Choosing the Rules for Consensus Standardization *

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Abstract

Consensus standardization often involves bargaining without side payments or substantive compromise, creating a war of attrition that selects through delay. We investigate the tradeoff between screening and delay when this process selects for socially valuable but privately observed quality. Immediate random choice may outperform the war of attrition, or vice versa. Allowing an uninformed neutral player to break deadlocks can improve on both mechanisms. Policies that reduce players’ vested interest, and hence delays, can strengthen the \textit{ex ante} incentive to improve proposals. JEL Codes: L15, C78, D71, D83.

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1 Introduction

In Standard Setting Organizations (SSOs) firms seek voluntary consensus on aspects of product design so that different products can work together. This process mingles technical discussion and political negotiation, in contrast to the race for market share and battle for expectations typical of de facto standards wars. Engineers often hope, and sometimes think, that the process is a collaborative search for the best solution. But firms often have strong vested interests in particular technologies, and there may be no effective mechanism for compromise. This makes it hard to achieve consensus, and turns the coordination problem into a war of attrition: proponents argue for their preferred solution, or simply hold out, until one side concedes.

We show how the war of attrition can select for the technically best solution, but imposes delays when there is vested interest. We then explore policy implications. First, in our model, it is often efficient to interrupt the war of attrition and make a random choice, either immediately or after a period of screening. If commitment to random choice is institutionally awkward, an SSO could give an uninformed but neutral participant a tie-breaking vote. Second, policies that reduce vested interest, including intellectual property licensing rules, can reduce delays while preserving screening, and need not weaken players’ ex ante incentive to improve proposals.

While motivated by compatibility standards, our model could be applied to any institution that seeks consensus among parties with private information who cannot use side-payments or substantive compromise to reach an agreement. Examples include legislatures, multi-national fora such as the World Trade Organization or United Nations, and the various not-for-profit consortia used to create metrics for product quality, safety, and corporate labor or environmental practices.
1.1 Standard Setting Organizations

SSOs promote technical coordination by replacing (or complementing) the bandwagon \textit{de facto} standards process with an orderly explicit search for consensus.\footnote{We use the term SSO to include both formally accredited standards developing organizations (SDOs) and less formal standards forums and industry consortia.} Active participants incur direct costs such as membership, staff support and travel.\footnote{Estimates of these costs vary widely. (Siegel, 2002, page 227) suggests that “a medium to large end-user corporation will invest $50,000 per year” per SSO in salary, travel, and membership fees; and that firms who “contribute” technology may spend considerably more. Both Hewlett Packard and Sun Microsystems belonged to more than 150 SSOs in 2003 (see “Major Standards Players Tell How They Evaluate SSOs,” Andrew Updegrove, Consortium Standards Bulletin, June 2003). Chiao et al. (2007) cite a Forbes magazine article suggesting that IBM spent $500 million on standards-related activities in 2005.} This can induce a free-rider problem, encouraging over-representation by firms with vested interests in a particular solution (Weiss and Toyofuku, 1996; Osborne et al., 2000). For compatibility standards, these private benefits are often linked to leads in product development, the presence of an installed base or (increasingly) intellectual property rights.\footnote{The trade press contains numerous accounts of vested interest, such as “Zapping the Competition: How companies are using obscure standards-setting bodies to cripple new technologies and hog-tie rivals” (Scott Woolley, Forbes, October 2, 2006) or “UWB Standards Group Calls it Quits,” (Mark Hachman, Computerworld, January 19, 2006). (Eisenman and Barley, 2006, page 4) quote one participant on the matter of development lead times: “… there is tremendous resistance from other companies, who say ‘Why should I endorse something that you’re ready to ship and give you a time-to-market lead?’” The March 2007 Consortium Standards Bulletin (http://consortiuminfo.org/bulletins) also offers a practitioner’s view of intellectual property issues.}

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In principle, an SSO could choose any decision mechanism to select a standard: taking into account members’ information, their incentives to participate, the level of vested interest, and the need to produce standards that will attract widespread support. However, SSOs generally seek consensus, which the International Organization for Standardization (ISO) defines as:

“general agreement, characterized by the absence of sustained opposition to substantial issues by any important part of the concerned interests and by a process that involves seeking to take into account the views of all parties concerned and to reconcile any conflicting arguments.” (ISO/IEC Directives 2004, page 26)

While economists might view monetary transfers as a natural route to consensus, SSOs do not encourage that. Since consumers are seldom at the table, antitrust issues could arise if firms paid each other to agree to proposed standards — especially
when they implicate participants’ intellectual property. Explicit side-payments could also lead to many weak proposals from firms hoping to get “bought out” during negotiations.

Substantive compromise may provide an alternative to side-payments. For example, many standards include “vendor specific options” to allow for product differentiation. But there is no guarantee that a technical compromise will be efficient. At some point, participants are often forced to make a clear choice between competing proposals.\(^4\)

When the consensus principle enables each participant to hold up an agreement, and side-payments or compromise are unavailable or ineffective, bargaining becomes a war of attrition that selects a winner through costly delay. Thus, Davies (2004) describes consensus decision-making at the Internet Engineering Task Force:

“A single vocal individual or small group can be a particular challenge to Working Group progress and the authority of the chair. The IETF does not have a strategy for dealing effectively with an individual who is inhibiting progress, whilst ensuring that an individual who has a genuine reason for revisiting a decision is allowed to get his or her point across.”

Similarly, Sun Microsystems’ director of standards (Cargill, 2001) claims that “[the] formal process — with its Byzantine structures, considerable delays for review, reconsideration, and re-vote... can take up to 48 months to complete, resulting in lost market opportunity.”\(^5\)

1.2 Outline of the Paper

We introduce and solve a model of the war of attrition with private information on the quality of sponsors’ proposals. In Section 2, we show that any symmetric equilibrium

\(^4\)Similar issues arise in other settings. Roth (2007) describes a variety of cases where ethical norms or “repugnance” present a significant obstacle to monetary transactions. More colorfully, the 1993 film “Indecent Proposal” explores the cost of explicit side-payments in romantic competition. Powell (2004) surveys a literature that emphasizes dynamic commitment problems as a source of bargaining failures in government formation.

\(^5\)While many SSOs have adopted “fast track” procedures and strive to reduce administrative delays, most still take several years to approve a significant standard. For example, the 1999 Annual Report of the International Electrotechnical Commission (IEC) claimed that 20 percent of standards were developed in less than three years, “in direct response to industry requests that we speed up the standardization process.” (By comparison, the 1991 Annual Report indicated a mean development time of 87 months (page 6)).
of the war of attrition picks the better proposal, and calculate the delay involved; the key parameter is a measure of vested interest. Section 3 explores the welfare tradeoff between selection and delay, showing when immediate random choice outperforms the war of attrition, and that a delayed random choice may outperform either. We also show that SSOs might implement delayed random choice by granting the pivotal vote to an uninformed neutral participant. Section 4 analyzes implications for sponsors’ incentives to develop and improve their proposals, finding that the uninterrupted war of attrition provides relatively weak incentives, but (because improving the losing proposal causes longer delays) that even those weak incentives may be excessive. We also show that policy changes to reduce vested interest can strengthen the incentives to develop good proposals in the first place.

1.3 Related Literature

Farrell and Saloner (1988) first modeled consensus standard setting as a war of attrition. Their complete-information analysis predicts that a consensus process is more likely to achieve coordination than decentralized adoption, but that on average it is slow, and in a symmetric setting is outperformed by immediate random choice. Random choice, however, picks a random standard, while (as we show) a war of attrition could select for technological quality. To address that possibility we introduce private information on the quality of proposals, and evaluate how well, and how promptly, the process selects one.

David and Monroe (1994) also use the war of attrition to study SSOs, but in their model there is no trade-off between delay and the quality of outcomes. Our setup is closer to Bolton and Farrell (1990), who study entry in natural monopoly markets when firms have private information on costs. They find that decentralized entry selects the low cost firm more often than immediate random choice, but also leads to delays and duplication, so neither mechanism is always preferred. Unlike that model, we assume an element of common interest — all payoffs depend (positively) on the winner’s type — and show that delays will disappear as vested interests approach
zero. We also use our model to analyze the role of third parties in consensus decision-making, and to evaluate *ex ante* incentives to improve proposals.

The war of attrition has been used to describe biological competition (Smith, 1974; Riley, 1980), labor strikes (Kennan and Wilson, 1989), the decision to exit a market (Fudenberg and Tirole, 1986), the road to war (Fearon, 1994), and a variety of picturesque applications (Bliss and Nalebuff, 1984). Bulow and Klemperer (1999) and Myatt (2005) use standard-setting as a motivating example of the war of attrition, but do not focus on the issues we examine. Krishna and Morgan (1997) make the formal link between a war of attrition and the second-price all-pay auction. We study an all-pay auction with positive externalities (Jehiel and Moldovanu, 2000) in a setting where “revenues” (i.e. delays) are costly.\(^6\)

There is a public-choice literature on consensus decision rules, typically modeled as unanimity (e.g. Buchanan and Tullock, 1965). Li et al. (2001) study the incentive for committee members to distort private information under a consensus rule, and Maggi and Morelli (2006) show that unanimity may be optimal in a repeated game with complete-information binary voting against the status quo. We evaluate the performance of a consensus rule when players have private information about competing alternatives, each of which is known to dominate the status quo, and vested interests make cheap-talk impossible.

