Optimal traffic grooming for
wavelength-division-multiplexing rings
with all-to-all uniform traffic

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We consider the problem of traffic grooming in wavelength-division-multiplexing rings with all-to-all uniform traffic. Our objective is to
minimize the total number of electronic add–drop multiplexers (ADMs)
required. We derive explicit optimal solutions for two special cases: one
with a traffic granularity of 4 and the other with 16. When the traffic
grainularity is equal to 4, we also show that the minimum number of
ADMs can be achieved either with the minimum number of wavelengths,
a result that was conjectured in earlier research [J. Lightwave Technol.
18, 2 (2000)], or with ADMs uniformly placed among nodes. Our results
here are among the few in which analytically tractable optimal solutions
are obtained for the traffic grooming problem. Our solutions provide
insight as well as valuable tools for evaluating other approximate and
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1. Introduction

Wavelength division multiplexing (WDM) is now being widely used for expanding
capacity in optical networks. In this paper we consider the traffic grooming problem
for WDM ring networks with all-to-all uniform traffic. In a WDM network each fiber
link can carry high-rate traffic at many different wavelengths; thus multiple channels
can be created within a single fiber. At each node of a WDM ring, electronic add–
drop multiplexers (ADMs) are used to multiplex lower-rate traffic circuits onto
individual wavelengths with higher rates. The number of lower-rate traffic circuits
that can be multiplexed onto a single wavelength is called the traffic granularity. For
example, 64 OC-3 (155 Mbit/s) circuits can be multiplexed onto a single wavelength
with capacity OC-192 (10 Gbit/s), and in this case the traffic granularity is 64 (OC
represents optical carrier). Since the cost of ADMs often makes up a significant
portion of the total cost for a WDM ring, one of the most important issues in the
design of a WDM ring network is deciding how to multiplex lower-rate traffic circuits
onto different wavelengths so that the number of ADMs required is minimized. This
problem is referred to as the traffic grooming problem. In general, an ADM for a
wavelength is needed at a particular node only when the wavelength is dropped
at the node (i.e., when a circuit originating from the node is multiplexed onto
the wavelength), and it is not needed when the wavelength only passes through
the node. There are two types of ring network: unidirectional path-switched ring
(UPSR) and bidirectional line-switched ring (BLSR). An excellent introduction to
UPSR and BLSR can be found in Ref. 1. In this paper we consider only UPSR.

The problem of traffic grooming for WDM rings has been studied by several
researchers.2–14 In this paper we focus on WDM rings with all-to-all uniform traffic.
The all-to-all traffic model is one in which a traffic circuit exists between each pair
of nodes and all circuits have the same size. WDM rings with such a traffic model have been considered in Refs. 4, 7, 9, 13, and 14, in which various heuristic algorithms have been developed and in some cases bounds are established on the number of ADMs and wavelengths required. Our research in this paper is closely related to Ref. 4. We derive optimal traffic grooming solutions for two special cases: one with traffic granularity of 4 and the other with 16. For the first case our optimal solution in fact achieves the lower bound provided in Ref. 4. Furthermore, we show that the minimum number of ADMs can be achieved with the minimum number of wavelengths, a result that is first conjectured in Ref. 4. In addition, we show that the minimum number of ADMs can also be achieved with ADMs evenly distributed among nodes. When the traffic granularity is 16, we are able to prove that our solutions are optimal only for rings with less than 15 nodes. We believe that our analysis can be extended; however, operation could become extremely tedious and cumbersome for rings with 15 nodes or more. On the basis of our solutions, we propose a lower bound on the number of ADMs required, which is better than that provided in Ref. 4. Finally, we discuss whether it is still possible in this case to achieve the minimum number of ADMs with the minimum number of wavelengths. Our analysis suggests that it may not be.

Arguably, the all-to-all traffic model is often not realistic in most applications. The significance of our research is that it provides analytically tractable optimal solutions, which so far are almost nonexistent for the traffic grooming problem (an exception is found in Ref. 4 where the optimal solutions are derived for rings with hubbed uniform traffic). The analytical solutions provide us with valuable insight. For example, by comparing the optimal solutions for the two cases, we can see how the traffic granularities would affect the minimum number of ADMs required: If the traffic size is doubled (equivalently, the traffic granularity is reduced by half), then the minimum number of ADMs does not need to be doubled; in fact, it needs to increase by only \(\sim 50–60\%\). In addition, the optimal solutions provided here can be used to evaluate approximate and heuristic algorithms. Without optimal solutions (or at least good bounds on optimal solutions), such evaluation would be an extremely difficult task.

