Ideologues Beat Idealists

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Dec 2011

Our model considers a majority election between two candidates – an ideologue committed to a fixed policy and an idealist who implements the ex-post choice of the majority. Voters are aware that their individual rankings of policies may change after the election according to common or idiosyncratic shocks. We show that in equilibrium the ideologue often beats the idealist, even when this choice hurts all voters. Inefficiency arises both for sincere and for strategic voters; we also show that it is more pervasive in the latter case. Groups may be inflexible even when each individual has a preference for flexibility. (JEL C72, D72)

Voters are often aware that their ranking of policies might change after an election. The following question then arises naturally: When will they cast their ballots for an ideologue who always chooses a fixed policy rather than for an idealist who chooses a policy in line with public opinion after the election? One of our points of departure from the usual model is that candidates and policies are not synonymous. We demonstrate how the standard model of voting can, in this setting, generate counterintuitive conclusions.

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For ease of exposition consider a concrete scenario. An election is in the offing, and the result hinges on one issue: How best to respond to an adversary that will threaten national security if it has weapons of mass destruction (henceforth WMDs). A simple majority election is held between two candidates $B$ and $U$, whose electoral platforms will be described shortly. When they go to the polls, voters are divided on how to respond to the potential WMDs. Some support a direct confrontation (policy $0$), while the less hawkish voters prefer a diplomatic response (policy $1$). Initially more people are hawkish. Henceforth the “type” of a voter at any date is defined to be his preferred policy ($0$ or $1$) at that date. However voters are aware that their types may change after the election in response to shocks described below. The game ends with the elected candidate picking one of the two policies $0$ and $1$. A voter gets a unit of utility if and only if the chosen policy coincides with his new type.

We now describe the shocks. With some probability conclusive scientific evidence will come to the fore, either for or against the said opponent being a threat, and cause all voters to agree on what the right policy is; this we call a common shock. To illustrate, if it is discovered that the enemy was close to arming missiles even the pacifist prefer war; similarly, everybody prefers a diplomatic response if scientists find that the adversary was incapable of producing WMDs (say the country was technologically unable to refine uranium).

But it is quite likely that there won’t be such conclusive evidence; in this case voters experience either idiosyncratic ranking shocks or no shocks (in the latter case types are the same before and after the election). Idiosyncratic shocks, if any, may run in either direction: If the left-wing media runs editorials condemning the war, a type $0$ who reads it could change to type $1$. We model this as follows: With probability $\theta_1$ an idiosyncratic shock causes type-0 voters to become type 1 with a small probability $p$ independently of one another. Similarly an idiosyncratic shock towards 0 has conditional probability $\theta_0$ and makes each type 1 into a type 0 with probability $p$ independently.

Finally we come to the candidates. $B$, is the ideologue/biased candidate who implements policy 0 irrespective of the post-shock rankings; $U$, is the idealist/unbiased candidate who credibly promises to pick the policy eventually preferred by the majority. Two comment are in order. First, the word idealist can take on various shades of meanings, and in an extreme form may be confounded with someone who subscribes to an ideology; we use it to mean one given to ideals (raising social welfare, in this case)
even when it is not a realistic goal.\textsuperscript{1} Second, how does $U$ know the final rankings? One solution is for $U$ to call for a referendum once the final types are drawn; in the final period each voter would have a weakly dominant strategy to reveal his true type.

Who stands a better chance of winning the (simple) majority election? One might expect $U$ to be the natural choice, especially when there is a significant probability that policy 0 will be bad for \textit{all} voters. (Note that in the case of a common shock, $U$ always implements what everyone prefers.) We find, perhaps counterintuitively, that voters may prefer $B$, thereby committing to a policy rather than waiting to learn their final rankings.

The next two paragraphs explain the intuition for the rational voter.\textsuperscript{2} For clarity of exposition, we make two simplifications. First, assume shocks are towards 1 only, i.e. either there is a common shock making all voters type 1s, or idiosyncratic shocks change each type 0 voter into type 1 with a small probability $p$ independently of the other voters. Idiosyncratic shocks are more likely than a common shock. Second, rule out by assumption the event where there are no shocks.

Strikingly enough, it can be an equilibrium for type 1s to vote $U$ and type 0s to vote $B$, who is more likely to win. Consider a rational type 0. He is aware that his vote matters only when he is pivotal, i.e. the election is close. In this case, no matter how small $p$ is, idiosyncratic shocks are enough to swing the majority from type 0 to type 1. Each type-0 voter thinks that, conditional on idiosyncratic shocks, he himself won’t change (because $p$ is small) while sufficiently many other voters will flip and lead $U$ to choose policy 1 at the next date whereas our voter would still want policy 0. He therefore votes for $B$, who is committed to policy 0, to guard against other type-0 voters changing their views later. When we compare our work to previous work on commitment bias, this will emerge as one of our key insights: This inefficiency can hurt \textit{both} groups within the population, including the current majority. Even when a voter of type 0 is better off under $U$ he could vote for $B$ because $U$ \textit{is bad precisely in those cases where his vote matters}.

Our model admits of an alternative interpretation as the timing of a public decision. $B$ stands for an immediate commitment to the alternative that is currently leading (policy 0), whereas choosing $U$ amounts to deferring the decision until the final types are

\textsuperscript{1}The \textit{Oxford English Dictionary}, Second edition, 1989: “A person who creates, aspires to, or pursues ideals. Also depreciatively: one who entertains visionary or unpractical notions.”

\textsuperscript{2}The rational voters’ strategies constitute an equilibrium, whereas sincere voters ignore strategic considerations in a way that will be clear later. Proofs in Subsection A of Section III cover the case of the sincere voter.
drawn. Public referenda fit this formulation quite naturally, as the following incident illustrates. In October 1992, the Swedish Nuclear Fuel and Waste Management Company (SKB), an organization charged with the responsibility of safely disposing nuclear waste, proposed to conduct a study to determine the feasibility of locating a repository. One of the towns to be evaluated was Storuman, in northern Sweden. The findings of the SKB would not be binding on the city and if Storuman were deemed feasible it would still be up to the city council to decide, in keeping with public opinion and the interest of the city, whether to allow SKB to actually build a nuclear waste dump. A 1995 referendum asked “whether SKB should be allowed to continue the search for a final repository location in Storuman”. The outcome was an overwhelming ‘no’ (70.5%): Voters opted to reject it outright rather than allow more information to be disclosed by a non-binding scientific study.

Existing explanations of the inefficiency of groups, discussed in detail later, rely critically on a high probability that a substantial part of the current majority will change its mind. We believe many situations are better described by assuming that this probability is small. In the context of the above example, our model suggests that voters might have been driven by the fear that even if they themselves are not persuaded, others could change their minds.

After describing the related literature in Section I, we present the model in Section II; here Subsections A and B consider the sincere and the rational voter in turn; Subsection C establishes robustness of our equilibrium and Subsection D shows that our insights extend to small populations as well. Section III discusses further implications of the model.

