On the Generic (Im)possibility of Full Surplus Extraction in Mechanism Design*

Aviad Heifetz† Zvika Neeman‡

This Version: July 2004

Abstract

A number of studies, most notably Crémer and McLean (1985, 1988), have shown that in Harsanyi type spaces of a fixed finite size, an uninformed principal can generically design mechanisms that extract all the surplus from privately-informed players. In contrast, we show that within the set of common priors on the universal type space, the subset of priors that permit the extraction of the players' full surplus is non-generic: It is a proper face of the convex set of priors, with an infinite co-dimension. In particular it is shy. Shyness is a notion of smallness for convex subsets of infinite-dimensional topological vector spaces (in our case, the set of common priors), which generalizes the usual notion of zero Lebesgue measure in finite-dimensional spaces.

Journal of Economic Literature Classification numbers: D44, D82, H41.

Keywords: Surplus extraction, information rents, mechanism design, private information, universal type space, genericity, prevalence.

*Acknowledgements to be added.

†Department of Economics and Management, The Open University of Israel; URL http://www.openu.ac.il/Personal_sites/Aviad-Heifetz/; Email aviadhe@openu.ac.il.

‡Department of Economics, Boston University, and The Center for Rationality and Department of Economics, The Hebrew University of Jerusalem; URL http://people.bu.edu/zvika; Email zvika@bu.edu

1 Introduction

Does relevant private information typically confer a positive economic rent to those who possess it? Surprisingly, the answer given by the literature to this question is negative. A number of studies, including, most notably, Crémer and McLean (1985, 1988), have shown that under standard assumptions – the existence of a common prior, a fixed finite number of types, risk neutrality, and no limited liability – an uninformed principal facing privately-informed players can generically implement any decision rule he could implement were that private information accessible to him (d'Aspremont et al. 2003, Kosenok and Severinov 2004). An uninformed seller, for example, should generically be able to extract the full surplus of any number of privately-informed bidders in an auction. As these "full-surplus-extraction" results imply that the players' private information is (generically) irrelevant, they have been said to "cast doubt on the value of the current mechanism design paradigm as a model of institutional design" (McAfee and Reny, 1992, p. 400).

Since full-surplus-extraction results make heavy use of the assumption that the type spaces have a fixed finite size, it is natural to ask to which extent this assumption is crucial for obtaining these results. This assumption is problematic because there is no a priori finite bound on the number of types needed for modeling a situation involving asymmetric information. Indeed, the universal type space (Mertens and Zamir, 1985) that embeds all such models is infinite, and its consistent subspaces can be arbitrarily large. Supposing that any common prior on this universal type space could just as well serve as a plausible model of a situation involving asymmetric information, is it "typically" the case that full-surplus-extraction is possible? This is the question addressed in this paper.[1] The other two important assumptions necessary for full surplus extraction, namely, risk neutrality and no limited liability, are maintained throughout the analysis.

If beliefs are endowed with the minimal topology that allows for the formulation of each player's beliefs about the state of nature and the beliefs of other players (the topology of weak convergence), then the set of common priors with finite support is dense in the space of all common priors[2]. Combining this observation with the results of Crémer and McLean (1985, 1988) for finite type spaces implies that the set of priors that permit full-surplus-extraction is dense in the space of all possible common priors.

Recently, Neeman (2004) showed that full-surplus-extraction is possible only if the type space has the "beliefs determine preferences" property, which requires that almost every possible belief of every player about other players' types is associated (with probability one)

[1] The reason we confine our attention to subspaces with a common prior is twofold. First, confining attention to such consistent subspaces (in the sense of Harsanyi 1967-68) of the universal type space and their associated common priors is standard practice in information economics. Indeed, this practice, which has been called the Harsanyi doctrine, is considered by some to be a hallmark of Bayesian rationality (see the discussion in Aumann 1998, Gul 1998 and Morris 1995). Second, the universal type space has a product structure – it includes all the possible combinations of players' beliefs and private signals. Hence, as should become clear below when the terms are defined, beliefs do not determine preferences in the universal type space, and consequently full-surplus-extraction is generically impossible there. Thus the question is settled in this case.

