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Effective fiber-coupling of entangled photons for quantum communication

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Abstract

We report on theoretical and experimental demonstration of high-efficiency coupling of two-photon entangled states produced in the nonlinear process of spontaneous parametric down conversion into a single-mode fiber. We determine constraints for the optimal coupling parameters. This result is crucial for practical implementation of quantum key distribution protocols with entangled states.

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1. Introduction

Entangled-photon pairs generated in the nonlinear process of spontaneous parametric down conversion (SPDC) are proven to be a highly desirable means [1] for practical quantum cryptography [2]. The main difficulty of practical utilization of such system usually stems from a relatively low photon collection efficiency because of the complex spatial distribution of SPDC radiation and due to the broad spectral width of entangled-photon wave packets.

The problem of coupling entangled photons into a fiber has been considered before by Kurtsiefer et al. [3]. Assuming the pump to be a plane wave the emission angle of the SPDC has been calculated as a function of the wavelength. The waist of the focused pump beam has been chosen to maximally overlap the "impression" of the Gaussian mode of a single-mode fiber on the

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crystal [3]. It has been pointed out that the coupling efficiency may be significantly affected by transverse walk-off.

In this article we present an approach that includes crystal parameters as well optical propagation parameters, allowing us to achieve a highefficiency coupling of the SPDC pairs into single mode fibers. In particular, we demonstrate how the pump beam waist, crystal length, optical system magnification and the fiber mode field diameter (MFD) must obey a precise joint relation in order to ensure high coupling efficiency. We describe a specific model that allows us to determine a scaling law accounting for all the real experimental parameters. The obtained experimental data follow the general trend of the proposed model.

2. Rates and coupling efficiency

The two-photons state can be written as [4]

$$|\Psi\rangle_{\text{SPDC}} = \int d\omega_{\text{o}} d\mathbf{q}_{\text{o}} \int d\omega_{\text{e}} d\mathbf{q}_{\text{e}} \widetilde{\Phi}(\mathbf{q}_{\text{o}}, \omega_{\text{o}}; \mathbf{q}_{\text{e}}, \omega_{\text{e}})$$
$$\cdot \hat{a}_{\text{o}}^{\dagger}(\omega_{\text{o}}, \mathbf{q}_{\text{o}}) \hat{a}_{\text{e}}^{\dagger}(\omega_{\text{e}}, \mathbf{q}_{\text{e}})|0\rangle. \tag{1}$$

The function $\widetilde{\Phi}(\mathbf{q}_{o}, \omega_{o}; \mathbf{q}_{e}, \omega_{e}) = \widetilde{\mathscr{E}}_{p}(\mathbf{q}_{o} + \mathbf{q}_{e}, \omega_{o} + \omega_{e}) \widetilde{\chi}^{(2)}(\mathbf{q}_{o}, \omega_{o}; \mathbf{q}_{e}, \omega_{e})$ accounts for the phase matching conditions. $\widetilde{\mathscr{E}}_{p}(\cdot)$ represents the amplitude of the plane-wave expansion of the pump field and $\widetilde{\chi}^{(2)}(\mathbf{q}_{o}, \omega_{o}; \mathbf{q}_{e}, \omega_{e}) = \int_{-L}^{0} dz \exp[-i\Delta k(\mathbf{q}_{o}, \omega_{o}; \mathbf{q}_{e}, \omega_{e})z]$ with $\Delta k = k_{p} - k_{oz} - k_{ez}$. Inside the crystal the z-component of the wave-vector is defined as $k_{z}(\mathbf{q}, \omega) = \sqrt{[\omega n(\mathbf{q}, \omega)/c]^{2} - |\mathbf{q}|^{2}}$. All the information on the state is given by the amplitude $\mathscr{A}_{1,2}(\mathbf{x}_{1}, t_{1}; \mathbf{x}_{2}, t_{2})$ [5,6] of detecting the SPDC two-photons in space-time events at (\mathbf{x}_{1}, t_{1}) and (\mathbf{x}_{2}, t_{2}) . The Fourier transform with respect to t_{1} and t_{2} of the two-photon amplitude $\mathscr{A}_{1,2}(\mathbf{x}_{1}, t_{1}; \mathbf{x}_{2}, t_{2})$ is given by

