Using OLS to Estimate and Test for Structural Changes in Models with Endogenous Regressors*

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Abstract

We consider the problem of estimating and testing for multiple breaks in a single equation framework with regressors that are endogenous, i.e., correlated with the errors. We show that even in the presence of endogenous regressors, it is still preferable, in most cases, to simply estimate the break dates and test for structural change using the usual ordinary least-squares (OLS) framework. Except for some knife-edge cases, it delivers estimates of the break dates with higher precision and tests with higher power compared to those obtained using an IV method. Also, the OLS method avoids potential weak identification problems caused by weak instruments. To illustrate the relevance of our theoretical results, we consider the stability of the New Keynesian hybrid Phillips curve. IV-based methods only provide weak evidence of instability. On the other hand, OLS-based ones strongly indicate a change in 1991:1 and that after this date the model loses all explanatory power.

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1 Introduction

Both the statistics and econometrics literature contain a vast amount of work on issues related to structural changes with unknown break dates (see, Perron, 2006, for a detailed review). For the problem of multiple structural changes, Bai and Perron (1998, 2003) provided a comprehensive treatment of various issues in the context of multiple structural change models: consistency of estimates of the break dates, tests for structural changes, confidence intervals for the break dates, methods to select the number of breaks and efficient algorithms to compute the estimates. Perron and Qu (2006) extended the analysis to the case where arbitrary linear restrictions are imposed on the coefficients of the model. In doing so, they also considerably relaxed the assumptions used in Bai and Perron (1998). Bai, Lumsdaine and Stock (1998) considered asymptotically valid inference for the estimate of a single break date in multivariate time series allowing stationary or integrated regressors as well as trends with estimation carried using a quasi maximum likelihood (QML) procedure. Also, Bai (2000) considered the consistency, rate of convergence and limiting distribution of estimated break dates in a segmented stationary VAR model estimated again by QML when the break can occur in the parameters of the conditional mean, the variance of the error term or both. Qu and Perron (2007) considered a multivariate system estimated by quasi maximum likelihood which provides methods to estimate models with structural changes in both the regression coefficients and the covariance matrix of the errors. Kejriwal and Perron (2009, 2010) provide a comprehensive treatment of issue related to testing and inference with multiple structural changes in a single equation cointegrated model. Zhou and Perron (2007) considered the problems of testing jointly for changes in regression coefficients and variance of the errors.

More recent work considered the case of a single equation with regressors that are endogenous, i.e., correlated with the errors. Perron and Yamamoto (2012) provide a very simple proof of the consistency and limit distributions of the estimates of the break dates by showing that using generated regressors, the projection of the regressors of the space spanned by the instruments, to account for potential endogeneity implies that all the assumptions of Perron and Qu (2006) (or those of Bai and Perron, 1998) obtained with original regressors contemporaneously uncorrelated with the errors, are satisfied. Hence, the results of Bai and Perron (1998) carry through, though care must be exercised when the reduced form contains breaks not common to those of the structural form. For an earlier, more elaborate though less comprehensive treatment, see also Hall et al. (2012) and Boldea et al. (2012).

In this paper, we show that even in the presence of endogenous regressors, in general it
is still preferable to simply estimate the break dates and test for structural change using the usual ordinary least-squares (OLS) framework. The idea is simple yet compelling. First, except for a knife-edge case, changes in the true parameters of the model imply a change in the probability limits of the OLS parameter estimates, which is equivalent in the leading case of regressors and errors that have a homogenous distribution across segments. Second, one can reformulate the model with those probability limits as the basic parameters in a way that the regressors and errors are contemporaneously uncorrelated. We are then back to the framework of Bai and Perron (1998) or Perron and Qu (2006) and we can use their results directly to obtain the relevant limit distributions. More importantly, the OLS framework involves the original regressors while the IV framework involves as regressors the projection of these original regressors on the space spanned by the instruments. This implies that the generated regressors in the IV procedure have less quadratic variation than the original regressors. Hence, in most cases, a given change in the true parameters will cause a larger change in the conditional mean of the dependent variable in the OLS framework compared to the corresponding change in an IV framework. Accordingly, using OLS delivers consistent estimates of the break fractions and tests with the usual limit distributions and also improves on the efficiency of the estimates and the power of the tests in most cases. This is shown theoretically and also via simulations. Also, the OLS method avoids potential weak identification problems inherent when using IV methods. Some care must, however, be exercised. Upon a rejection, one should verify that the change in the probability limit of the OLS parameter estimates is not due to a change in the bias term and carefully assess the values of the changes in the structural parameters and the bias terms. In most applications, there will be no change in the bias but given the possibility that it can occur one should indeed be careful to assess the source of the rejection. This is easily done since after obtaining the OLS-based estimates of the break dates one would estimate the structural model based on such estimates. The relevant quantities needed to compute the change in bias across segments are then directly available.

To illustrate the relevance of our theoretical results, we consider the stability of the New Keynesian hybrid Phillips curve as put forward by Gali and Gertler (1999). The results show that IV-based methods only provide weak evidence of structural instability. On the other hand, an OLS-based sup-Wald test for one break is highly significant, with 1991:1 as the estimate of the break date (with a very tight confidence interval (1990:4 to 1991:2)). The higher discriminatory power of OLS-based methods over IV-based ones occurs despite the fact that the instruments are highly correlated with future inflation. The estimates for the
period 1960:1-1991:1 are close to the full sample estimates reported earlier and support Gali and Gertler’s (1999) conclusion. On the other hand, the estimates for the period 1991:2-1997:4 are all very small and insignificantly different from zero. Hence, the hybrid New Keynesian Phillips curve specification has lost any explanatory power for inflation. This forecast breakdown of the New Keynesian Phillips curve for inflation is interesting and can be traced back to the change in the behavior of inflation (e.g., Stock and Watson, 2007).

The structure of the paper is as follows. Section 2 introduces the model and summarizes the relevant result about the limit distribution of the estimates of the break dates using IV procedures. Section 3 shows how using the OLS framework is not only valid but also preferable in that it delivers estimates of the break dates with higher precision and tests with higher power. Section 4 substantiates our theoretical results via simulations and shows their practical importance. Section 5 presents our empirical illustration related to the New Keynesian hybrid Phillips curve. Section 6 provides brief concluding remarks.

2 The Model and Results about the IV Procedure

Consider a multiple linear regression model with \( m_x \) breaks occurring at \( \{T^x_1, ..., T^x_{m_x}\} \),

\[ y = \bar{X}\delta + u, \tag{1} \]

where \( y = (y_1, ..., y_T)' \) is the dependent variable and \( \bar{X} = \text{diag}(X_1, ..., X_{m_x+1}) \) is a \( T \) by \( (m_x + 1)p \) matrix with \( X_i = (x_{T_i-1+1}, ..., x_{T_i})' \) for \( i = 1, ..., m_x + 1 \) with the convention that \( T_0 = 0 \) and \( T_{m_x+1} = T \). We denote the true break dates with a 0 superscript, i.e., \( \{T^0_1, ..., T^0_{m_z}\} \) with the associated diagonal partition of \( \bar{X} \) by \( \bar{X}^0 \) and the true value of the coefficient vector \( \delta^0 \). Note that this framework includes the case of a partial structural change model by introducing linear restrictions \( R\delta = r \) with \( R \) a \( k \) by \( (m_x + 1)p \) matrix with rank \( k \) and \( r \) a \( k \) dimensional vector of constant. It will be useful also to define the break fractions \( (\lambda^0_1, ..., \lambda^0_{m_z}) = (T^0_1/T, ..., T^0_{m_z}/T) \) with corresponding estimates \( (\hat{\lambda}^x_1, ..., \hat{\lambda}^x_{m_z}) = (\hat{T}^x_1/T, ..., \hat{T}^x_{m_z}/T) \).

We allow for some regressors to be correlated with the errors and assume that there exists a set of \( q \) variables \( z_t \) that can serve as instruments, and we define the \( T \) by \( q \) matrix \( Z = (z_1, ..., z_T)' \). We consider a reduced form linking \( Z \) and \( X \) that itself exhibits \( m_z \) changes, so that

\[ X = \bar{Z}^0\theta^0 + v, \tag{2} \]

with \( \bar{Z}^0 = \text{diag}(Z^0_1, ..., Z^0_{m_z+1}) \), the diagonal partition of \( Z \) at the break dates \( (T^0_1, ..., T^0_{m_z}) \) and \( \theta^0 = (\theta^0_1, ..., \theta^0_{m_z+1}) \). Also, \( v = (v_1, ..., v_T)' \) is a \( T \) by \( q \) matrix, which can be cor-
related with \( u_t \) but not with \( z_t \). Given estimates \((\hat{T}_1^z, ..., \hat{T}_{m_z}^z)\) obtained in the usual way using the method of Bai and Perron (1998, 2003), one can construct the diagonal partition \( \hat{Z} = \text{diag}(\hat{Z}_1, ..., \hat{Z}_{m_z+1}) \), a \( T \) by \((m_x+1)q\) matrix with \( \hat{Z}_l = (z_{T_{l-1}^z+1}, ..., z_{T_l^z})' \) for \( l = 1, ..., m_z + 1 \). Let \( \hat{\theta} \) be the OLS estimate of the parameter in a regression of \( X \) on \( \hat{Z} \).

The instruments are then \( \hat{X} = \hat{Z}\hat{\theta} = \text{diag}(\hat{X}_1, ..., \hat{X}_{m_z+1})' \) where \( \hat{X}_l = \hat{Z}_l(\hat{Z}'_l\hat{Z}_l)^{-1}\hat{Z}_l\hat{X}_l \) with \( \hat{X}_l = (x_{T_{l-1}^z+1}, ..., x_{T_l^z})' \), so that its value in regime \( l \) is obtained using only data from that regime. The relevant IV regression is then

\[
y = \hat{X}^*\delta + \tilde{u},
\]

subject to the restrictions \( R\delta = r \), where \( \hat{X}^* = \text{diag}(\hat{X}_1, ..., \hat{X}_{m_z+1}) \), a \( T \) by \((m_x+1)p\) matrix with \( \hat{X}_j = (\hat{x}_{T_{j-1}^z+1}, ..., \hat{x}_{T_j^z})' \) for \( j = 1, ..., m_x + 1 \). Also, \( \tilde{u} = (\tilde{u}_1, ..., \tilde{u}_T)' \) with \( \tilde{u}_t = u_t + \eta_t \) where \( \eta_t = (x_t' - \hat{x}_t')\delta_j \) for \( T_{j-1}^x + 1 \leq t \leq T_j^x \). The estimates of the break dates are then

\[
(\hat{T}_1^x, ..., \hat{T}_{m_x}^x) = \arg\min_{T_1, ..., T_{m_x}} \text{SSR}_T^R(T_1, ..., T_{m_x}),
\]

where \( \text{SSR}_T^R(T_1, ..., T_{m_x}) \) is the sum of square residuals from the restricted OLS regression (3) evaluated at the partition \( \{T_1, ..., T_{m_x}\} \). We also define the break fractions \((\lambda_1^{x_0}, ..., \lambda_{m_x}^{x_0}) = (T_{i-1}^x/T, ..., T_{m_x}^x/T)\) with corresponding estimates \((\hat{\lambda}_1^{x_0}, ..., \hat{\lambda}_{m_x}^{x_0}) = (\hat{T}_i^x/T, ..., \hat{T}_{m_x}^x/T)\).