I. Related Literature

Our work is related to several strands of literature. The first link is to a long literature documenting why groups choose inefficiently. The second link is to a long literature on electoral competition and pandering, including the recent work of Maskin and Tirole (2004) and Callander (2008). Finally this forms part of the extensive literature on pivotal voting and information aggregation.

Hao (2001) shows that conservatism may be a way to increase private incentives to gather evidence and improve the quality of the group decision. This is very different from our model, where information is available at no cost. Like us, Bueno de Mesquita and Friedenberg (2008) argues that ideologically committed candidates can often do
better than pragmatic ones, but the models and mechanisms are very different.

The status quo bias in reform is well documented – welfare-improving reforms are defeated by the status quo; see for example Samuelson and Zeckhauser (1988). Fernandez and Rodrik (1991) [henceforth FR] provides an explanation in the context of trade reforms, to our knowledge the first that does not appeal to risk-aversion. Their explanation is based on the identity of the winners being unknown at the time of voting. All voters of the majority group are ex-ante identical and hence maximize the expected value of the group, behaving in effect like the representative voter of the group.

In this paper the strategic interaction of voters generates inefficiency, whereas FR’s inefficiency is independent of strategic considerations. Indeed, both rational and sincere voting generate the same results in FR because each voter has a weakly dominant strategy to vote for the policy he prefers in expectation whether or not he is strategic.

This allows us to contrast the predictions for sincere and rational voters. We find that an electorate with strategic voters does no better than one with sincere voters, and indeed strictly worse in many situations. Strategies of others are not relevant for a voter in FR; effectively, each voter has a choice between two lotteries that are independent of others’ types and strategies. In contrast to FR, our rational voter takes others’ equilibrium strategies into account because candidate U’s policy depends on the electorate’s final rankings. Even when we are faced with the same shocks, it is possible that the electorate chooses B but any individual would choose U in isolation. Each voter is wary that others might change their minds. The second substantive distinction is that inefficiency survives in our model even when the “idiosyncratic” shock is unlikely to precipitate a large change, i.e. when $p$ is small. Indeed our contrasting predictions for sincere and rational voters is one of our distinctions from the contemporaneous and independent work of Strulovici (2010) on the inefficiency of group decision-making in infinite-horizon bandit problems.\footnote{We thank a referee for drawing our attention to this paper.} Strulovici is about experimentation while ours is about the advantage an ideologue might enjoy. Neither model can be mapped into the other: the “safe arm” of the bandit problem does not correspond to our $B$, because the utility from $B$ is also stochastic. In Strulovici the group’s experimentation is always sub-optimal, whereas we find conditions under which we have inefficiency. We also show that the inefficiency survives even when voters are allowed to mix.

Strategic voting was first introduced in Theory of Voting by Farquharson (1969). More recently this has been exploited in several papers – for example Austen-Smith and
Banks (1996), Feddersen and Pesendorfer (1997), and Meirowitz (2006) – to analyze information aggregation. Feddersen and Pesendorfer (1997) finds that in a large election a vanishingly small proportion of the electorate votes informatively,\(^4\) but information is almost perfectly aggregated and the efficient decision is taken. We show that the equilibrium may not be efficient, even when all agents vote informatively. In our model being pivotal is informative about the distribution of rankings at the next date, and therefore about \(U\)'s policy. It is worth pointing out that one can interpret our model as showing that efficiency results might not survive some natural variations.

Our work may also be linked to models of candidate location, notably the pioneering work of Downs and Hotelling. In our model a candidate with an extreme position beats an unbiased candidate. Hansson and Stuart (1984) have a very different explanation for non-convergence of policy platforms. While the valence aspect of voting has long been discussed informally in political science (Stokes (1963) is a standard reference), Callander (2008) proposed the idea that candidates wish to be perceived as ideological. In Kartik and McAfee (2007) candidates do not converge to the median to avoid being seen as pandering to voters. Details are contained later. Our model provides an endogenous explanation why commitment to an ideology may be an advantage.

II. The Model

We present below a simple model, chosen with analytical tractability in mind, that captures the features mentioned in the introduction. There are two dates \((\tau = 0, 1)\), an odd number\(^5\) of voters indexed \(1, 2, \ldots, 2n + 1\) \((n > 1)\), and a binary policy space \(A = \{0, 1\}\). Define voter \(i\)'s type \(t_\tau^i \in A\) at date \(\tau\) as the policy he ranks higher at date \(\tau\). This definition assumes for simplicity that there are no ties.

At date 0, nature draws each voter’s initial type \(t_0^i \in \{0, 1\}\) from a Bernoulli distribution with \(\Pr\{t_0^i = 0\} = q \in (0.5, 1)\); this corresponds to there being more hawks than doves at the initial date. Although a voter does not know the exact types of the others, he has a general sense of the dispersion in opinion: voter \(i\)'s type \(t_0^i\) is private information, but \(q\) is common knowledge. An election between two candidates is held.

\(^4\)To vote informatively is to condition the vote on one's signal. Although voting games have equilibria where all voters cast their ballots for the same candidate, it is standard practice to look at informative symmetric equilibria. We do the same here; later it is shown our equilibrium can be selected using various robustness criteria.

\(^5\)Our calculations would be different if we also considered an even number of voters, because then we would also have to deal with ties. However the intuition should go through.
at date 0 as well. After the election, each voter’s type changes to $t^1_i$ according to a stochastic process described shortly. Figure 1 below shows the temporal structure of the game.

![Timeline Diagram]

**Figure 1: Timeline**

The election is not over policies, but between two candidates – $U$ and $B$. Candidate $B$ is known to be an ideologue committed to policy 0. The idealist $U$ credibly promises to implement the ex-post social optimum policy if elected.\(^6\) If policy $a \in A$ is implemented at date 1, voter $i$’s utility is given by

$$u_i(a; t^1_i) = \begin{cases} 1 & \text{if } a = t^1_i \\ 0 & \text{otherwise} \end{cases}.$$  

While in reality not all mismatches between preference and policy have identical utility consequences, we can modify our model to take this into account. Utilities are earned at date 1; there is no discounting. When we interpret our model as one of timing of group decisions, $B$ is an immediate decision while $U$ stands for deferring the decision until more information arrives. It is important to note that our voters can use Bayesian reasoning, and our result do not arise from their inability to foresee that they may change.

\(^6\)For example, $U$ could choose by referendum at date 1, when each voter has a (weakly) dominant strategy to vote for his preferred alternative. Since there are no strategic effects in this referendum, we omit it from our discussion and use the word “election” to refer to the contest between $B$ and $U$.  

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Voters’ types change over time in response to new events (shocks). With probability \( \delta \), a common shock hits; all voters then prefer policy 0 with probability \( \pi \) or policy 1 with probability \( 1 - \pi \). With probability \( 1 - \delta \), there is no common shock. Conditional on this event there are idiosyncratic shocks towards policy 0 with probability \( \theta_0 \) and towards policy 1 with probability \( \theta_1 \), and with probability \( 1 - \theta_0 - \theta_1 \) there is no change i.e. \( t^1_i = t^0_i \forall i \). An idiosyncratic shock towards 0 makes each type-1 voter become type 0 with probability \( p \) independently,\(^7\) while type-0 voters are unaffected; if the idiosyncratic shock is towards 1 then all type 1s stay put but each type 0 changes to

\(^7\)Independence is merely for tractability, and could be replaced by a weak correlation between changes in types; we thank Robert Aumann and Marco Battaglini for this observation.
type 1 with probability $p$ independently of the others. Figure 2 illustrates the shocks. In a realistic case one would expect $\delta < 0.5$, because conclusive evidence that persuades everyone is much less likely than idiosyncratic shocks.