[2] Mertens, Sorin and Zamir (1994), p. 156, thm. 3.1.

with a unique valuation or private signal of the player. Neeman (2004) also showed that arbitrarily close to any consistent finite type space, there is another consistent finite type space in which beliefs do not determine preferences, and consequently full surplus cannot be extracted. It thus follows that both the set of priors that allow for full-surplus-extraction (henceforth, FSE priors), *and* its complement – the set of priors that do not allow for full-surplus-extraction (henceforth, NFSE priors), are *dense* in the space of all finite-support priors, and hence also in the space of all priors. In particular, with the topology of weak convergence, neither of these sets is open and dense.

However, just as both the rationals and the irrationals are dense in the set of real numbers although the set of irrationals is larger in other senses (cardinality, Lebesgue measure), it is also conceivable that one of the subsets of common priors above is larger than the other in some meaningful sense.

One may think of two general approaches that may permit such a sharper result. First, it may be argued that the topology of weak convergence on priors is not the natural topology to apply in a strategic setting. Intuitively, two priors can be said to be "close" if and only if they induce "similar" equilibrium behavior.[3] Indeed, preliminary results (Kajii and Morris, 1998) suggest that such a notion of strategic proximity may induce a stronger topology on the set of priors. However, a characterization of strategic proximity in terms of beliefs is not yet available. If and when such a characterization is obtained, then it may turn out that the resulting stronger topology renders one of the subsets of common priors above both open and dense, while its complement would not be dense. The results that are obtained in this paper imply that the set of FSE priors would not be open in such a stronger topology, and so only the complementary set of NFSE remains a candidate to become open and dense.

Second, it is also possible to consider non-topological notions of genericity, such as the measure-theoretic notion of full Lebesgue measure. However, the notion of full Lebesgue measure cannot be applied directly because it can meaningfully capture the idea of a set being "large" only in finite-dimensional spaces. The space of common priors on the universal type space (or even the smaller space of common priors with a finite support) is not only infinite but also infinite-dimensional. It is therefore necessary to consider a measure-theoretic notion of genericity that can be applied in infinite-dimensional topological vector spaces. Such an appropriate notion, called *prevalence,* was originally conceived by Christensen (1974) and Hunt et al. (1992) and further developed by Anderson and Zame (2001) as an extension of the idea of full Lebesgue measure to infinite-dimensional spaces. The complement of a prevalent set is called *shy.* The collection of shy sets in a finite-dimensional space is identical to the collection of sets with Lebesgue measure zero. In an infinite-dimensional space, shy sets retain the properties of zero-probability events: no open set is shy, and the collection of shy sets is closed under subsets, under translations, and under countable unions.

We show that the subset of FSE priors is shy within the set of common priors on the universal type space. It therefore follows that the complement of the set of FSE priors, or the subset of NFSE priors, is generic. The same result also obtains if attention is restricted to the subset of priors with finite-support.

[3] See Morris (2002) for such an argument and Dekel et al. (2004) for two possible definitions.

In fact, the set of FSE priors is non-generic even in a more direct and intuitive sense: It is a proper face of the convex body of priors that has an infinite co-dimension, i.e. this set "misses" infinitely many dimensions of the entire set of common priors.

The proof of this result is based on the following argument. Only mixtures of FSE priors yield FSE priors, while a mixture of a NFSE and any other prior yields a NFSE prior. This asymmetry "in favor" of the NFSE priors delivers the result. What makes this result mathematically non-trivial is the fact that the set of FSE priors is nevertheless dense (with the topology of weak convergence) in the set of all priors on the universal space.

The rest of the paper proceeds as follows. For simplicity, we begin in the next section with the consideration of the classic problem of a seller of an object who designs an auction for n risk neutral bidders with private valuations, with the goal of maximizing his expected revenue. In Section 3, we explain how our results can be applied to any mechanism design problem with interdependent types. Section 4 surveys the related literature, and section 5 concludes. Proofs are relegated to the appendix.