Let the union of the break dates in the structural and reduced form be denoted by \((T_1^0, ..., T_m^0)\) with the corresponding regime indexed by \( i \) for \( i = 1, ..., m \). Note that \( m \) need not equal \( m_x + m_z \) if some break dates are common. It will be convenient to state the set of assumptions used, which will permit us to define the notation to be used throughout. These are the same used in Perron and Yamamoto (2012) following Perron and Qu (2006).

- **Assumption A1:** Let \( w_t = (x_t', z_t')' \). For each \( i = 1, ..., m + 1 \), let \( \Delta T_i^0 = (T_{i+1}^0 - T_i^0) \), then

\[
(1/\Delta T_i^0) \sum_{t=T_{i-1}^0}^{T_i^0+\Delta T_i^0} w_t w_t' \rightarrow_p Q_i(s) = \begin{bmatrix} Q_{XX}(s) & Q_{XZ}(s) \\ Q_{ZX}(s) & Q_{ZZ}(s) \end{bmatrix},
\]

a non-random positive definite matrix uniformly in \( s \in [0, 1] \).

- **Assumption A2:** There exists an \( n_0 > 0 \) such that for all \( n > n_0 \), the minimum eigenvalues of \((1/n) \sum_{t=T_{i-1}^0}^{T_i^0+\Delta T_i^0} z_t z_t' \) and of \((1/n) \sum_{t=T_{i-1}^0}^{T_i^0-\Delta T_i^0} z_t z_t' \) are bounded away from zero \((i = 1, ..., m)\).

- **Assumption A3:** \( \text{rank}[E(x_t z_t')] = p \) and the matrix \( \sum_{t=k_1}^{k_2} z_t z_t' \) is invertible for \( k_1 - k_2 \geq \epsilon T \) for some \( \epsilon > 0 \).
• **Assumption A4:** Let the $L_r$-norm of a random matrix $A$ be defined by $\|A\|_r = (\sum_i \sum_j E|A_{ij}|^r)^{1/r}$ for $r \geq 1$. (Note that $\|A\|$ is the usual matrix norm or the Euclidean norm of a vector.) With $\{\mathcal{F}_t : t = 1, 2, \ldots\}$ a sequence of increasing $\sigma$-fields, we assume that $\{z_t u_t, \mathcal{F}_t\}$ forms a $L^r$-mixingale sequence with $r = 2 + \varepsilon$ for some $\varepsilon > 0$. That is, there exist nonnegative constants $\{c_t : t \geq 1\}$ and $\{p_j : j \geq 0\}$ such that $p_j \downarrow 0$ as $j \to \infty$ and for all $t \geq 1$ and $j \geq 0$, we have: (a) $E(z_t u_t | \mathcal{F}_{t-j}) \leq c_j p_j$, (b) $\sup_t \|z_t u_t - E(z_t u_t | \mathcal{F}_{t-j})\|_r \leq c_j^2$. Also assume (c) $\max_t c_t \leq K < \infty$, (d) $\sum_{j=0}^{\infty} j^{1+k} p_j < \infty$, (e) $\|z_t\|_2 \leq K < \infty$ and $\|u_t\|_2 < N < \infty$ for some $K, M, N > 0$.

• **Assumption A5:** $\{z_t v_t, \mathcal{F}_t\}$ also satisfies Assumption A4 and $\sup_t \|z_t z_t^T\| = O_p(\log^{1/2} T)$.

• **Assumption A6:** $T_i^0 = [T \lambda_i^0]$, where $0 < \lambda_1^0 < \ldots < \lambda_m^0 < 1$.

• **Assumption A7:** The minimization problem defined by (4) is taken over all possible partitions such that $T_i - T_{i-1} \geq \epsilon T$ for some $\epsilon > 0$.

• **Assumption A8:** Let $\Delta T_j = \delta_{i+1} - \delta_i$, Assume $\Delta T_j = v_T \Delta_j$, for some $\Delta_j$ independent of $T$ where $v_T > 0$ is a scalar satisfying either a) $v_T$ is fixed, or b) $v_T \to 0$ and $T^{1/2-h} v_T \to \infty$ for some $h \in (0, 1/2)$.

• **Assumption A9:** Let $\Delta T_i^0 = T_i^0 - T_{i-1}^0$, for $i = 1, \ldots, m$, and $w_t = (x_t', z_t')'$. Then, as $\Delta T_i^0 \to \infty$, uniformly in $s \in [0, 1]$:  

1. $(\Delta T_i^0)^{-1} \sum_{t=T_{i-1}^0}^{T_i^0} w_t w_t' \to_p s Q^i 
= \begin{bmatrix} Q_{XX}^i & Q_{XZ}^i \\ Q_{ZX}^i & Q_{ZZ}^i \end{bmatrix}$, 
2. $(\Delta T_i^0)^{-1} \sum_{t=T_{i-1}^0}^{T_i^0} u_t u_t' \to_p s \sigma_i^2$, 
3. $(\Delta T_i^0)^{-1} \sum_{t=T_{i-1}^0}^{T_i^0} (z_t z_t' \tilde{u}_t \tilde{u}_t')' \to_p s \Omega_z^i$, 
4. $(\Delta T_i^0)^{-1/2} \sum_{t=T_{i-1}^0}^{T_i^0} z_t \tilde{u}_t \Rightarrow B_{ZU}^i(s)$ with $B_{ZU}^i(s)$ a multivariate Gaussian process on $[0, 1]$ with mean zero and covariance $E[B_{ZU}^i(s)B_{ZU}^j(s)'] = \min\{s, r\} \Omega_{ZU}^i$.

Perron and Yamamoto (2012) provide a short proof about the consistency, rate of convergence and limit distribution of the estimates of the break dates. These are summarized in the following Proposition.

**Proposition 1** 

a) **Under Assumptions A1-A8:** for every $\epsilon > 0$, there exists a $C < \infty$, such that for all large $T$, $P(\{|Tv_T^2(\lambda_j - \lambda_0)| > C\} > \epsilon$ for every $j = 1, \ldots, m_x$. b) Let $j = 1, \ldots, m_x$ index the $m_x$ break dates of the structural form and let $i$ denote the position of the $j^{th}$ break in the structural form amongst the $m$ total break dates in both the structural and reduced forms.
(note that $i$ can range from 1 to $m$). Also suppose that the $i^{th}$ overall regime corresponds to regime $l$ in the reduced form and the $(i+1)^{th}$ overall regime corresponds to regime $l^*$ in the reduced form. Under Assumptions A1-A9, we have:

$$\frac{(\Delta_i^j Q_{H \bar{H}}^i \Delta_j)^2}{\Delta_j^i Q_{H \bar{H}}^j \Delta_j} v_T(\hat{T}_j^x - T_j^x) \rightarrow_d \arg \max_s V_H^{(i,j)}(s),$$

where $V_H^{(i,j)}(s) = W_1^{(i)}(-s) - |s|/2$ if $s \leq 0$ and

$$V_H^{(i,j)}(s) = \sqrt{\xi^i_H (\phi^{i,2}_H/\phi^{i,1}_H)} W_2^{(i)}(s) - \xi^i_H |s|/2$$

if $s > 0$,

with $W_1^{(i)}$ and $W_2^{(i)}$ independent Wiener processes defined on $[0, \infty]$, $(\phi^{i,1}_H)^2 = \Delta_i^i Q_{H \bar{H}}^i \Delta_i/\Delta_i^j Q_{H \bar{H}}^j \Delta_i$, $(\phi^{i,2}_H)^2 = \Delta_i^i Q_{H \bar{H}}^{(i+1),i^*} \Delta_i/\Delta_i^j Q_{H \bar{H}}^{(i+1),j^*} \Delta_i$ and $\xi^i_H = \Delta_i^j Q_{H \bar{H}}^{(i+1),j^*} \Delta_i/\Delta_i^j Q_{H \bar{H}}^j \Delta_i$.

They also show that all results pertaining to the problem of testing the null hypothesis of no structural change using equation (3) remain the same as in Bai and Perron (1998), although we do not restate these explicitly.

3 Estimating and testing using OLS estimates

In this section, we show that, in most cases, it is preferable to estimate the break dates using the standard OLS method rather than an IV procedure even in the presence of endogenous regressors. The reasons are very simple. Assume for simplicity that the break dates are known and let $Q_{XX}^i$ be defined in Assumption A1, $p \lim_{T \rightarrow \infty} E(X_i u) = \phi_i$ for $i = 1, \ldots, m+1$, then the probability limit of the OLS estimate $\hat{\delta}$ from (1) is given by, under A9,

$$\hat{\delta}^* = \delta^0 + p \lim_{T \rightarrow \infty} (\hat{X}_0' \hat{X}_0)^{-1} \hat{X}_0' u = \delta^0 + \text{diag}(Q_{XX}^1, ..., Q_{XX}^{m+1})^{-1} (\phi_1, ..., \phi_{m+1})'.

