PROBLEM SET NO. 2

1. Apply the Index Theorem to show that competitive equilibrium must be unique in an economy where every household $i$ has a utility function of the form $U_i = \phi_i(x_{1i}, x_{2i}, \ldots, x_{L-1,i}) + x_{Li}$, where $\phi_i$ is a strictly monotone and strictly concave function.

2. Consider an exchange economy with $L$ commodities where for every household $i$ there is a good $l(i)$ on which household $i$ spends all its income. Under what additional conditions will competitive equilibrium in this economy be unique?

3. Consider an intertemporal borrowing and lending economy with $T$ dates and a single consumption good, where a forward market operates at $t = 0$ for delivery of the good at any of the subsequent dates $t = 1, \ldots, T$. Household $i$ has an endowment $\omega_i > 0$ of the consumption good at date $t$ and utility function $U_i = \sum_{t=1}^{T} \delta^{t-1} \frac{c_{it}^{1-\rho}}{1-\rho}$, where $\rho \in (0, 1)$. In the forward market each household operates with an intertemporal budget constraint $\sum_{t=1}^{T} p_t (c_{it} - \omega_i) \leq 0$. Show that this economy has a unique competitive equilibrium (Hint: check that excess demand functions satisfy the gross substitute property).