Lerner and Tirole (2006) develop an alternative model of SSOs in their role as a certifier, rather than a forum for reaching consensus. In their theory, SSOs differ in their degree of sympathy for technology vendors relative to end-users. Vendors choose the friendliest SSO whose certification will persuade end-users to adopt the standard. Chiao et al. (2007) used data on SSO procedures to test that model of forum shopping. But aside from case studies,\(^7\) empirical research on SSOs remains scant. Weiss and

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\(^6\)There is an extensive literature on bargaining delays. In addition to time-based signalling, as in the war of attrition, bargaining delays can occur when disagreement payoffs are endogenous (Fernandez and Glazer, 1991; Busch and Wen, 1995), when there are externalities (Jehiel and Moldovanu, 1995), when the size of the pie is stochastic (Merlo and Wilson, 1995), or when there is learning about bargaining power (Yildiz, 2004).

\(^7\)Case studies of compatibility standard setting include Besen and Johnson (1988), Besen and Saloner (1989), Foray (1995), Lehr (1995) and Eisenman and Barley (2006). Combining case research with descriptive data analysis, Bekkers et al. (2002) show how the patent portfolios and alliance network of firms participating in the European...

2 A Model of Consensus

Two proponents offer solutions; each \(i = 1, 2\) has a privately known quality \(q_i\). The game ends when one player concedes, agreeing to the other’s proposal. We assume there is no scope for compromise or side payments.

Player \(i\)’s strategy is a concession time \(t_i\), meaning that he will concede at \(t_i\) if \(j \neq i\) has not yet conceded. If \(t_1 < t_2\) then player 1 actually concedes at time \(t_1\), and as of that date gets a payoff of \(Lq_2\), while player 2 gets \(Wq_2\), where \(L > 0\) and \(W > 0\) measure the “loser’s” and “winner’s” shares of the total surplus \(q = q_2\).\(^8\) If \(t_1 = t_2\) each player wins with probability \(\frac{1}{2}\).

When \(L = W\), both players want the type with lower quality to concede.\(^9\) More realistically, as we assume henceforth, \(W > L\): there is “vested interest.” Thus, each player would like its proposal adopted, even if its rival has somewhat higher quality, though each would concede if it knew that a rival’s quality was more than \(W/L\) times better than its own. While players with a high \(q_i\) would like to reveal that information, vested interest makes a rival skeptical.\(^10\)

Players share a discount rate \(r\), and flow payoffs are zero until agreement is reached. We assume it is common knowledge that each \(q_i > 0\), so that both players prefer either

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\(^8^{ See Section 4 considers policies that may influence \(W\) independently of \(L\) (e.g. licensing rules that allow the winner to capture more consumer surplus), so we do not normalize \(W + L = 1\).

\(^9^{ As a result, they could resolve the issue through cheap talk. We show below that in the limit as \(W\) and \(L\) converge, the war of attrition also achieves full screening with no delay.

\(^10^{ Information on customer behavior or demand for a particular technology may be important but hard to verify. Even technical performance, which might seem easy to observe, can be controversial when there are many relevant dimensions (e.g. speed, size, cost, and power consumption); leading to many degrees of freedom in design, testing and measurement.
standard to the status quo. We consider a symmetric model and focus on symmetric
perfect Bayesian equilibrium: a concession-time strategy \( t(\cdot) \), such that a player of
type \( q \) finds it optimal to concede at time \( t(q) \) if its rival has not yet conceded and if it
believes its rival is also using the strategy \( t(\cdot) \). The resulting mechanism resembles a
second-price all-pay auction with a consolation-prize: each player “bids” a concession
time, both pay a waiting cost determined by the smaller bid, and both receive prizes
proportional to the winner’s type.

2.1 Screening and Equilibrium

If proposal qualities are independently distributed, it is well known that equilibrium
concession strategies \( t(\cdot) \) are increasing in proposal quality. Proofs not contained
in the text are collected in an appendix.)

Lemma 1 When qualities are independently distributed, every rationalizable Bayesian
strategy is weakly increasing.

Lemma 1 is intuitive: a player’s expected payoff from concession is independent
of its own type (quality), while its expected payoff from holding out longer increases
with its type. If qualities are drawn from a continuous distribution function \( F(\cdot) \),
symmetric equilibrium concession strategies will be one-to-one. Thus, we have

Proposition 1 When qualities are independently and continuously distributed, the
symmetric equilibrium of an uninterrupted war of attrition selects the better proposal.

It is helpful to define the expected type of a player whose type is at least \( x \):

\[
G(x) \equiv E[y|y \geq x] = \frac{\int_x^{\infty} s \, dF(s)}{1 - F(x)}.
\]

11There is often also an asymmetric equilibrium in which player 1 never concedes and player 2 concedes immediately.
This equilibrium must be supported by out-of-equilibrium beliefs of the same form: if player 2 does not immediately
concede, player 1 must expect it to do so at the next instant. There must also be enough vested interest so that if
player 1 knows its quality is extremely low it will still prefer to let player 2 concede. In ex ante symmetric situations,
the symmetric equilibrium seems a natural model, but if one player is seen as a focal standard-setter, the asymmetric
equilibrium may well be played. Historically, IBM may have been focal in the computer industry, and AT&T in US

12Versions of this result are in Bliss and Nalebuff (1984, Theorem 2), Fudenberg and Tirole (1991, page 217) and
To characterize equilibrium when concession strategies are strictly monotonic, define $\beta(t(q)) = q$, the inverse of the $t(\cdot)$ function, and consider a type $q$ player who, in equilibrium, will concede at time $t(q)$, yielding (as of then) expected payoff $LG(q)$. If this player were instead to hold out a short time $dt$ longer before conceding, its expected (present-value) payoff as of $t(q)$ would be

$$[h(q)\beta'(t)dt]Wq + e^{-rdt}[1 - h(q)\beta'(t)dt]LG(q + \beta'(t)dt),$$

where $h(q)$ is the hazard rate $f(q)[1 - F(q)]^{-1}$. The first-order condition, equating marginal costs and benefits of delay (and suppressing arguments for brevity), is:

$$rLG = h\beta'Wq + LG'\beta' - h\beta'LG \quad (1)$$

and substituting $G'(q) = h(q)[G(q) - q]$ into equation (1) yields

$$rLG = h\beta'[W - L]q \quad (2)$$

Intuitively, the marginal cost of delay is $rLG$, while the marginal benefit is the probability that a rival concedes in the next instant ($h\beta'$) multiplied by the change in payoffs, given that concession reveals the rival to have type $q$.

To solve for the equilibrium strategies, define a measure of vested interest $v \equiv \frac{(W - L)}{L} \geq 0$, and a quantity $K(x) \equiv G(x)[1 - F(x)] = \int_x^\infty s dF(s)$. Note that $G(q_{min}) = K(q_{min}) = \mu$, the ex ante average quality, and that $K'(s) = -sf(s)$. Rearranging equation (2) yields

$$\frac{1}{\beta} = \frac{dt}{dq} = \frac{vh(q)}{rG(q)} = \frac{vf(q)}{rK(q)} \quad (3)$$

This differential equation, together with the boundary condition $t(q_{min}) = 0$, defines a unique symmetric Bayesian equilibrium,

$$t(q) = \frac{v}{r} \int_{q_{min}}^q \frac{sf(s)ds}{K(s)} = \frac{v}{r}[\log \mu - \log K(q)] \quad (4)$$
or in terms of the time value of delay until $q$ concedes,

$$\delta(q) \equiv e^{-rt(q)} = \left[ \frac{K(q)}{\mu} \right]^v$$

3 Performance: War-of-Attrition vs. Random Intervention

We now ask whether the war-of-attrition’s screening properties make it a desirable standard setting mechanism, given the costs of delay. We measure performance as the players’ expected payoffs. From our equilibrium it immediately follows that $r$ affects the time to agreement but not present-value payoffs as of $t = 0$.

**Proposition 2** The ex post (and hence interim and ex ante) performance of the symmetric equilibrium of the war of attrition is independent of $r$. Delays are increasing in $v$. When $v$ approaches zero (no vested interest), so do delays, and performance approaches first-best.

In our model, for low $v$ the war of attrition selects the best proposal at low cost. Appendix A describes several policies that SSOs might use to lower $v$. These include starting early, before firms become attached to a particular solution; endorsing partial or “incomplete” standards that allow for product differentiation; and adopting disclosure rules that lead participants to reveal their relevant patents.

Though Proposition 2 generalizes somewhat (see Appendix B), it may be too optimistic when willingness to wait depends on factors other than quality. Moreover, for large $v$, our model predicts long delays. Ex post performance is especially poor when both players have good proposals, yielding long delays and only a slightly better outcome than rapid random choice, which even the winner might prefer if qualities were known. But might participants favor a well-timed random choice rule ex ante, while they are behind a veil of ignorance?

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13 This approach does not explicitly account for non-participants’ welfare. In Appendix A we argue that the interests of participants and non-participants are broadly but not perfectly aligned.
To model random intervention, we introduce a neutral player that shares the proponents’ discount rate $r$ and whose present-value payoff once consensus is reached is $\alpha q$. This player knows the game and parameters, but not the proposal qualities, and can only decide how long ($T$) to let the war of attrition run before choosing a winner at random. Two special cases are the war of attrition itself (never intervene, $T = \infty$), and immediate random choice (intervene immediately, $T = 0$).