Any change in the parameter $\delta^0$ will imply a change in the limit value of the OLS estimate $\hat{\delta}^*$, except for a knife-edge case such that the change in the bias term $(Q_{XX})^{-1} \phi$ exactly offsets the change in $\delta^0$. Hence, one can still identify parameter changes using OLS estimates. The second feature is the well known inequality $\|P_Z X\| \leq \|X\|$ so that using an IV procedure leads to regressors that have less quadratic variation than when using OLS. These facts imply that the estimates of the break dates will, in general, be less precisely estimated using an IV procedure. The main cause for this is the fact that a change in the parameter $\delta^0$ will, in general, cause a larger change in the conditional mean of the dependent variable in the OLS framework compared to the corresponding change in an IV regression.
To make the above arguments more precise, consider writing the DGP (1) as

\[ y = \bar{X}_0 \delta^0 + P_{X_0} u + (I - P_{X_0}) u \]
\[ = \bar{X}_0 (\delta^0 + (\bar{X}_0' \bar{X}_0)^{-1} \bar{X}_0' u) + (I - P_{X_0}) u = \bar{X}_0 \delta^*_T + u^* , \]

where \( u^* = (I - P_{\tilde{X}_0}) u \) and \( \delta^*_T = [\delta^0 + (\bar{X}_0' \bar{X}_0)^{-1} \bar{X}_0' u] \) for which \( \delta^*_T \to_{p} \delta^* \). So we can consider a regression in terms of the population value of the parameters, viz., \( y = \bar{X}_0 \delta^* + u^* \). It is clear that in this framework \( \tilde{X}_0 \) is uncorrelated with \( u^* \) so that the OLS estimate, say \( \hat{\delta}^* \), will be consistent for \( \delta^* \). This suggests estimating the break dates by minimizing the sum of squared residuals from the following regression

\[ y = \bar{X} \delta^* + u^* . \]  

Then, the estimates of the break dates are \((\hat{T}_1^*, ..., \hat{T}_m^*) = \arg \min_{T_1, ..., T_m} SSR_{T}^*(T_1, ..., T_m)\), where \( SSR_{T}^*(T_1, ..., T_m) = (y - \bar{X} \hat{\delta}^*)'(y - \bar{X} \hat{\delta}^*) \). We then obtain directly from Perron and Qu (2006) that, under standard conditions, the estimates of the break fractions are consistent and have the same convergence rate as in the usual OLS framework with regressors contemporaneously uncorrelated with the errors. The limit distribution of the estimates of the break dates is given in the following proposition.

**Proposition 2** Under Assumptions A1–A4 and A6–A9 with A2–A4 and A9 stated with \( x_t \) instead of \( z_t \) or \( w_t \), A4 and A9 stated with \( u_t^* \) instead of \( u_t \) and \( \tilde{u}_t \) (using the notations \( \sigma^*_i, \Omega_{XU}^i, \Omega_{ZU}^i, B_{XU}^i, B_{ZU}^i \)), with A8 stated in term of \( \Delta_i^* = \delta_{i+1}^* - \delta_i^* \) instead of \( \Delta_i = \delta_{i+1}^0 - \delta_i^0 \) and \( \Delta_i^* \neq 0 \), we have, for \( i = 1, ..., m \):

\[
\frac{(\Delta_i^* Q_{XX}^i - \Delta_i^*)^2}{\Delta_i^* Q_{XU}^i, \Delta_i^*} v_T(\hat{T}_i^* - T_i^0) \to_d \arg \max_s V(i)(s) \\
\]

where \( V(i)(s) = W_1(i)(-s) - |s|/2 \) if \( s \leq 0 \) and

\[
V(i)(s) = \sqrt{\xi_i} (\phi_{i,2}/\phi_{i,1}) W_2(i)(s) - \xi_i |s|/2 \text{ if } s > 0, \\
\]

with \( \phi_{i,1} = \Delta_i^* Q_{XX}^i, \phi_{i,2} = \Delta_i^* Q_{XX}^{i+1} Q_{XX}^i, \Delta_i^* = \Delta_i^* Q_{XX}^{i+1} Q_{XX}^i, \Delta_i^* = \Delta_i^* Q_{XX}^{i+1} Q_{XX}^i, \Delta_i^* = \Delta_i^* Q_{XX}^{i+1} Q_{XX}^i \).

The proof of the above result is a simple consequences of the model (5) and Perron and Qu (2006), hence is omitted. Note that the limit distribution depends on \( \Delta_i^* = \delta_{i+1}^0 - \delta_i^0 \). But since the OLS estimates are consistent for \( \delta^* \) this quantity can still be consistently estimated. One can then compare the limit distributions of the estimates of the break dates using OLS.
or the IV procedure. For simplicity, consider the case where the instruments, regressors and errors have a constant distribution throughout the sample so the moment matrices do not change across regimes. Also, suppose that the errors are uncorrelated. Then, the limit distribution of the break fractions based on the IV procedure, denoted $\hat{\lambda}_{IV}$, is given by

$$\frac{\Delta t_i Q_{HH}\Delta t_i}{\sigma^2} T(\hat{\lambda}_{i,IV} - \lambda_i^0) \rightarrow^d \arg \max_s V^i_H(s),$$

where $V^i_H(s) = W^i_1(-s) - |s|/2$ if $s \leq 0$, $V^i_H(s) = (\phi_H^{i,2}/\phi_H^{i,1})W^i_2(s) - |s|/2$ if $s > 0$ with $(\phi_H^{i,2})^2 = \Delta_i^1\Omega_H^{i} \Delta_i$, $(\phi_H^{i,1})^2 = \Delta_i^1\Omega_H^{i+1} \Delta_i$, and that of the break fractions based on the OLS procedure, denoted $\hat{\lambda}_{OLS}$, is

$$\frac{\Delta t_i Q_{XX}\Delta t_i}{\sigma^2} T(\hat{\lambda}_{i,OLS} - \lambda_i^0) \rightarrow^d \arg \max_s [W(s) - |s|/2],$$

where $\sigma^2 = \lim_{T \rightarrow \infty} T^{-1}\tilde{w}^t \tilde{u}^* = \lim_{T \rightarrow \infty} T^{-1}\gamma(y - \hat{X} \delta^*)'(y - \hat{X} \delta^*)$. Note that, even in this case, the limit distribution of the IV estimate will not be symmetric since $\phi_H^{i,2} \neq \phi_H^{i,1}$ even with homogenous distributions for the errors and regressors. This makes difficult a precise comparison of the relative efficiency of the two estimates. Note, however, that the component that contributes most to the variability of each estimate is the scaling factor on the left-hand side. Hence, to gain some insights about the relative efficiency of the OLS and IV estimates, we shall analyze the conditions for which $Q_{XX}/\sigma^2 - Q_{HH}/\sigma^2$ is positive in the scalar case with one regressor. We then have $y_t = x_t \delta_i + u_t$ and $x_t = z_t \theta + v_t$ with $Q_{HH} = \theta^2 \sigma^2_z$, $Q_{XX} = \theta^2 \sigma^2_z + \sigma^2_v$, $\tilde{\sigma}_i = \sigma^2 + 2\phi \delta_i + \delta_i^2 \sigma^2_v$, $\sigma^* = \sigma^2 - 2\phi(\theta^2 \sigma^2_z + \sigma^2_v)^{-1}$ and $\phi = E(u_t v_t)$ such that $\phi^2 \leq \sigma^2 \sigma^2_v$. Finally,

$$D \equiv Q_{XX} \tilde{\sigma}_i^2 - Q_{HH} \sigma^2,$$

$$= (\theta^2 \sigma^2_z + \sigma^2_v)(\sigma^2 + 2\phi \delta_i + \delta_i^2 \sigma^2_v) - \theta^2 \sigma^2_z(\sigma^2 - \phi^2(\theta^2 \sigma^2_z + \sigma^2_v)^{-1}),$$

$$= 2\theta^2 \sigma^2_z \phi \delta_i + \theta^2 \sigma^2_z \delta_i^2 \sigma^2_v + 2\phi \delta_i \sigma^2_v + \delta_i^2 \sigma^4_v + \theta^2 \sigma^2_v \phi^2(\theta^2 \sigma^2_z + \sigma^2_v)^{-1}.$$
\[ \tilde{\sigma}_i^2 = \sigma_i^2 + \delta_i^2 \sigma_v^2 \geq \sigma^2 = \sigma_v^2 \] so that OLS again dominates IV. When \( \delta_i \neq 0 \), define \( a = \theta^2 \sigma_z^2 \) and \( x_i = \phi/\delta_i \) so that \( D/\delta_i^2 = 2ax_i + a + (\sigma^2/\delta_i^2) + 2x_i + 1 + (a/(a + 1))x_i^2 \). Using the fact that \( \phi^2 \leq \sigma^2 \) a sufficient condition for \( D \geq 0 \) is that \( D^* \geq 0 \) where

\[
D^* = 2ax_i + a + x_i^2 + 2x_i + 1 + (a/(a + 1))x_i^2.\
\]

After some algebra we get

\[
\frac{a + 1}{2a + 1} D^* = x_i^2 + 2\left(\frac{a + 1}{2a + 1}\right)x_i + \frac{(a + 1)^2}{2a + 1} = \left[ x_i + \frac{a + 1}{2a + 1} \right] [x_i + a + 1]
\]

Hence, \( D^* \geq 0 \) when \( x_i \leq -(a + 1) \) or \( x_i \geq -(a + 1/(2a + 1)) \). Using \( R^2 = \theta^2 \sigma_z^2/(\theta^2 \sigma_z^2 + \sigma_v^2) \) these conditions can be expressed as, without the normalization \( \sigma_v^2 = 1 \),

\[
R^2 \leq \max \left\{ 1 + \frac{\delta_i \sigma_v^2}{\phi}, -1 - \frac{\delta_i \sigma_v^2}{\phi} \right\}.
\]

Accordingly, OLS dominates IV except for a narrow cone centered around \( \delta_i \sigma_v^2/\phi = -1 \). Though this case can possibly occur, it is unlikely in practice. For example, if \( \sigma_v^2 = 1 \), it requires that the correlation between the errors and regressors (\( \phi \)) be exactly the negative of the value of the parameter in the first regime (\( \delta_i \)). We nevertheless consider the finite sample properties of the estimates in that case in Section 4.1.

In summary, it is, in most cases, preferable to estimate the break dates using the simple OLS-based method. As we shall see below, the loss in efficiency can be especially pronounced when the instruments are weak as is often the case in applications. Of course, the ultimate goal is not to get estimates of the break dates per se but of the parameters within each regime, one should then use an IV regression but conditioning on the estimates of the break dates obtained using the OLS-based procedure. Their limit distributions will, as usual, be the same as if the break dates were known since the estimates of the break fractions converge at a fast enough rate. Using the same logic, we can expect tests for structural change to be more powerful when based on the OLS regression rather than the IV regression. This will be investigated through simulations in the next section.

