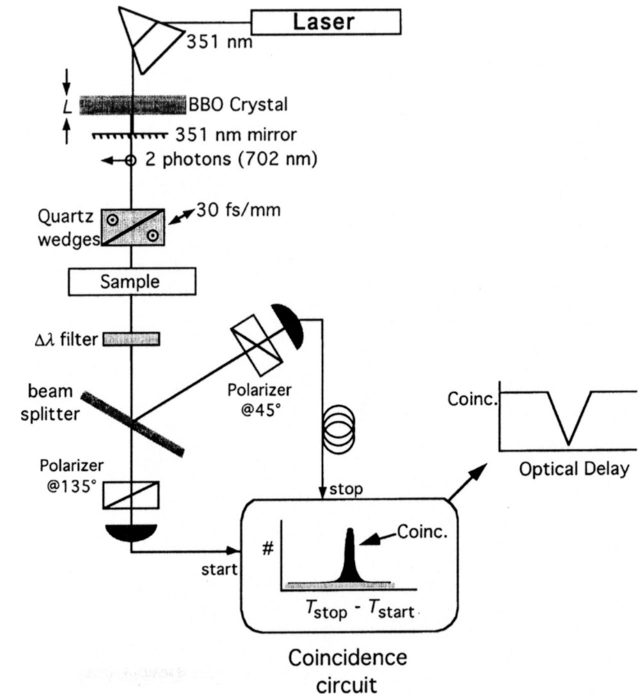

Fig. 18. Apparatus for high-precision PMD measurements (Dauler, Jaeger, Muller, Migdall and Sergienko [1999]).

of the down-conversion crystal, and a triangular-shaped interference feature is seen. This occurs because the effect is to convolve two rectangular two-photon wavefunctions. The shift of the center of this interference feature is identical to the PMD of the sample. The uncertainty limit of the method was determined by how well the center of that feature was determinable. This was found to be as low as 0.15 fs.

Most recently, the two-photon interferometer has been modified to produce a modified interferogram, with additional "internal fringing" (see fig. 19) (Branning, Migdall and Sergienko [2000]). This feature of "fringing in the HOM dip" is introduced by moving the additional variable delay line of the first arrangement for PMD measurement after the first beamsplitter (see fig. 20). Using this improved technique allows one to measure the PMD with a precision of 8 attoseconds.

In the quantum-informational context, decoherence-free subspaces within multiple-photon Hilbert spaces have been a subject of interest. They could be

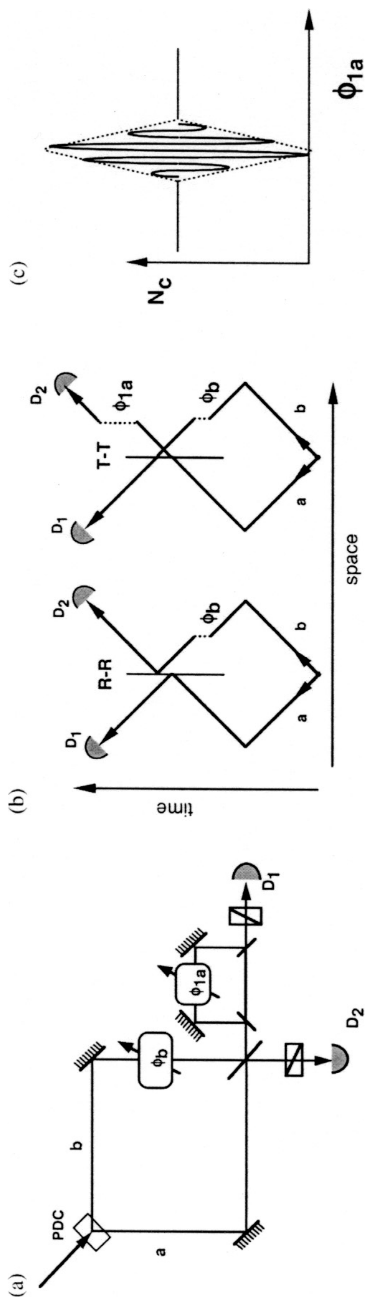

Fig. 19. Ultra-high-precision PMD measurements (cf. figs. 13, 14) (Branning, Migdall and Sergienko [2000]).

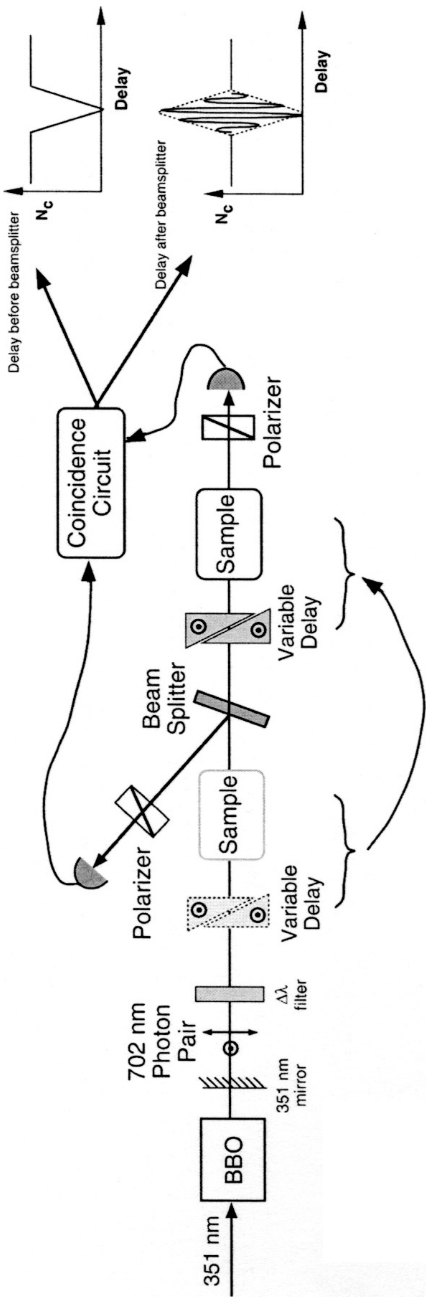

Fig. 20. Ultra-high-precision PMD measurements (cf. fig. 18) (Branning, Migdall and Sergienko [2000]).