In reality, the neutral player could be an SSO participant with a pivotal vote. Many SSOs have policies to encourage neutral participants, and allow a specified super-majority to approve standards. The neutral player could also represent a bystander with enough clout to bypass the SSO and jump-start market adoption, such as a large end-user, a “platform leader” (Cusumano and Gawer, 2002) or a government agency that would defer to a timely consensus recommendation from industry.

3.1 Equilibrium with random intervention

The proponents’ concession strategies will generally respond to anticipations of $T$. Nevertheless, we begin by analyzing the neutral player’s choice of $T$ as if proponents would continue playing the strategies derived in the uninterrupted war of attrition.

**Fixed Concession Strategies (FCS):** If proponents expect a random choice at $T$, then their equilibrium concession strategies remain unchanged (i.e., as if $T = \infty$).

Assumption FCS would of course be true if random intervention were not anticipated. As we show below, FCS is also true when the proponents expect an intervention at $T = t^*$, the optimal time for a surprise intervention.

Suppose that screening ends at $t$: the earlier of $T$ or the concession time. The neutral player’s expected payoff is $\alpha e^{-rt}G(\beta(t))$, so his optimal strategy (under FCS)

\[14\] To promote a “balance of interests,” ANSI’s Essential Requirements stipulate that “no single interest category may constitute a majority of the membership of a consensus body” and describes several broad categories of participant, including producers, users, consumers, and general interest (e.g. academic) members (ANSI, 2006, page 6). Of course, each of these groups may have their own rent-seeking incentives (see Appendix A). Examples of super-majority voting include ISO, which requires a two-thirds majority of “P-member” votes (IEC/ISO Directives, part 1, section 2.4.3.), and OASIS (a consortium that develops software standards), which allows two-thirds of the membership to approve a proposal provided no more than twenty-five percent object.
will put mass only on the value(s) of \( t \) that maximizes that payoff. The expected value of waiting slightly longer before a random intervention is

\[
\alpha e^{-rt}\{G'(\beta(t))\beta'(t) - rG(\beta(t))\}
\]  

(5)

Substituting \( \beta' \) from (3) reveals that a neutral player prefers a bit more screening to immediate intervention if and only if

\[
G(q) - (1 + v)q > 0
\]

(6)

If (6) holds for all \( q \), then (under FCS) a neutral player will never want to intervene, because the war of attrition screens so efficiently. If (6) never holds, the war of attrition screens so slowly that delay is never desirable, and the neutral player should make an immediate random choice. Both of these extreme cases are possible:\footnote{Details of this and subsequent examples, all based on the Pareto Distribution, are in the appendix. The Pareto distribution is a power law with support \( x \geq 1 \) and a dispersion parameter \( a > 0 \): it has \( F(x) = 1 - x^{-(1+a)} \).}

**Example 1** With the Pareto distribution and under assumption FCS, a neutral player makes an immediate random choice if \( av > 1 \) and allows an uninterrupted war of attrition if \( av < 1 \).

In general, neither (6) nor its reverse will hold globally. When \( G(q) - (1 + v)q \) crosses the axis from above exactly once, at say \( q^\ast \), the neutral player’s expected payoff (under FCS) is single-peaked as a function of \( T \), so he prefers the war of attrition until \( T = t^\ast \equiv t(q^\ast) \) and immediate random choice thereafter. Bagnoli and Bergstrom (2005, Theorem 6) prove that \( G(q) - q \) is decreasing when either \( f(q) \) or \( 1 - F(q) \) is log-concave, which holds for many standard probability distributions, including the uniform, normal, exponential and logistic.\footnote{With \( v > 0 \), the Bagnoli-Bergstrom result implies that \( G(q) - (1 + v)q \) is strictly decreasing. In failure analysis \( G(q) - q \) is called the mean-residual-life function. It gives the expected time-to-failure of a machine that has already run for \( q \). If this function is decreasing, then a machine “ages” or wears out over time.} Thus we have:

**Lemma 2** Under FCS, if either \( f(q) \) or \( 1 - F(q) \) is log-concave, there is a unique time \( t^\ast \geq 0 \) such that a neutral player prefers screening via the war of attrition at all
That is, for log-concave quality distributions, screening is efficient at small values of \( q \) (or \( t \)), but not large values. To see how the social benefits of screening relate to the private benefits of delay, re-write the sponsor’s first-order condition (1) as

\[
L(G' \beta' - rG) + h \beta'(Wq - LG) = 0
\]  

The first term is proportional to the marginal social benefits of delay, as in (5). The second term reflects a sponsor’s vested interest “wish to win” as of time \( t \); it is the probability that the other sponsor concedes in the next instant, times the change in (interim) expected private payoff if that happens. Perhaps surprisingly, this interim “wish to win” is negative when \( Wq < LG(q) \), since winning brings news that rival quality is much worse than expected. And since (7) is a sponsor’s first-order condition, the two terms must cancel for the type meant to concede at \( t \). Thus, we have:

**Lemma 3** In a symmetric equilibrium, at time \( t(q) \), the marginal type \( q \) prefers concession to immediate victory if and only if screening is locally efficient: \( G'(q) \beta'(t(q)) > rG(q) \) if and only if \( Wq < LG(q) \).

As promised above, we now justify assumption FCS by showing that it holds if players expect a random choice at \( T = t^* \).

**Proposition 3** The following strategies are a perfect Bayesian equilibrium: Sponsor-types \( q < q^* \) use the concession strategy derived in Section 2.1. Types \( q > q^* \) never concede. The neutral player waits until \( t^* \) before making a random choice (and would also intervene at all times \( t > t^* \)).

To understand this result, note that by Lemma 3, \( Wq^* = LG(q^*) \): the type \( q^* \) who is meant to concede at \( t^* \) is (at that time) indifferent between concession and an immediate random choice. Players of lower type would prefer to concede. If (contrary to equilibrium) they waited until \( t^* \), they would then scurry to concede rather than
risk “winning”, because for them, \( Wq < LG(q^*) \). Players of higher type, for whom \( Wq > LG(q^*) \), would rather win than concede at \( t^* \), so they will not want to pre-empt the random intervention. Other potential deviations are unprofitable just as they were without the prospect of random choice: their marginal costs and benefits have not changed. Finally, if the game has continued (out-of-equilibrium) past \( t^* \), remaining types will not concede, since the neutral player’s specified strategy is then immediate random intervention, which they prefer to concession.

Before proceeding, we offer three remarks about this equilibrium. First, while log-concavity guarantees a unique \( t^* \), Proposition 3 holds for general quality distributions. Second, the neutral player’s threat of intervention at all \( t > t^* \) is credible even if sponsors have weaker off-equilibrium beliefs. For example, a neutral player should intervene after \( t^* \) if the war of attrition would otherwise continue indefinitely (as it would under FCS), rather than “stalling” at \( t^* \) because sponsors expect a random choice at any moment. And third, totally differentiating (6) shows that \( \frac{d\hat{q}^*}{dv} < 0 \): the amount of screening is decreasing in vested interest.

### 3.2 Alternative deadlines

Proposition 3 shows that random intervention at \( t^* \) is an equilibrium: it is optimal given that proponents expect it. We now show that for log-concave quality distributions, intervention at \( t^* \) is the unique pure-strategy perfect Bayesian equilibrium.

To begin, consider a sponsor that anticipates random intervention at \( T \neq t^* \). This player could wait for the random choice, attempt a “last minute” concession\(^{17}\) just before \( T \), or drop out earlier. Monotonicity (Lemma 1) implies that these strategies are ranked: the lowest types concede according to their first-order conditions (derived in Section 2.1), intermediate types may try a last minute concession at \( T \), and the highest types wait for a random choice.

\(^{17}\)We assume the neutral player can accept concessions that occur at \( T \), and will select a winner at random when there are more than one.
Lemma 4 If proponents expect a random intervention at $T < \infty$, their symmetric equilibrium concession strategies will have thresholds $\underline{q} \leq \overline{q}$, such that all types $q < \underline{q}$ play the strategies $t(q)$ derived in Section 2.1; all types $q \in (\underline{q}, \overline{q})$ concede just before $T$; and all types $q > \overline{q}$ wait for the random intervention.

Now suppose that $T < t^*$. Since there is not enough time for screening (as described by equation (3)) to reach type $q^*$, we must have $q < q^*$. Log-concavity and Lemma 3 imply that types just above $q$ prefer concession to a delayed random choice. Since $q$ is the last type to concede strictly before $T$ (by Lemma 4), there must be an atom or “flurry” of concessions at $T$. Finally, since type $q^*$ is indifferent between concession and random choice when all lower types have dropped out – and in this hypothetical equilibrium, types between $\underline{q}$ and $\overline{q}$ have not – the highest type in the atom of concessions must be less than $q^*$.

When $T > t^*$, types below $q^*$ have enough time to concede according to their first-order conditions. Log-concavity and Lemma 3 imply that types above $q^*$ strictly prefer a random choice to immediate concession. Since $q > q^*$, there must be a gap between $t(q)$ and $T$, and there will be no atom of concessions at $T$.

Proposition 4 For log-concave quality distributions, if proponents anticipate $T < t^*$, then $\underline{q} < \overline{q} < q^*$; screening does not reach $q^*$, and there is an atom in the distribution of concession times at $T$. If proponents anticipate $T > t^*$, then $q^* < \underline{q} = \overline{q}$ and $t(q) < T$; there is a gap in concession times after $t^*$, but no atom at $T$.