4 Simulation evidence

In this section, we provide simulation evidence to assess the advantages of estimating the break dates and constructing tests using an OLS regression compared to using an IV regression. Three scenarios are possible for a change between regimes \( i \) and \( i - 1 \), say: 1) \( \delta_i^* \neq \delta_{i-1}^* \)
and $\delta^0_i \neq \delta^0_{i-1}$; 2) $\delta^*_i \neq \delta^*_{i-1}$ and $\delta^0_i = \delta^0_{i-1}$; 3) $\delta^*_i = \delta^*_{i-1}$ and $\delta^0_i \neq \delta^0_{i-1}$. The first is the most interesting since one can test and form confidence intervals using either the OLS or IV method. We discuss this scenario in Section 4.1 with the leading case of regressors that have a homogenous distribution across segments and with the correlation between regressors and errors also invariant. In Section 4.2, we consider the case for which either can be changing, a case not covered by our theory. In Section 4.3, we consider the second scenario. The third possibility is a knife-edge case that would require a change in the bias term to exactly offset the change in the structural parameters. Even though in practice this case is a remote possibility, we nevertheless discuss it in Section 4.4.

4.1 Homogenous distributions across segments

The data are generated by $y_t = x_t \delta_t + u_t$ where $y_t$, $x_t$ and $u_t$ are scalars. We consider, for simplicity, the case of a single break in the parameter $\delta_t$ occurring at mid-sample, so that

$$\delta_t = \begin{cases} c & \text{for } t \in [1, T/2]; \\ -c & \text{for } t \in [T/2 + 1, T]. \end{cases}$$

First, define the following random variables: $u_t$ and $v_t$ are i.i.d. $N(0, \sigma_u^2)$ and i.i.d. $N(0, \sigma_v^2)$ with $E(u_tv_t) = \phi$; also $\xi_t \sim i.i.d. N(0, \sigma_v^2)$ and $\varepsilon_t \sim i.i.d. N(0, \sigma_e^2)$, mutually uncorrelated and also uncorrelated with $u_t$ and $v_t$. The regressor $x_t$ is kept the same throughout the specifications and is generated by $x_t = \mu + \xi_t + v_t$ and, again for simplicity, we consider the case of single instrument $z_t$ generated by $z_t = \sqrt{\gamma} \xi_t + \sqrt{2 - \gamma} \varepsilon_t$ with $0 \leq \gamma \leq 2$. Note that $u_t$ and $v_t$ are correlated so the regressors $x_t$ are correlated with the errors $u_t$. The variable $\xi_t$ is a component common to $x_t$ and $z_t$ and the parameter $\gamma$ controls the extent of the correlation between the regressors and the instruments. The component $\varepsilon_t$ is added in the generation of $z_t$ to keep the variability of the instrument constant when varying $\gamma$. The data-generating process can also be expressed as $y_t = x_t \delta_t + u_t$ with $x_t = \mu + \xi_t + v_t$ and $z_t = \sqrt{\gamma} \xi_t + \sqrt{2 - \gamma} \varepsilon_t$, so that the reduced form can be expressed as:

$$x_t = \mu + (1/\sqrt{\gamma}) z_t + (v_t - \sqrt{(2 - \gamma)/\gamma} \varepsilon_t),$$

$$= \mu + [(1/\sqrt{\gamma}) - \sqrt{(2 - \gamma)/\gamma} \lim_{T \to \infty} (\sum_{t=1}^T z_t \varepsilon_t)(\sum_{t=1}^T z_t^2)^{-1}] z_t$$

$$+ [v_t - \sqrt{(2 - \gamma)/\gamma} \varepsilon_t + \sqrt{(2 - \gamma)/\gamma} \lim_{T \to \infty} (\sum_{t=1}^T z_t \varepsilon_t)(\sum_{t=1}^T z_t^2)^{-1} z_t],$$

$$= \mu + \beta z_t + e_t.$$
The error $e_t$ is then not correlated with $z_t$ by construction and it is a valid reduced form representation. The limit of the first stage $R^2$ is $R^2 \to Var(\beta z_t)/Var(x_t)$, where

$$Var(\beta z_t) = \left[ \frac{1}{\gamma} - \frac{2}{\sqrt{\gamma}} \frac{2-\gamma}{\gamma} Cov(z_t, e_t) Var(z_t)^{-1} + \frac{2-\gamma}{\gamma} Cov(z_t, e_t)^2 Var(z_t)^{-2} \right] Var(z_t),$$

and $Var(x_t) = Var(\xi_t) + Var(v_t)$. Using $Cov(z_t, e_t) = \sqrt{2-\gamma}\sigma_{\xi}^2$, $Var(z_t) = 2\sigma_{\xi}^2$, $Var(v_t) = \sigma_v^2$, and $Var(x_t) = \sigma_{\xi}^2 + \sigma_v^2$, then the limit of the $R^2$ can be simplified to $R^2 \to \gamma \sigma_{\xi}^2/[2(\sigma_{\xi}^2 + \sigma_v^2)]$. Throughout the simulations $\gamma$ is adjusted according to the specified value of $R^2$. Since $\gamma$ is bounded above by 2, the highest possible $R^2$ is $\sigma_{\xi}^2/(\sigma_{\xi}^2 + \sigma_v^2)$. Specifically, we set $\sigma_{\xi}^2 = 5\sigma_v^2$ so that the maximum $R^2$ is 0.83. Here, we set $\sigma_{\xi}^2 = 1$, $\sigma_v^2 = 1$, $\mu = 1$ and $\phi = .5$. We also consider a stronger endogeneity parameter $\phi = 1.0$. We report results for $R^2 = .8, .5, .3$ and .1, and $c = .25, .5$, and 1.0. The sample size is $T = 100$ and the number of replications is 3000. The results are presented in Figure 1 for the cumulative distribution functions.

The OLS-based estimates are always more precise than the IV-based estimates. The extent to which the IV-based estimates perform badly increases as $R^2$ decreases, and even for a value as high as 0.5, which is larger than what can be obtained in most applications, the IV-based estimates show much higher variability than the OLS-based estimates. In the case of weak instruments ($R^2 = .1$), the IV-based estimates are not informative. Also, when the magnitude of change increases, the precision of the OLS-based estimate increases noticeably. The increase in the precision of the IV-based estimate is, however, not as substantial as $c$ increases. Hence, even with large breaks, the OLS-based estimates are far superior. The results are robust when we consider stronger endogeneity with $\phi = 1.0$. The simulations show that it is indeed highly preferable to estimate the break dates using the standard OLS framework even when the regressors are contemporaneously correlated with the errors.

We also simulated the power of the sup-Wald test for a single break of Quandt (1958, 1960) as developed by Andrews (1993), given by $sup W = sup_{[T]} \sup_{T_b < (1-\epsilon)T} [SSR_T - SSR(T_b)]/[SSR(T_b)/(T - 1)]$, where $SSR_T$ is the restricted sum of squared residuals from a regression assuming no break, and $SSR(T_b)$ is the sum of squared residuals when allowing for a break at date $T_b$. The trimming parameter is set to $\epsilon = .15$ and we report results for tests with a 5% nominal size. We also considered the UDmax test of Bai and Perron (1998), for which there is no need to pre-specify the number of breaks. The results were qualitatively similar and are not reported. To get the power functions, we varied $c$ between 0 and .5. The results, presented in Figure 2, show that the OLS-based test has the highest power. The power of the IV-based test is noticeably inferior and decreases as the instrument gets more weakly correlated with the regressor. The results are again robust to the stronger endogeneity.
Hence, testing should also be performed using the usual OLS-based methods.

### 4.1.1 The case when IV dominates OLS

As argued in Section 3, there is a region of the parameter space for which IV may dominate OLS in large samples. It is given by (6) and depicted in Figure 3 and corresponds to a narrow cone around $\delta^2 \sigma_v^2 / \text{\phi} = -1$. It remains to be seen whether using OLS in such cases leads to noticeably inferior estimates in finite samples. To that effect, we report simulations setting $y_t = x_t \delta_t + u_t$, $x_t = z_t \theta + v_t$ then choose parameter values $\delta_i \sigma_v^2 / \phi = -1$ to maximize the chances that IV is superior. Again, we consider a single break and we use the following parameter values: $\delta_1 = 1$, $\theta = (R^2/(1 - R^2))^{1/2}$, $\sigma_z = \sigma = \sigma_v = 1$ and $\phi = -1$ so that $\delta_1 \sigma_v^2 / \phi = -1$ is satisfied. The sample size is $T = 100$, and the parameter $\delta$ has one time break at mid-sample such that $\delta_1 = 1$ for $t \leq 50$ and $\delta_2 = 0.5$ afterwards. Again 3,000 replications are used to construct the cdf of the break date estimates for four different values of the first stage $R^2$. These are reported in Figure 4. They show that indeed IV can dominate OLS if the first stage $R^2$ is high ($R^2 = 0.5$). The differences are, however, small. When the $R^2$ is smaller, OLS dominates IV and with weak instruments the IV estimate is uninformative, as expected. Note also the pronounced skewness of the distribution of the IV estimate. We tried different parameter values within the set defined by Figure 3 and the results were qualitatively similar. In particular, slight deviations from values satisfying $\delta^2 \sigma_v^2 / \phi = -1$ lead to having OLS-based estimates be more precise than IV-based ones. This is important for two reasons. First, it shows the relevance and adequacy of our large sample approximation to obtain sufficient conditions for IV to be superior to OLS. Second, it shows how restricted, indeed unlikely, is the set of parameters for which IV dominates OLS. Hence, these results suggest that the cost in finite samples of using OLS when IV is superior to OLS using large sample arguments is small, especially compared to the substantial gains of using OLS in most of the parameter space, as shown above.