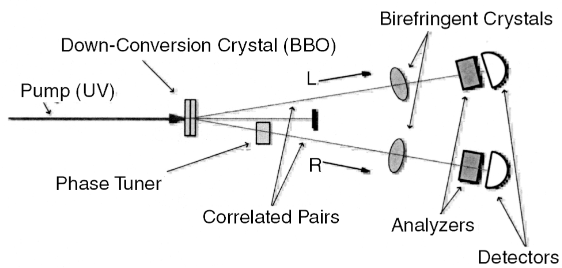

Fig. 21. Preserving a two-photon decoherence-free polarization subspace (Berglund [2000]).

useful, for example, in the redundant coding of quantum information (Ekert, Palma and Suominen [2000]). One could, for example, encode the states $|\tilde{0}\rangle$, $|\tilde{1}\rangle$ as follows:

$$|\tilde{0}\rangle = |0, 1\rangle, \qquad |\tilde{1}\rangle = |1, 0\rangle, \tag{3.5}$$

where information-bearing states built from the double eigenstate are chosen not to be susceptible to decoherence in a way that those constructed using the single eigenstate might be. The decoherence-free (DF) subspaces of the Hilbert space of hyperentangled polarization states have recently begun to be studied (Berglund [2000] and Kwiat, Berglund, Altepeter and White [2000]).

In particular, it has been shown that, while the energy correlations required by down-conversion phase-matching conditions can render two-photons susceptible to decoherence under the influence of an environment where frequency–polarization coupling is present, the DF subspace can be readily preserved (fig. 21). By appropriately symmetrizing the induced phase errors for the antisymmetric polarization state $|\psi^-\rangle$, its decoherence-free character can be demonstrated (Zanardi [1997]).

The Bell states

$$|\psi^\pm\rangle = \tfrac{1}{\sqrt{2}}\left(|\mathrm{H}, \mathrm{V}\rangle \pm |\mathrm{V}, \mathrm{H}\rangle\right), \qquad |\phi^\pm\rangle = \tfrac{1}{\sqrt{2}}\left(|\mathrm{H}, \mathrm{H}\rangle \pm |\mathrm{V}, \mathrm{V}\rangle\right), \tag{3.6}$$

are initially considered, with identical birefringent crystals placed across both paths L and R; denoting the thickness of the nonlinear crystal as $\mathcal{L}$, the phase difference between the two arms will be $\omega\frac{\mathcal{L}\Delta n}{c}$. The off-diagonal elements of the density matrix for $|\psi^\pm\rangle$ approach zero as the crystal thickness surpasses the coherence length of the down-conversion photons, which is proportional to $c/\delta\omega$, with $\delta\omega$ the width of the frequency spectrum. In this configuration, $|\phi^\pm\rangle$ do not undergo decoherence, while $|\psi^\pm\rangle$ do.

In particular, at the analyzer, $|\phi^{\pm}\rangle$ is transformed to

$$|\phi'^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{H},\mathrm{H}\rangle \pm \exp\left[\mathrm{i}\frac{\omega_0}{2}\frac{\mathcal{L}\Delta n}{c}\right]|\mathrm{V},\mathrm{V}\rangle\right), \tag{3.7}$$

which has no effect on the magnitude of off-diagonal density matrix elements. However, these states do not generate a DFS, since in the left/right-circular polarization basis they are written

$$\left|\phi^{\pm}\right\rangle = \tfrac{1}{2}\left(|\mathrm{L},\mathrm{R}\rangle \pm |\mathrm{R},\mathrm{L}\rangle\right), \tag{3.8}$$

and thus can lose phase information in crystals with eigenmodes $|\mathrm{L}\rangle, |\mathrm{R}\rangle$. By contrast, the antisymmetric state is rotationally invariant, so there is hope for recovering its DFS. By rotating the nonlinear crystal in one arm by 90 degrees, the states $|\phi^{\pm}\rangle$ and $|\psi^{+}\rangle$ will be seen to decohere, while the state $|\psi^{-}\rangle$ will not: its state at the analyzers will be

$$|\psi'^{-}\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{H},\mathrm{V}\rangle - \exp\left[\mathrm{i}\frac{\omega_0}{2}\frac{\mathcal{L}\Delta n}{c}\right]|\mathrm{V},\mathrm{H}\rangle\right). \tag{3.9}$$

Quantifying the fidelity of the transmission process, $F = \mathrm{Tr}(\rho_{\mathrm{in}}\rho_{\mathrm{out}})$ or, for a mixed input state, $F = \left[\mathrm{Tr}(\sqrt{\sqrt{\rho_{\mathrm{in}}}\,\rho_{\mathrm{out}}\sqrt{\rho_{\mathrm{in}}}})\right]^2$, decoherence-free subspaces will have $F = 1$, which is the case for $|\psi^{-}\rangle$.

The currently most advanced form of quantum information experimentation is taking place in quantum cryptography – more precisely, quantum key distribution (QKD). QKD is the distribution of a secret key (bit sequence) between two interested parties, usually called Alice and Bob. This key can be used to encrypt and decrypt secret messages using the safe one-time pad method of encryption. The security of QKD is not based on complexity, but on quantum mechanics, since it is generally not possible to measure an unknown quantum system without altering it. Any eavesdropping introduces physical errors in the transmitted data (see also § 3.1.4 of ch. 1 in this volume).

The basic QKD protocols are the BB84 scheme (Bennett and Brassard [1984]) and the Ekert scheme (Ekert [1991]). BB84 uses single photons transmitted from sender (Alice) to receiver (Bob), which are prepared at random in four partly orthogonal polarization states: 0, 45, 90 and 135 degrees. When an eavesdropper, Eve, tries to obtain information about the polarization, she introduces observable bit errors, which Alice and Bob can detect by comparing a random subset of the generated keys. The Ekert protocol uses entangled pairs and a Bell-type inequality. In that scheme, both Alice and Bob receive one particle of the entangled pair. They perform measurements along at least three different directions on each side, where measurements along parallel axes are used for key generation, and those along oblique angles are used for security verification.

Several innovative experiments have been made using entangled photon pairs to implement quantum cryptography in the recent period, 1999–2000 (Sergienko, Atatüre, Walton, Jaeger, Saleh and Teich [1999], Jennewein, Simon, Weihs, Weinfurter and Zeilinger [2000] and Tittel, Brendel, Zbinden and Gisin [2000]). Quantum cryptography experiments have had two principal implementations: weak coherent state realizations of QKD and those using two-photons. The latter approach made use of the nonlocal character of polarization Bell states generated by spontaneous parametric down-conversion. The strong correlation of photon pairs, entangled in both energy–time and momentum–space, eliminates the problem of excess photons faced by the coherent-state approach, where the exact number of photons actually injected is uncertain. In the entangled-photon technique, one of the pair of entangled photons is measured by the sender, confirming for the sender that the state is the appropriate one. It has thus become the favored experimental technique.