We now argue that proponents cannot reasonably anticipate $T \neq t^*$. Proposition 4 shows that $t^*$ dominates “late” intervention ($T > t^*$) for the neutral player, since equilibrium screening after $t^*$ is no faster, and after $t(q)$ is strictly slower, than under FCS. While “early” intervention might seem desirable ex ante, it is not credible. Following a flurry of concessions at $T < t^*$, the neutral player would be tempted to screen a bit more, since proponents in $(\overline{q}, q^*)$ screen efficiently in a symmetric equilibrium, and would concede even faster if they expected an immediate random choice. Thus, in a perfect Bayesian equilibrium, $T$ must satisfy the neutral player’s
first-order condition (5), even though we have not maintained assumption FCS.\textsuperscript{18}

**Proposition 5** For log-concave quality distributions, random intervention at \( t^\ast \) is the unique pure-strategy perfect Bayesian equilibrium.

Finally, since intervention at \( t^\ast \) dominates all \( T > t^\ast \), even if the neutral player can make a public \textit{ex ante} commitment to \( T \), we have

**Corollary 1** For log-concave quality distributions, a neutral player will intervene no later than \( t^\ast \).

Our analysis leaves open the possibility that a neutral player with commitment power could improve on the outcome of Proposition 3 by choosing a deadline strictly below \( t^\ast \).\textsuperscript{19} Unfortunately, the analysis of this case is complex and we do not know whether that possibility can occur. In practice, it may be difficult for SSOs to commit to an arbitrary deadline, particularly when an explicitly random decision would jeopardize \textit{ex post} adoption.\textsuperscript{20} In contrast, allowing neutral participants to break deadlocks can reduce delay while preserving a (less demanding) version of consensus.

### 4 Innovation Incentives

Above, we took the distribution of \( q \) to be exogenous. We now ask whether policies that reduce screening (random choice) or delays (lower \( v \)) would reduce sponsors’ incentives to develop better solutions in the first place.

\[\textsuperscript{18}\text{For general quality distributions, the set of perfect Bayesian equilibria will correspond to the (local) peaks of the neutral player’s objective under FCS. In that case, } t^\ast \text{ might still provide a natural focal point, since it is the neutral player’s global optimum.}\]

\[\textsuperscript{19}\text{Of course, a neutral player with commitment power might do even better by choosing a more complex mechanism that specifies waiting times and the probability of winning as a function of sponsors’ reported types. We hope to pursue that approach in future work.}\]

\[\textsuperscript{20}\text{In our model the neutral player need only be willing to wait a while and then toss a coin. Taken literally, Proposition 3 suggests that an } N \text{-player standards committee should accept any proposal that can gather two or more votes; since any sponsor who has already conceded could serve as the tie-breaker. However, a naive “two-vote rule” would be easy to manipulate, since competing proposals are typically sponsored by coalitions that vary in size and influence. And while random choice may seem tempting when the standards process is painfully slow, formalizing that decision rule could encourage the submission of low quality proposals.}\]
4.1 War of attrition versus random choice

A player’s interim expected payoff in the uninterrupted war of attrition can be found by accounting for outcomes as a function of its rival’s type:

\[ u(q) = Wq \int_{q_{\text{min}}}^{q} \delta(s) \, dF(s) + \delta(q)G(q) \left[ 1 - F(q) \right] \] (8)

If this player’s rival cannot observe its quality improvements, its interim incentive to improve \( q \) is \( u'(q) \). From the envelope theorem, this equals the gain from an increase in \( q \) holding its own (as well as its rival’s) concession strategies fixed:

\[ u'(q) = W \int_{q_{\text{min}}}^{q} \delta(s) \, dF(s) \] (9)

This describes the incentive to un-observably improve quality that is now \( q \). For a simple model of \textit{ex ante} incentives, suppose that players choose effort \( e \) that increases quality by \( e \) whatever the realization of \( q \), so the cumulative quality distribution becomes \( F^*(q; e) \equiv F(q - e) \). Since the marginal benefit of effort is equivalent to a small increase in quality \( dq \), taking expectations over \( q \) measures the (gross) \textit{ex ante} innovation incentive in an uninterrupted war of attrition: \( I^{WOA} \equiv E[u'(q)] \). \(^{22}\)

Now consider incentives to improve \( q \) under several interpretations of random choice. First, a predetermined standard-setter’s payoff is \( Wq \), so its incentive to improve quality is \( I^{PS} = W \). Its rivals have no incentive to develop or propose technologies.\(^{23}\) Second, if random choice represents a “rough and ready” selection process, where each player wins immediately half of the time, then each player has \textit{ex ante} innovation incentive \( I^{RC} = \frac{1}{2} W \). Finally, if a neutral player will break deadlocks through random intervention at \( t^* \), then interim incentives to un-observably improve quality

\(^{21}\)An alternative derivation of \( u(q) \) starts from (9) and uses the boundary condition \( u(q_{\text{min}}) = L\mu \). Integrating by parts, \( u(q) = L\mu + W \int_{q_{\text{min}}}^{q} \delta(s)f(s)[q - s] \, ds \). Thus, the “information rents” in this model are a weighted average of differences between a player’s own type \( q \) and all worse types.

\(^{22}\)Myatt (2005) finds that the asymmetric war of attrition strongly favors players with a (stochastically) better proposal, suggesting that innovation incentives will be strong when players can observe \( e \), though we do not model this case.

\(^{23}\)Given the pre-determined standard setter, this demotivation of others is efficient in the model. In general it can be desirable to give one player a strong incentive to innovate, and others none, if innovation is costly but needs no imagination. In other cases it may be better to give some incentive to each of many potential innovators.
(IRI) are a combination of the previous cases: sponsor-types \( q < q^* \) have the same incentives they would in a war of attrition, while those with \( q \geq q^* \) will face a delayed symmetric random choice whenever their rival will also wait till \( t^* \). The following proposition compares innovation incentives under these four different arrangements.

**Proposition 6** A predetermined standard setter has the strongest incentive to innovate, followed by firms facing an immediate random choice. Firms that anticipate random intervention at a finite \( t^* > 0 \) have weaker incentives than under immediate random choice, but stronger than under an uninterrupted war of attrition:

\[
I_{PS} = W > I_{RC} = \frac{1}{2}W \geq I_{RI} \geq I_{WOA}.
\]

A proof is in the appendix, but intuitively, a player gains from an increase in its own \( q \) if and only if it is the winner. This accounts for the comparison between a predetermined standard setter and an immediate random choice. Next, the war of attrition gives each player the same *ex ante* probability of winning as random choice, but delays payoffs (and by longer if it is uninterrupted) thus reducing the expected slope of \( e^{-rt}Wq \) conditional on eventual victory. One might expect offsetting rent-seeking “competition” in quality, but in the model each player could act as if its quality were higher (conceding later and winning more often) without actually changing its quality. So the war of attrition produces weaker incentives than immediate or delayed random choice.\(^{24}\)

### 4.2 Reducing vested interest

We now consider policies that would reduce delay without interrupting the war of attrition. Proposition 2 says that delays are increasing in \( v \). However, innovation incentives depend on \( W \) and \( L \), not only via \( v \), and policies may affect both \( W \) and \( L \). For instance, allowing a winning sponsor to exploit its patents without limits after a standard is locked in will transfer surplus from end-users to the sponsor, and may

\(^{24}\) Cabral and Salant (2008) develop a model where standardization may occur *before* firms have a chance to innovate, and improvements to a shared standard benefit all players equally. In their model, firms may delay choosing a standard to prevent free-riding in R&D.
also hurt other participants, but plausibly not as much as the winning sponsor gains. We model such changes as a shift in payoffs from \((W, L)\) to \((W + dW, L - k dW)\) for some \(k \geq 0\).25

In our model, increasing \(W\) raises private returns to innovation directly, while increasing \(v\) reduces them by increasing delay. On balance, increasing vested interest can either raise or lower incentives to innovate:

**Example 2** *In the uninterrupted war of attrition with a Pareto distribution, \(I^{WOA}\) is increasing in \(W\) if and only if \(k < \frac{a+2}{a(v+1)}\).*

For the Pareto distribution, a pure transfer from consumers to the winning sponsor \((k = 0)\) will increase the *ex ante* incentive to improve proposals. With small enough \(v\), even policies that re-distribute from losers to winners \((k = 1)\) will encourage innovation. But when \(v\) is large, policies that reduce \(L\) will discourage innovation by increasing delay.

**Proposition 7** *In the uninterrupted war of attrition, policies that reduce vested interest may raise or lower ex ante incentives to improve the distribution of \(q\).*

### 4.3 Private and social incentives to innovate

Finally, we compare private to social innovation incentives in the uninterrupted war of attrition. Conditional on player 1’s type \(q\), the total expected surplus in a war of attrition is

\[
s(q) = (W + L) \left\{ q \int_{q_{\min}}^{q} \delta(y) dF(y) + \delta(q) \int_{q}^{\infty} y dF(y) \right\}
\]

By (9), the change in this from slightly improving \(q\) is:

\[
s'(q) = (W + L) \left\{ \frac{u'(q)}{W} - vq\delta(q) f(q) \right\}
\]

\[25\text{The case } k > 1 \text{ corresponds to where stronger patent rights reduce total industry rents, as may occur if the patent thicket problem (Shapiro, 2001) is severe. Appendix A comments briefly on the issue of patents in standards.} \]
As (10) suggests, under a war of attrition interim social returns to improving a low-quality proposal can be negative: improving an eventually losing proposal merely delays its concession. By contrast, as (9) shows, interim private incentives to improve quality go to zero but remain positive as $q \to q_{\text{min}}$. Improving an already high-quality proposal, on the other hand, confers a positive externality: the other player, already likely to lose, loses to a better proposal. Thus interim private incentives are smaller than is socially desirable for high-quality proposals. Taking expectations over $q$, we can ask which of these effects dominates \textit{ex ante}.