### 4.2 Non-homogenous segments : Scenario 1

We now consider the case for which the distribution of the regressors and the correlation between the regressors and errors can vary across segments so that the change in the pseudo limit values $\Delta^*_i = \delta^*_i - \delta^*_i$ need not equal the change in the true population values $\Delta_i = \delta^0_i - \delta^0_i$. Given the nature of the limit distributions stated in Propositions 1-2, it is difficult to provide theoretical results; hence, we resort to simulations. The data generating process is similar to the one used in the previous section except that we allow some key parameters
to change across regimes. More specifically, we now have \( x_t = \mu_t + \xi_t + \nu_t, \nu_t \sim i.i.d. N(0, \sigma_{\nu_t}^2), \xi_t \sim i.i.d. N(0, \sigma_{\xi_t}^2) \) and \( E(\xi_t\nu_t) = \phi_t \). We consider three cases: 1) a change in \( Q_{XX} \) induced by a change in the variance of the regressor such that \( \sigma_{\nu_t}^2 = \sigma_{\xi_t}^2 \) for \( t \in [T/2 + 1, T] \); 2) a change in the correlation between the regressor and the errors such that \( \phi_t = \phi_1 \) for \( t \in [1, T/2] \) and \( \phi_t = \phi_2 \) for \( t \in [T/2 + 1, T] \); 3) a change in \( Q_{XX} \) induced by a change in the mean of the regressor such that \( \mu_t = \mu_1 \) for \( t \in [1, T/2] \) and \( \mu_t = \mu_2 \) for \( t \in [T/2 + 1, T] \). The other specifications are the same. Throughout the simulations, we again set \( \gamma = (12/5)R^2 \) so that \( R^2 \) is fixed for the whole sample. Note that \( \gamma = (12/5)R^2 \) is also fixed. Again, in all cases, the sample size is \( T = 100 \), the number of replications is 3000 and we report results for \( R^2 = .8, .5, .3 \) and .1. The results reported are the cumulative distribution functions of the IV and OLS-based estimates. In all cases, we consider changes such that the value of the relevant parameter is smaller or greater in the first regime compared to the second, since the distributions are not symmetric for equal changes occurring in the two segments. The change in the parameter \( \delta \) is given by (7) with \( c = 0.25 \). We use \( \sigma_{\nu_t}^2 = 1, \sigma_{\xi_t}^2 = 1, \sigma_{\nu_t}^2 = 5\sigma_{\xi_t}^2, \phi = 0.5, \) and \( \mu = 1 \) unless otherwise stated.

Consider first case (1) with a smaller variance of the regressor in the first regime so that \( \sigma_{\nu_t}^2 = 1 \) and \( \sigma_{\xi_t}^2 = a \) with \( a = 0.8, 0.5 \) and 0.25. The results are presented in Figure 5 (left column). Here \( \Delta_1^* \) is greater in absolute value than \( \Delta_1 \), so we can expect an increase in the efficiency of the OLS-based estimate of the break date. When comparing with the base case in Figure 1, this is indeed the case and more so as \( a \) decreases. The results in Figure 5 (right column) pertain to the case of a smaller variance in the second regime so that \( \Delta_1^* \) is smaller in absolute value than \( \Delta_1 \). The specifications used are \( \sigma_{\nu_t}^2 = 1 \) and \( \sigma_{\xi_t}^2 = b \) with \( b = 0.8, 0.5 \) and 0.25. One can see a reduction in the efficiency of the OLS-based estimate. It nevertheless remains more efficient than the IV-based estimate and again more so as the extent of the correlation between the regressor and instrument decreases.

Consider now a change in the correlation between the errors and the regressors across regimes. The first case is one for which the correlation is smaller in the first regime given by \( \phi_2 = 1 \) and \( \phi_1 = a = 0.5, 0.25 \) and 0.1, for which \( \Delta_1^* \) is smaller in absolute value than \( \Delta_1 \). As depicted in Figure 6 (left column), this translates into a reduction in the precision of the OLS-based estimate of the break date. We have some instances for which the IV-based estimate is marginally more precise than the OLS-based one. This, however, requires a very high correlation between the regressor and instrument and a very large change in the correlation between the regressor and the errors. The results with a correlation higher in the first regime are presented in Figure 6 (right column) for the specifications \( \phi_1 = 1 \) and
\( \phi_2 = b = 0.5, 0.25 \) and \( 0.1 \). Here \( \Delta^*_1 \) is greater in absolute value than \( \Delta_1 \) and the results show that in all cases the OLS-based estimate is more precise than the IV-based one.

The last case considered has the mean of the regressor changing across regimes. Figure 7 (left column) presents the results when the mean is higher in the first regime, specified by \( \mu_2 = 1 \) and \( \mu_1 = a = 1.5, 2.0 \) and 5.0. In this case, \( \Delta^*_1 \) is smaller in absolute value than \( \Delta_1 \). The OLS-based estimate nevertheless remains more efficient unless the change in mean is very large \( (a = 5) \) and the correlation between the regressor and instrument is high, even then the differences are minor. When the mean of the regressor is higher in the second regime \( \Delta^*_1 \) is greater in absolute value than \( \Delta_1 \). The results in Figure 7 (right column) for the specifications \( \mu_1 = 1 \) and \( \mu_2 = b = 1.5, 2.0 \) and 5.0 show that the OLS-based estimate is more efficient than the IV-based one and more so as the change in mean increases.

Overall, the simulations presented showed that, in general, the OLS-based estimate is more precise than the IV-based one even when the distribution of the regressors or the correlation between the regressors and the errors change across regimes. There are cases for which this is not true, though with small differences, but they occur for unrealistically high values of the correlation between the regressors and instruments and very large changes in either the correlation between the regressors and errors or the means of the regressors. Hence, for all practical purposes, it is more beneficial to use an OLS-based method.

### 4.3 Scenario 2

In the second scenario \( \delta^*_i \neq \delta^*_{i-1} \) and \( \delta^0_i = \delta^0_{i-1} \), so that there are no changes in the structural parameters but there is a change in either the marginal distribution of the regressors \( x_t \) or in the correlation between these regressors and the errors \( u_t \) so that the limit value of the OLS estimates \( \delta^*_i \) changes. Since there are no changes in the structural parameters, it is uninformative to compare the relative efficiency of the estimate of the break dates. Hence, we concentrate on the rejection frequencies of the tests for structural change using the IV and OLS methods. The presumption is that the IV method would correctly identify no change, while the OLS method would incorrectly identify a change. We use simulations to address this issue and discuss afterwards the practical implications.

Consider first the case in which the correlation between the regressors and the errors changes across segments. Note first that the limit distribution of the sup-Wald OLS-based test does not have the stated limit distribution, while the limit distribution of the IV based test is not affected. Hence, we can expect size-distortions using the OLS-based method. Though these are minor for the sup-Wald test, we nevertheless also report the rejection
frequencies using critical values obtained using the fixed regressors bootstrap method of Hansen (2000). We use the same data-generating process as in Section 4.2, except that \( \delta_t = 1 \) throughout. We set \( \phi_2 = 0.5 \) and vary \( \phi_1 \) between -1 and 1. The results presented in Table 1 show the IV-tests to retain an exact size close to 5%, as expected, and the OLS-based test can have high rejection frequency if the induced change in bias is large. This is also the case when the bootstrap tests are used.

Consider now the case of a change in the marginal distribution of the regressors, either a change in the variance and a change in the mean of \( x_t \). Consider first a change in the variance of \( x_t \). Since the variance change in \( x_t \) and that in \( z_t \) (hence, in \( \hat{x}_t \)) occur, none of the IV or OLS-based sup-Wald tests have the same limit distributions. From results in Hansen (2000) (see also Kejriwal and Perron, 2010, in a related context) the size distortions of the sup-Wald test are minor in the presence of a change in the mean of the regressors but substantial when the variance of the errors change. In the IV case, the variance of the errors in the second stage regression \( \eta_t = (x_t - \hat{x}_t)\delta \) also changes with a change in the marginal distribution of the regressors \( x_t \). Hence, we also present results using the fixed regressors bootstrap methods of Hansen (2000) using the heteroskedastic version. To assess the finite sample rejection frequencies, we use the same data-generating process as in Section 4.2, except that \( \delta_t = 1 \) and we set \( \sigma^2_{2v} = 1 \) while varying \( \sigma^2_{1v} \) between 10 to 0.25. The results are presented in Table 2. They show the OLS-based test to exhibit liberal size distortions when the bias change is large. The IV-test has better size provided the heteroskedastic bootstrap method is used, though even then some size-distortions occur with a large change.

The last case is a change in the mean of the regressors \( x_t \). Here, again none of the IV or OLS-based sup-Wald tests have the stated limit distribution that applies when the distribution of the regressors is homogeneous. This is so for the IV-based method since a change in the marginal distribution of \( x_t \) implies a change in the marginal distribution of \( \hat{x}_t \). If the reduced form is stable, a change in the mean of \( x_t \) must be associated with a change in the mean of the instruments and accordingly in the mean of the fitted values \( \hat{x}_t \). If the reduced form is unstable but the breaks are properly accounted for, the mean of \( \hat{x}_t \) varies by construction. Again, in the IV case, the variance of the errors in the second stage regression change. Hence, we also present results using the fixed regressors bootstrap methods of Hansen (2000) using the heteroskedastic version. Given that the reduced form is unstable, for the IV-based tests we proceed as follows. We first estimate the break date in the reduced form and test whether the change is significant using a sup-Wald test. If it is, the break is accounted for when estimating the structural model using the full sample method of
Perron and Yamamoto (2012). Table 3 reports the rejection frequencies of the tests using the same DGP as in section 4.2, except again that \( \delta_t = 1 \) throughout and \( \mu_2 = 1 \) and \( \mu_1 \) varying between 0.1 and 10 (to obtain a range for the bias similar to the previous experiments). Here, things are quite different. As expected, the OLS-based test does have high power if the change in mean is large. The IV-based test using the standard asymptotic critical values has the correct size if the instrument is very strongly correlated with the regressor and the change is small. As the instrument gets more weakly correlated, the size of the test increases rapidly. The rejection frequencies of the IV-based test exceed those of the OLS-based test when the \( R^2 \) of the first stage regression is small. Intuitively, as the instruments get weaker, \( \hat{x}_t \) gets closer to a white noise process so that the change in \( \hat{x}_t \) cannot fully account for the change in the mean of \( y_t \) induced by the change in \( x_t \) without a concurrent change in either the regression coefficient, the intercept or both, thereby inducing a change in the parameters. With tests constructed using the heteroskedastic bootstrap, the size distortions are considerably reduced for the IV-based test, but can remain important when the correlation between the regressor and the errors is weak. As expected, the OLS-based tests have high power even when the heteroskedastic bootstrap version is used.