2 Surplus Extraction in Single Object Auctions with Private Values

We consider the problem of a seller who wishes to design an auction that would maximize the expected revenue he obtains by selling some object to one out of a set of n risk neutral bidders with private valuations for the object. Each bidder or player may refuse to participate in the seller's auction, but if she agrees to participate, then she is bound by the outcome of the auction.

Let $N = \{1, ..., n\}$ denote the set of bidders or players. Bidder i's valuation or willingness to pay for the object is denoted by $v_i \in V_i$. The set of bidder i's valuations V_i is assumed to be a complete, separable, metric space (in particular, V_i may be finite). The payoff to a bidder with valuation v_i who wins the object with probability q and who pays an expected amount m is given by $q \cdot v_i - m$. We refer to v_i as bidder i's *preference* or *preference type.* Let $V \equiv V_1 \times \cdots \times V_n$. The set V is the *basic space of uncertainty* for this problem.

The behavior of the bidders in the auction may obviously depend on their willingness to pay for the object. It therefore follows that the bidders' behavior may also depend on their beliefs about other bidders' willingness to pay, because such beliefs possibly convey important information about the way other bidders will behave in the auction. For the same reason, beliefs about beliefs are also important, and so are beliefs about beliefs about beliefs, and so on, ad infinitum. A complete analysis of the seller's problem therefore requires a model that allows for the specification of the bidders' entire infinite hierarchy of beliefs about beliefs about beliefs ... about whatever is relevant in the auction. Such infinite hierarchies of beliefs may be conveniently encoded in what is known as a *type space.*

2.1 Type Spaces

Bidder i's private information is captured by its type $\theta_i \in \Theta_i$. The sets of bidders' types Θ_i, $i \in N$, are assumed to be complete, separable, metric spaces. For every space X, let $\Delta(X)$ denote the space of probability measures over X. Each type $\theta_i \in \Theta_i$ is associated with a *preference type* $\widehat{v}_i(\theta_i) \in V_i$ which describes θ_i's willingness to pay for the object, and with a *belief-type* $\widehat{b}_i(\theta_i) \in \Delta(\Theta_{-i})$ which is a probability measure on the space of other bidders' types $\Theta_{-i} \equiv \prod_{j \neq i} \Theta_j$. The space of probability measures $\Delta(\Theta_{-i})$ is endowed with the topology of weak convergence.

Each type of each bidder is assumed to know its own willingness to pay for the object and its own beliefs. Since we focus our attention in this section on a private values model, each type θ_i's preference type $\widehat{v}_i(\theta_i)$ is defined independently of θ_i's belief type $\widehat{b}_i(\theta_i)$. This assumption is relaxed in the next section.[4]

A product space $\Theta \equiv \prod_{i \in N} \Theta_i$ of the players' type spaces is called a private values type space. Each profile of types $\theta \in \Theta$ is called a state of the world.

2.2 The Private Values Universal Type Space

Given the basic space of uncertainty $V \equiv V_1 \times \cdots \times V_n$ and the set of bidders N, there exists[5] a *private values universal type space*

$$T^{PV} = \prod_{i \in N} T_i^{PV}$$

into which every other private values type space can be mapped in a beliefs-preserving way. That is, for every type space $\Theta \equiv \prod_{i \in N} \Theta_i$ there exists a unique set of measurable mappings[6]

$$\mathbb{E}_i : \Theta_i \to T_i^{PV}, \quad i \in N$$

satisfying

$$\widehat{v}_i(\mathbb{E}_i(\theta_i)) = \widehat{v}_i(\theta_i)$$

and

$$\widehat{b}_i(\mathbb{E}_i(\theta_i))(A) = \widehat{b}_i(\theta_i)\left(\mathbb{E}_{-i}^{-1}(A)\right)$$

for every measurable set $A \subseteq T_{-i}^{PV}$, where

$$\mathbb{E}_{-i} : \Theta_{-i} \to T_{-i}^{PV}$$

is defined by

$$\mathbb{E}_{-i}\left((\theta_j)_{j \neq i}\right) = (\mathbb{E}_j(\theta_j))_{j \neq i}.$$

[4] The assumption that each type knows its own belief is captured by defining $\widehat{b}_i(\theta_i)$ as a probability measure over Θ_{-i} rather than over $\Theta_i \times \Theta_{-i}$. The implied presumption about the bidders' introspective ability is standard, and is maintained throughout the paper.