The rest of the paper is organized as follows. In Section 2 we introduce the traffic grooming problem and some basic definitions and terminologies. In Section 3 we consider the case in which the traffic granularity is 4, and in Section 4 we consider the case with traffic granularity of 16. In Section 5 we compare solutions for the two cases along with those for traffic granularity of 1 and 64. Finally, conclusions are given in Section 6.

2. Traffic Grooming Problem

Consider a WDM ring with \(N\) nodes, labeled 1, 2, \ldots, \(N\) clockwise. Throughout this paper we assume that the ring has all-to-all uniform traffic; i.e., we need to establish a traffic circuit between every pair of nodes, and all the circuits have the same size. Let \(g\) be the traffic granularity of the ring, which is defined as the total number of low-rate traffic circuits that can be multiplexed onto a single wavelength. For example, if each circuit is OC-3 and the wavelength capacity is OC-48, then the traffic granularity is 16. Although here we assume that there is only one circuit between each pair of nodes, we can easily deal with the case of multiple circuits. In fact, it is shown in Ref. 8 that as long as the number of circuits between each pair of nodes is divisible by \(g\), which is the case in our problem, then there is an optimal solution in which all the circuits between each pair of nodes are multiplexed onto the same wavelength. Therefore the multiple circuits between each pair of nodes
can be combined into one circuit, albeit with a higher rate.

In the traffic grooming problem of interest here, we need to decide where to place necessary ADMs for different wavelengths in the ring. In general, an ADM for a wavelength is needed at a particular node only when the wavelength is dropped at the node, and it is not needed when the wavelength only passes through the node.

It is clear that for the ring with $N$ nodes and all-to-all traffic there are a total of $N(N - 1)/2$ circuits. Therefore the minimum number of wavelengths needed is $\lceil N(N - 1)/2g \rceil$. However, we should point out that in general the minimum number of ADMs may not be achieved with the minimum number of wavelengths (e.g., see Refs. 4 and 14). In Ref. 4 it is conjectured that for the all-to-all uniform traffic model the minimum number of ADMs can be achieved with the minimum number of wavelengths. In Section 3 we prove that the conjecture is true for $g = 4$. However, our analysis in Section 4 suggests that for $g = 16$ it is likely that the conjecture does not hold.

Before closing this section, we introduce some terminology. We say that a wavelength covers a circuit (or a circuit is covered by a wavelength) if the circuit is multiplexed onto the wavelength. Similarly, we say that a wavelength covers a node (or a node is covered by a wavelength) if the wavelength needs to be dropped at the node. So if a wavelength covers a circuit, then it also covers the two end nodes associated with the circuit. Also, if a wavelength covers a node, then an ADM for the wavelength is needed at the node. Finally, we call a wavelength an $n$-node wavelength if it covers $n$ nodes.

### 3. Case of $g = 4$

Let $A^*$ denote the minimum number of ADMs required. We first present a lower bound on $A^*$, which is provided in Ref. 4.

**Lemma 1.** For $g = 4$, $A^* \geq N(N - 1)/2$.

In what follows we show that the above lower bound can in fact be achieved.

**Theorem 1.** For $g = 4$, $A^* = N(N - 1)/2$ when $N \geq 5$.

With Lemma 1 we need to show only that there exists a solution that needs only $N(N - 1)/2$ ADMs. Our proof provides a procedure on whose basis the optimal solution can be obtained. The basic idea of the proof is to divide nodes into different groups and then selectively combine some of them together. A similar idea is also used in Ref. 4 to develop one of the heuristic algorithms there.