The first of the recent innovative experiments using SPDC demonstrated a more flexible and robust method of quantum secure key distribution with type-II phase-matched two-photons, in an improved configuration (Sergienko, Atatüre, Walton, Jaeger, Saleh and Teich [1999]). The high contrast and stability of the fourth-order quantum interference, along with the available knowledge of the exact number of photons present in the quantum communication channel, clearly show the performance of EPR-state-based quantum key distribution to be superior to the coherent-state-based technique. The entangled-photon technique had previously used type-I phase-matched pairs and, as a result, suffered from low visibility (only up to 85%) and poor stability of the intensity interferometer. This has primarily been due to the need in previous experiments for the synchronous manipulation of interferometers well separated in space. The intervention of any classical measurement apparatus (eavesdropping) will cause an immediate reduction of the visibility to 70.7%, so high visibility is required to ensure key security. Only an undisturbed EPR state can produce 100% interference visibility.

Previous attempts to demonstrate the feasibility of quantum key distribution using EPR photons had failed to attain the high-visibility coincidences. A double, strongly unbalanced, distributed polarization intensity interferometer was used to avoid the simultaneous spatial manipulation that compromised previous attempts. A frequency-doubled femtosecond Ti:sapphire laser was used to generate 80-fs pulses at 541.5 nm that were sent through a 0.1-mm-thick BBO crystal, oriented so as to yield collinearly propagating type-II phase-matched EPR pairs. The

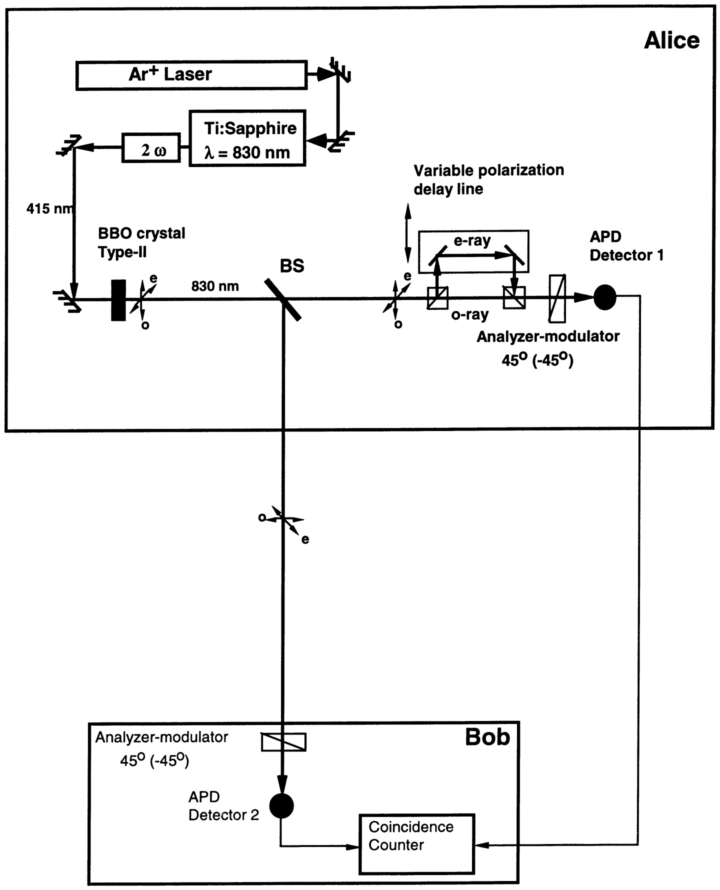

Fig. 22. Two-photon entangled-state QKD scheme 2000 (Sergienko, Atatüre, Walton, Jaeger, Saleh and Teich [1999]).

photons entered two spatially separated interferometer arms via a polarization-insensitive 50–50 beamsplitter BS, which allowed photons of both ordinary and extraordinary polarization to be reflected and transmitted with equal probability. One output port led to a controllable polarization-dependent optical delay – the e-ray/o-ray loop – then to detector 1. The other led, through an optical channel, to detector 2. Polarization analyzers were placed in front of each photon-counting detector and were oriented at 45° or 315°. Coincidence counts between the two detectors were monitored as a function of the optical delay between the orthogonally polarized photons. In this quantum key-distribution arrangement, the first

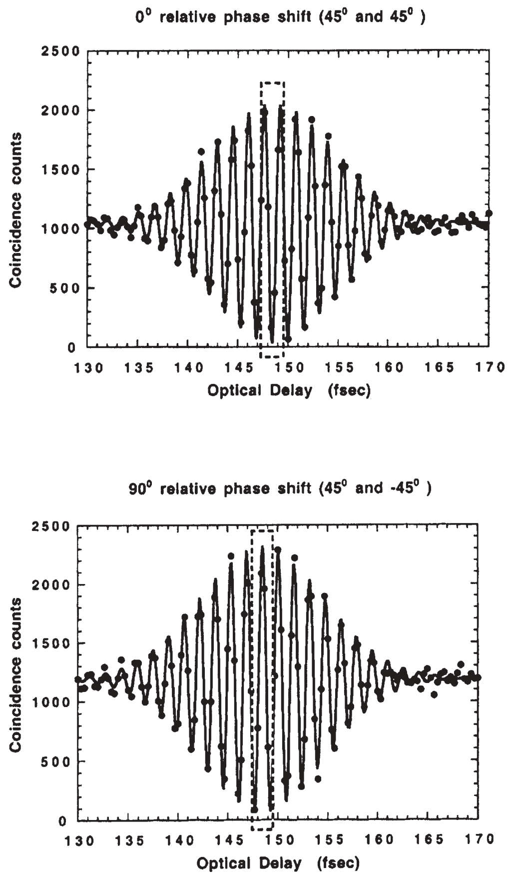

Fig. 23. Two-photon quantum cryptographic signals (Sergienko, Atatüre, Walton, Jaeger, Saleh and Teich [1999]).

beamsplitter is located with the quantum key sender (Alice), while one of the output beamsplitters is located at a distance with the receiver (Bob), as in fig. 22.