A few comments on our model are in order. There are alternative ways to model an idiosyncratic shock towards 0. One is to have changes occurring in both directions and letting $p$ be the net shock towards 0. In this formulation each type changes into the other with a net movement of $p$. We chose the other formulation of unidirectional shocks, where the algebra is cleaner; our results should go through with the first formulation.\footnote{One can imagine yet another formulation, where the shocks are zero mean. This interpretation of idiosyncratic change as a random noise is not what we have in mind, but our qualitative implications would not be sensitive to this assumption.}

We also focus on pure-strategy symmetric Nash equilibria [henceforth PSNE]; we discuss this later in more detail. One might wonder what roles the two types of shocks, common and idiosyncratic, play; this will be taken up once the main results have been presented.

\textbf{A. The Sincere Voter}

The sincere voter does not necessarily play a best response to the strategies of the other voters, but instead picks the candidate who is more likely to agree with him at date 1 given the unconditional distribution of types. When he is not pivotal his vote does not affect the result; when he is indeed pivotal, his vote may differ from that of the rational voter. While falling short of rationality in one of many ways, the sincere voter provides the most natural and useful benchmark against which to compare the rational voter; this also facilitates comparison with previous work. An interesting point emerges from the comparison: When voters are strategic the outcome could be worse than when they are sincere. As we shall see, the sincere voter can only generate one of two forms of inefficiency. Another important motivation is that our sincere voter is similar to Harsanyi’s rule-utilitarian voter [12].

Since $q > 0.5$, the type-0 voter determines the outcome of the election in a large population. Proposition 1 below shows that type 0s vote $U$ when either (1) type 0s are expected to be in a majority at date 1, or (2) type 1s are expected to be in a majority at date 1, and any particular type-0 voter is very likely to switch to type 1 at the next date i.e. $p$ is ‘high’. He votes $B$ only when the majority is likely to be at 1 at the next date but he is very likely to stay put i.e. $p$ is ‘low’. In other words, he prefers to commit and safeguard his interests today if and only if he will very likely be in the minority at the next date.
PROPOSITION 1 (Sincere Voter) Let \( p^* := 0.5 \left[ 1 - \delta \left( 1 - \pi \right) \right] / \left\{ (1 - \delta) \theta_1 \right\} \) and \( q \in (0.5, 1) \). We can find a function \( n : (0, 1) \to \mathbb{N} \) such that

(i) If \( q(1 - p) > 1/2 \), sincere type-0 agents prefer to vote U for \( n \geq n(p) \);

(ii) If \( q(1 - p) < 1/2, p < p^* \), sincere type-0 agents strictly prefer to vote B for \( n \geq n(p) \);

(iii) If \( q(1 - p) < 1/2, p > p^* \), sincere type-0 agents strictly prefer to vote U for \( n \geq n(p) \);

(iv) The sincere type-1 voters always prefer to vote for U.

When \( q(1 - p) < 1/2 \) and \( p < p^* \), the probability that B wins goes to 1 as \( n \to \infty \).

PROOF Let \( u_i(C, t^0_i) \) denote the expected utility of voter \( i \) when candidate \( C \in \{B, U\} \) wins and \( i \)'s date 0 type is \( t^0_i \in \{0, 1\} \).

SINCERE TYPE-0 VOTER: The expected utility of a sincere type-0 voter \( i \) when B is elected is given by

\[
u_i(B, 0) = 1 - (1 - \delta) \theta_1 p - \delta (1 - \pi) .
\]

Voter \( i \) gets 1 except when he changes to type 1, which can happen (a) w.p. \( (1 - \delta) \theta_1 p \) as a result of an idiosyncratic 1-shock, and (b) w.p. \( \delta (1 - \pi) \) as a result of a common 1-shock.

In order to derive the utility \( u_i(U, 0) \) we first define the following probabilities. For any pair \( (a, b) \in A \times A \), let \( Q^a_b(1) \) be the probability that the date 1 majority is at \( a \), conditional on an idiosyncratic 1-shock and \( t^1_i = b \). Similarly \( Q^a_b(0) \) is the probability that the majority is at \( a \), conditional on an idiosyncratic 0-shock and \( t^1_i = b \). Note that the superscript denotes the majority, the subscript denotes the voter, and the policy in parentheses is the direction of the idiosyncratic shock. In our notation,

\[
Q^a_b(1) := \Pr \left\{ \left| \left\{ j \neq i : t^1_j = 0 \right\} \right| \geq n + 1 \right\} \text{ idiosyncratic 1-shock, } t^1_i = 1 \\
= \sum_{j=n+1}^{2n} \binom{2n}{j} \gamma^j (1 - \gamma)^{2n-j},
\]

where \( \gamma = q(1-p) \) is the probability that, conditional on an idiosyncratic shock towards 1, any \( j \neq i \) will be type 0 at date 1. To derive this note that any \( j \) is a type 0 at date 1 if he is initially a type 0 and he doesn’t change; furthermore these events are
independent over agents $j \neq i$. Similarly we have

$$Q_0^1(1) := \Pr \left\{ \left| \{ j \neq i : t_j^1 = 1 \} \right| \geq n + 1 \mid \text{idiosyncratic 1-shock, } t_i^1 = 0 \right\}$$

$$= \frac{1}{n} \sum_{j=0}^{n-1} \binom{2n}{j} \gamma^j (1 - \gamma)^{2n-j}.$$  

Conditional on an idiosyncratic shock towards 0, any $j \neq i$ will be type 0 at date 1 with probability $\psi = q + (1 - q)p$ independently of the others because he can be type 0 either if he is type 0 at date 0 or if he is type 1 but switches in response to the shock; the independence follows from the independence of the initial draws and the conditional independence of the idiosyncratic changes. Therefore,

$$Q_0^1(0) := \Pr \left\{ \left| \{ j \neq i : t_j^1 = 0 \} \right| \leq n - 1 \mid \text{idiosyncratic 0-shock, } t_i^0 = t_i^1 = 0 \right\}$$

$$= \frac{1}{n} \sum_{j=0}^{n-1} \binom{2n}{j} \psi^j (1 - \psi)^{2n-j}.$$  

Finally, the probability $Q_1^1$ that type 1's are in a majority at date 0 conditional of voter $i$ being of type 0 is given by

$$Q_0^1 := \Pr \left\{ \left| \{ j \neq i : t_j^0 = 0 \} \right| \leq n - 1 \mid t_i^0 = 0 \right\}$$

$$= \frac{1}{n} \sum_{j=0}^{n-1} \binom{2n}{j} q^j (1 - q)^{2n-j}.$$  

When $U$ is elected, the expected utility of voter $i$ of type-0 is

$$u_i(U, 0) = 1 - (1 - \delta) \theta_1 p Q_0^0(1) - (1 - \delta) \theta_1 (1 - p) Q_0^1(1)$$

$$- (1 - \delta) \theta_0 Q_0^1(0) - (1 - \delta) (1 - \theta_0 - \theta_1) Q_1^0.$$  