**Proof.** In Ref. 15 explicit solutions with $N(N - 1)/2$ ADMs are obtained for $N \leq 11$ (because of space limitations, these solutions have to be left out here). So we need to prove only the result for $N \geq 12$. We use induction and assume that the result is true for rings with $<N$ nodes. For $N(\geq 12)$ we consider the following four cases (where $m \geq 3$):

1. $N = 4m$. Divide the nodes into two groups $V_1$ and $V_2$, each with $2m$ nodes. It is clear that the circuits can be divided into three groups based on $V_1$ and $V_2$: (i) the circuits with both end nodes in $V_1$, (ii) the circuits with both end nodes in $V_2$, and (iii) the cross circuits with one end node in $V_1$ and the other one in $V_2$. From the inductive assumption we can find a solution for the circuits in (i) with $m(2m - 1)$ ADMs. We can do the same for the circuits in (ii). Furthermore, we group the nodes in $V_i$ ($i = 1, 2$) into $m$ pairs of nodes. If we choose two pairs of nodes, one from $V_1$ and the other from $V_2$, and put them...
together, there are $m^2$ different combinations. Each combination contains four nodes with four cross circuits among them. These four cross circuits can be covered by a four-node wavelength. Therefore we can cover all the cross circuits between $V_1$ and $V_2$ by using $m^2$ four-node wavelengths, with a total of $4m^2$ ADMs. Putting these circuits together, we have a solution that needs a total of $2m(2m - 1) + 4m^2$ ADMs. Note that $2m(2m - 1) + 4m^2 = N(N - 1)/2$.

2. $N = 4m + 1$. Divide the nodes into three groups: $V_0$ with one node, $V_1$ with $2m$ nodes, and $V_2$ with $2m$ nodes. The circuits can also be divided into three groups: (i) the circuits with both end nodes in $V_0 \cup V_1$, (ii) the circuits with both end nodes in $V_0 \cup V_2$, and (iii) the cross circuits with one end node in $V_1$ and the other one in $V_2$. Again, using the inductive assumption, we can derive a solution with $m(2m + 1)$ ADMs for all the circuits in (i) (note that there are $2m + 1$ nodes in $V_0 \cup V_1$). The same is also true for the circuits in (ii). Using the same argument as in case 1, we can show that the cross circuits in (iii) can be covered by $m^2$ four-node wavelengths with $4m^2$ ADMs. Putting the three groups of circuits together, we have a solution with $2m(2m + 1) + 4m^2 = N(N - 1)/2$ ADMs.

3. $N = 4m + 2$. Divide the nodes into two groups: one with $2m$ nodes ($V_1$) and the other one with $2m + 2$ nodes ($V_2$). As with case 1, we can show that there exists a solution that requires $m(2m + 1) + (m + 1)(2m + 1) + 4m(m + 1) = N(N - 1)/2$ ADMs.

4. $N = 4m + 3$. Divide the nodes into three groups: the first one with one node ($V_0$), the second one with $2m$ nodes ($V_1$), and the third one with $2m + 2$ nodes ($V_2$). Similar to with case 2, we can show that there is a solution in which the number of ADMs required is $m(2m + 1) + (m + 1)(2m + 3) + 4m(m + 1) = N(N - 1)/2$. This completes the proof. \(\square\)

In Ref. 4 it is conjectured that for rings with all-to-all uniform traffic the minimum number of ADMs can be achieved with the minimum number of wavelengths. In what follows, we prove that the conjecture is true for $g = 4$.

**Theorem 2.** For $g = 4$ the minimum number of ADMs, $N(N - 1)/2$, can be achieved with the minimum number of wavelengths, which is equal to $\lceil N(N - 1)/8 \rceil$.

**Proof.** This proof is similar to that for Theorem 1. First, the result holds for $N \leq 11$ (see explicit solutions provided in Ref. 15). For $N = 12$ we can divide the nodes into two groups, each with six nodes, and obtain a solution with 66 ADMs and 17 wavelengths; hence the result holds. For $N \geq 13$ we again use induction, and we assume that the result is true for rings with $<N$ nodes. We consider the following two cases (where $m \geq 1$):