### 4.4 Scenario 3

The third scenario pertains to a knife-edge case of \( \delta_i^0 \neq \delta_i^{0-1} \) but \( \delta_i^* = \delta_i^{*-1} \) so that the change in the structural parameters is exactly offset by the bias change induced by a change in the moments of the regressors or in the correlation between the regressors and the errors. This is arguably a case of lesser practical relevance but we still include a discussion about it for completeness. We again consider three cases; 1) a change in the variance of the regressor, 2) a change in the correlation between the regressor and the errors, and 3) a change in the mean of the regressor. We use \( \sigma_{u}^2 = 1 \), \( \sigma_{v}^2 = 5\sigma_{v}^2 \), \( \mu = 1 \), \( \sigma_{v}^2 = 1 \) and \( \phi = 1 \) unless otherwise stated. The changes occur at \( T/2 \). We present the distributions of the break date estimates at \( \delta_i^* = \delta_i^{*-1} \) (but \( \delta_i^0 \neq \delta_i^{0-1} \)) in the left column of Figure 8. We also provide power functions of the sup-Wald test with a 5% nominal size using the asymptotic critical values with \( c \) ranging from 0 to 1 in the right column of Figure 8.

For the first case, we set the change from \( \sigma_{1v}^2 = 10 \) to \( \sigma_{2v}^2 = 1 \) (the largest change considered in Section 4.3). However, even with such a large variance change, the effective break in the structural parameter is small (\( c = 0.063 \)). Figure 8 (the top panel in the left column) shows that the OLS-based estimate is, as expected, not informative. However, the cumulative distribution functions of the IV-based estimates are also quite flat. What is
more instructive is to look at the relative performance of both methods in a neighborhood of the knife-edge case. To that effect, we computed the power functions for this case with $c$ ranging from 0 to 1. The results are in the top right panel of Figure 8. Both the OLS and IV-based tests have important size distortions at $c = 0$, consistent with the results presented in Section 4.3 (though these could be reduced using the bootstrap method). The power of the OLS-based test decreases up to $c = 0.063$ (when $\delta_{i}^* = \delta_{i-1}^*$), but it increases rapidly afterwards. The OLS method can dominate the IV-based one when the instrument is weak.

We next consider the second case with $\phi_1 = -1$ and $\phi_2 = 1$. This represents an (unrealistically) large change that requires the correlations to vary from perfect negative correlation to perfect positive correlation at mid-sample. Such a large change is needed to have at least a modest bias change with magnitude 0.286. The distribution of the OLS estimate of the break date with $c = 0.143$ (corresponding to $\delta_{i}^* = \delta_{i-1}^*$) shows that it is uninformative. The IV estimates are more precise if the correlations between the regressors and the instruments are strong, however, it becomes less informative as the instruments get weaker. Looking at the neighborhood around the knife-edge case, the power function of the OLS-based test has again a U-shape around $c = 0.143$ but the power quickly increases as $c$ increases.

For the third case, we again need to consider a large change from $\mu_1 = 5$ to $\mu_2 = 1$ along with a maximum correlation between the regressors and the errors. These yield a bias change of 0.133 so that the knife-edge case occurs at $c = 0.067$. The results presented in the bottom left panel of Figure 8 show that the OLS method is, as expected, uninformative but also that the IV-based estimate is barely superior irrespective of the strength of the instruments. Looking at the power function around the knife-edge case, the power of the OLS-based test dominates the IV-based test regardless of the strength of the instruments (power is above size when $c = 0.067$ because of distortions arising from the very large mean shift).

### 4.5 Practical implications

Our analyses showed the OLS-based procedure to outweigh, by possible large margins, the IV-based procedure when a change in the structural form occurs (more precise estimate and tests with higher power). There are exceptions but they all apply to so-called knife-edge cases that arguably are not likely to occur in practice, and even in such cases IV is only marginally better than OLS. We also discussed how OLS-based methods can spuriously detect a change in the structural form when none occurs due to possible changes in the marginal distributions of the regressors or the correlation between the errors and regressors. These are of clear relevance in practical applications. Should the fact that the OLS-based
tests reject the null hypothesis of no change when $\delta^*_i \neq \delta^*_{i-1}$ and $\delta^0_i = \delta^0_{i-1}$ be of a concern and a reason to abandon it in favor of the IV-based test? The answer is clearly no but with the caveat that care must be exercised in practice. Upon a rejection, one should verify that it is not due to a change in the bias term and carefully assess the values of $\delta^0_i$ and the bias terms $(Q^i_{XX})^{-1} \phi_i$. In most applications, there will be no change in the bias but given the possibility that it can occur one should indeed be careful to assess the source of the rejection.

The price of discarding the OLS-based method in favor of the IV-based one is too high. First, in most cases the power of the IV-based method is lower and much more so as the instruments gets weaker, a common feature in applications. Second, as illustrated by the case with a change in the mean of the regressors the IV-based test can suffer from serious size distortions, which are, however, reduced to a great extent using Hansen’s (2000) heteroskedastic bootstrap test. Since these are finite sample problems, one would not be inclined to question the source of the rejection and would reach the wrong conclusion. The fact that we could have $\delta^*_i \neq \delta^*_{i-1}$ and $\delta^0_i = \delta^0_{i-1}$ does not invalidate the OLS-based method, it simply calls for a more careful analysis. This is easily done since after obtaining the estimates of the break dates one would estimate the structural model based on such estimates. The relevant quantities needed to compute the change in bias across segments are then directly available. Given that the residuals from the IV regression are given by $\tilde{u}_t = u_t + (x'_t - \hat{x}'_t)\delta_j$ for $T^0_{i-1} + 1 \leq t \leq T^0_i$, an estimate of the change in bias from regime $(i - 1)$ to regime $i$, can be obtained as

\begin{equation}
\begin{aligned}
(\sum_{t=T_{i-1}+1}^{T_i} x_t x'_t)^{-1} \sum_{t=T_{i-1}+1}^{T_i} \left( x_t \tilde{u}_t - x_t (x'_t - \hat{x}'_t)\delta_i \right) \\
-(\sum_{t=T_{i-2}+1}^{T_{i-1}} x_t x'_t)^{-1} \sum_{t=T_{i-2}+1}^{T_{i-1}} \left( x_t \tilde{u}_t - x_t (x'_t - \hat{x}'_t)\delta_{i-1} \right)
\end{aligned}
\end{equation}

where $\hat{T}_i$ are the estimates of the break dates obtained using the OLS-based method, $\tilde{u}_t$ and $\delta_i$ are the estimated residuals and coefficient estimates, respectively, from the estimated IV regression.

5 Empirical example

To illustrate the relevance of our results, we consider the stability of the New Keynesian hybrid Phillips curve as put forward by Gali and Gertler (1999). The basic specification is

\begin{equation}
\pi_t = \mu + \gamma \pi_{t-1} + \kappa x_t + \beta \bar{E}_t \pi_{t+1} + u_t
\end{equation}
where \( \pi_t \) is the inflation rate and \( E_t \) is the expectation operator conditional on information available at time \( t \). The variable \( x_t \) is a real determinant of inflation usually taken as a measure of real economic activity such as the output gap, though Gali and Gertler (1999) argue that using a measure of real marginal cost is preferable. The key ingredient is the component \( E_t \pi_{t+1} \) which makes the Phillips-curve forward looking. The one-period lag of inflation is usually introduced on the ground that it improves the fit of the model but can be rationalized by supposing that a proportion of firms use backward-looking rules to set prices. In this specification, one expects \( \beta \) and \( \gamma \) to be positive and that \( \beta \) is substantially larger than \( \gamma \) so that the forward looking component dominates. Also, \( \kappa \) is expected to be positive so that an increase in real activity is associated with an increase in inflation. As argued by Gali and Gertler (1999), the estimate of \( \kappa \) is usually negative when using the output gap but is positive, though quite small, when using real marginal cost.

We follow the approach taken by Gali and Gertler (1999) and use basically the same specifications and data. The data was obtained from Andre Kurmann’s website (and used in Kurmann, 2007). It is for the U.S.A. and quarterly for the period 1960:1-1997:4 as in Gali and Gertler (1999). The inflation rate \( \pi_t \) is the quarterly change in the GDP deflator and \( x_t \) is either 1) labor income share as a proxy of real unit labor cost (nonfarm business unit labor cost deflated by the GDP deflator), or 2) detrended real GDP assuming a quadratic linear trend as a measure of the output gap. The only difference is that we use as instruments only one lag of the following variables: inflation, labor income share, the output gap, the interest rate spread (10 years US treasury bill rate minus the 3 years bill rate), wage inflation (quarterly change in the nonfarm business nominal wage rate) and the commodity price inflation (quarterly change in the spot market price index). Using four lags as in Gali and Gertler (1999) would imply too many instruments so that testing for changes in the reduced form, needed for the IV-based methods, becomes impossible with the tests having no power.

Gali and Gertler (1999) estimated the model using a non-linear IV procedure given that their model implies restrictions across the coefficients which are functions of some basic parameters. We use a linear IV method of estimation. The results are, however, in line with those of Gali and Gertler (1999) for the full sample specification and doing so does not affect our conclusions. The parameter estimates for the full sample are presented in Table 4 for both cases in which \( x_t \) is specified as the real marginal labor cost or as the output gap. The results are in close agreement with those of Gali and Gertler (1999). The coefficient \( \beta \) on future expected inflation is large and significant and about twice as large as the coefficient \( \gamma \) on lagged inflation, indicating that the forward-looking behavior is dominant. The estimate
of $\kappa$ is positive when using the real marginal cost of labor and negative when using the output gap, though the values are small and insignificant. Note also that the instruments are quite strongly correlated with future inflation, the $R^2$ of the first-stage regression being 0.73. The results are qualitatively similar using four lags of each variables as instruments.

In order to proceed with the IV-based methods, we need to consider the stability of the reduced form. This is not needed for the OLS-based method as long as tests are constructed using the fixed regressors bootstrap method of Hansen (2000). The results presented in Table 5 points to the presence of 1 or 2 breaks. Applying the Bai and Perron (1998) sequential method, the test for one break is significant at the 1% level using the asymptotic critical values and at the 10% level using the critical values from Hansen’s (2000) bootstrap method. The test for 2 versus 1 break is again significant at the 1% level using the asymptotic critical values but not using the bootstrap method, a difference possibly due to the large number of regressors. In the following, we consider both cases with 1 and 2 breaks.

We now turn to the issue of the stability of this hybrid New-Keynesian Phillips curve. Consider first the OLS-based method. When using the labor income share, the test for one break is significant at the 1% level using the asymptotic critical values and at the 10% level using the bootstrap ones. The estimate of the break date is at 1991:1 with a very tight confidence interval (1990:3 to 1991:3). There is no evidence for the presence of an additional break. When using the output gap, the test for one break is again significant at the 1% level using the asymptotic critical values but not significant using the bootstrap ones. The estimate of the break date is again 1991:1 with an even tighter confidence interval (1990:4 to 1991:2). There is again no evidence for the presence of a second break.