The high-frequency carrier that resides under the HOM-type interference feature reflects the period of the UV pump wavelength rather than that of individual waves, and arises from the nonlocal entanglement of the twin beams. As shown in fig. 23a, a 90º phase shift of one of the analyzers modifies the quantum interference pattern so that the central fringe is constructive rather than destructive. The contrast is very high, 98%, as is evident from fig. 23b. This demonstrates that cryptographic key qubits – one value corresponding to each of the two sorts of interference – can be sent with a high degree of fidelity using this

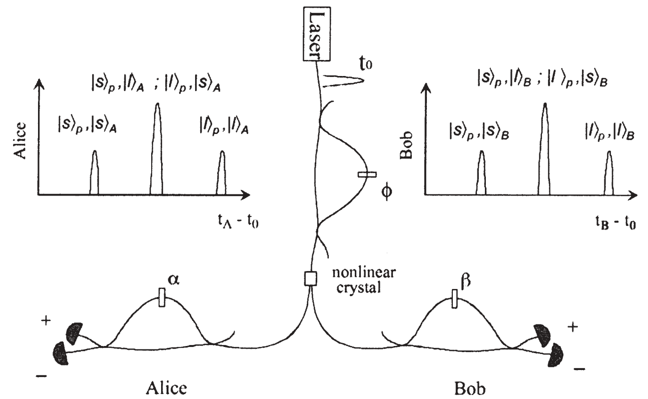

Fig. 24. Time–energy entanglement quantum key distribution scheme (Tittel, Brendel, Zbinden and Gisin [2000]).

apparatus. Key distribution works as follows. The polarizations of the photons are randomly modulated by switching each analyzer–modulator in the rectilinear basis (45º and 315º), providing 0º or 90º relative phase shift between them.

In order to fully complete the procedure of quantum key distribution, it would also be necessary to randomly switch the polarization parameters of the two-photon entangled state between two nonorthogonal polarization bases, such as rectilinear and circular polarization. This could be accomplished using fast Pockels-cell polarization rotators. These sets of randomly selected angles force the mutual measurements by Alice and Bob to be destructive (a binary “0”) or constructive (a binary “1”) with a 50–50 probability, depending on the mutual orientation of the modulators on both sides. Communications between Alice and Bob, which give the set of polarizer orientations selected during each measurement but not the measurement outcomes themselves, are then to be sent over a public classical communication channel. Other protocols may be devised to endow this configuration with the full security that has been added to other configurations.

A second experiment uses a scheme that combines using photon pairs and energy–time entanglement (Tittel, Brendel, Zbinden and Gisin [2000]). This scheme realizes the initial concept of using photon-pair correlations (Ekert [1991]) for QKD (fig. 24). However, it implements Bell states, and the robustness of energy–time entanglement allows the information produced using this second method to be preserved over long distances. In this scheme, a light pulse sent at time t_0 enters an initial interferometer imposing a large path length difference relative to the pulse length. The pulse is split in two, so that the subpulses leave time-separated but with a definite phase difference.

The 655 nm, 80 MHz pulses entered a down-converting ($KnbO_3$) crystal creating two-photons described by

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|s\rangle_p \, |s\rangle_p + e^{i\phi} \, |\ell\rangle_p \, |\ell\rangle_p \right), \tag{3.10}$$

where $|s\rangle_p$ and $|\ell\rangle_p$ are photon states created by a pump photon that traveled via the short or the long arm of the initial fiber interferometer, respectively. The full set of Bell states are thus achievable by the appropriate choosing of and/or interchange of a short or long photon state for one of the photons. The photons were separated and sent, one to the Alice wing and one to the Bob wing. Each photon then traveled through another fiber interferometer, introducing the same path difference through one or the other arm as the initial, pre-crystal interferometer.

In order to use this system for QKD, Alice and Bob are to carry out the following procedure. The photons arrive at Alice's detectors in one of three time slots relative to t_0. The time-of-arrival to Alice does not give a full description of the two-photon state, since the path of the photon traveling to Bob is unknown to Alice. To obtain the quantum key, Alice and Bob then use a classical channel to find those events where both detect a photon in a side peak, without revealing the detector. The other half of the events are discarded, leaving them with correlated detection times. Finally, they assign bit value "0" to the short cases and "1" to the long cases. When both find the photon in the central temporal position, by choosing appropriate phase settings, Alice and Bob will always find perfect correlations in the output ports. Either both detect the photons in their "+" detector (bit value "0"), or both in their "–" detector ("1"). Bit rates of roughly 33 bps and bit error rates of around 4% were achieved.

A third recent QKD experiment (fig. 25) used $|\psi^-\rangle$-state two-photons created in BBO to approximate a single photon source (Jennewein, Simon, Weihs, Weinfurter and Zeilinger [2000]). It implemented a novel key distribution scheme using the Wigner-version Bell-type inequality (Wigner [1970]) to test the security of the quantum channel, as well as a variant of the BB84 protocol (Bennett and Brassard [1984]). To use the Wigner inequality analogous to the CHSH inequality in the Ekert protocol, observer Alice chose between two polarization measurements along the axes $\boldsymbol{x}$ and $\boldsymbol{u}$, corresponding to angles χ and ψ, with the possible results 1 and -1, on photon A; Bob chose between measurements along $\boldsymbol{u}$ and $\boldsymbol{v}$, corresponding to angles ψ and ω, on photon B. When the polarization was parallel to the analyzer axis the result was 1; with polarization orthogonal

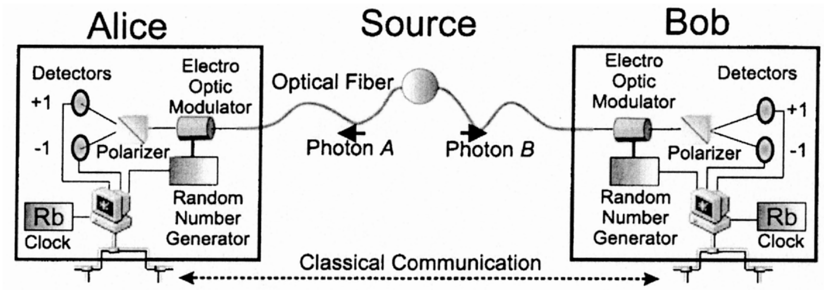

Fig. 25. Realization of two-photon QKD over a long distance (Jennewein, Simon, Weihs, Weinfurter and Zeilinger [2000]).

to the analyzer axis the result was −1. By assuming premeasurement values for properties along $\boldsymbol{x}$, $\boldsymbol{u}$ and $\boldsymbol{v}$ and perfect anticorrelation of measurements along parallel axes, the probabilities for obtaining 1 on both sides obey the inequality

$$p_{++}(\chi,\psi)+p_{++}(\psi,\omega)-p_{++}(\chi,\omega)\geqslant 0. \tag{3.11}$$