1. $N = 8m + k$, where $k = 0, 1, 2, 3, 4$. For $k = 0, 2, 4$ we divide the nodes into two groups: $V_1$ with $8(m - 1)$ nodes and $V_2$ with $8 + k$ nodes. For $k = 1, 3$ we divide the nodes into three groups: $V_0$ with one node, $V_1$ with $8(m - 1)$ nodes, and $V_2$ with $7 + k$ nodes. The rest of the proof is essentially similar to that of Theorem 1. In what follows, we consider only $k = 0, 2, 4$, for illustration. From the inductive assumption we can find one solution for the circuits in $V_1$ with $4(m - 1)(8m - 9)$ ADMs and $(m - 1)(8m - 9)$ wavelengths and another solution for the circuits in $V_2$ with $(8 + k)(7 + k)/2$ ADMs and $[(8 + k)(7 + k)/2]$ wavelengths. The cross circuits between $V_1$ and $V_2$ can be covered by $2(m - 1)(8 + k)$ four-node wavelengths with $8(m - 1)(8 + k)$ ADMs, as demonstrated
in the proof of Theorem 1. Putting the three solutions together, we have a solution with $4(m−1)(8m−9)+(8+k)(7+k)/2+8(m−1)(k+8) = N(N−1)/2$ ADMs and $(m−1)(8m−9)+2(m−1)(8+k)+[(8+k)(7+k)/8] = [N(N−1)/8]$ wavelengths. Hence the result holds.

2. $N = 8m + k$, where $k = 5, 6, 7$. For $k = 6$ we divide the nodes into two groups: $V_1$ with $8m$ nodes and $V_2$ with $k$ nodes. For $k = 5, 7$ we divide the nodes into three groups: $V_0$ with one node, $V_1$ with $k−1$ nodes, and $V_2$ with $8m$ nodes. Again, the rest of the proof is the same as in Theorem 1.

In addition to the minimum number of wavelengths, the following result shows that the minimum number of ADMs can also be achieved with all the ADMs uniformly distributed among the nodes.

**Theorem 3.** For $g = 4$ the minimum number of ADMs can be achieved with ADMs uniformly placed among the nodes in the sense that (1) when $N$ is an odd number, each node has $(N−1)/2$ ADMs and (2) when $N$ is an even number, half the nodes have $N/2$ ADMs and the other half have $N/2−1$ ADMs.

**Proof.** Again, the proof is similar to that of Theorem 1. Assuming that the result holds for rings with less than $N$ nodes, we consider three different cases.

1. $N = 4m$. We divide the nodes and the circuits in the same way as we did for case 1 of Theorem 1. From the inductive assumption, we can find a solution for the circuits in (i) with $m(2m−1)$ ADMs that are evenly distributed among $V_1$; i.e., half the nodes in $V_1$ have $m$ ADMs, and the other half have $m−1$ ADMs. The same solution can be found for the circuits in (ii). Furthermore, the circuits in (iii) can be covered by $m^2$ wavelengths with $4m^2$ ADMs that are evenly distributed among all the nodes (i.e., each node has $m$ ADMs). When three groups of ADMs are put together, it is clear that in this solution half the nodes have $2m$ ADMs, whereas the other half have $2m−1$.

2. $N = 4m + 2$. We divide the nodes into two groups, one with $2m$ nodes ($V_1$) and the other one with $2m + 2$ nodes ($V_2$), and then use the same argument as in case 1.

3. $N$ is an odd number. In this case we divide the nodes into three groups: $V_0$ (with one node), $V_1$ (with $N−3$ nodes), and $V_2$ (with two nodes). From the inductive assumption, we have a solution for all the circuits within $V_0 ∪ V_1$ such that each node in $V_0 ∪ V_1$ has $(N−3)/2$ ADMs. We can then use a three-node wavelength to cover the three circuits between nodes in $V_0 ∪ V_2$ and $(N−3)/2$ four-node wavelengths to cover the cross circuits between $V_1$ and $V_2$, which results in $(N−1)/2$ ADMs for the two nodes in $V_2$ and one additional ADM for each node in $V_0 ∪ V_1$. Therefore we have a solution in which each node needs $(N−1)/2$ ADMs. This completes our proof.

