Objective of problem: We have discussed optimization over time for individual decision-makers and for a social planner. We now consider a revenue-maximizing government’s optimal money creation decisions, when it is faced with a forward-looking demand for money. This revenue maximization differs in important ways from the others that we have faced, due to forward-looking constraints. In week 3 of the class, we will study recursive methods for optimal policy design in such settings, but now we begin with a simple and direct approach.

Problem #2: The Revenue from Money Creation. Suppose that a government seeks to maximize the revenue that it derives from inflation, within a model with purely flexible prices ($P_t$ is the price level at date $t$). In real terms, the government’s revenue at date $t$ is

$$z_t = m_t - \frac{1}{\pi_t} m_{t-1}$$

where $\pi_t$ is the inflation rate – defined as $\pi_t = P_t/P_{t-1}$ – and $m_t$ is the amount of real balances held by agents in period $t$.\(^1\)

The demand for real money balances is given by

$$m_t = \beta f(\pi_{t+1})$$

which is assumed to be positive, but declining in the inflation rate ($f(\pi) \geq 0$ for all $\pi \geq 0$ and $f_\pi < 0$). The parameter $\beta$ satisfies $0 < \beta < 1$. In some of the analysis below, it is also assumed that the money demand function takes the particular functional form

$$m_t = \kappa \pi_{t+1}^{-\alpha}$$

with $\kappa$ and $\alpha$ being positive parameters.

(a) What inflation rate maximizes steady-state rate revenue,

$$m(1 - \frac{1}{\pi})$$

subject to the particular money demand function $m = \kappa \pi^{-\alpha}$? How does revenue-maximizing this inflation rate depend on $\kappa$ and $\alpha$?

\(^1\)This revenue may be derived as follows. First, the nominal money stock in period $t$ is $M_t$ and the newly printed money in period $t$ is $M_t - M_{t-1}$. The real value of this newly printed money is $(M_t - M_{t-1})/P_t$, with $P_t$ being the price level. Hence, the real revenue is as specified in the body of the question, if $\pi_t = P_t/P_{t-1}$.
(b) Consider next a government that maximizes the present discounted value of its revenue,

$$\sum_{t=0}^{\infty} \beta^t z_t$$  \hspace{1cm} (5)

and assume that the government can commit to a series of inflation rates at dates $t = 0, 1, 2, \ldots$. Form a dynamic Lagrangian for the government’s revenue maximization problem, treating the money demand function as an inequality of the form

$$m_t \leq \beta f(\pi_{t+1})$$  \hspace{1cm} (6)

t = 0, 1, 2, \ldots$. That is: assume that the government can pick a "tax base" for the inflation tax which is no larger than the real balances that individuals are willing to hold. Call the multiplier on this constraint $\phi_t$.

(c) Find the first-order conditions for optimal choice of $m_t, \pi_t$, and $\phi_t$ for all dates $t = 0, 1, 2, \ldots$. Record these as follows

$$\pi_0 :$$  \hspace{1cm} (7)

$$m_0 :$$  \hspace{1cm} (8)

$$\phi_0 :$$  \hspace{1cm} (9)

for $t > 0$.

$$\pi_t :$$  \hspace{1cm} (10)

$$m_t :$$  \hspace{1cm} (11)

$$\phi_t :$$  \hspace{1cm} (12)

(d) Using the economics of the problem and the first-order conditions, explain why the government has sharply different inflation incentives at date $t=0$ and date $t > 0$.

(e) Work out the stationary level of the revenue from money creation. How does the optimizing government’s revenue compare to the answer from part (a)? Why?