The quantum-mechanical prediction for arbitrary analyzer settings α with Alice and β with Bob given the linear polarization singlet Bell state Ψ^- is

$$p_{++}^{\mathrm{QM}}(\alpha,\beta)=\tfrac{1}{2}\sin^2(\alpha-\beta). \tag{3.12}$$

Maximum violation of the inequality is thus obtained for $\chi = -30^{\circ}$, $\psi = 0^{\circ}$, $\omega = 30^{\circ}$, when the l.h.s. reaches −1/8. To send the quantum key Alice and Bob randomly change their analyzer settings: Alice between −30º and 0º, Bob between 0º and 30º. Four combinations of analyzer settings can thus occur: the three oblique settings allow a test of Wigner's inequality, the remaining combination of parallel settings allows key generation using perfect anticorrelations. When the probabilities violated Wigner's inequality, then the generated key was taken to be secured.

The second QKD realization of this experiment implemented a variant of the BB84 protocol with entangled photons, with the same $|\psi^-\rangle$-state polarization-entangled photon pairs approximating the single-photon realization of BB84. Alice and Bob randomly changed their polarizer settings between 0º and 45º. They observed perfect anticorrelations whenever their analyzers were parallel. They obtained identical keys by simply inverting all resulting bit values. Whenever Alice made a measurement on photon A, photon B was projected into the orthogonal state that was analyzed by Bob, or vice versa. After the initial bit distribution, key security could be checked by classically comparing a small subset of their keys to check the security via the error rate.

The nonlinear crystal used was again BBO, producing polarization-entangled photon pairs at a wavelength of 702 nm from cw pump light of 351 nm at a power of 350 mW. The photons were each coupled into 500 m long optical fibers and transmitted to "Alice" and "Bob", respectively, who were separated by 360 m. Wollaston polarizing beamsplitters were used as polarization analyzers. The users generated raw keys at rates of 400–800 bps, with bit error rates of approximately 3%.

§ 4. Higher multiple-photon entanglement

The entanglement of three or more photons has been a subject of great interest since the proposals in 1989 and 1990 of Greenberger, Horne, Zeilinger and Shimony to test locality, reality and completeness assumptions of EPR using entangled three-particle states (Greenberger, Horne and Zeilinger [1989], Greenberger, Horne, Shimony and Zeilinger [1990], Bernstein, Greenberger, Horne and Zeilinger [1993], Klyshko [1993], Aravind [1997]). Bell's inequality had provided a test of these assumptions using statistical correlations, with the most striking results involving Bell states. The GHZ theorem provided a test involving perfect correlations without the use of inequalities, through the use of "GHZ states". The GHZ states can be written as

$$
\begin{aligned}
|\Phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle|\mathrm{H}\rangle|\mathrm{H}\rangle \pm |\mathrm{V}\rangle|\mathrm{V}\rangle|\mathrm{V}\rangle\right),\\
|\Psi_1^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|\mathrm{V}\rangle|\mathrm{H}\rangle|\mathrm{H}\rangle \pm |\mathrm{H}\rangle|\mathrm{V}\rangle|\mathrm{V}\rangle\right),\\
|\Psi_2^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle|\mathrm{V}\rangle|\mathrm{H}\rangle \pm |\mathrm{V}\rangle|\mathrm{H}\rangle|\mathrm{V}\rangle\right),\\
|\Psi_3^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle|\mathrm{H}\rangle|\mathrm{V}\rangle \pm |\mathrm{V}\rangle|\mathrm{V}\rangle|\mathrm{H}\rangle\right),
\end{aligned}
\tag{4.1}
$$

in the three-polarization case. Among other reasons, such states are interesting because, as in the Bell states (3.6), the polarization of each photon is indeterminate while the three particles are certainly perfectly correlated in polarization. A similar, beam-entangled set of states has also been introduced (Greenberger, Horne, Shimony and Zeilinger [1990]). These states allow entanglement to be studied in a less trivial context than that of the traditional two particles. Interferometric studies subsequently sought to exhibit these correlations and to use them for various means.

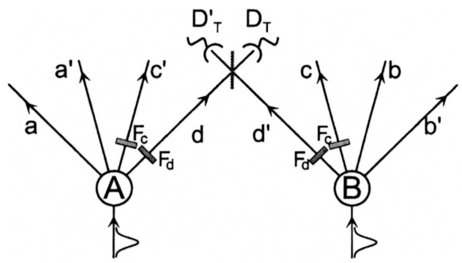

Fig. 26. Three-photon beam-entanglement source (Zeilinger, Horne, Weinfurter and Zukowski [1997]).

In 1997, Zeilinger, Horne, Weinfurter and Zukowski proposed a scheme for generating GHZ states using the concept of quantum erasure, following an earlier direction of investigation initiated by Yurke and Stoler [1992] that they developed further (Zukowski, Zeilinger, Horne and Ekert [1993], Zukowski, Zeilinger and Weinfurter [1995], Zeilinger, Horne, Weinfurter and Zukowski [1997]). This approach allows one to achieve entanglement while avoiding the problematic need for particle interaction, as had previously been used for this end; its use was explicated for both polarization and beam entanglement. This scheme begins with two independent sources of two-photons, followed by the "erasing" of source information of one of the four photons at a beamsplitter (see, for example, figs. 26 and 28).

This was first done using a pair of laser pulses. An illustration of this principle is shown in fig. 26 for the case involving beam entanglement. The states of the initial down-conversion pairs can be written

$$\frac{1}{\sqrt{2}}\left(|\mathrm{a}\rangle|\mathrm{d}\rangle \pm |\mathrm{a}'\rangle|\mathrm{c}'\rangle\right), \qquad \frac{1}{\sqrt{2}}\left(|\mathrm{d}'\rangle|\mathrm{b}'\rangle \pm |\mathrm{c}\rangle|\mathrm{b}\rangle\right). \tag{4.2}$$

After one of the four photons triggers a detector in the source, three-photon states arise for the remaining three particles, yielding the entangled state of beam-direction eigenvectors

$$\frac{1}{\sqrt{2}}\left(|\mathrm{a}\rangle|\mathrm{b}\rangle|\mathrm{c}\rangle + \mathrm{e}^{\mathrm{i}\phi}|\mathrm{a}'\rangle|\mathrm{b}'\rangle|\mathrm{c}'\rangle\right). \tag{4.3}$$

The possibilities represented in this state then can be made to interfere in an apparatus such as that shown schematically in fig. 27, with triple-incidences, for example, at detectors $\mathrm{D_A}$, $\mathrm{D_B}$, $\mathrm{D_C}$, that vary sinusoidally in $\phi_\mathrm{a} + \phi_\mathrm{b} + \phi_\mathrm{c}$.