From the proofs for Theorems 2 and 3, it is quite clear that the two optimal solutions for achieving the minimum number of wavelengths and for having ADMs evenly distributed among nodes are usually different. This is particularly true when $N$ is an odd number, in which case one may need to have many three-node wavelengths for even distribution of ADMs among nodes. This in turn can significantly increase the total number of wavelengths needed. In some applications the number of ADMs that can be placed at each node is limited; hence distributing ADMs evenly among nodes would be more advantageous.
4. Case of $g = 16$

For $g = 4$ we derived the optimal traffic grooming solutions and showed that they can be achieved with either the minimum number of wavelengths or ADMs evenly distributed among nodes. Finding optimal solutions for $g = 16$ proves to be much more difficult. Instead of trying to find optimal solutions for any $N$, we focus our attention on some special cases of $N$. From the mixed integer linear programming (MILP) formulation developed in Ref. 8, we first obtain the best possible solutions we can find for $N \leq 20$ by using the cplex software package (for solving linear programming problems). We then show for $N \leq 14$ that these solutions are indeed optimal. Although our proof on optimality can potentially be extended for larger values of $N$, we will not attempt to do it here, since it could become extremely tedious. In addition to verifying the optimality of our solutions, more importantly, the proof itself provides some insight on the structural properties of the optimal solutions. We also provide a tight lower bound on the minimum number of ADMs required and develop a heuristic algorithm that can produce good solutions.

In Table 1 we first present the number of ADMs required for the best possible solutions that we were able to find on the basis of the MILP formulation provided in Ref. 8 and the cplex software package (because of space limitations, we choose not to present detailed solutions here). We should point out that we were able to use cplex to verify the optimality of these solutions only for very small values of $N$ ($N \leq 10$; the optimality of a solution provided by cplex is verified when cplex terminates by itself). For large values of $N$ (e.g., $N = 14$) cplex kept running even after days; so we had to terminate the program manually, and the solution obtained at the time of termination is the best possible one that we could find.

<table>
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<th>$N$</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
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<th>17</th>
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<th>19</th>
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<td>14</td>
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<td>20</td>
<td>26</td>
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<td>54</td>
<td>62</td>
<td>70</td>
<td>78</td>
<td>88</td>
</tr>
</tbody>
</table>

In what follows we prove that the numbers provided in Table 1 are indeed the minimum numbers of ADMs for $N \leq 14$. However, we need to caution the reader that for $15 \leq N \leq 20$ there is no guarantee that the numbers in Table 1 are the minimum numbers, though we strongly believe that they are very close to, if not equal to, the minimum numbers (see Ref. 8 for testing examples).

**Theorem 4.** For $N \leq 14$ the numbers of ADMs presented in Table 1 are equal to the minimum numbers of ADMs required.

Clearly, to prove that the numbers of ADMs provided in Table 1 represent the minimum numbers of ADMs required, we need to show that they are lower bounds as well. We should point out that for $N \leq 10$ the conclusion has already been verified by the cplex program, and in theory, if we had more computation power, it could be used for large $N$ as well. However, the conclusion provides little insight. Our analytical proof reveals more about the structural properties of the optimal solutions, which, for example, can be used to construct near-optimal solutions for large $N$. We should emphasize that although we provide the optimality proof only for $N \leq 14$, it can potentially be extended to larger $N$. Because of space limitations, here we provide only a summary of the proof, and its details can be found in Ref. 15.
Summary of the proof for Theorem 4. Essentially, we need to consider each $N \leq 14$ individually. We can show that lower bounds on the minimum numbers of ADMs are achieved by traffic grooming solutions with the following:

- For $N = 7$, three nodes need one ADM and four nodes need two ADMs.
- For $N = 8$, two nodes need one ADM and six nodes need two ADMs.
- For $N = 9$, all nine nodes need two ADMs.
- For $N = 10$, all ten nodes need two ADMs.
- For $N = 11$, seven nodes need two ADMs and four nodes need three ADMs.
- For $N = 12$, four nodes need two ADMs and eight nodes need three ADMs.
- For $N = 13$, three nodes need two ADMs and ten nodes need three ADMs.
- For $N = 14$, one node needs two ADMs and thirteen nodes need three ADMs.

Clearly, these lower bounds are the same as the numbers provided in Table 1 for $N \leq 14$.

As with $g = 4$ we can observe from the above summary that there exists an optimal solution in which ADMs are allocated uniformly among nodes. In fact, this uniform property has also been observed for larger $N$. However, we are unable to prove it in general. Hence we present it in the following conjecture.

**Conjecture 1.** For $g = 16$ there exists an optimal solution in which ADMs are uniformly allocated among the nodes; i.e., if we let $k = \lfloor A^*/N \rfloor$ (recall that $A^*$ is the minimum number of ADMs required), then there are $A^* - kN$ nodes that need $k + 1$ ADMs and $(k + 1)N - A^*$ nodes that need $k$ ADMs.