$$\widetilde{\mathscr{A}}_{1,2}(\mathbf{x}_1, \omega_{\mathrm{o}}; \mathbf{x}_2, \omega_{\mathrm{e}}) = \int d\mathbf{q}_{\mathrm{o}} d\mathbf{q}_{\mathrm{e}} \widetilde{\varPhi}(\mathbf{q}_{\mathrm{o}}, \omega_{\mathrm{o}}; \mathbf{q}_{\mathrm{e}}, \omega_{\mathrm{e}})$$
$$\cdot \mathscr{H}_1(\mathbf{x}_1; \mathbf{q}_{\mathrm{o}}, \omega_{\mathrm{o}}) \mathscr{H}_2(\mathbf{x}_2; \mathbf{q}_{\mathrm{e}}, \omega_{\mathrm{e}}),$$
(2)

 $\mathscr{H}_{j}(\mathbf{x}_{j};\mathbf{q},\omega)$ (j = 1,2) being the Fourier transform of impulse response functions $h_{j}(\mathbf{x}_{j},\mathbf{x};\omega)$ of the optical systems [5] through which the two photons propagate from the output face of the crystal to the detection plane.

The coupling of the photon pairs into fibers can be considered a problem of maximizing the overlap between the two-photon amplitude $\mathscr{A}_{1,2}$ of entangled-photon state in the detector plane with the field mode profiles of single-mode fibers [3]. Assuming a quasi-monochromatic and quasi-plane wave travelling in the z-direction and defined on the twodimensional continuous space of the detector planes, we can express the electromagnetic field operator $\hat{E}^{(+)}(\mathbf{x},\omega)$ in terms of a linear superposition of electromagnetic field operators, $\hat{c}_{lm}(\omega)$ and $\hat{c}_k(\omega)$ associated with a *complete orthonormal* set of functions [7,8]. Choosing conveniently the noncontinuous set of guided modes $\psi_{lm}(\mathbf{x}, \omega)$ and the continuous set of the radiation modes $\psi_k(\mathbf{x}, \omega)$, respectively [8], the field operator can be decomposed as $\hat{E}^{(+)}(\mathbf{x},\omega) = \sum_{l,m} \psi_{lm}(\mathbf{x},\omega) \hat{c}_{lm}(\omega) + \int dk \psi_k(\mathbf{x},\omega)$ $\hat{c}_k(\omega)$ where the new operators $\hat{c}_{\alpha}(\omega)$ are defined as $\hat{c}_{\alpha}(\omega) = \int d\mathbf{x} \psi_{\alpha}^{*}(\mathbf{x},\omega) \hat{E}^{(+)}(\mathbf{x},\omega)$ with $\alpha = (l,m)$ or k, and obey the usual bosonic commutation relations $[\hat{c}_{\alpha}(\omega), \hat{c}^{\dagger}_{\beta}(\omega)] = \delta_{\alpha,\beta}$. The amplitude in Eq. (2) can be expanded in terms of guided and radiation modes. The coefficients of the expansion for two guided modes, (l, m) and (l', m'), are given by

$$\widetilde{\mathscr{A}}_{lm,l'm'}^{(1,2)}(\omega_{o},\omega_{e}) = \int d\mathbf{x}_{1} d\mathbf{x}_{2} \widetilde{\mathscr{A}}_{1,2}(\mathbf{x}_{1},\omega_{o};\mathbf{x}_{2},\omega_{e})$$
$$\cdot \psi_{lm}^{(1)*}(\mathbf{x}_{1},\omega_{o}) \psi_{l'm'}^{(2)*}(\mathbf{x}_{2},\omega_{e}). \tag{3}$$