In general, when three-particle interference visibilities surpass 50%, a violation of a Bell locality can be demonstrated (Mermin [1990b]). A polarization

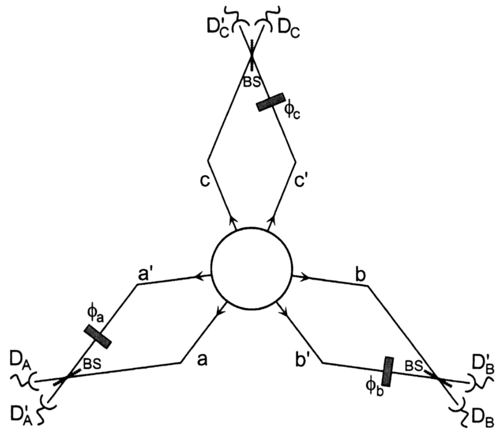

Fig. 27. Three-photon beam-entanglement interferometer (Zeilinger, Horne, Weinfurter and Zukowski [1997]).

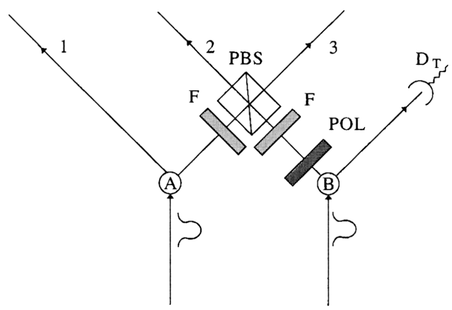

Fig. 28. Three-photon polarization-entanglement source (Pan and Zeilinger [1998]).

GHZ-state source analogous to that for beams in fig. 26 is shown in fig. 28. A GHZ-state analyzer (see fig. 29) can be constructed, just as a Bell-state analyzer (see fig. 16) can, in a manner that can also be extended to construct an n-particle entangled state analyzer. For three photons in the modes A, B and C, the scheme of fig. 29 will give rise to triple-incident detections when the photons are in GHZ states $|\Phi^{\pm}\rangle$. These two states can then be distinguished because, after the half-wave plates (HWPs), $|\Phi^{+}\rangle$ results in one or three photons with polarization H and zero or two photons with polarization V, while $|\Phi^{-}\rangle$ results in just the opposite situation.

The first experimental proof of entanglement of more than two spatially separated particles was only recently produced (Bouwmeester, Pan, Daniell, Weinfurter and Zeilinger [1999]). In this demonstration, the first two photon pairs

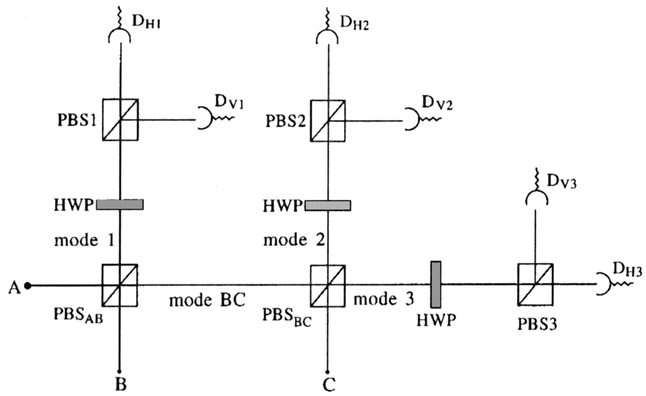

Fig. 29. GHZ-state analyzer (Pan and Zeilinger [1998]).

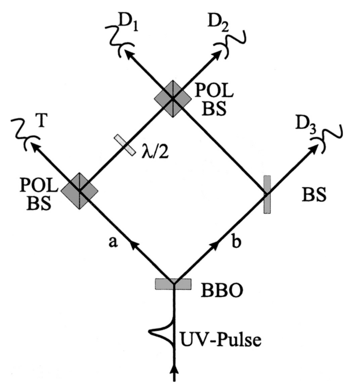

Fig. 30. Three-photon polarization-entanglement source (Bouwmeester, Pan, Weinfurter and Zeilinger [1998]).

were generated from a single PDC source (BBO) using a 394 nm pulsed laser pump (see fig. 30) to create the state

$$\frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle_{\mathrm{a}}|\mathrm{V}\rangle_{\mathrm{b}} - |\mathrm{V}\rangle_{\mathrm{a}}|\mathrm{H}\rangle_{\mathrm{b}}\right). \tag{4.4}$$

Narrow-bandwidth filters made the coherence time of the photons (500 fs) more than twice as long as that of the initial UV pulse (200 fs).

In arm **a**, a polarizing beamsplitter reflected only vertical polarizations, which were subsequently rotated 45° by an HWP; in arm **b**, an ordinary 50–50 beamsplitter reflected both polarizations with equal likelihood. The two arms were arranged so as to meet at a polarizing beamsplitter from opposite sides. The events of interest were those in which two photon pairs were created simultaneously. The GHZ state in those cases,

$$\left|\Psi_3^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle|\mathrm{H}\rangle|\mathrm{V}\rangle + |\mathrm{V}\rangle|\mathrm{V}\rangle|\mathrm{H}\rangle\right), \tag{4.5}$$

was then post-selected by considering only events where four simultaneous detections were made at detectors D_1, D_2, D_3 and the trigger detector T. Though the detection efficiency for such joint events was only 10^{-10}/pulse, the pulse rate was nearly 10^8/s and the observed coincidence detection rate was nearly one detection every 2.5 minutes. The ratio of desired versus undesired detections (by polarization state) for the joint detections was 12 : 1. Coherent superposition, as opposed to an undesirable mixture, of the two desired states was verified by measuring the first photon in the 45º polarization state and finding the second and third photons to be entangled by virtue of their polarizations being seen to be identically polarized along the 45º direction, as predicted for "entangled entanglement" (Krenn and Zeilinger [1996]).