\textbf{Example 3} If $F(\cdot)$ has a Pareto distribution, then $E[s'(q)] < E[u'(q)]$ if and only if $v(v+2)(a+1) > 1$: private incentives to improve quality are too high if there is strong vested interest.

Thus we have:

\textbf{Proposition 8} Given $v$ and the uninterrupted war-of-attrition structure, \textit{ex ante} private incentives to improve the distribution of $q$ may be stronger or weaker than is socially optimal. Interim incentives are too strong for improving low-quality proposals and too weak for improving high-quality proposals.

Conditional on the distribution of quality, increasing $v$ does not improve screening but increases delay. This suggests that SSOs should set $v$ below the level that would be optimal if delays were not an issue. (Starting from that point, a small decrease in $v$ would reduce delays without (to first order) changing \textit{ex ante} incentives.) Moreover, if $E[u'(q)] > E[s'(q)]$, a small reduction in $v$ that reduces incentives $E[u'(q)]$ would bring them closer to the social return from quality improvement, as well as reducing delays.

5 Conclusions

Motivated by the formal standards process, we study bargaining between privately informed parties with competing proposals whose adoption requires consensus. While
the economics literature on bargaining stresses side payments and substantive compromise as routes to agreement, those paths are not always used. In some institutions, including many standards fora, participants hold out for their preferred solution until one side concedes. We proposed a model in which to evaluate the resulting delay, relative to random choice.

In our simple model, the war of attrition discriminates well, but is slow. With strong vested interest, it can be more efficient to stress speed even at the expense of screening for quality — perhaps relaxing the concept of consensus and encouraging neutral participants to break deadlocks. The model also suggests that when vested interest is strong, policies that limit vested interest may even increase the incentive to improve proposals.

We addressed these issues using a simple model of what SSOs actually do. Future work might also pursue a complementary methodology, along the lines of mechanism design, that would lay out the informational and incentive constraints in more detail and characterize optimal incentive-compatible decision-rules.
References


Proofs and Examples

Proof of Lemma 1

When qualities are independently distributed, every rationalizable Bayesian strategy is weakly increasing.

Proof: Consider two possible types of player 1, $q_L < q_H$. Suppose that $q_L$ puts positive probability weight on conceding at time $t_L$, and $q_H$ puts positive weight on conceding at time $t_H$. Let $\Gamma$ be player 1’s (perceived) distribution function of $t_2$, the time when player 2 concedes (if player 1 does not previously do so). Let $E[q|t] = E[q_2|t_2 = t]$ be player 1’s expected value of player 2’s quality $q_2$ given that player 2 concedes at $t$. (This conditional expectation does not depend on player 1’s own type by assumption of independence.) We now state player 1’s incentive compatibility constraints.

Since type $q_L$ is willing to concede at $t_L$, we have:

\[
W_{q_L} \int_0^{t_L} e^{-rt} d\Gamma(t) + Le^{-rt_L} \int_{t_L}^{\infty} E[q|t] d\Gamma(t) \geq W_{q_L} \int_0^{t_H} e^{-rt} d\Gamma(t) + Le^{-rt_H} \int_{t_H}^{\infty} E[q|t] d\Gamma(t).
\]

The same argument leads to an incentive compatibility constraint for type $q_H$ (replace $q_L$ with $q_H$ and reverse the inequality in the previous expression). Adding these two inequalities yields:

\[
\int_0^{t_L} e^{-rt} d\Gamma(t) \leq \int_0^{t_H} e^{-rt} d\Gamma(t)
\]

Thus, if $t_L > t_H$, there is zero concession probability for player 2 between $t_H$ and $t_L$. But then no type of player 1 should concede at $t_L$ (it would be better to concede at or just after $t_H$), contradicting our assumption that $q_L$ can optimally concede at that time. Thus, we conclude that $t_L \leq t_H$.□

Proof of Proposition 1

When qualities are independently and continuously distributed, the symmetric equilibrium of an uninterrupted war of attrition selects the better proposal.

Proof: Suppose there is an atom in the distribution of concession times at $T$, and let $\bar{q}$ ($\underline{q}$) be the supremum (infimum) of the set of types who can optimally concede at that time. Since, for a given strategy, a player’s expected payoff is continuous in its type, type $\bar{q}$ can also optimally concede at $T$.

If $\bar{q} < \infty$, then by waiting until just after $T$, type $\bar{q}$ would (strictly) increase its own probability of winning without reducing the expected quality of the system that emerges. This contradicts the
statement that type $\bar{q}$ can optimally concede at $T$. If $\bar{q} = \infty$, all types $q \in (q, \infty)$ concede at $T$ (by Lemma 1) and receive an expected payoff $\frac{1}{2}(Wq + LG(q))$. This is also a contradiction, since types $q > \frac{LG(q)}{W}$ could do strictly better by waiting an instant to get $Wq$ for sure.

Now suppose there is a gap in the distribution of concession times between $t_L$ and $t_H$. This also leads to a contradiction: the lowest type that can optimally concede at $t_H$ would do better to concede just after $t_L$. Thus, if quality is continuously distributed, the symmetric equilibrium strategies in an uninterrupted war of attrition must be one-to-one. The better system wins because concession times are strictly increasing in $q$. □

Proof of Lemma 2

Under FCS, if either $f(q)$ or $1 - F(q)$ is log-concave, there is a unique time $t^* \geq 0$ such that a neutral player prefers screening via the war of attrition at all $t < t^*$ and immediate random choice thereafter.

Proof: There are several cases to consider. First, if $G(q_{\text{min}}) = \mu < (1 + v)q_{\text{min}}$ then (6) implies that a neutral player prefers immediate random choice at $t = 0$. Since log-concavity implies that $G(q) - (1 + v)q$ is monotone decreasing, the neutral player will also prefer a random intervention at all $t > 0$; so $t^* = 0$.

If $\mu > (1 + v)q_{\text{min}}$ then screening is initially worthwhile. Suppose the support of $F(\cdot)$ is $[q_{\text{min}}, \infty)$. Log concavity implies that $G(q) - q$ is bounded above by $\mu - q_{\text{min}}$. Since $vq$ grows without bound as $q \to \infty$, continuity and the intermediate value theorem imply the existence of a unique $q^*$ such that $G(q^*) - q^* = vq^*$. For all $t > t(q^*)$ the neutral player prefers random intervention.

If the support of $F(\cdot)$ is $[q_{\text{min}}, \overline{q}]$, then $G(q)$ must converge to $\overline{q}$ as $q$ approaches the upper bound. Since $v\overline{q} > 0$, continuity and the intermediate value theorem imply a unique $q^* < \overline{q}$. □

Proof of Proposition 3

The following strategies are a perfect Bayesian equilibrium: Sponsor-types $q < q^*$ use the concession strategy derived in Section 2.1. Types $q > q^*$ never concede. The neutral player waits until $t^*$ before making a random choice (and would also intervene at all times $t > t^*$).

Proof: To begin, note that Lemma 3 implies $Wq^* = LG(q^*)$, so the last type to concede before a random intervention must be indifferent between immediate victory and concession. We need to show that two kinds of deviations are unrewarding for the players. First, in the candidate equilibrium, a sponsor with type $q > q^*$ will wait until its rival concedes or the random choice is made; might
this player deviate by conceding early, before $t^*$? Second, a sponsor with type $q < q^*$ is meant to concede before the random choice at $t^*$; might it deviate by waiting till $t^*$ and having a good chance of winning?

First, by Lemma 3, at $t^*$ a sponsor with $q > q^*$ prefers random choice to concession. Since the type $q^*$ sponsor at least weakly prefers concession at $t^*$ to concession at any $t < t^*$, Lemma 3 implies that the same is true of any higher type. Thus, types $q > q^*$ do not have an incentive to deviate from equilibrium play. We must also check that they would not deviate from the prescribed out-of-equilibrium play. That is, if at $t > t^*$ there has still been no random choice, but they expect one imminently, would a high-type try to concede? No: by Lemma 3, they would prefer an immediate random choice.

Second, a player with $q < q^*$ could, by waiting till $t^*$, obtain a fifty percent chance of winning. Couldn’t that be a tempting deviation? No—by Lemma 3, for a low quality sponsor, concession just before $t^*$ dominates random choice at $t \geq t^*$: the expected benefits of choosing a rival system exceed those of being the winner.

Finally, consider the neutral player’s random intervention at $t^*$. By the definition of $t^*$, this player prefers to allow screening at all $t < t^*$ but wants to intervene then. Since the out-of-equilibrium expectations are that no vendors will concede at any $t > t^*$, if no actual concession has happened by such a time, the same calculation says that the neutral player wants to intervene now. Of course, if a concession has happened, there is no incentive question to resolve. □

**Proof of Lemma 4**

*If proponents expect a random intervention at $T < \infty$, their symmetric equilibrium concession strategies will have thresholds $\underline{q} \leq \overline{q}$, such that all types $q < \underline{q}$ play the strategies $t(q)$ derived in Section 2.1; all types $q \in (\underline{q}, \overline{q})$ concede at $T$; and all types $q > \overline{q}$ wait for the random intervention.*

**Proof:** Suppose $\underline{q}$ is the supremum of the set of types that concede strictly before $T$. Types below $\underline{q}$ will also concede before $T$, Lemma 1. Since random intervention at $T$ has no impact on the marginal costs or benefits of delay for types below $\underline{q}$, they will concede according to first-order conditions, which imply the strategies in equation (4).