Coupling into a single-mode fiber can be quantified by the *coupling efficiency* parameter $\eta_{\rm fc}$ defined as the ratio of the probability to find two photons in the guided modes over the square root of the product of the probability to find one photon in a guided mode independently of the detection of the other photon [9,10]. In the case of a single-mode fiber, when only the *linearly-polarized* fundamental mode $LP_{01}(\psi_{01}^{(j)})$ is allowed [8], the coupling efficiency takes the simple form

$$\eta_{\rm fc} = \frac{\mathscr{P}^{(1,2)}}{\sqrt{\mathscr{P}^{(1)} \cdot \mathscr{P}^{(2)}}}.\tag{4}$$

The numerator of this expression is given by $\mathscr{P}^{(1,2)} = \int d\omega_o d\omega_e |\widetilde{\mathscr{A}}_{01,01}^{(1,2)}(\omega_o, \omega_e)|^2$ where $\widetilde{\mathscr{A}}_{01,01}^{(1,2)}(\omega_o, \omega_e)$ is given by Eq. (3). The contributions at

the denominator of Eq. (4) are given by $\mathscr{P}^{(1)} = \int d\omega_0 d\omega_e \int d\mathbf{x}_2 | \int d\mathbf{x}_1 \widetilde{\mathscr{A}}_{1,2}(\mathbf{x}_1, \omega_0; \mathbf{x}_2, \omega_e) \psi_{01}^{(1)*}(\mathbf{x}_1, \omega_0)|^2$ and analogous expression for $\mathscr{P}^{(2)}$. The maximum coupling, i.e., $\eta_{\rm fc} = 1$, is reached when the two-photon amplitude in Eq. (2) is the product of single-mode field profiles of two fibers. The rate of the coincidences and the singles on each detector are proportional to the quantities $\mathscr{P}^{(1,2)}$ and $\mathscr{P}^{(j)}$ (j = 1, 2), respectively.

We now consider a model that includes propagation of both fields through two equal infinite ideal lenses without an aperture limit. We also assume that the output plane of the crystal, at a distance d_{bl} from the lenses, is imaged on the fiber plane, at a distance $d_{\rm al}$ from the lenses, i.e., $1/d_{\rm bl} + 1/d_{\rm al} = 1/f$. The amplitude (2) becomes $\mathscr{A}_{1,2}(\mathbf{x}_1, \omega_{\mathrm{o}}; \mathbf{x}_2, \omega_{\mathrm{e}}) \propto \Phi(\mu \mathbf{x}_1, \omega_{\mathrm{o}}; \mu \mathbf{x}_2, \omega_{\mathrm{e}})$ where $\mu =$ $d_{\rm bl}/d_{\rm al} = d_{\rm bl}/f - 1$ is the inverse of the magnification and $\Phi(\mathbf{x}', \omega_{o}; \mathbf{x}'', \omega_{e})$ is the 2-D inverse Fourier transform of the matching function $\Phi(\mathbf{q}_0, \omega_0)$; $\mathbf{q}_{e}, \omega_{e}$), which we calculate in the paraxial and quasi monochromatic approximation. Inside the crystal, such approximations allow us to consider only first terms in the exponential expansion of expression $\widetilde{\chi}^{(2)}(\mathbf{q}_{o},\omega_{o};\mathbf{q}_{e},\omega_{e})$. For a type-II noncollinear configuration and assuming a pump field to be factorable in terms of frequency and wave vectors $\mathscr{E}_{p}(\mathbf{q}_{o} + \mathbf{q}_{e}, \omega_{p}) = \mathscr{E}_{p}(\omega_{p}) \mathbf{E}_{p}(\mathbf{q}_{o} + \mathbf{q}_{e})$, we can calculate the inverse Fourier transform with respect to the frequencies and obtain $\Phi(\mathbf{x}', \omega_{o})$; $\mathbf{x}'', \omega_{\rm e} \Pi_{DL}(t) \mathscr{E}_{\rm p}(T - \Lambda t/D) \operatorname{E}_{\rm p}[(\mathbf{x}' + \mathbf{x}'' - \mathbf{A}t/D)/2]$ $\delta(\mathbf{x}' - \mathbf{x}'' - \mathbf{B}t/D)$ where we have introduced $t = t_1 - t_2$ and $T = (t_1 + t_2)/2$. The function $\Pi_{DL}(t)$ has value 1 for $0 \le t \le DL$ and zero elsewhere. The vectors $\mathbf{A} = 2\mathbf{M}_{p} - \mathbf{M}, \ \mathbf{B} = \mathbf{M} + 2\mathbf{Q}/\bar{K}$ depends on M and M_p , which are the spatial walk-off vectors for the extraordinary fields [11] at the generation and pump frequency, respectively. The walk-off vector **M** is defined as $\mathbf{M} = M\mathbf{e}_M$, where \mathbf{e}_M is a vector pointing in the direction of the projection of the optical axes in the plane perpendicular to pump beam propagation direction, and $M = \delta \ln n_{\rm e}(\theta, \lambda) / \delta \theta$, evaluated at the optical axes cut angle and at the wavelength of the radiation as discussed in [11]. Q is the transverse wavevector associated with perfect phase matching along the intersection of cones, and $\bar{K} = 2K_{\rm o}K_{\rm e}/$