The four negative terms correspond to the potential sources of loss under $U$: (i) when an idiosyncratic 1-shock hits and voter $i$ changes to type 1, while type 0's are a majority; (ii) when an idiosyncratic 1-shock hits and voter $i$ remains 0 but the majority is at 1 at date 1; (iii) the majority is at 1 after an idiosyncratic 0-shock while voter $i$ remains type 0; and (iv) the majority is at 1 in the initial draw given that $i$ is type 0 and there is no shock. In all other situations, including that of a common shock, $U$ gives utility

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Note that there is no negative term with the factor $Q_0^1(0)$ because types 0s are not affected by an idiosyncratic 0-shock, and hence $Q$ cannot have subscript 1.
1. The type-0 voter casts the ballot for $B$ if

$$u_i(B, 0) > u_i(U, 0)$$

$$\Leftrightarrow (1 - \delta) \theta_1 p + \delta (1 - \pi) < (1 - \delta) \theta_1 p Q_1^0(1) + (1 - \delta) \theta_1 (1 - p) Q_0^1(1)$$

$$+ (1 - \delta) \theta_0 Q_1^0(0) + (1 - \delta) (1 - \theta_0 - \theta_1) Q_0^1.$$  

(2)

When the above inequality is reversed they vote for $U$. First note that

$$q > 0.5 \Rightarrow \lim_{n \to \infty} Q_0^1(0) = 0 = \lim_{n \to \infty} Q_0^1,$$

these limits being independent of the value of $p$. Taking the limit of (2) as $n \to \infty$ and substituting from (3), we get the asymptotic condition for type 0s to vote $B$:

$$\begin{align*}
(1 - \delta) \theta_1 p + \delta (1 - \pi) < (1 - \delta) \theta_1 p & \lim_{n \to \infty} Q_1^0(1) + (1 - \delta) \theta_1 (1 - p) \lim_{n \to \infty} Q_0^1(1) \\
& + (1 - \delta) \theta_0 Q_1^0(0) + (1 - \delta) (1 - \theta_0 - \theta_1) Q_0^1.
\end{align*}$$

(4)

Now let us calculate the two limits in (4). Conditional on an idiosyncratic shock towards 1, the probability that an arbitrary voter is type 0 at date 1 is $q(1 - p)$. Define the random variable $X_0(1) \sim \text{Binomial}(2n, q(1 - p))$ as the number of $j$ out of $2n$ (other than $i$) who have $t^1_j = 0$ after an idiosyncratic 1-shock; then

$$\Pr \{X_0(1) \geq n + 1\} = \Pr \left\{ \frac{1}{2n} X_0(1) \geq \frac{1}{2} + \frac{1}{2n} \right\}.$$ 

By the Weak Law of Large Numbers,

$$\lim_{n \to \infty} \Pr \left\{ \left| \frac{1}{2n} X_0(1) - q(1 - p) \right| < \epsilon \right\} = 1 \text{ for any } \epsilon > 0.$$ 

Case 1 : If $q(1 - p) > 0.5$, there exists a small enough $\epsilon > 0$ so that

$$Q_0^1(1) = \Pr \left\{ \frac{1}{2n} X_0(1) > \frac{1}{2} + \frac{1}{2n} \right\} \geq \Pr \left\{ \left| \frac{1}{2n} X_0(1) - q(1 - p) \right| < \epsilon \right\} \to 1.$$ 

It follows that

$$q(1 - p) > 0.5 \Rightarrow \lim_{n \to \infty} Q_0^1(1) = 1 \text{ and } \lim_{n \to \infty} Q_0^1(1) = 0.$$ 

Substituting these limits into (4), asymptotically $i$ strictly prefers to vote $B$ iff $\delta (1 - \pi) < 0$, which is never the case.
Case 2: When $q(1 - p) < 0.5$, the date 1 majority is expected to be at 1 and by a logic similar to Case 1 above it follows that

$$q(1 - p) < 0.5 \Rightarrow \lim_{n \to \infty} Q^0_1(1) = 0 \text{ and } \lim_{n \to \infty} Q^0_0(1) = 1.$$  

From (4), if $p < (>) p^*$, voter $i$ strictly prefers to vote for $B(U)$ for large $n$.

SINCERE TYPE-1 VOTER: A type-1 voter’s expected utility from $B$ is given by

$$u_i(B, 1) = \delta \pi + (1 - \delta) \theta_0 p.$$  

He gets a utility of 1 iff he switches to 0 himself, in response to either an idiosyncratic or a common 0-shock. Define the following quantities – $P^0_0(0)$ is the probability that the majority is at $a$ following an idiosyncratic 0-shock and $t^1_i = b; P^1_0(1)$ is the corresponding probability when the idiosyncratic shock is towards 1 rather than 0. His utility from voting $U$ is

$$u_i(U, 1) = \delta + (1 - \delta) \theta_0 p P^0_0(0) + (1 - \delta) \theta_0 (1 - p) P^1_1(0) + (1 - \delta) \theta_1 P^1_1(1) + (1 - \delta) (1 - \theta_0 - \theta_1) Q^1_0.$$  

As before, $Q^1_0$ is the probability that the initial draw has more type 1s. By arguments similar to those for the sincere type-0 voter above,

$$q > 0.5 \Rightarrow \lim_{n \to \infty} P^0_0(0) = 1 \text{ and } \lim_{n \to \infty} P^1_1(0) = 0 \text{ and } \lim_{n \to \infty} Q^1_0 = 0.$$  

Substituting the above limits into (4), we see that asymptotically a sincere voter $i$ of type 1 casts his ballot for $B(U)$ when

$$\delta \pi \begin{cases} > & \delta + (1 - \delta) \theta_1 \lim_{n \to \infty} P^1_1(1). \end{cases}$$

Case (i): When $q(1 - p) > 0.5$, we have $\lim_{n \to \infty} P^1_1(1) = 0$. Voter $i$ strictly prefers $B(U)$, for large $n$, according as $\delta \pi > (,) \delta$. Since $\delta (1 - \pi) \geq 0$, he always (weakly) prefers to vote for $U$.

Case (ii): When $q(1 - p) < 0.5$, we have $\lim_{n \to \infty} P^1_1(1) = 1$. For large $n$, he votes for $B(U)$ according as $\delta \pi > (,) \delta + (1 - \delta) \theta_1$. Since $\delta (1 - \pi) + (1 - \delta) \theta_1 \geq 0$, he always (weakly) prefers to vote for $U$. Note that $\delta (1 - \pi) > 0$ is a sufficient condition for $i$ to
strictly prefer to vote for $U$.