To see that $\overline{q}$ is finite, note that $t(q) \to \infty$ as $q$ approaches $\infty$ (or any finite upper bound). Moreover, since $F(\cdot)$ and the payoffs are continuous in $q$, type $\overline{q}$ will be indifferent between conceding

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26This would break down if the sponsors believe that types $q \in (q^*, Q)$ would have already conceded by now (after $t^*$), so that concession looks better (relative to random choice) because the opponent’s expected quality is now $G(Q) > G(q^*)$. But this is not an issue, since we are specifying that each believes that the other is playing “never concede after $t^*$,” and there can be no evidence to falsify that belief: the only available evidence would be concession, after which the question doesn’t arise.
at $t(q)$ and $T$.

The remaining types $q > q$ have two options; they can concede at $T$ — and win with probability $\frac{1}{2}$ if the other player also concedes at that time — or “wait” (an instant) for the random intervention. Lemma 1 says that if $\overline{q}$ is the supremum in the set of types that can optimally concede at $T$, then any proposals between $q$ and $\overline{q}$ must also concede at that time. Types greater than $\overline{q}$ wait for the random choice by construction. □

**Proof of Proposition 4**

*For log-concave quality distributions, if proponents anticipate $T < t^*$, then $q < \overline{q} < q^*$; screening does not reach $q^*$, and there is an atom in the distribution of concession times at $T$. If proponents anticipate $T > t^*$, then $q^* < q = \overline{q}$ and $t(q) < T$; there is a gap in concession times after $t^*$, but no atom at $T$.***

**Proof:** Suppose $T < t^*$. Lemma 4 says that types up to $\underline{q}$ concede according to (3), so we have $q \leq \beta(T) < q^*$. Lemma 3 says that types just above $\underline{q}$ will prefer concession at $t(q)$ to an immediate (or delayed) random choice. Since types just above $\underline{q}$ cannot concede before $T$ (by Lemma 4), but prefer concession to random choice, they must be part of an atom of concessions at $T$. And since types just above $q^*$ strictly prefer random choice to concession (once all lower types have conceded) we must have $\overline{q} < q^*$, or the highest types in the atom would deviate by waiting for the random choice.

Now suppose $T > t^*$ and there is an atom of concession at $T$. We just established that the highest type in any atom of concessions is less than $q^*$; so we must have $\underline{q} < \overline{q} < q^*$, which implies a delay between the final concession and $T$. In this (hypothetical) equilibrium, type $\underline{q}$ must be indifferent between conceding at $t(q)$ and $T$. Concession at $t(q)$ yields the same expected payoff as in the uninterrupted war of attrition: $LG(q)$. Concession at $T > t(q)$ imposes a delay cost, which is offset by the fact that type $\underline{q}$ will defeat rivals in the interval $(\underline{q}, \overline{q})$ with probability $\frac{1}{2}$. However, in the uninterrupted war of attrition, type $\underline{q}$ could deviate to the concession time $t(\overline{q}) < T$ and pay a strictly smaller delay cost to defeat the same rival-types with certainty. Since this deviation does not lead to a higher expected payoff in the uninterrupted game, concession at $t(q)$ must also be a dominant strategy for type $\underline{q}$ when there is random intervention at $T$, which contradicts our hypothetical equilibrium and the assumed atom of concessions.

Since there is no atom when $T > t^*$, we must have $\underline{q} = \overline{q} > q^*$. If not, there would be some types just above $\underline{q}$ that prefer concession to random choice. Finally, since $\underline{q} > q^*$, there must be a gap in concession times between $t(q)$ and $T$; so that $\underline{q}$ is indifferent between immediate concession and
Proof of Proposition 5

For log-concave quality distributions, random intervention at $T = t^*$ is the unique perfect Bayesian equilibrium.

Proof: Proposition 3 establishes that intervention at $T = t^*$ is a perfect Bayesian equilibrium. Proposition 4 shows that when $T > t^*$, there is a gap (but no atom) in the distribution of concession times. Thus, screening up to $q$ is no faster, and after $q$ is strictly slower, than under FCS. Since the marginal benefits of screening are negative after $t^*$ under FCS, intervention at $T$ will be strictly dominated by intervention at $t^*$ in the (hypothetical) equilibrium where proponents anticipate an intervention at $T > t^*$.

When proponents anticipate $T < t^*$, Proposition 4 says there will be an interval of types $(q, q^*)$ that do not concede by $T$. Since these types are below $q^*$, they would screen efficiently in a symmetric equilibrium where $q$ was the lowest remaining type. This implies that a neutral player would prefer to renege on the random intervention at $T$. (While this temptation would not exist if the remaining sponsors would screen very slowly, we can apply Lemma 4 to the sub-game starting at $T$ to show that for any off-equilibrium beliefs about the timing of a subsequent random intervention, the lowest remaining sponsor-types will concede at least as quickly as they would in an uninterrupted war of attrition.) Since it is impossible to sustain the sponsors’ belief in random intervention at $T < t^*$, the only perfect Bayesian equilibrium is random intervention at $T = t^*$. □

Proof of Proposition 6

A predetermined standard setter has the greatest incentive to innovate, followed by firms facing an immediate random choice. Firms that anticipate a random intervention at some finite $t^* > 0$ have weaker incentives than under immediate random choice, but stronger than those under an uninterrupted war of attrition: $I_{PS} = W > I_{RC} = \frac{1}{2}W \geq I_{RI} \geq I_{WOA}$

Proof: Since $I_{WOA} = E[u'(q)]$, equation (9) implies that

$$I_{WOA} = W \int_{q_{min}}^{\infty} \int_{q_{min}}^{x} \delta(y) dF(y) dF(x) \leq W \int_{q_{min}}^{\infty} \int_{q_{min}}^{x} dF(y) dF(x) = \frac{W}{2} = I_{RC}$$

and the inequality is strict for $v > 0$, since $\delta(y) < 1$ for all $y > q_{min}$ (by Proposition 1).
Random intervention is equivalent to an immediate random choice as of $t^*$, and the incentive comparison is no different since both proposals must be better than $q^*$ for random intervention to make any difference in outcomes. Formally, we have

$$ I^{RI} - I^{WOA} = \int_{q^*}^{\infty} \left( \frac{1}{2} W \delta(q^*) - W \int_{q^*}^{x} \delta(y) dF(y) \right) dF(x) $$

Since $\delta(y) < \delta(q^*)$ for all $y > q^*$, we can factor out $\delta(q^*)$ and integrate the previous expression to show that $I^{RI} - I^{WOA} > \frac{1}{2} W \delta(q^*) [1 - F(q^*)] > 0$.

Calculations for Examples 1-3

Example 1

All of the examples are based on the Pareto distribution: $F(x) = 1 - x^{-(1+a)}$ for $x \geq 1$ and $a > 0$.

Example 1 uses the following calculation:

$$ G(x) = \frac{\int_{x}^{\infty} x f(s) ds}{1 - F(x)} = -\frac{(1 + a) \int_{x}^{\infty} s^{-(1+a)} ds}{x^{-(1+a)}} = -\frac{(1 + a) s^{-a}}{ax^{-(1+a)}} = \frac{(a+1)x}{a} $$

Example 2

To find $E[u'(q)]$, start with $u'(q)$ from equation (9). From the calculations for Example 1, we know $\mu = \frac{(a+1)}{a}$ and $K(x) = \frac{(a+1)x^{-a}}{a}$. Thus, we have

$$ u'(q) = W \int_{q}^{\infty} \left[ \frac{K(s)}{\mu} \right] v dF(s) = W(1 + a) \int_{1}^{q} s^{-a(v+1)-2} ds = \frac{W(1 + a) [1 - q^{-a(v+1)-1}]}{a(v+1) + 1} $$

and taking expectations yields

$$ E[u'(q)] = \frac{W(1 + a)}{a(v+1) + 1} \int_{1}^{\infty} [1 - s^{-a(v+1)-1}] (1 + a)s^{-a-2} ds = \frac{LW(a+1)}{Wa + L(a+2)} $$

where the final equality is found by substituting $v = \frac{W-L}{L}$ and simplifying. In Example 2 we differentiate this expression with respect to $w$ where (by assumption) $\frac{dW}{dw} = 1$ and $\frac{dL}{dw} = -k$, and ask whether the result is greater than zero. This yields

$$ \frac{dI^{WOA}}{dw} = \frac{(a+1)[L^2(a+2) - kaW]}{(Wa + L(a+2))^2} $$

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and the numerator is positive if and only if $k < \frac{L^2(a+2)}{W^2a} = \frac{a+2}{a(v+1)^2}$

**Example 3**

Integrating equation (10) yields $E[s'(q)]$ as a function of $E[u'(q)]$. Thus, using $E[u'(q)]$ from Example 2, we have

$$E[u'(q)] < E[s'(q)] \iff \frac{L}{W} E[u'(q)] > (W + L) E[qf(q)v\delta(q)] \iff \frac{L(a + 1)}{a(v + 2) + 2} > (W + L) E[qf(q)v\delta(q)]$$