 $(K_{\rm o} + K_{\rm e})$ represents a mean value of wave-vector for generated photons inside the crystal. $|\mathbf{Q}|$ can be evaluated from the expression $|\mathbf{Q}| \simeq \bar{K} \sin \phi_{\text{int}} \simeq$ $2\pi/\lambda \sin \phi_{\text{ext}}$, where ϕ_{int} and ϕ_{ext} are the internal and external emission angles. We have also introduced $D = 1/u_{o} - 1/u_{e}$ and $\Lambda = 1/u_{p} - (1/2u_{o} +$ $1/2u_{\rm e}$) where u_i is the group velocity for the *i*-polarization. The expression has a simple physical meaning. Due to the locality of the interaction, as dictated by the delta function, photons are created in pairs at each point of the crystal illuminated by the pump field, $E_p(\mathbf{x})$. After their birth, photons propagate in the dispersive nonlinear crystal environment experiencing longitudinal D and spatial walk-off, M. They spread with respect to each other in time and in transverse direction, according to the travel distance, z. The transverse spread contributes through two distinct processes when multiplied by the crystal length L. The product $|\mathbf{A}|L$ represents the shift between the generated pairs and the pump field, which is also extraordinary and hence has a walk-off. $|\mathbf{B}|L$ is the spread between the two entangled photons generated from the same pump photon. This vector contains a contribution due to a spatial-walk-off, $|\mathbf{M}|L$, and one more term, $2|\mathbf{Q}|L/\bar{K}$, which represents the transverse distance between pairs generated at the input face of the crystal with respect to the ones generated at the output face, due to the geometry of optical propagation inside the crystal. Those effects contribute to a failure of the ideal imaging of photons emanating from all emission planes inside the crystal onto a single mode fiber plane and hence a reduction of coupling efficiency in the case of long crystal. The effects described above become negligible in case when a thin nonlinear crystal is utilized.