We now establish a good lower bound on $A^*$. As pointed out in Ref. 4, the most efficient way of multiplexing circuits for $g = 16$ is to use a six-node wavelength to cover 15 circuits; i.e., the maximum average number of circuits that can be supported per ADM is 2.5. Therefore $A^* \geq \lceil N(N-1)/5 \rceil$. However, it is clear that $\lceil N(N-1)/5 \rceil$ is not a tight lower bound (in most cases there is a 10% gap between it and the solutions provided in Table 1). We note that the second most efficient way of multiplexing circuits is to use a seven-node wavelength to cover 16 circuits (16/7 circuits per ADM), and the third most efficient way is to use either an eight-node wavelength to cover 16 circuits or a five-node wavelength to cover 10 circuits (2 circuits per ADM). Furthermore, it is in general impossible to use only six-node wavelengths to cover all circuits except when $N = 6$. In fact, we have observed that on average one cannot improve upon seven-node wavelengths, i.e., 16/7 circuits per ADM. We are unable to prove that 16/7 is the maximum ratio; so we present the following conjecture:

**Conjecture 2.** For $g = 16$, $A^* \geq \lceil 7N(N-1)/32 \rceil$.

We can easily verify that this lower bound is satisfied by all the solutions provided in Table 1, and it is very tight (in some cases it in fact coincides with the optimal solutions). Since for $g = 16$ we are unable to obtain analytical tractable optimal solutions as for $g = 4$, in what follows we propose an algorithm for obtaining a class of feasible solutions. The algorithm is similar to the second heuristic algorithm developed in Ref. 4; however, our algorithm gives better solutions (see examples below). The basic idea of the algorithm is to use solutions for small rings as building blocks to construct solutions for large rings. To help reinforce how the algorithm works, let us consider the following eight cases (where $m \geq 1$):
1. $N = 8m$. Divide nodes into $m$ groups, $V_1, V_2, \ldots, V_m$, each with eight nodes. For 28 circuits within each group we have a grooming solution, which requires 14 ADMs (see Table 1). For 64 cross circuits between $V_i$ and $V_j$ ($i, j = 1, 2, \ldots, m, i \neq j$) they can be covered with 4 eight-node wavelengths. This leads to a solution with $2m(8m - 1)$ ADMs.

2. $N = 8m + 1$. Divide nodes into $m + 1$ groups: $V_0$ with one node and $V_i$ with eight nodes ($i = 1, 2, \ldots, m$). From Table 1 we have a solution with 18 ADMs for circuits within each group $V_0 \cup V_i$ ($i = 1, 2, \ldots, m$), and we can also cover cross circuits between $V_i$ and $V_j$ ($i, j = 1, 2, \ldots, m, i \neq j$) as in case 1. Hence we have a solution with $2m(8m + 1)$ ADMs.

3. $N = 8m + 2$. Divide nodes into $m + 1$ groups: $V_0$ with two nodes and $V_i$ with eight nodes ($i = 1, 2, \ldots, m$). From Table 1 we have a solution with 20 ADMs for circuits within each group $V_0 \cup V_i$ ($i = 1, 2, \ldots, m$), and we can do the same for cross circuits, which gives us a solution with $4m(4m + 1)$ ADMs.

4. $N = 8m + 3$. Divide nodes into $m + 1$ groups: $V_0$ with three nodes and $V_i$ with eight nodes ($i = 1, 2, \ldots, m$). As with case 2, a solution with $2m(8m + 5)$ ADMs can be obtained.

5. $N = 8m + 4$. Divide nodes into $m + 1$ groups: $V_0$ with four nodes and $V_i$ with eight nodes ($i = 1, 2, \ldots, m$). We have a solution with 32 ADMs for circuits within $V_0 \cup V_1$ (12 nodes) and a solution with 14 ADMs for circuits within $V_i$ ($i = 2, \ldots, m$). As for cross circuits, we need 6 eight-node wavelengths for those between $V_0 \cup V_i$ and $V_i$ ($i = 2, \ldots, m$) and 4 eight-node wavelengths for those between ($i, j = 2, \ldots, m, i \neq j$). Therefore we have a solution with $16m^2 + 14m + 2$ ADMs.

