1. Consider a Walrasian economy with $L$ goods, $I$ identical households each with excess demand function $z(p)$, and $J$ identical firms each with a constant returns to scale production set $Y \in \mathbb{R}^L$. Show that the Walrasian equilibrium aggregate production vector $\Sigma_{j=1}^J y^j$ is unique.

2. Verify the Slutsky equation for excess demand function $Z_i(p, W_i)$ of a given household $i$ whose wealth equals the sum of the value of its commodity endowments $p, \omega_i$ and lumpsum income $W_i$:

$$\frac{\partial Z_{il}}{\partial p_k} = S_{lk}^i - Z_{ki} \frac{\partial Z_{ik}}{\partial W_i}$$

where $p_k$ denotes the price of good $k$, and $S_{lk}^i$ denotes the Slutsky substitution effect between goods $l$ and $k$. (You can invoke the relevant Slutsky equation for the optimal consumption demand of household $i$.)

3. Suppose that each household $i$ has a homothetic utility function, resulting in unit income elasticity of demand for every good (i.e., its optimal consumption demand vector can be expressed as $X_i = x_i(p)[p, \omega_i + W_i]$). If in addition if each household’s endowment vector $\omega_i$ is proportional to the economy wide endowment vector $\omega \equiv \sum_I \omega_i$ (i.e., there exists a scalar $\alpha_i \in (0, 1)$ such that $\omega_i = \alpha_i \omega$; then (using (1)) show that the Jacobian matrix of the aggregate excess demand function of the economy is negative definite at every price vector.

**Additional (Optional) Problem**

4. Consider an economy with an excess demand function $Z(p; q)$ for the first $L-1$ goods, where $p$ is the vector of prices of these goods relative to the $L$-th good, and $q$ is a real number that enters as an exogenous parameter in the demand function. Suppose that this function is continuously differentiable in both $p$ and $q$, and that its Jacobian with respect to $p$ is negative definite at every $p, q$. Suppose in addition that at some parameter value $q^*$,
\[ \frac{\partial Z_k(p, q^*)}{\partial q} \] is strictly positive for some commodity \( k = l \), and zero for all other commodities \( k \neq l \). Show that a local increase in \( q \) from \( q^* \) must increase the equilibrium price of commodity \( l \).