Further calculations show that

$$E[qf(q)v\delta(q)] = v(1 + a)^2 \int_1^\infty s^{-a(2+v)} ds = \frac{v(1 + a)^2}{a(v + 2) + 2}$$

and substituting this expression into the previous inequality, yields the result stated in the text.

$$E[u'(q)] < E[s'(q)] \iff \frac{L(a + 1)}{a(v + 2) + 2} > \frac{(W + L)v(1 + a)^2}{a(v + 2) + 2} \iff 1 > v(v + 2)(a + 1)$$
Appendix A: Additional Discussion

A-1 Non-participants’ Interests

Observers of formal standardization often fear that the interests of users, who typically do not themselves participate in the standards process, may be poorly served. In our notation, the participants’ payoffs in aggregate are equal to \((W + L)e^{-rt}\); nonparticipants’ might be represented as 
\((1 - W - L)f(q)e^{-rt}\), where the function \(f(\cdot)\) might be nonlinear. Thus the two groups’ incentives differ in several possible ways:

A-1.1 Direct costs

Our model assumed that delay leads to opportunity costs, i.e. not getting the benefits of a standard. In fact, participants bear the direct costs of the process and nonparticipants do not, potentially creating several disagreements. First, there may be a public-good or free-rider problem: if participants’ rents are too small, nobody may shoulder the costs of participating. We assume that self-selection leads to participation by the those with the strongest vested interests. Second, active participants will be biased against standardizing in advance, which increases direct costs because they are borne earlier and are often borne when delay might reveal that there will be no market. Third, if working faster has a greater flow cost, the model suggests that participants should be roughly indifferent to this (since it is screening that determines time to agreement), while nonparticipants will urge more speed.

A-1.2 Rent shifting

Participants want both \(W\) and \(L\) to be large, although they also care about the balance between them; nonparticipants (at least once the systems are developed) prefer both \(W\) and \(L\) to be small, as well as wanting \(v\) to be small (i.e. \(W \approx L\)).

How might participants design or influence the process so as to increase their joint rents \(W + L\)? They might set substantial license fees for the standard or for technology embodied in it. They might favor a technology that (given its quality) has a demand structure that lets an imperfectly competitive industry extract a relatively large fraction of the social surplus. They might use the standards process to exclude rent-destroying new technology. Many such actions, if jointly undertaken in order to increase joint rents, would presumably be antitrust violations. Since the standards community is generally apprehensive about antitrust, they will have some motive to steer clear of blatantly anti-competitive acts. But in practice, the distinction between technical necessity and rent-shifting...
can be subtle.\textsuperscript{27}

\textbf{A-1.3 Value of additional quality}

If $f(\cdot)$ does not take the form $f(q) \equiv f q$ (i.e. if it is not linear through the origin) or if participants and nonparticipants have different discount rates, then the two groups face different tradeoffs between quality and delays. For instance, if $f(\cdot)$ is not very steep, nonparticipants want some standard but do not care very much about its quality relative to participants. This is an incidence question: where do the incremental rents from higher quality accrue, and does this differ according to the level of quality? The answer will depend on the details of user tastes and of competition among vendors.

Closely allied to this question is the tradeoff between speed and completeness of a standard. If an incomplete standard (or “model”) can be adopted relatively easily and therefore quickly, is it worth the extra delay in order to make the standard more complete, and thus either ensure full inter-operability or at least reduce the cost and increase the quality of converters? Again, this is an incidence question: who bears the costs of converters? For instance, if the costs of converters fall primarily on users while the costs of delays are shared between vendors and users, we would suspect that the vendors are inclined to set the rules in such a way as to over-use converters and under-use standardization, relative to the overall efficient solution. The side that has higher \textit{proportional} losses is less ready to accept speedy incomplete standardization.

\textbf{A-2 Policies to Reduce Delay}

Broadly speaking, policies that reduce $v$, or change the structure of the game (e.g. by introducing side-payments or substantive compromise) ought to speed up the process. Reforms that leave vested interest and the underlying war-of-attrition structure untouched are unlikely to have an impact.

\textbf{A-2.1 Anticipatory standards}

Vested interests are growing all the time as installed bases grow or proprietary knowledge develops. As when one sets off on a commute just before rush-hour, every delay in starting means a bigger delay in finishing. Thus, some observers urge “anticipatory standardization” in advance of the market, before vested interests grow strong. Standardizing in advance seems likely to reduce $v$, and hence

\textsuperscript{27}For example, cellular handsets use standardized “system operator codes” to determine whether they are eligible to utilize a particular carrier network. By “locking” these codes, the wireless carriers — who helped write the standards — could raise customers’ switching costs by effectively turning off their existing phone when they change networks. (Such an increase in switching costs may well, though it does not necessarily, increase prices and profits of the wireless carriers: see Farrell and Klemperer (2007)). A November 2006 ruling by the U.S. Library of Congress’ Copyright Office made it legal for consumers to break these software locks and use their phones on a competing carrier’s network.
delays, but to reduce both the opportunities for product development and the reliability of screening for quality. (One well-known example of an anticipatory standard that failed to anticipate future demand is the physical-address space in IPv4 — i.e. a computer’s 32-bit Internet address.) It could thus be viewed as a move towards random choice.

Some practitioners suggest that parallel efforts can limit the cost of technological short-sightedness and preserve the benefits of starting before vested interest sets in. For example, Updegrove describes the development of multiple wireless standards:28

“The difficulty of predicting the future when setting anticipatory standards is clearly demonstrated by the example of early wireless data standards development. HomeRF disappeared, Bluetooth found a home in one set of short-range applications, and Wi-Fi predominated for longer distances...over time, it became clear which standard was optimally useful within areas of overlap.”

When multiple efforts do not overlap, the parallel approach may preserve technological flexibility — though it is likely to create a future demand for converters. However, when rival firms commit themselves to a particular specification, parallel development efforts can increase vested interests in the final push for a common standard. For example, the IEEE’s 802.15.3a committee disbanded after three years of work when rival factions, each with its own consortium, could not reach a compromise on protocols for ultra-wideband wireless networking.29

A-2.2 Incomplete standards

In practice, formal standards do not always ensure compatibility: not every two “conforming” products are inter-operable. In particular, many standards include incompatible or “vendor-specific” options; partly in response to the uncertainty about feasibility, cost, and demand that results from standardizing early, but also as a way to reduce vested interest. For instance, Sirbu and Zwimpfer (1985) describe how incompatible options were included in the X.25 packet switching standard in order to placate intractable vested interest (more than one system already had an installed base). Similarly, Kolodziej relates that an impasse in negotiating PC modem standards (V.42) was overcome by deciding “to put both protocols into the standard.”30

The result is a “model” — an incompletely-specified standard, or menu of choices — meant to ensure a baseline level of inter-operability. When firms make different choices from the menu, their products may not be fully compatible. Nevertheless, a model can be of great help in achieving

compatibility. The market, or other organizations, may find it easier to choose a profile within
a model than develop a standard from scratch: not all possible (or even proposed) options need
be included, and many uncontroversial issues may be standardized. Moreover, it is often easier,
cheaper, and more effective to patch together compatibility through converters within a model than
it would be if competing technologies did not respect a model. For example, while every Internet
device shares a common networking protocol (TCP/IP), the Internet’s various physical networks
use different transport protocols and communicate with each other through a series of converters
(gateway protocols) developed and maintained by the IETF.

A-2.3 Intellectual property rules

Many SSOs adopt intellectual property (IP) policies, whereby participants promise to disclose rele-
vant patents and license on “reasonable” terms any that are essential to comply with a standard.31
These rules can reduce the risk of hold-up, in which a firm fails to disclose its IP until others have
substantially committed to the standard, weakening them in license negotiations.32 Here, we stress
that IP rules also tend to lower \( v \), and thus lead to faster screening.

Disclosure rules will lower \( v \) if, in the absence of disclosure, participants will believe that rivals are
likely to hold patents (or patent applications) that are essential to implement their own proposals.
Licensing rules limit participants’ use of their IP once it is embedded in an industry standard,
typically requiring licensing on “reasonable and non-discriminatory” (RAND) terms.33 To the extent
that RAND rules commit SSO participants to liberal licensing (there is some ambiguity about the
legal definition of a “reasonable” price) these policies will lower \( W \), and likely raise \( L \), thus reducing \( v \).

Recently, a few SSOs (notably VITA and IEEE) have started to encourage \textit{ex ante} disclosure of
maximum allowable royalty rates, which already takes place in some cases (see for instance the FTC’s
complaint in \textit{NDData}). In our model, these commitments provide a negotiating tool that resembles
side-payments.

31Lemley (2002) and Chiao et al. (2007) survey SSOs’ IP policies; Teece and Sherry (2003) and Farrell et al. (2007)
discuss these policies in antitrust terms; see also American Bar Association (2003), or U.S. Department of Justice and
32The FTC has taken action against several firms for various versions of hold-up. In \textit{Dell Computer} (FTC No.
931-0097), Dell agreed to grant royalty-free licenses. In \textit{Rambus} (FTC Docket No. 9302), the commission ruled that
Rambus had fraudulently exploited the formal standards process, and placed royalty caps on relevant patents; the
Court of Appeals remanded this decision.
33ANSI requires sponsors to offer patent licenses either “without compensation” or “under reasonable terms and
conditions that are demonstrably free of any unfair discrimination.” (ANSI, 2006, Appendix I). Some SSOs require
Other SSOs require that licenses also be “fair”, creating the alternative acronym FRAND.
A-2.4 Meeting more often

Committees charged with developing a consensus standard typically meet only periodically. A natural initiative toward reducing delays is meeting more often. Presumably, this provides more time to work out technical issues; and, once consensus emerges, more frequent plenary or official meetings to finish the process can help. But to the extent that, as in the model, the work is largely bargaining, the time to agreement is determined by screening constraints, and meeting more often is unlikely to reduce delays (unless it noticeably increases flow costs of delay to the participants). Indeed, our model assumes perpetual meetings; but the delays persist.