6. $N = 8m + 5$. Divide nodes into $m + 2$ groups: $V_0$ with one node, $V_1$ with four nodes, and $V_i$ with eight nodes ($i = 2, \ldots, m + 1$). We have a solution with 36 ADMs for circuits within $V_0 \cup V_1 \cup V_2$ (13 nodes) and a solution with 18 ADMs for circuits within $V_0 \cup V_i$ ($i = 2, \ldots, m + 1$). When we combine with eight-node wavelengths for cross circuits, we have a solution with $16m^2 + 18m + 2$ ADMs.

7. $N = 8m + 6$. Divide nodes into $m + 2$ groups: $V_0$ with two nodes, $V_1$ with four nodes, and $V_i$ with eight nodes ($i = 2, \ldots, m + 1$). As with case 6, we can derive a solution with $16m^2 + 20m + 5$ ADMs.

8. $N = 8m + 7$. Divide nodes into $m + 2$ groups: $V_0$ with three nodes, $V_1$ with four nodes, and $V_i$ with eight nodes ($i = 2, 3, \ldots, m + 1$). As with case 6, we can derive a solution with $16m^2 + 26m + 4$ ADMs.

The above procedure is flexible and can be easily improved. For example, instead of dividing nodes into 8-node groups, we can divide them into 12-node groups, which can lead to better solutions. In fact, the procedure works as long as the number of nodes is divisible by 4, and the larger the number of nodes in each group, the better solutions it produces. The heuristic algorithm developed in Ref. 4 essentially divides nodes into four-node groups; hence our algorithm provides better solutions. For example, for $N = 23$ our algorithm produces a solution with 120 ADMs, whereas the one produced in Ref. 4 has 131 ADMs. Also, it is worth pointing out that the numbers of ADMs produced by our algorithm are close to those in Table 1.
Finally, we examine whether we can still achieve the minimum number of ADMs with the minimum number of wavelengths for \( g = 16 \). It is obvious that if in the optimal solution all the wavelengths used were six-node, then the number of wavelengths needed would be larger than the minimum number, since a six-node wavelength can cover at most only 15 circuits. In fact, if there were more than 16 six-node wavelengths contained in a solution, then the number of wavelengths needed would exceed the minimum number. In general, our numerical results indicate that the number of six-node wavelengths in an optimal solution would increase as \( N \) increases. If this is indeed true, then the number of six-node wavelengths would eventually exceed 16 when \( N \) is large enough. This seems to suggest that for \( g = 16 \) the minimum number of ADMs is not likely to be achieved with the minimum number of wavelengths in general.

5. Effect of Traffic Granularity on Add–Drop Multiplexers

In this section we use the results obtained in Sections 3 and 4 to examine the effect of the traffic granularity \( g \) on the minimum number of ADMs required in ring networks. Table 2 summarizes the results derived in Sections 3 and 4. In addition, we also include the number of ADMs required for \( g = 1 \) and \( g = 64 \). For \( g = 1 \) it is quite clear that the minimum number of ADMs is simply equal to \( N(N - 1)/2 \), the number of circuits. For \( g = 64 \) the solutions were obtained on the basis of the MILP formulation in Ref. 8.

From the results in Table 2 one can observe that as the traffic granularity quadruples in size, the number of ADMs required is reduced by approximately half. In other words, when the size of the circuit quadruples, the number of ADMs required approximately doubles. We call this phenomenon economics of scale; i.e., when one increases the size of circuits (or the number of circuits) offered, one can reduce the unit cost (by \( \sim 40\text{–}50\% \)). Clearly, further study on this issue is needed, which is beyond the scope of the current paper.

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<th>( g = 4 )</th>
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6. Conclusion

We have studied the traffic grooming problem for UPSR rings with all-to-all uniform traffic. Two cases are considered: $g = 4$ and $g = 16$. For $g = 4$ we provided the optimal solution and also proved that the minimum number of ADMs can be achieved with either the minimum number of wavelengths or ADMs evenly distributed among nodes. For $g = 16$ we derived the optimal solution for rings with less than 15 nodes and developed a heuristic algorithm for larger rings. There are still some open issues for $g = 16$. Also, it would be interesting to extend the results in this paper to more-general cases. These are future research directions.

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References and Links