**THE WINNING CANDIDATE:**
Consider large $n$. When either (a) $q(1 - p) > 0.5$, or (b) when $q(1 - p) < 0.5$ and $p > p^*$, both the type $0$s and type $1$s vote $U$ and he wins with probability 1. When $q(1 - p) < 0.5$ but $p < p^*$, type $0$s vote $B$ while type $1$s vote $U$; since $q > 0.5$, $B$ wins with probability greater than one half. His probability of winning is

$$Pr\{B \text{ wins}\} = \sum_{k=n+1}^{2n+1} \binom{2n}{k} q^k (1-q)^{2n-k} \geq \frac{1}{2} \forall n.$$  

Furthermore, $Pr\{B \text{ wins}\} \to 1$ as $n \to \infty$.

Figure 3 captures how the type-0 voters behave. In regions II and IV, the type-0 voter elects $U$ because he expects to remain in the majority even if there is an idiosyncratic shock, and has nothing to lose by voting $U$; when there is an common 1-shock, he is better off with $U$ because $B$ will still continue to implement policy 0; with a common 0-shock, both $B$ and $U$ pick 0. In region I, a majority is expected to prefer policy 1 at date 1; since $p$ is high each type 0 expects to switch and be in the subsequent majority. So he votes for $U$. Finally in region III, the sincere type-0 voter picks the socially suboptimal candidate $B$ because the majority is likely to prefer 1 at the next date, but given that $p$ is small he would probably stay put at 0.
We have seen above that the sincere voter of type \( a \) maximizes the value of group-\( a \), because he is in effect the representative agent of the group. This is easy to see: All agents of a group are ex-ante identical, and the probabilities that appear in the decision of the sincere voter are the expected proportions in the representative agent’s decision problem. For example, the probability \( p \) of switching can be interpreted as the expected proportion of type 0s who switch when there is an idiosyncratic 1-shock. When we require the strategies to constitute a Nash equilibrium and there is a positive probability of being pivotal, a rational voter might not behave like the sincere voter. We conjecture the following Nash equilibrium in pure strategies: Each voter of type 0
votes $B$, and all type 1s vote $U$; then we solve for the range of parametric values where this is indeed the case.$^{10}$

**DEFINITION 1** Let $u_i(c | c_0, c_1; t_0^i, Piv)$ denote the utility of the pivotal voter $i$ of type $t_0^i$ when he votes for candidate $c$, the type 0s vote for candidate $c_0$ and the type 1s vote for candidate $c_1$.

A pivotal type-0 voter’s utility of voting for $B$ is the same as that for the sincere voter, and is given by equation (1):

$$u_i (B | B, U; 0, Piv) = 1 - \delta (1 - \pi ) - (1 - \delta ) \theta_1 p.$$  

His utility of voting for $U$ is different from that of the sincere voter. Electing $U$ gives him 1 whenever there is a common shock, no shock, or an idiosyncratic 0-shock; if an idiosyncratic 1-shock arrives, the type-0 voter gets utility 1 if (and only if) nobody switches or if he himself switches.

$$\therefore u_i (U | B, U; 0, Piv) = 1 - (1 - \delta ) \theta_1 (1 - p) \{1 - (1 - p)^n\}.$$  

Therefore voter $i$ strictly prefers $B$ when

$$u_i (B|B, U; 0, Piv) > u_i (U|B, U; 0, Piv)$$

$$\Leftrightarrow \delta (1 - \pi ) + (1 - \delta ) \theta_1 p < (1 - \delta ) \theta_1 (1 - p) \{1 - (1 - p)^n\}. $$

This inequality admits of an intuitive explanation. The LHS of (5) is the loss from voting for $B$ and getting him elected – the first term is the loss when the entire population switches to type 1 at date 1, the second is the loss when voter $i$ idiosyncratically switches (and finds himself on the wrong side vis-a-vis $B$). The RHS is the loss when $U$ is elected; this loss happens only when $i$ does not switch (probability $1 - p$) in response to an idiosyncratic 1-shock but at least one of the $n$ other types 0s does (probability $1 - (1 - p)^n$). The pivotal voter reacts very differently to the possibility of an idiosyncratic switch depending on who is in power – $B$ or $U$. When $B$ is in power, irrespective of what the others do, $i$ loses when he himself switches. When $U$ is in power, $i$ is no longer afraid of switching; instead he fears is staying put when others switch. Inequality (5) captures exactly this trade-off. We prove the asymptotic

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$^{10}$Subsection C, titled “Robustness”, considers equilibrium selection.
proposition first because this allows us to explicitly solve for a threshold level of \( p \); the result for finite \( n \) is in Proposition 6.

**PROPOSITION 2 (Asymptotic Result in \( n \))** If \( p < p^* \), \( \exists N \in \mathbb{N} \) such that for \( n \geq N \) we can find a PSNE where all type 0s vote B and all type 1s vote U. In this PSNE, B’s probability of victory tends to 1 as \( n \to \infty \).

**PROOF** Let \( p < p^* := 0.5 \left[ 1 - \delta \left( 1 - \pi \right) / \left( (1 - \delta) \theta_1 \right) \right] \). As \( n \to \infty \), \( (1 - p)^n \to 0 \); so a sufficient condition for (5) to hold is

\[
\delta(1 - \pi) + (1 - \delta) \theta_1 p < (1 - \delta) \theta_1 (1 - p),
\]

which is equivalent to \( p < p^* \). Then we can find a large enough integer \( N \) such that when \( n \geq N \), type-0 voters cast their ballots for B if type-1 voters conform to the conjectured equilibrium strategy. Since \( q > 0.5 \), the Weak Law of Large Numbers ensures that with a very high probability type 0s are a majority on election day, which in turn implies that B wins with almost probability 1 in a large population.

Finally to show that the above is indeed a Nash equilibrium, we show that the pivotal type 1s will vote \( U \). Expected utility from \( B \) is

\[
u_i(B \mid B, U; 1, \text{Piv}) = \delta \pi + (1 - \delta) \theta_0 p,
\]

noting that a pivotal type-1 voter \( i \) expects to get a utility of 1 iff \( t^1_i = 0 \); this can happen only if player \( i \) himself switches from type 1 to type 0 in response to either a common or an idiosyncratic shock.

The expected utility of a pivotal type-1 voter from \( U \) is

\[
u_i(U \mid B, U; 1, \text{Piv}) = (1 - \delta) (1 - \theta_0 - \theta_1) + \delta + (1 - \delta) \theta_1 + (1 - \delta) \theta_0 p.
\]

The first term is the probability that there are no shocks, common or mixed; the second is the probability that there is a common shock; the third is that there is an idiosyncratic shock towards 1; the fourth term is the probability that there is an idiosyncratic shock towards 0 but none of the types change, leading to type 1s staying in the majority at date 1; the final term captures that \( i \) himself changes in response to idiosyncratic shock towards 0, leading to types 0s being in a majority at date 1.
Type 1s always prefer $U$ as (7) and (8) imply that

$$(1 - \delta) (1 - \theta_0) + \delta (1 - \pi) > 0 \Rightarrow u_i (U \mid B, U; 1, Piv) > u_i (B \mid B, U; 1, Piv).$$

When $p \geq p^*$, we can use similar reasoning to conclude that all voters prefer $U$ to $B$. Candidate $B$ is valuable to voter $i$ of type 0 only in as much as he guards against the possibility of policy 1 being chosen while $i$ remains a type 0 because others have flipped. A large $p$ makes $i$ very likely to switch to an idiosyncratic shock; so he does not fear being left in the minority.