We now consider the results obtained using IV-based methods. We start with the sub-sample approach advocated by Hall et al. (2012), which looks at evidence for changes in the structural form using the sub-samples induced by the estimates of the break dates in the reduced form. The results in Table 6 show that one cannot find any statistically significant evidence for the presence of a break. This is true whether one considers a single or two breaks in the reduced form. Things are somewhat different when using the full-sample method advocated by Perron and Yamamoto (2012). Consider first the case with one break in the reduced form at date 1980:4. The results are the same using either the labor income share or the output gap. The test for one break is significant at the 5% level using the asymptotic critical values but not when using the bootstrap ones. There is no evidence for the presence of a second break and the estimate of the break date is 1967:2, very different from that obtained using the OLS-based method. Overall, the evidence for a break in the
structural form is weaker. Consider now the case with two breaks in the reduced form at dates 1973:1 and 1980:4. When using the labor income share the results are similar to the case of a single break in the reduced form. The test for one break is barely significant at the 10% level using the asymptotic critical values and not using the bootstrap ones. The estimate of the break date is again 1967:2. Things are different when using the output gap with results identical to those obtained using the OLS-based method. The test for one break is significant at the 1% level using the asymptotic critical values but not significant using the bootstrap ones. The estimate of the break date is again 1991:1 with a tight confidence interval (1990:3 to 1991:3). There is again no evidence for the presence of a second break.

It remains to be seen whether this change in 1991:1 is economically significant. To that effect we estimated the parameters of the model by IV using the same instruments allowing for a change in all parameters. The estimates are presented in Table 7. They are indeed very interesting. The estimates for the period 1960:1-1991:1 are close to the full sample estimates reported earlier and support Gali and Gertler’s (1999) conclusion. On the other hand, the estimates for the period 1991:2-1997:4 are all very small and insignificantly different from zero in both cases. Hence, both of these hybrid New Keynesian Phillips curve specifications have lost any explanatory power for inflation. Again, the results are qualitatively similar using four lags of each variables in the instrument list.

As discussed above, the full-sample IV-based method pointed to a break in 1967:2 when considering one break in the reduced form with either the labor income share or the output gap and also when considering two breaks in the reduced form when using the labor income share. To see which of 1967:2 or 1991:1 is the more appropriate break date, we present in Table 8 the parameter estimates of the structural form with a break in 1967:2. The estimates show no significant differences between the samples 1960:1-1967:2 and 1967:3-1997:4. The parameter $\beta$ shows a slight increase that is not significant, while $\gamma$ shows an increase (from negative to positive) when using the labor income share. With one break in the reduced form, however, it shows an increase when using the output gap. So the results are sensitive to the exact specification and also not significant as the standard errors are large in both subsamples. The changes when considering a break in 1991:1 are economically more important and statistically significant than those obtained with a break in 1967:2. This reinforces the conclusion that the OLS-based method provides more reliable results.

Overall, the results point to the following conclusions. With the OLS-based method, the evidence for a break in the structural form is strong and points to the same break date, 1991:1, whether using the labor income share or the output gap. When using the IV-based
methods, things are not as precise. First, the sub-sample approach yields no evidence for a break in the structural equation. The more powerful full-sample approach suggested by Perron and Yamamoto (2012) can deliver strong results in line with the OLS-based ones, as obtained when considering the output gap. But it can also be less powerful as in the case for which the labor income share is used. Note finally that the higher discriminatory power of OLS-based methods over IV-based ones occurs despite the fact that the instruments are highly correlated with future inflation. The OLS-based method is also much easier to implement without the need to worry about changes in the reduced form.

Finally, we need to assess that the rejection is not due to change in the bias term. To that effect we estimated the change in the bias from (8) with a break in the structural equation in 1991:1. For the case with one break in the reduced form in 1980:4, the estimated value was $0.0388$ with the labor income share and $0.0513$ with the output gap. For the case with two breaks in the reduced form in 1973:1 and 1980:4, the estimated value was $0.0107$ with the labor income share and $0.0212$ with the output gap. These compare to a change in coefficient of $1.011$ with the labor income share and $1.052$ with the output gap. Hence, the change in the bias is negligible compared to the change in the parameters so that a rejection using the OLS-based sup-Wald test genuinely reflects a change in the structural parameters.

This forecast breakdown of the New Keynesian Phillips curve for inflation is interesting and can be traced back to the change in the behavior of inflation. As argued by Stock and Watson (2007) even though inflation has become more stable after the mid-80’s it also has become more difficult to forecast (see also Rossi and Sekhposyan, 2010). Our evidence is consistent with theirs, though stronger perhaps because of the different break date identified.

6 Conclusions

In this paper, we considered the problem of multiple structural changes in a single equation framework with regressors that are endogenous, i.e., correlated with the errors. We showed that even in the presence of endogenous regressors, it is still preferable to simply estimate the break dates and test for structural change using the usual ordinary least-squares framework. The reasons are simple. First, changes in the true parameters of the model imply a corresponding change in the probability limits of the OLS parameter estimates, except for a possible knife-edge case. Second, one can reformulate the model with those probability limits as the basic parameters in a way that the regressors and errors are contemporaneously uncorrelated. We are then simply back to the framework of Bai and Perron (1998) or Perron and Qu (2006) and we can use their results directly to obtain the relevant limit distributions.
Since the OLS framework involves the original regressors, while the IV framework involves as regressors the projection of these original regressors on the space spanned by the instruments, this implies that the generated regressors in the IV procedure have less quadratic variation than the original regressors. Accordingly, using OLS not only delivers consistent estimates of the break fractions and tests with the usual limit distributions, it also improves on the efficiency of the estimates and the power of the tests in the vast majority of cases. Lastly, it is important to note that the OLS procedure avoid potential weak identification problems when estimating and testing for structural changes.
References


Perron P, Yamamoto Y. 2012. A note on estimating and testing for multiple structural changes in models with endogenous regressors via 2SLS. Forthcoming in *Econometric Theory*.


Figure 1: Cumulative distribution functions of the estimates of the break date;
Figure 2: Power functions of the sup-Wald structural change test;
Figure 3: Set of parameter values with which IV estimates can be more efficient than OLS estimates;

Figure 4: Finite sample distributions of break date estimates under the condition $\frac{\delta}{\phi} \sigma_v^2 = -1$;
Figure 5: Change in the variance of the regressor;
(Left: $\sigma_{2u} = 1$ and $\sigma_{1u} = a$, Right: $\sigma_{1u} = 1$ and $\sigma_{2u} = b$)
Figure 6: Change in the correlation between the regressor and errors;
(Left: $\phi_2 = 1$ and $\phi_1 = a$, Right: $\phi_1 = 1$ and $\phi_2 = b$)
Figure 7: Change in the mean of the regressor;
(Left: $\mu_2 = 1$ and $\mu_1 = a$, Right: $\mu_1 = 1$ and $\mu_2 = b$)
Figure 8: The cases of $\delta_i^* = \delta_{i-1}^*$ and $\delta_i^0 \neq \delta_{i-1}^0$;
(Left: distributions of the break date estimate,  
Right: power functions of the sup-Wald test)

Change in the variance of the regressor

Change in the correlation between the regressor and the errors

Change in the mean of the regressor
Table 1. Rejection frequencies of the sup-Wald tests
Change in the correlation between the regressor and errors

<table>
<thead>
<tr>
<th></th>
<th>asymptotic test</th>
<th>bootstrap test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>1.0 0.1 −0.5 −1.0</td>
<td>1.0 0.1 −0.5 −1.0</td>
</tr>
<tr>
<td>bias change</td>
<td>0.07 −0.06 −0.14 −0.21</td>
<td>0.07 −0.06 −0.14 −0.21</td>
</tr>
<tr>
<td>OLS</td>
<td>0.137 0.133 0.338 0.687</td>
<td>0.088 0.119 0.394 0.651</td>
</tr>
<tr>
<td>IV($R^2=0.8$)</td>
<td>0.069 0.067 0.070 0.112</td>
<td>0.032 0.023 0.105 0.080</td>
</tr>
<tr>
<td>IV($R^2=0.5$)</td>
<td>0.064 0.072 0.068 0.081</td>
<td>0.013 0.024 0.166 0.094</td>
</tr>
<tr>
<td>IV($R^2=0.3$)</td>
<td>0.059 0.065 0.067 0.075</td>
<td>0.015 0.020 0.128 0.124</td>
</tr>
<tr>
<td>IV($R^2=0.1$)</td>
<td>0.063 0.057 0.069 0.062</td>
<td>0.031 0.014 0.050 0.145</td>
</tr>
</tbody>
</table>

Table 2. Rejection frequencies of the sup-Wald tests
Change in the variance of regressor

<table>
<thead>
<tr>
<th></th>
<th>asymptotic test</th>
<th>bootstrap test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{1v}$</td>
<td>10.0 5.0 2.0 0.25</td>
<td>10.0 5.0 2.0 0.25</td>
</tr>
<tr>
<td>bias change</td>
<td>−0.06 −0.06 −0.03 0.13</td>
<td>−0.06 −0.06 −0.03 0.13</td>
</tr>
<tr>
<td>OLS</td>
<td>0.619 0.360 0.116 0.455</td>
<td>0.318 0.094 0.092 0.269</td>
</tr>
<tr>
<td>IV($R^2=0.8$)</td>
<td>0.239 0.138 0.083 0.109</td>
<td>0.052 0.065 0.084 0.016</td>
</tr>
<tr>
<td>IV($R^2=0.5$)</td>
<td>0.202 0.123 0.073 0.094</td>
<td>0.057 0.114 0.005 0.011</td>
</tr>
<tr>
<td>IV($R^2=0.3$)</td>
<td>0.167 0.114 0.077 0.082</td>
<td>0.118 0.115 0.020 0.017</td>
</tr>
<tr>
<td>IV($R^2=0.1$)</td>
<td>0.138 0.092 0.073 0.068</td>
<td>0.219 0.115 0.027 0.090</td>
</tr>
</tbody>
</table>

Table 3. Rejection frequencies of the sup-Wald tests
Change in the mean of the regressor