As this reasoning suggests, SDOs may have had more success in accelerating post-consensus administrative processing than in speeding consensus. In its 1994 Annual Report (page 4), the IEC noted that, “Time for the fundamental part of standards production — preparatory and technical development stages...has remained substantially the same, while time for the latter stages of approval and publication...has been brought down by 60 percent.”

Appendix B: Generalizing the War of Attrition

B-1 Correlated proposals

If proposals are correlated, Lemma 1 does not hold, and the symmetric equilibrium need not screen for quality. For example, perfect correlation leads to a complete-information game, whose symmetric equilibrium involves mixed strategies, and thus delay, but no screening. In general, with positive correlation, increasing privately observed \( q \) increases the benefits of winning (encouraging players to hold out longer), and also the expected quality of a rival (encouraging players to concede earlier). The following proposition provides a sufficient condition for the first effect to dominate, so there is still perfect screening. The proof is similar to Krishna and Morgan (1997), who analyze a second-price all-pay auction with affiliated private values.

**Proposition B-1** If proposal qualities are drawn from a symmetric and continuous probability distribution \( f(\cdot, \cdot) \) and the function \( \phi(x, y) = \frac{xf(y|x)}{K(y|x)} \) is strictly increasing in \( x \), then there is a unique symmetric Bayesian equilibrium with increasing strategies: \( t(q) = \frac{
}{
} \int_{q_{min}}^{q} \phi(s, s) \, ds \)

**Proof:** This proof has two steps: (1) Use first-order conditions to find the symmetric equilibrium strategies, and (2) show that the payoff function is single-peaked, so that the necessary first-order conditions are also sufficient.
**Step 1**: Suppose player 1 chooses the strictly increasing concession strategy \( t(\cdot) \), with inverse \( \beta(\cdot) \), and consider the stopping problem faced by player 2 (with type \( q \)). If the game has reached time \( t \), player 2’s expectation of a rival’s quality is

\[
G(\beta(t)|q) \equiv \mathbb{E}[y|y > \beta(t), q] = \frac{1}{1 - F(x|q)} = \frac{\int_{-\infty}^{\infty} sf(s,q) \, ds}{\int_{-\infty}^{\infty} f(s,q) \, ds}
\]

Thus, player 2’s expected payoff from concession at \( T \) is

\[
\Pi(T; q) = Wq \int_{0}^{T} e^{-rT} f(\beta(s)|q) \beta'(s) \, ds + [1 - F(\beta(T)|q)] e^{-rT} L G(\beta(T)|q)
\]

and the first-order condition for this stopping problem is

\[
Wq f(\beta(T)|q) \beta'(T) - L \beta(T) f(\beta(T)|q) \beta'(T) - rL K(\beta(T)|q) = 0
\]

Since type \( q \) is meant to concede at \( T \), symmetry implies that \( \beta(T) = q \). Substitution yields

\[
(W - L)q f(q|q) \beta'(T) = rL K(q|q)
\]

Finally, since the lowest type \( q_{min} \) concedes immediately and \( \beta' \equiv \frac{d\beta}{dq} \) we have

\[
t(q) = \frac{v}{r} \int_{q_{min}}^{q} \frac{xf(x|x)}{K(x|x)} \, dx \tag{B-1}
\]

**Step 2**: We now verify that this strategy is optimal if one’s opponent plays \( t(q) \). Consider the payoff to a type \( q \) sponsor who deviates from (B-1) by choosing the concession time meant for type \( z \):

\[
\Pi(t(z); q) = Wq \int_{0}^{z} e^{-rt(s)} f(s|q) \, ds + L e^{-rt(z)} K(z|q) \tag{B-2}
\]

The second term in (B-2) can be written as an integral:

\[
L e^{-rt(z)} K(z|q) = L e^{-rt(q_{min})} K(q_{min}|q) + \int_{q_{min}}^{z} \left[ L e^{-rt(s)} K'(s|q) - rL e^{-rt(s)} K(s|q) t'(s) \right] ds
\]

We know that \( t(q_{min}) = 0 \), \( K(q_{min}|q) = G(q_{min}|q) \equiv E[q_1|q_2 = q] \) and (from equation (B-1)) \( t'(s) = \frac{v sf(s|s)}{rK(s|s)} \). Thus, substitution yields
\[ L e^{-rt(z)} K(z|q) = LG(q_{min}|q) - L \int_{q_{min}}^{z} e^{-rt(s)} \left[ sf(s|q) + \frac{vsK(s|q)f(s|s)}{K(s|s)} \right] ds \]

If we plug this back into (B-2) and rearrange terms, we arrive at the following expression

\[ \Pi(t(z); q) = LG(q_{min}|q) + \int_{q_{min}}^{z} e^{-rt(s)} K(s|q) \left[ \left( \frac{Wq - Ls}{q} \right) qf(s|q) \frac{K(s|q)}{K(s|s)} - (W - L) \frac{s f(s|s)}{K(s|s)} \right] ds \]

If \( \phi(x, y) = \frac{s f(y|x)}{K(y|x)} \) is increasing in \( x \), then the term in square brackets (and therefore the entire integrand) is positive for all \( s < q \) and negative for all \( s > q \). This implies that the optimal choice of \( z \) is \( q \), so there is no profitable deviation from the equilibrium strategies derived in Step 1. □

**B-2 Additional players**

A second way to generalize our model is by adding more players. Bulow and Klemperer (1999) study the N-player war of attrition and prove that when concession does not alter a participant’s flow costs, the equilibrium concession rate does not depend on the number of remaining players, i.e. participants choose an exit time and stick to it.\(^{34}\) In their model, strategies are not influenced by \( N \), but expected delays still increase with the size of a committee, since they depend on a second-order statistic. The following proposition shows that the same logic applies to our model, where flow costs are primarily the opportunity cost of delayed implementation.

Consider a symmetric standards committee with \( N \geq 3 \) proposals, where \( N - 1 \) concessions are needed to reach a consensus. Each player’s quality is private information, and is independently distributed with cumulative distribution \( F(\cdot) \). The winner’s payoff (as of the final concession) is \( Wq \), and all other players receive \( Lq \). We claim that it is a symmetric Bayesian Nash equilibrium for all \( N \) committee members to adopt the concession strategy derived in Section 2.1:

\[ t(q) = \frac{v}{r} \int_{q_{min}}^{q} \frac{s f(s)}{K(s)} ds = \frac{v}{r} [\log \mu - \log K(q)] \]  \hspace{1cm} (B-3)

**Proposition B-2** For a symmetric N-player standards committee, it is a Bayesian Nash equilibrium for all players to adopt the concession strategy (B-3): the exit rate is identical to a two-member

\(^{34}\)In Bulow and Klemperer (1999), payoffs are linearly separable from participation costs, which is a natural assumption when costs are “direct” (and in technical terms, facilitates a comparison to the all-pay auction). Our model’s multiplicative structure is motivated by the importance of opportunity costs in formal standardization. A second difference is our inclusion of the “consolation prize” \( Lq \).
Proof: We show that if all $N-1$ other players concede according to (B-3), any possible deviation for the remaining player (with proposal-quality $q_N$) is weakly dominated by (B-3), which is sufficient to establish this strategy as a symmetric Nash equilibrium.

Since (B-3) is strictly increasing in $q$, the lowest type of player $1\ldots N-1$ remaining at time $t$ is $\beta(t)$. Now consider any sub-game, starting at time $t$, in which only two players remain. Since this sub-game is identical to the war of attrition analyzed in Section 2, there is a unique symmetric equilibrium. Moreover, if $q_N < \beta(t)$ then player $N$ must drop out immediately; otherwise, they must play according to (B-3) with $q_{\min} = \beta(t)$.

Now suppose that three or more players remain, and player $N$ deviates to some concession time $\hat{t} > t(q_N)$ (i.e. they stay in for “too long”). There are two possible outcomes. If at least two other players remain at $\hat{t}$, this deviation makes no difference in outcomes. However, if at any time $t \in (t(q_N), \hat{t}]$ all but one of the other players have dropped out, then player $N$ must concede immediately (since this is the two-player sub-game considered above). Thus, deviation to $\hat{t}$ cannot lead to any improvement in $q_N$’s the expected payoff.

If type $q_N$ deviates to an “early” concession time $\tilde{t} < t(q_N)$, there are also two possibilities. If at least two other players remain at $t(q_N)$, this deviation is irrelevant. However, if the final concession occurs at some time $t \in (\tilde{t}, t(q_N)]$, then deviating made player $N$ worse off (in expectation), since they would choose to wait for some strictly positive length of time in the unique equilibrium of the two-player sub-game starting at $t$. Thus, there is no deviation that leads to a strictly higher payoff. □

Combining the two generalizations leads to an $N$-player model with correlated proposals. Participants will update their beliefs with each concession. When $q$ is an affiliated (positively correlated) random variable, beliefs about the quality of remaining proposals grow more pessimistic with each exit, so the concession rate declines over time.