We now show by contradiction that there is no equilibrium where type 1s vote for $B$. The expected utility of a pivotal type 1 from electing $B$ is

$$u_i (B \mid U, B; 1, Piv) = \delta \pi + (1 - \delta) \theta_0 p,$$

noting that a pivotal type-1 voter $i$ expects to get a utility of 1 iff $t^1_i = 0$; this can happen only if player $i$ himself switches from type-1 to type-0 in response to either a common or an idiosyncratic shock. The expected utility of a pivotal type-1 voter from $U$ is

$$u_i (U \mid U, B; 1, Piv) = (1 - \delta) (1 - \theta_0 - \theta_1) + \delta + (1 - \delta) \theta_1 + (1 - \delta) \theta_0 (1 - p)^{n+1} + (1 - \delta) \theta_0 p.$$ 

The first term is the probability of no shock; the second, of a common shock (in either direction); the third, of an idiosyncratic 1-shock; the fourth, of an idiosyncratic 0-shock that leaves everyone unaffected; the fifth, an idiosyncratic 0-shock that makes voter $i$ a type 0 and makes the type 0s a majority. Comparing the two expressions term by term, we find that

$$u_i (B \mid U, B; 1, Piv) < u_i (U \mid U, B; 1, Piv) \forall p \in (0, 1),$$

thereby ruling out the equilibrium where types 0s and 1s vote for $U$ and $B$ respectively.

For large $n$, there are two forms of inefficiencies illustrated above. The first, which corresponds to region III of Figure 3 and is exhibited by both sincere and rational voters, was discussed in Section A. The difference between the sincere and the rational voter is in region IV: The rational voter prefers the committed candidate even when ex-ante he expects to remain in the majority following an idiosyncratic shock. This
happens because conditional on being pivotal, the value of $q$ is irrelevant. In contrast to the decision of the sincere voter for large $n$, his decision depends only on the value of $p$ and not that of $q$. However the socially optimal choice depends on $q$; hence the inefficiency. The interaction among rational voters enters through the size of the population: When $n$ is larger it is more likely the case that, starting from a situation where his vote matters, a type-0 voter will be in the minority if he does not switch following an idiosyncratic shock. The corollary below summarizes this.

**COROLLARY 3** Let $p < p^* := 0.5[1 - \delta (1 - \pi) /\{(1 - \delta) \theta_1\}]$ and $p < 1 - q/2$. All sincere voters vote $U$, whereas the pivotal type-0 and type-1 voters cast their ballots for $B$ and $U$ respectively when $n$ is large. Therefore, sincere voting results in election of $U$ while pivotal voting most likely results in the committed candidate $B$ being elected. If $B$ wins, he implements the sub-optimum policy with probability at least $\delta (1 - \pi) > 0$.

**REMARK 1** For all $n > 2$, not necessarily large, and $q > 0.5$ the probability of an inefficient decision is bounded below by $\delta (1 - \pi)/2$. This follows since $B$ wins with a probability bounded below by $0.5$ for all $n > 2$.

Let us explain the role of the two shocks in our model. The idiosyncratic shocks are essential to our story – a type 0 votes for $B$ because he is worried others might switch to type 1 while he is left behind. What, then, is the role of the common shock?

First, in the examples we gave and in potential applications it is natural to consider a model with both types of shocks. We provide realistic examples of both types of shocks in the introduction to this paper. Second, while $B$ continues to win inefficiently even when we remove the common shock, the inefficiency is very small in large populations. This understates the magnitude of the problem. Let us explain this point carefully.

Recall that $q > 0.5$ is the probability with which a each player is drawn to be type 0. When idiosyncratic shocks towards 1 hit, each type 0 changes to type 1 with probability $p$ (changes are i.i.d.). Suppose $p$ is small and $q(1 - p) > 0.5$. Inefficiency happens because the type 0s vote for $B$ and there is a positive probability that after the shocks (of whatever type) type 1s will be a majority. A common shock towards 1 inevitably gives rise to this inefficiency; therefore the probability of this inefficiency is at least that of the common shock towards 1. Now suppose there are no common shocks. Even then idiosyncratic shocks can make type 1s a majority and lead to inefficiency if $B$ is elected. In a large population it’s very likely that the actual proportion of type

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11 We are grateful to an anonymous referee for the suggestion.
1s after an idiosyncratic 1 shock is \( q(1-p) \), which is greater than 0.5 by assumption. Therefore \( B \)'s probability of making a mistake is small but positive. In other words, inefficiency becomes a very low (but still positive) probability event without a common shock towards 1. The common shock towards 0 is unimportant and included to clarify that the the result is not an artifact of one-sided shocks.

C. Robustness

The equilibrium in which type 0s vote \( B \) and type 1s vote for \( U \) is said to be informative as the vote carries information about the date-0 type or signal. This equilibrium is a strict equilibrium, which satisfies a stringent test of robustness due to Okada (1981):

An equilibrium \( \sigma \) is said to be strictly perfect if \( \exists \epsilon > 0 \) such that if \( \sigma^k \to \sigma \) as \( k \to \infty \), then

1. each \( \sigma^k \) is totally mixed,
2. \( \sigma^k \to \sigma \) as \( k \to \infty \),
3. \( |\sigma^k_j - \sigma_j| < \epsilon \) \( \forall k, j \),

then \( \sigma_i \) is a best response to \( \sigma^k \) \( \forall i, k \).

PROPOSITION 4 (Strict Perfection) The equilibrium in Proposition 3 is strictly perfect.

PROOF As in the above definition, take a sequence of slightly trembled strategy profiles \( \sigma^k \) converging to the informative equilibrium \( \sigma \). Fix a type-0 voter \( i \). Let \( N_0 \) be the number of voters other than \( i \) who are type 0 at date 0, and let \( N_B \) be the number of voters other than \( i \) who vote for \( B \) at date 0. Then,

\[
\Pr \{ N_0 = n \mid N_B = n, \sigma^k \} = \frac{\Pr \{ N_B = n \mid N_0 = n, \sigma^k \} \Pr \{ N_0 = n \}}{\sum_m \Pr \{ N_B = n \mid N_0 = m, \sigma^k \} \Pr \{ N_0 = m \}}.
\]

For \( m \neq n \) all probabilities of the form \( \Pr \{ N_B = n \mid N_0 = m \} \) go to 0 as the trembles go to 0. Hence \( \Pr \{ N_0 = n \mid N_B = n, \sigma^k \} \to 1 \). What makes \( B \) attractive to type 0 is the insurance \( B \) provides against an idiosyncratic 1-shock. When the trembles are small, Bayes' rule implies that conditional on the event \( \{ N_B = n \} \) it is very likely the case that \( \{ N_0 = n \} \). But we already know that the conditional on the event \( \{ N_0 = n \} \) the pivotal type-0 voter will cast his ballot for \( B \). Hence it is a best response for player \( i \).
to vote for \( B \) when the others follow the trembled strategies. It is easy to check that type 1s vote for \( U \) when type 0s vote for \( B \) with a high probability.