With the arrangement of fig. 30, only the triple coincidences (of particles 1, 2 and 3) predicted by quantum mechanics were observed and none of those predicted by local realism were found, within experimental uncertainties. Entangled three-particle states were created with a purity of 71%. An interference visibility of 75% was obtained. The optimal Bell-type inequality for three particles was derived by Mermin [1990a] to be

$$|\langle xyy\rangle + \langle yxy\rangle + \langle yyx\rangle - \langle xxx\rangle| \leqslant 2, \tag{4.6}$$

with x the outcome for measurements in the basis $\{|\mathrm{x}_\pm\rangle = \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle \pm |\mathrm{V}\rangle)\}$, and y that for a measurement in $\{|\mathrm{y}_\pm\rangle = \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle \pm \mathrm{i}|\mathrm{V}\rangle)\}$. This inequality requires only a visibility >50% to be violated. With this arrangement, the l.h.s. of eq. (4.6) was found to reach 2.83±0.09 (Bouwmeester, Pan, Daniell, Weinfurter and Zeilinger [2000]), in clear violation of local realism.

Another recent experimental discovery was that two particles, each initially entangled with one other's partner particle, can be placed in an entangled state by making a Bell measurement, giving rise to "entanglement swapping" (Bennett, Brassard, Crepeau, Jozsa, Peres and Wootters [1993], Zukowski, Zeilinger, Horne and Ekert [1993] and Bose, Vedral and Knight [1998]). In a recent experiment, a single nonlinear crystal was used as the source of a pair of two-photons to be used in entanglement swapping (Pan, Bouwmeester, Weinfurter and Zeilinger [1998]). In that experiment, calling the photons from one source (I) 1 and 2, and those from the other source (II) 3 and 4, a Bell-state measurement was made of two

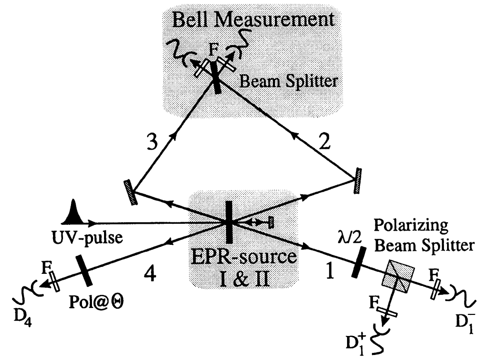

Fig. 31. Experimental arrangement for entanglement swapping (Bouwmeester, Pan, Weinfurter and Zeilinger [1998]).

photons from different sources, say photons 2 and 3 (see fig. 31). The result was that the state of the other two photons, 1 and 4, was projected onto an entangled state. This can be seen as follows. The initial state of the two photon pairs is

$$
\begin{aligned}
|\Xi\rangle &= \tfrac{1}{2}\left(|\mathrm{H}\rangle|\mathrm{V}\rangle - |\mathrm{V}\rangle|\mathrm{H}\rangle\right)_{12}\left(|\mathrm{H}\rangle|\mathrm{V}\rangle - |\mathrm{V}\rangle|\mathrm{H}\rangle\right)_{34} \\
&= \tfrac{1}{2}\left(|\Psi^{+}_{14}\rangle|\Psi^{+}_{23}\rangle + |\Psi^{-}_{14}\rangle|\Psi^{-}_{23}\rangle + |\Phi^{+}_{14}\rangle|\Phi^{+}_{23}\rangle + |\Phi^{-}_{14}\rangle|\Psi^{-}_{23}\rangle\right).
\end{aligned}
\tag{4.7}
$$

Photons 2 and 3 are then measured, projecting their state onto one of the Bell states,

$$
\begin{aligned}
\left|\Psi^{\pm}\right\rangle_{23} &= \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle|\mathrm{V}\rangle \pm |\mathrm{V}\rangle|\mathrm{H}\rangle\right), \\
\left|\Phi^{\pm}\right\rangle_{23} &= \frac{1}{\sqrt{2}}\left(|\mathrm{H}\rangle|\mathrm{H}\rangle \pm |\mathrm{V}\rangle|\mathrm{V}\rangle\right).
\end{aligned}
\tag{4.8}
$$

As a result, the pair of photons 1 and 4 are in the state that was found in measurement, as can be seen from the expansion of $|\Xi\rangle$.

Similarly, in the so-called quantum polarization teleportation process, a laser pump-pulse was used to provide the opportunity to create two pairs of photons: on the path from left to right the pulse creates an entangled pair, the so-called "ancillary" pair of photons 2 and 3 (see fig. 32). One of these photons is passed on to Alice and the other one to Bob, who receives the polarization state. On the return path from the mirror the pulse again creates a pair of photons (photons 1 and 4). The second photon of that pair (photon 4) is sent to a trigger detector, p, that is used to reject all events in which this second pair was not created. In the experiment the entangled photons, photons 2 and 3, were produced in the

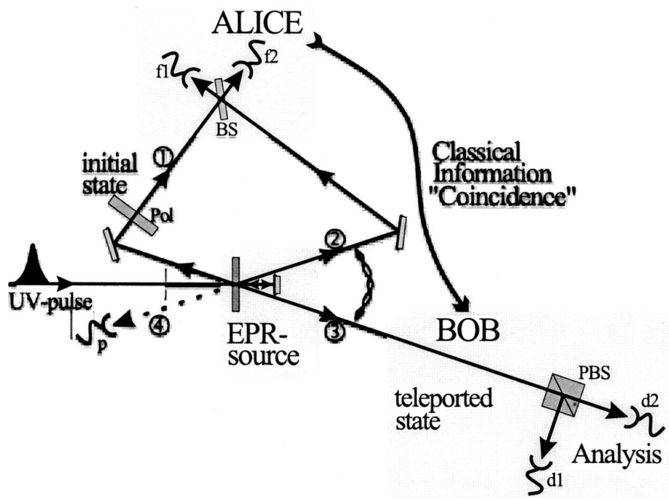

Fig. 32. Experimental arrangement for polarization teleportation (Bouwmeester, Pan, Weinfurter and Zeilinger [2000]).

antisymmetric state $|\Psi_{23}^{-}\rangle$. Alice subjects both the photon to be teleported and her ancillary photon to a partial Bell-state measurement using a beamsplitter. Observation of a coincidence at the Bell-state analyzer detectors, f1 and f2, then informs Alice that her two photons were projected into the antisymmetric state $|\Psi_{12}^{-}\rangle$. This then implies that Bob's photon is projected by Alice's Bell-state measurement onto the original state completing the process.