<table>
<thead>
<tr>
<th></th>
<th>asymptotic test</th>
<th>bootstrap test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>0.1 1.5 5.0 10.0</td>
<td>0.1 1.5 5.0 10.0</td>
</tr>
<tr>
<td>bias change</td>
<td>0.01 −0.01 −0.06 −0.07</td>
<td>0.01 −0.01 −0.06 −0.07</td>
</tr>
<tr>
<td>OLS</td>
<td>0.063 0.066 0.337 0.798</td>
<td>0.076 0.057 0.035 0.176</td>
</tr>
<tr>
<td>IV($R^2=0.8$)</td>
<td>0.036 0.024 0.180 0.600</td>
<td>0.093 0.066 0.033 0.012</td>
</tr>
<tr>
<td>IV($R^2=0.5$)</td>
<td>0.031 0.023 0.249 0.737</td>
<td>0.080 0.012 0.036 0.028</td>
</tr>
<tr>
<td>IV($R^2=0.3$)</td>
<td>0.048 0.035 0.368 0.869</td>
<td>0.075 0.016 0.061 0.060</td>
</tr>
<tr>
<td>IV($R^2=0.1$)</td>
<td>0.156 0.101 0.685 0.967</td>
<td>0.222 0.009 0.101 0.158</td>
</tr>
</tbody>
</table>
Table 4. IV full sample coefficient estimates for the hybrid Philips curve

<table>
<thead>
<tr>
<th>x_t</th>
<th>$\mu$ (const)</th>
<th>$\gamma$ ($\pi_{t-1}$)</th>
<th>$\kappa$ ($x_t$)</th>
<th>$\beta$ ($E_t\pi_{t+1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>labor income share</td>
<td>0.000</td>
<td>0.367</td>
<td>0.003</td>
<td>0.619</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.103)</td>
<td>(0.008)</td>
<td>(0.116)</td>
</tr>
<tr>
<td>GDP gap</td>
<td>0.000</td>
<td>0.260</td>
<td>-0.010</td>
<td>0.745</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.162)</td>
<td>0.009</td>
<td>(0.190)</td>
</tr>
</tbody>
</table>

Table 5. Tests for structural changes in the reduced form equation

<table>
<thead>
<tr>
<th>number of breaks</th>
<th>test</th>
<th>break dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\text{SupF}=43.7^{**<em>}$[</em>]</td>
<td>1980:4</td>
</tr>
<tr>
<td>2</td>
<td>$F(2</td>
<td>1)= 52.0^{***}$</td>
</tr>
<tr>
<td>3</td>
<td>$F(3</td>
<td>2)= 21.0$</td>
</tr>
</tbody>
</table>

Table 6. Tests for structural changes in the hybrid Philips curve equation

1. One break in the reduced form (1980:4)
   (a) $x_t$ is labor income share

|              | SupF  | F(2|1) | break date | 95% C.I.    |
|--------------|-------|-------|------------|------------|
| IV(full sample) | 18.2^{**}     | 14.5 | 1967:2     | 1964:4, 1969:4 |
| IV(segment 1) | 5.93  | -    | 1976:3     | 1976:2, 1976:4 |
| IV(segment 2) | 5.96  | -    | 1991:1     | 1990:3, 1991:3 |

(b) $x_t$ is the GDP gap

|              | SupF  | F(2|1) | break date | 95% C.I.    |
|--------------|-------|-------|------------|------------|
| IV(full sample) | 21.0^{***}    | 12.6 | 1967:2     | 1965:3, 1969:1 |
| IV(segment 1) | 10.8  | -    | 1967:2     | 1965:3, 1969:1 |
| IV(segment 2) | 8.7   | -    | 1991:1     | 1990:3, 1991:3 |

2. Two breaks in the reduced form (1973:1, 1980:4)
   (a) $x_t$ is labor income share

|              | SupF  | F(2|1) | break date | 95% C.I.    |
|--------------|-------|-------|------------|------------|
| IV(segment 1) | 9.3   | -    | 1967:2     | 1967:1, 1967:3 |
| IV(segment 2) | 3.5   | -    | 1974:1     | 1969:3, 1978:3 |

(b) $x_t$ is the GDP gap

|              | SupF  | F(2|1) | break date | 95% C.I.    |
|--------------|-------|-------|------------|------------|
| IV(segment 1) | 7.1   | -    | 1968:3     | 1968:2, 1968:4 |
| IV(segment 2) | 2.7   | -    | 1977:2     | 1974:3, 1980:1 |
| IV(segment 3) | 8.7   | -    | 1991:1     | 1990:3, 1991:3 |
Table 7. IV sub-sample estimates for the hybrid Phillips curve equation (break in 1991:1)

1. One break in the reduced form (1980:4)
   (a) $x_t$ is labor income share
<table>
<thead>
<tr>
<th></th>
<th>$\mu$ (const)</th>
<th>$\gamma$ ($\pi_{t-1}$)</th>
<th>$\kappa$ ($x_t$)</th>
<th>$\beta$ ($E_t\pi_{t+1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960:1-1991:1</td>
<td>0.000</td>
<td>0.351</td>
<td>0.000</td>
<td>0.624</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.111)</td>
<td>(0.008)</td>
<td>(0.107)</td>
</tr>
<tr>
<td>1991:2-1997:4</td>
<td>0.001</td>
<td>0.130</td>
<td>0.042</td>
<td>-0.328</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.157)</td>
<td>(0.036)</td>
<td>(0.385)</td>
</tr>
</tbody>
</table>
   (b) $x_t$ is the GDP gap
<table>
<thead>
<tr>
<th></th>
<th>$\mu$ (const)</th>
<th>$\gamma$ ($\pi_{t-1}$)</th>
<th>$\kappa$ ($x_t$)</th>
<th>$\beta$ ($E_t\pi_{t+1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960:1-1991:1</td>
<td>0.000</td>
<td>0.310</td>
<td>-0.006</td>
<td>0.674</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.129)</td>
<td>(0.008)</td>
<td>(0.132)</td>
</tr>
<tr>
<td>1991:2-1997:4</td>
<td>0.007</td>
<td>0.047</td>
<td>-0.042</td>
<td>-0.340</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.170)</td>
<td>(0.018)</td>
<td>(0.363)</td>
</tr>
</tbody>
</table>

2. Two breaks in the reduced form (1973:1 and 1980:4)
   (a) $x_t$ is labor income share
<table>
<thead>
<tr>
<th></th>
<th>$\mu$ (const)</th>
<th>$\gamma$ ($\pi_{t-1}$)</th>
<th>$\kappa$ ($x_t$)</th>
<th>$\beta$ ($E_t\pi_{t+1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960:1-1991:1</td>
<td>0.000</td>
<td>0.302</td>
<td>0.001</td>
<td>0.683</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.070)</td>
<td>(0.007)</td>
<td>(0.067)</td>
</tr>
<tr>
<td>1991:2-1997:4</td>
<td>0.001</td>
<td>0.130</td>
<td>0.042</td>
<td>-0.328</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.157)</td>
<td>(0.036)</td>
<td>(0.385)</td>
</tr>
</tbody>
</table>
   (b) $x_t$ is the GDP gap
<table>
<thead>
<tr>
<th></th>
<th>$\mu$ (const)</th>
<th>$\gamma$ ($\pi_{t-1}$)</th>
<th>$\kappa$ ($x_t$)</th>
<th>$\beta$ ($E_t\pi_{t+1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960:1-1991:1</td>
<td>0.000</td>
<td>0.279</td>
<td>-0.007</td>
<td>0.712</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.076)</td>
<td>(0.007)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>1991:2-1997:4</td>
<td>0.007</td>
<td>0.047</td>
<td>-0.042</td>
<td>-0.340</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.170)</td>
<td>(0.018)</td>
<td>(0.363)</td>
</tr>
</tbody>
</table>
Table 8. IV sub-sample estimates with a break in 1967:2

1. One break in the reduced form (1980:4)
   (a) $x_t$ is labor income share
   \[
   \begin{array}{cccc}
   \mu \text{ (const)} & \gamma (\pi_{t-1}) & \kappa (x_t) & \beta (E_t\pi_{t+1}) \\
   \hline
   1960:1-1967:2 & 0.000 & -0.028 & 0.006 & 0.554 \\
   & (0.004) & (0.374) & (0.019) & (0.036) \\
   1967:3-1997:4 & -0.003 & 0.167 & 0.023 & 0.807 \\
   & (0.002) & (0.136) & (0.013) & (0.129) \\
   \hline
   \end{array}
   \]

   (b) $x_t$ is the GDP gap
   \[
   \begin{array}{cccc}
   \mu \text{ (const)} & \gamma (\pi_{t-1}) & \kappa (x_t) & \beta (E_t\pi_{t+1}) \\
   \hline
   1960:1-1967:2 & 0.004 & 0.218 & 0.055 & 0.646 \\
   & (0.001) & (0.327) & (0.026) & (0.360) \\
   1967:3-1997:4 & 0.000 & 0.158 & -0.008 & 0.831 \\
   & (0.001) & (0.162) & (0.009) & (0.158) \\
   \hline
   \end{array}
   \]

2. Two breaks in the reduced form (1973:1 and 1980:4)
   (a) $x_t$ is labor income share
   \[
   \begin{array}{cccc}
   \mu \text{ (const)} & \gamma (\pi_{t-1}) & \kappa (x_t) & \beta (E_t\pi_{t+1}) \\
   \hline
   1960:1-1967:2 & -0.002 & -0.358 & 0.014 & 0.970 \\
   & (0.003) & (0.248) & (0.013) & (0.230) \\
   1967:3-1997:4 & -0.003 & 0.243 & 0.024 & 0.723 \\
   & (0.002) & (0.081) & (0.011) & (0.075) \\
   \hline
   \end{array}
   \]

   (b) $x_t$ is the GDP gap
   \[
   \begin{array}{cccc}
   \mu \text{ (const)} & \gamma (\pi_{t-1}) & \kappa (x_t) & \beta (E_t\pi_{t+1}) \\
   \hline
   1960:1-1967:2 & 0.001 & -0.215 & 0.004 & 0.716 \\
   & (0.002) & (0.302) & (0.022) & (0.394) \\
   1967:3-1997:4 & 0.000 & 0.251 & -0.007 & 0.729 \\
   & (0.000) & (0.083) & (0.007) & (0.078) \\
   \hline
   \end{array}
   \]

Notes for Tables 4-8:

1. In parentheses below the coefficient estimates are the heteroskedasticity robust estimates of the standard errors.

2. The Sup-F test uses a 10% trimming. *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively, using asymptotic critical values. In brackets are the significance level using the fixed regressors bootstrap method of Hansen (2000).

3. The confidence intervals of the estimates of the break date are based on the two-sided 95% nominal level symmetric method described in Bai (1997).