In particular this property (\textit{strict perfecion}) implies that the informative equilibrium is \textit{trembling hand-perfect} a la’ Selten (1975).\footnote{Wu and Jiang (1962) defined an equilibrium of a game as \textit{essential} if any game with slightly perturbed payoffs has an equilibrium close to that of the original game. This is an even stronger refinement than strict perfection. It can be shown that ours is the only essential equilibrium.} When the above equilibrium exists, the one where everyone votes for \( U \) is not strictly perfect. However our analysis has so far looked at pure strategy equilibria. Mixed strategy equilibria would not satisfy the strong robustness criterion proposed above. The above thus suggests that the informative equilibrium is a natural and robust construction.

Although finding all the mixed strategy equilibria of voting games is usually a daunting if not impossible task, we are able to prove that asymptotically efficient mixed equilibria as defined below do not exist. Let the typical mixed strategy as a function of \( n \) be denoted by \( (\alpha^n, \beta^n) \), where \( \alpha^n(\beta^n) \) is the probability that a type 0(1) votes for \( B \). If \( \exists \) a sequence of probabilities \( \langle \alpha^n, \beta^n \rangle \) such that \( \sigma^n := (\alpha^n, \beta^n) \) is an equilibrium for each \( n \) and the probability of \( U \) winning is less than \( 0.5 \) for at most finitely many values of \( n \in \mathbb{N} \), we say that \( \langle \sigma^n \rangle_{n \geq 1} \) is \textit{asymptotically efficient}. The corresponding sequence of equilibria is, with some abuse of terminology, referred to as an \textit{asymptotically efficient mixed equilibrium}.

**PROPOSITION 5 (No Efficient Mixed Equilibria)** Mixed asymptotically efficient equilibria do not exist when

\[
p < 1 - \frac{1}{2q} \; \text{i.e.} \; q(1 - p) > \frac{1}{2} \; \text{and} \; p < p^*.
\]

**PROOF** Fix a typical pivotal voter, say voter \( 2n + 1 \). Without loss of generality, number the other voters such that \( 1, \ldots, n \) vote for \( B \) and \( n + 1, \ldots, 2n \) vote for \( U \); denote this pivotal event by \( Piv \). Define \( x^n_B \) and \( x^n_U \) respectively as the probabilities that a voter \( j \neq 2n + 1 \) is of type 0 conditional on the fact that he voted for \( B(U) \). Among voters \( 1, \ldots, n \) let

\[
T^n_B := n - \sum_{i \leq n} t^n_i, \quad T^n_U := n - \sum_{k \geq n+1} t^n_k
\]

denote, respectively, the actual number of type 0s who voted for \( B \) and \( U \) respectively. Then \( T^n := T^n_B + T^n_U \) denotes the total number of type-0 voters. If \( 1 \leq i < j \leq 2n \), the
types of \( i \neq j \) are independent conditional on both having voted for \( B \) or both having voted \( U \). In other words,

\[
\Pr\{t_i^0 = 0, t_j^0 = 0 \mid i, j \text{ voted } B\} = \Pr\{t_i^0 = 0 \mid i, j \text{ voted } B\} \times \Pr\{t_j^0 = 0 \mid i, j \text{ voted } B\},
\]

etc., which follow from explicitly calculating the conditional probabilities. Also note that the conditional variances of the random variables \( t_i^0 \) are bounded above by 1 irrespective of the value of \( \alpha^n \) and \( \beta^n \). All expectations and probabilities below are conditioned on the event \( Piv \). From Chebyshev’s inequality it then follows that

\[
\Pr\{|T^n_B/n - x^n_B| > \epsilon\} \leq \frac{1}{n\epsilon^2},
\]

which implies that

\[
\Pr\{|T^n_B/n - x^n_B| > \epsilon\} < \epsilon \quad \forall n > N_1.
\]

Similarly,

\[
\Pr\{|T^n_U/n - x^n_U| > \epsilon\} < \epsilon \quad \forall n > N_1.
\]

Therefore if \( n > N_1 \) the actual proportion \( T^n/n \) of type 0 voters (excluding our pivotal voter \( 2n + 1 \)) lies within 2\( \epsilon \) of \( x^n_B + x^n_U \) w.p. at least \( 1 - 2\epsilon \).

Let \( Z^n \) denote the actual proportion of voters who will be type 0 at date 1, conditional on \( Piv \) and an idiosyncratic 1-shock; henceforth probabilities are conditioned on the intersection of these two events. We have

\[
E(Z^n) := \frac{x^n_B + x^n_U}{2} \cdot (1 - p) =: z^n,
\]

As above, Chebyshev’s inequality implies that \( \exists N_2 > N_1 \) such that

\[
\Pr\{|Z^n - z^n| > \epsilon\} < \epsilon, \forall n > N_2.
\]

Take any \( n > N_2 \).

Case 1: If \( z^n > 0.5 + \epsilon \), the actual proportion of type 0s at date 1 will be greater than 0.5 with probability at least 1 − \( \epsilon \). So if \( \epsilon < p/2 \) is small enough a type-0 pivotal voter would not be worried about idiosyncratic shocks and would therefore strictly prefer to vote for \( U \). This then reduces to the PSNE where everyone votes \( U \).

Case 2: If \( z^n < 0.5 - \epsilon \), the actual proportion of type 0s at date 1 will be below 0.5 with probability at least 1 − \( \epsilon \). So a pivotal type-0 voter is in a very similar situation to
the pivotal type-0 voter in Proposition 2, as he would be in the minority with probability approximately $1 - p$ if an idiosyncratic shock hits. He therefore strictly prefers to vote for $B$. This case therefore reduces to the PSNE of Proposition 2.

Case 3: Thus the only case when a type $0$ might be willing to mix (for $n > N_2$) is if $0.5 - \epsilon \leq z^n \leq 0.5 + \epsilon$. Using the expression for $z^n$ from (9) we have

$$0.5 - \epsilon \leq z^n \Rightarrow x_B^n + x_U^n > (1 - 2\epsilon) / (1 - p).$$

But then the expected proportion of type-0 voters conditional on an idiosyncratic 0-shock is at least $(1 - 2\epsilon) / [2(1 - p)] + p/2$; since this is greater\(^{13}\) than 0.5 for large $n$ it very likely the case that a pivotal type 1 will get a utility of 1 from electing $U$ if she changes in response to an idiosyncratic shock towards 0. Thus, as in Proposition 2, her utility from voting for $U$ is strictly higher than that from voting for $B$; therefore a pivotal type-1 voter will not mix.

Suppose each type 0 votes for $B$ with probability $\alpha$. Conditional on $Piv$ and given that type 1s vote for $U$, the expected proportion of type 0 voters is $\alpha/2$.