If we assume a pump beam with Gaussian profile, $E_p(\mathbf{x}) = \exp(-|\mathbf{x}|^2/2r_p^2)$, as well as a mode field profile, $\psi_j(\mathbf{x}) = \exp(-|\mathbf{x}|^2/2w^2)/\sqrt{\pi}w$, we can derive a closed expression for the quantities $\mathscr{P}^{(1,2)}$ and $\mathscr{P}^{(j)}$ with j = 1, 2:

$$\mathcal{P}^{(1,2)} \propto L \frac{4}{(2+\xi^2)^2} \frac{\operatorname{erf}(\sigma_c)}{\sigma_c}; \mathcal{P}^{(j)} \propto L \frac{1}{(1+\xi^2)} \frac{\operatorname{erf}(\sigma_j)}{\sigma_j},$$
(5)

where $\xi = w\mu/r_p$ and

$$\sigma_c = \frac{L}{r_p} \sqrt{\frac{(\alpha_1 + \alpha_2)\xi^2 + \beta}{\xi^2 (2 + \xi^2)}};$$

$$\sigma_j = \frac{L}{r_p} \sqrt{\frac{\alpha_j}{1 + \xi^2}}.$$
(6)

The parameters $\alpha_1 = |\mathbf{M}_p|^2 + |\mathbf{Q}|^2/\bar{K}^2$, $\alpha_2 = |\mathbf{M}_p - \mathbf{M}|^2 + |\mathbf{Q}|^2/\bar{K}^2$, and $\beta = |\mathbf{M}|^2 + 4|\mathbf{Q}|^2/\bar{K}^2$ are determined only by the crystal parameters and phase matching geometry. The coupling efficiency becomes, namely

$$\eta_{\rm fc} = 4 \frac{(1+\xi^2)}{(2+\xi^2)^2} \frac{\operatorname{erf}(\sigma_c)}{\sigma_c} \sqrt{\frac{\sigma_1}{\operatorname{erf}(\sigma_1)} \frac{\sigma_2}{\operatorname{erf}(\sigma_2)}}.$$
 (7)

The expression for $\eta_{\rm fc}$ depends on the size of fibers effectively imaged onto the crystal plane, $w\mu$, and the crystal length, *L*, scaled to the pump beam waist into the crystal, $r_{\rm p}$.

3. Experiment

The experimental verification of the above model has been performed using an actively modelocked Ti:Sapphire laser, which emitted pulses of light at 830 nm. After a second harmonic generator, a 100-fs pulse (FWHM) was produced at 415 nm, with a repetition rate of 76 MHz and a maximum average power of 200 mW. The UV-pump radiation was focused to a beam diameter of 150 µm inside a BBO crystal cut for a non-collinear type-II phase-matching using a f = 75 cm quartz lens (see Fig. 1). Two points of intersection of the ordinary and extraordinary cones [12] (3.5° relative to the pump direction) were imaged into a single mode fiber with a MFD of 4.2 µm using two f = 15.4 mm coupling lenses placed 78 cm from the output face of the non-linear crystal.

From the rate of the single counts, $R_{1,2}$, and the rate of coincidence counts, R_c , we obtain a measured overall coupling efficiency $\eta_{\text{meas}} = R_c / \sqrt{R_1 R_2}$. Using in front of the detectors interference filters with 11 nm bandwidth, we obtained a coupling efficiency of $\sim 18\%$ with a crystal of 3 mm length. We achieved $\sim 29\%$ coupling efficiency using a 1 mm crystal and $\sim 30\%$ using a 0.5 mm crystal in the same experimental setup. During our experimentation with 0.5 mm crystal length we observed a rate of coincidence counts of 19650 cps for 100 mW of pump power, and hence we were able to reach a rate of 393 photon pairs/s per mm of crystal length and mW of pump power. This is at the level of best results available in the literature [3]. The probability of double pairs generation is negligible at this level of pump power. The coupling efficiency of Eq. (7) can be estimated from the measured one using the relation $\eta_{
m meas} \sim$ $\eta_{\rm d}\eta_{\rm opt}\eta_{\rm fc}$, where $\eta_{\rm d}$ is the detectors quantum efficiency and η_{opt} is the efficiency due to the optical losses in the setup. Taking into account the approximately 50% detector efficiency (η_d) and 85% transmittance of filters and optical losses (η_{opt}), one can determine that effective fiber coupling coefficients can reach 42% and 68%, respectively



Fig. 1. Sketch of the fiber coupling experiment.

