Solutions to Problem Set No. 3

(a) Graph the Lorenz curves before and after the change and check that they cross. (Plot cumulative income shares of successive deciles, i.e., the poorest 10%, 20%, 30% etc. of the population, and connect successive points with a straight line).

See Graph 1.

(b) Calculate the Gini coefficient and coefficient of variation of the two distributions. Do the inequality rankings offered by the two measures agree? Can you explain why?

Per capita income before and after investment are:

\[
\mu_1 = \frac{1}{n} \sum_{j=1}^{m} n_j y_j = \frac{7,000,000 \times 1,000 + 3,000,000 \times 2,000}{10,000,000} = 1300\text{(dollars)}
\]

\[
\mu_2 = \frac{1}{n} \sum_{j=1}^{m} n_j y_j = \frac{5,000,000 \times 1,000 + 5,000,000 \times 2,000}{10,000,000} = 1500\text{(dollars)}
\]

The Gini coefficient before and after investment are:
\[ G_1 = \frac{1}{2n^2 \mu_1} \sum_{j=1}^{m} \sum_{k=1}^{m} n_j n_k |y_j - y_k| \]
\[ = \frac{(7,000,000 \times 3,000,000)|1,000 - 2,000| + (3,000,000 \times 7,000,000)|2,000 - 1,000|}{2 \times 10,000,000^2 \times 1300} \]
\[ (\text{When } j = k, |y_j - y_k| = 0) \]
\[ = 0.16 \]
\[ G_2 = \frac{1}{2n^2 \mu_2} \sum_{j=1}^{m} \sum_{k=1}^{m} n_j n_k |y_j - y_k| \]
\[ = \frac{(5,000,000 \times 5,000,000)|1,000 - 2,000| + (5,000,000 \times 5,000,000)|2,000 - 1,000|}{2 \times 10,000,000^2 \times 1500} \]
\[ (\text{When } j = k, |y_j - y_k| = 0) \]
\[ = 0.17 \]

The coefficient of variation before and after investment are:
\[ C_1 = \frac{1}{\mu_1} \sqrt{\sum_{j=1}^{m} \frac{n_j}{n} (y_i - \mu_1)^2} = \frac{\sqrt{0.7(1000 - 1300)^2 + 0.3(2000 - 1300)^2}}{1300} \]
\[ = 0.35 \]
\[ C_2 = \frac{1}{\mu_2} \sqrt{\sum_{j=1}^{m} \frac{n_j}{n} (y_i - \mu_2)^2} = \frac{\sqrt{0.5(1000 - 1500)^2 + 0.5(2000 - 1500)^2}}{1500} \]
\[ = 0.33 \]

So these two measures give different answers to the question whether inequality rises or falls. The two Lorenz curves cross each other, which is why they give different answers. As a consequence, it is difficult to form any judgment about how inequality has changed.

(c) Suppose the poverty line is $1500 per month. Compute the head-count ratio, poverty gap ratio and income gap ratio before and after the change.
Head count ratio before and after the change are:

\[ HCR_1 = \frac{7,000,000}{10,000,000} = 0.7 \]
\[ HCR_2 = \frac{5,000,000}{10,000,000} = 0.5 \]

Poverty gap ratio before and after are:

\[ PGR_1 = \frac{\sum_{y_i < p} (p - y_i)}{n\mu_1} = \frac{7,000,000 \times (1500 - 1000)}{10,000,000 \times 1300} = 0.27 \]
\[ PGR_2 = \frac{\sum_{y_i < p} (p - y_i)}{n\mu_2} = \frac{5,000,000 \times (1500 - 1000)}{10,000,000 \times 1500} = 0.17 \]

Income gap ratio before and after are:

\[ IGR_1 = \frac{\sum_{y_i < p} (p - y_i)}{pHC_1} = \frac{7,000,000 \times (1500 - 1000)}{7,000,000 \times 1500} = 0.33 \]
\[ IGR_2 = \frac{\sum_{y_i < p} (p - y_i)}{pHC_2} = \frac{5,000,000 \times (1500 - 1000)}{5,000,000 \times 1500} = 0.33 \]

(d) Provide a brief verbal assessment of the development experienced by this economy as a result of the new investments in the modern sector, in light of the above facts.

After investment, 2 million people’s income increase by $1000. As a result, per capita income increases. But it is unclear how inequality has changed. Intuitively, it has increased the gap between those who moved out of the rural sector and those who remained there. But on the other hand it reduced the gap between those who moved and those who were in the urban sector to start with. It is difficult to say whether inequality as a whole has gone up or not. The Gini coefficient and the coefficient of variation give different answers.

Poverty decreases, as measured by the head count ratio: fewer people live below the poverty line after the change. The faller poverty gap ratio says that a smaller fraction of the economy’s resources are needed to eliminate poverty via redistributive transfers. The income gap ratio remains constant, on the other hand: the extent of poverty among those below the poverty line has not changed.