.$z^n = \frac{1 - p}{2\alpha}$

$\Rightarrow 0.5 - \epsilon \leq z^n \leq 0.5 + \epsilon$

$\Rightarrow \frac{1 - p}{1 + 2\epsilon} \leq \alpha \leq \frac{1 - p}{1 - 2\epsilon}.$

(10)

The probability of a vote for $B$ in this $(\alpha, 1)$ equilibrium is the product of the probability $q$ of a type 0 being drawn at date 0 and of the type 0 voting for $B$, i.e. $q\alpha$, which goes to $q(1 - p)$ as $\epsilon \to 0$ from inequality (10). Since $q(1 - p) > 1/2$, the probability of $B$ winning approaches 1, and any mixed equilibrium would therefore asymptotically resemble our equilibrium in Proposition 2.

D. Finite Population Properties

Instead of an asymptotic result in $n$, the following proposition shows, as a function of $p$, exactly how many voters are required for the ideologue to win. It also establishes that when the probability of a common shock is small, such an equilibrium will always exist for 5 or more voters.

\(^{13}\)Note that $(1 - 2\epsilon) / [2(1 - p)] + p/2 = [1 - 2\epsilon + p(1 - p)] / [2(1 - p)] = (1 + p) / 2 + (p - 2\epsilon) / [2(1 - p)].$ Given that $p > 2\epsilon$, the second term is positive and the fraction is strictly greater than one-half.
PROPOSITION 6 (Finite n Result) Define $k(\delta, \pi, \theta_1) := \delta (1 - \pi) / ((1 - \delta) \theta_1)$.

(i) All type 0s voting for B and all type 1s voting for U is a PSNE when $n + 1 > \ln (1 - 2p - k) / \ln (1 - p)$, provided the expression on the right is well defined; B is then more likely to win the election than U.

(ii) When the probability of a common shock is small enough, for any $n \geq 2$ there are values $p, \overline{p} \in (0, 1)$ such that the above is an equilibrium for $p < p < \overline{p}$.

(iii) The set of values of $p$ for which the above equilibrium exists is monotonic in $n$.

PROOF Type-0 voters cast their ballot for B when

$$\delta (1 - \pi) + (1 - \delta) \theta_1 p < (1 - \delta) \theta_1 (1 - p) \{1 - (1 - p)^n\}. \tag{11}$$

Part (i): First note that, as in Proposition 2, the type-1 voters do not want to vote B. This part of the proposition now follows from rearranging the inequality.

Part (ii): When there is no common shock the condition above reduces to

$g(p) := 1 - 2p - (1 - p)^{n+1} > 0.$

Note that $g(0) = 0$ and $g'(0) = n - 1 > 0$ for $n \geq 2$; $g(p) > 0$ for all $p \in (0, p^*)$ for some $p^*$. In other words, B is the more likely winner for this range of values of $p$. The second part of the proposition follows from the continuity of the LHS and the RHS of inequality (11).

Part (iii): For given values of the parameters other than $n$, the RHS of (11) is increasing in $n$. Hence the inequality holds for $n_2 > n_1$ if it holds for $n_1$.

III. Discussion

Interpreting the model as a choice between deciding now or later suggests why some groups push for an early vote even if they might be able to learn something useful by delaying the vote. This logic survives even if voters foresee that they might be committing to a policy possibly disastrous for society. For example, newspapers document the case where Republicans, seeing an opportunity, forced a quick vote and swift rejection of Democratic lawmaker John Murtha’s call for an immediate troop withdrawal from Iraq. Our model suggests one possible explanation: What might have led Republicans to swiftly put it to vote is the very fear that delaying might lead to small yet critical defections.
In an article published in the Op-Ed section of the *L.A. Times*, Bruce Schulman argues that changing sides has been costly in American politics of late. Candidates spend resources trying to explain away changes in their stand on key issues, from affirmative action to foreign policy. Even fairly incontrovertible evidence of having changed does not dissuade them for arguing otherwise. Schulman suggests that political candidates do not wish to come across as opportunists who pander to the electorate for political gain.

This line of reasoning is also explored by Callander and then Kartik and McAfee, who modify the Hotelling/Downs model of electoral competition to give voters with an explicit preference for candidates with *character*. Our paper offers a different explanation for why political candidates might prefer to commit to an ideology rather than update their stands as new information becomes available. In an environment with changing preferences, the best conceivable flip-flopper, one who adjusts his position to what is best for society at large, cannot expect to win against an ideologue. Office seeking candidates might therefore prefer to be perceived as having ideological biases even when the electorate does not *intrinsically* value this trait.

Even decades after Farquharson (1969) introduced the concept of pivotal voting in his classic monograph, its predictions are debated in the literature. On the one hand there is work – for example Austen-Smith and Banks (1996), and Feddersen and Pesendorfer (1997) – using the pivotal calculus; recent studies such as [2] find their predictions consistent with the data. On the other hand, Margolis (2008) and others have argued that the amount of sophistication needed to sustain some of the equilibria is too much to expect. Surely voters don’t calculate a complicated probability of being pivotal, and mix with the exact probability required to make others play their role! Myerson (1998) expresses concerns about mixed asymmetric strategies.

The previous sections assumed that the voting rule used at date 0 is the same as the decision rule used at date 1 by the unbiased candidate $U$. Recall that an interpretation of our framework, one that we mention earlier, is the choice between acting now or waiting; with this interpretation it is indeed natural to suppose that the voting rule at 0 and $U$’s rule at 1 are the same. But if we think of it as an electoral contest, one is naturally led to investigate the properties when the two rules are different. Suppose now that $B$ and $U$ contest in an election where $B$ wins if he receives a fraction $m > 1/2$ or more of the votes and $U$ wins otherwise; as before, the electorate is assumed to be

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large. As one would expect, increasing $m$ from $1/2$ makes it difficult for $B$ to win, and helps mitigate the inefficiency generated by the pivotal voter. However the inefficient equilibrium can be shown to be robust to a large range of $(m, p)$ values.

IV. Conclusion

The electoral system is prone to widespread inefficiency when we relax the assumption that voters’ rankings of policies are unchanged from the time they vote and the time a policy is implemented. This paper illustrates two forms of inefficiency. When voters who are in a majority today are more likely to be in a minority tomorrow, they oppose social-welfare improving policies. This inefficiency requires a probability of idiosyncratic switching large enough to reduce the ex-ante majority to an ex-post minority.

However, a large range of electoral situations is better described by assuming that the probability of voters changing idiosyncratically is small. In contrast to the existing literature, we find that even in this case there is a stark inefficiency – In the unique informative symmetric pure strategy Nash equilibrium voters prefer to elect the ideologue rather than elect an idealist who waits for all information to be revealed and thereafter takes the optimal decision. The key to understanding this paradoxical result is that the pivotal voter finds himself in a fragile majority that is easily overturned; even though such a situation is (unconditionally) unlikely, he bases his vote on this situation and commits to the alternative that he currently prefers. This continues to hold even if there is a sizeable chance that everybody will dislike the committed candidate’s choice due to a common shock.

References