(same results have been obtained with both 10 nm bandwidth filters centered at 830 nm and passband filters). This result is illustrated in Fig. 2, where a solid line indicates theoretical curve obtained from our model Eq. (7). The parameters used have been calculated for crystal dispersion at 830 nm: $|\mathbf{M}_{\rm p}| = 0.076$, $|\mathbf{M}| = 0.072$, and $Q/\bar{K} =$ $|\mathbf{M}|/2 = 0.036$. The fiber parameter is $w = \text{MFD}/2\sqrt{2} \sim 1.48 \ \mu\text{m}$, the pump beam radius $r_{\rm p} \sim 53 \ \mu\text{m}$, and $\mu \sim 49$. Examining the dependence of $\eta_{\rm fc}$ vs. crystal length (see Fig. 2) for different values of parameter μ one can notice, for example, that one cannot reach a coupling efficiency greater than $\sim 50\%$ for $r_{\rm p} \sim 53 \ \mu\text{m}$ and for crystals longer than 2 mm.

Another important test is to evaluate the degree of polarization entanglement of the photon pairs coupled in the fibers. We performed a standard polarization correlation measurement using a 1 mm thick crystal. Two polarization analyzers, not showed in Fig. 1 where placed in front of the fiber couplers: one polarization analyzer has been set at -45° while the other one has been scanned over 360°. To compensate for the longitudinal walk-off (different group velocity between e- and orays) we placed in each arm a $\lambda/2$ to rotate the polarization of 90° and a 0.5 mm thick crystal with the same cut angle as the one used for the generation [12]. For this test in front of the detectors were placed two interference filters with wavelength



Fig. 2. Coupling efficiency η_{ic} from Eq. (7) vs. the crystal length *L* for different values of parameter μ . The solid line indicates the theoretical curve obtained using the parameters described in the text. The three points correspond to the experimental data.



Fig. 3. Polarization correlation measurement with a polarization analyzer set to $\theta_1 = -45^{\circ}$ and rotating the other polarization analyzer (θ_2). The high visibility (0.972 ± 0.002) proves the high degree of polarization entanglement of the photon pairs coupled into the single-mode fibers.

centered at 830 nm and bandwidth 11 nm. The results of such measurement is illustrated in Fig. 3. The visibility of measured coincidence modulation is 0.972 ± 0.002 with no subtraction of accidental coincidences, $R_{\rm c}^{\rm acc}$. The accidental coincidence rate estimated using the relation $R_c^{acc} \sim R_1 R_2 \Delta t_c$, is shown to be 6 cps for a coincidence window $\Delta t_c \sim 10$ ns and single rates $R_1 \sim 22$ kcps and $R_2 \sim 26.3$ kcps. The high visibility obtained in this test with pulsed pump suggests that the bandwidth of the coupled photons is reduced more then expected using 11 nm bandwidth filters [13]. In [3] has been showed that the coupling into single mode fibers reduce the bandwidth of the coupled photon pairs to ~ 4 nm. Considering that the modulation in the singles counting rate was less than 2%, this demonstrates the high degree of polarization entanglement of SPDC photon pairs coupled in single-mode fibers. Additional tests confirming that the degree of entanglement is preserved during the single-mode fiber propagation will be reported elsewhere.

4. Conclusion

We have evaluated the dependence of coupling efficiency of SPDC pairs into single-mode fiber on several major experimental parameters. We obtained an analytical expression for the coupling efficiency η_{fc} that allows us to characterize the importance of the spatial walk-off and to choose appropriate values of experimental parameters to reach high coupling efficiency. Theoretical predictions have been verified experimentally using femtosecond-pumped parametric down conversion in nonlinear crystals of variable thickness. Numerical inspection of the general expression (4) with a more sophisticated model of the impulse response function of our system can allow us to further increase the coupling efficiency.

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