Some work has also been done to assess the quality of quantum state teleportation (Bouwmeester, Pan, Mattle, Eibl, Weinfurter and Zeilinger [1997]). In particular, it has been pointed out that teleportation fidelity and teleportation efficiency are distinct. The quality of a quantum teleportation procedure can be evaluated on the basis of three properties:

(i) How well any arbitrary quantum state that it was designed to transfer can be teleported (fidelity of teleportation);

(ii) How often it succeeds when given an input it was designed to teleport (efficiency of teleportation);

(iii) How well it rejects a state it was not designed to teleport (cross-talk rejection efficiency).

The aim of the above experiment was to teleport with high fidelity a quantum bit of information, i.e., a two-dimensional quantum state, given by the polarization state of a single photon. When the teleportation system does not output a single photon carrying the desired qubit it is similar to an absorption process in a communication channel; after renormalization of a two-dimensional state, the original state, the qubit, is obtained again without any influence on fidelity. In the above experiment, any incoming UV pump pulse had two chances to create pairs of photons. So those cases where only one pair resulted are rejected, since only those situations are accepted in which the trigger detector p fires together with both Bell-state analyzer detectors f1 and f2. Similarly, any cases where more than two pairs are created is ignored in the experiment since the likelihood of creating one pair per pulse in the modes detected corresponds to less than one event per day.

Further, a three-fold coincidence p–f1–f2 has only two explanations. First, teleportation of the initial qubit could be properly encoded in photon 1 as was demonstrated for the 5 polarizer settings H, V, +45º, –45º and R (circular). These bases involve quite different directions on the Poincaré sphere, proving that teleportation works for arbitrary superpositions. Second, both photon pairs could be created by the pulse on its return trip; in that case no teleported photon arrives at Bob's station and teleportation does not happen, but Alice still records a coincidence count at her Bell-state detector. This leads to a high intrinsic cross-talk rejection efficiency. Nonetheless, only one of the four Bell states was identified, i.e. the protocol works in only one out of every four possible situations. However, this only reduces the efficiency of the scheme, not the fidelity of the teleported qubit.

Another method of teleportation avoids the problem of performing a joint Bell measurement on two particles, following an initial proposal of Popescu [1995]. This is done (see fig. 33) by encoding the two quantum states to be measured by Alice on two degrees of freedom of one particle (Boschi, Branca, De Martini, Hardy and Popescu [1998]). The price that is paid for this ability is that the preparer must select a pure quantum state (here a polarization state), rather than an arbitrary state, and give it directly to the EPR photon of Alice. Both a linear polarization state and an elliptical polarization state were teleported using this method and a 200 mW cw pump laser of 351 nm, with interference visibilities exceeding 80%.

Another subject in fundamental quantum theory of multiple photons that can be investigated using SPDC is attempted quantum cloning. Ideally, a quantum cloning machine could be constructed that creates an arbitrary number of high-fidelity copies of an arbitrary quantum state of a given quantum system. While it has long been known that such a device cannot be constructed as a matter of principle (Wootters and Zurek [1982], Dieks [1982]) – it would allow one to send signals faster than light (Herbert [1982]) – a device can be constructed that makes imperfect copies (Bužek and Hillery [1996, 1998], Bruss, DiVincenzo, Ekert, Macchiavello and Smolin [1998], Gisin and Massar [1997], Bruss, Ekert and

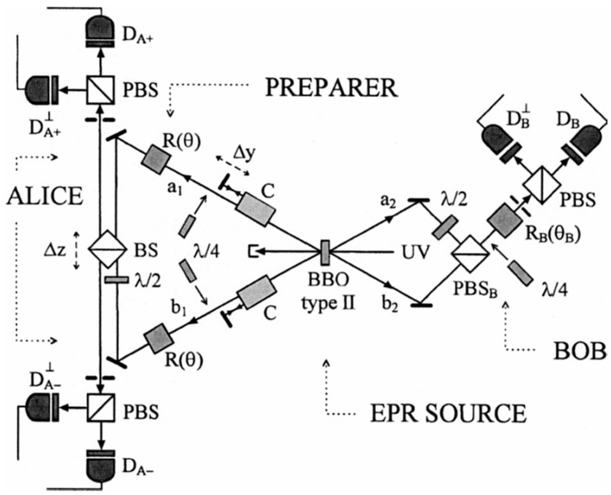

Fig. 33. Experimental arrangement for polarization teleportation (Boschi, Branca, De Martini, Hardy and Popescu [1998]).

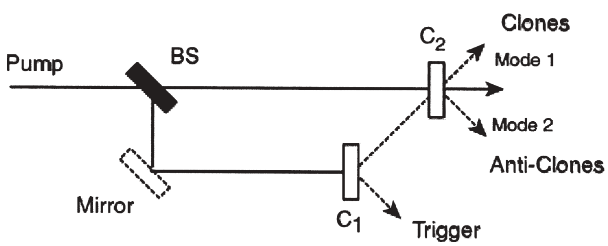

Fig. 34. Schematic for quantum cloning using down-conversion (Simon, Weihs and Zeilinger [2000]).

Macchiavello [1998]). Recently, Simon et al. investigated the question of such universal cloning via parametric down-conversion (Simon, Weihs and Zeilinger [2000]). They considered type-II phase-matched parametric down-conversion with pulsed light input for polarization-entangled two-photon singlet-state output (see fig. 34). Utilizing quasi-collinear outputs and cloning one photon of an entangled pair, they entangled all three output photons.

By considering cloning beginning with N identical photons, i.e., the initial state $|\psi_i\rangle = \left[\left(a^{\dagger}_{\mathrm{V1}}\right)^N/(N!)^{1/2}\right]|0\rangle$, the portion of the output state containing a fixed number of photons in mode 1 is proportional to

$$\sum_{l=0}^{M-N} (-1)^l \begin{pmatrix} M-l \\ N \end{pmatrix}^{1/2} |M-l\rangle_{\mathrm{V1}}\, |l\rangle_{\mathrm{H1}}\, |l\rangle_{\mathrm{V2}}\, |M-N-l\rangle_{\mathrm{H2}}\,, \tag{4.9}$$

which is the output of an optimal cloning machine for the initial state. These workers also investigated the practicality of creating such an apparatus. In the laboratory, pair-production probabilities of 0.004 were achieved using a 76 MHz pulse rate at a UV power of 0.3 W and a 1 mm BBO crystal (a situation designed for maximum overlap of photons from different down-conversion pairs). Assuming a realistic detection of 10%, this would allow for a two-pair detection every few seconds. By changing to a 300 kHz laser system an improvement in detection rate of more than an order of magnitude could be expected.

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