[5] The proof of existence follows from a slight adaptation of the arguments contained in Mertens and Zamir (1985), Brandenburger and Dekel (1993), or Heifetz (1993).

[6] which are in fact also continuous

The universal type set T_i^{PV} of bidder $i \in N$ is isomorphic to the product space

$$V_i \times \Delta\left(T_{-i}^{PV}\right)$$

by the mapping

$$\tau_i \to \left(\widehat{v}_i(\tau_i), \widehat{b}_i(\tau_i)\right).$$

Thus in what follows we use the terms T_i^{PV} and $V_i \times \Delta\left(T_{-i}^{PV}\right)$ interchangeably.

Finally, for the rest of this section we drop the superscript "PV" from the notation whenever there is no risk of confusion.

2.3 Priors

A probability measure p_i on a private values type space $\Theta = \prod_{i \in N} \Theta_i$ is called a prior for bidder i if bidder i's belief-types $\widehat{b}_i(\theta_i)$ are the posteriors of p_i. That is, p_i is a prior for bidder i if for every real-valued continuous function $\varphi : \Theta \to \mathbb{R}$

$$\int_{\Theta_i} \left(\int_{\Theta_{-i}} \varphi\left(\theta_i, \tilde{\theta}_{-i}\right) d\widehat{b}_i(\theta_i)\left(\tilde{\theta}_{-i}\right) \right) dp_{i|\Theta_i}(\theta_i) = \int_{\Theta} \varphi(\theta)\, dp_i(\theta) \qquad (1)$$

where $p_{i|\Theta_i}$ is the marginal of p_i on Θ_i.

A probability measure p on Θ is called a common prior, or prior for short, if it is a prior for every bidder $i \in N$. The support of a prior p is called a Harsanyi-consistent subspace.

It is immediate from the definition that the set of priors for bidder i is convex: If $p_i, p_i' \in \Delta(\Theta)$ are two priors for bidder i, then so is $\alpha p_i + (1-\alpha) p_i'$ for every $\alpha \in [0,1]$. It follows that the set of common priors is also convex.

Since every private values type space can be embedded in the private values universal type space, no loss of generality is implied by restricting attention to priors on the private values universal type space T. We thus take the set of priors on the private values universal space, denoted $\mathcal{P}^{PV}$, to be the set of relevant "environments" for our study.

Definition 1. A prior $p \in \Delta(T)$ satisfies the *Beliefs-Determine-Preferences* (BDP) property[7] for bidder $i \in N$ if there exists a measurable subset $T_i^p \subseteq T_i$ such that the marginal $p_{|T_i}$ of p on T_i assigns probability 1 to T_i^p,

$$p_{|T_i}\left(T_i^p\right) = 1,$$

and no pair of distinct types $\tau_i \neq \tau_i'$ in T_i^p hold the same beliefs –

$$\hat{b}(\tau_i) \neq \hat{b}(\tau_i')$$

for every two different types $\tau_i, \tau_i' \in T_i^p$.

[7] The notion of beliefs-determine-preferences generalizes the one in Neeman (2004) and is closely related to Bergemann and Morris (2003) one-to-one property and to d'Aspremont et al.'s (2003, 2004) notion of no free beliefs.

A prior p that satisfies the beliefs-determine-preferences property for bidder i is called a BDP prior for bidder i. A prior p that is a BDP prior for every bidder $i \in N$ is called a BDP prior. Since any pair of distinct types $\tau_i \neq \tau_i'$ in the private values universal space differ either by their belief-type or by their preference-type,[8] there is no pair of distinct types in T_i^p who hold identical beliefs but different preferences. In other words, a prior p satisfies the BDP property for bidder i if there exists a p-probability 1 set $V^i \times B_i^p$ where $B_i^p \subseteq \Delta(T_{-i})$, and a function that maps bidder i's beliefs to its willingness to pay

$$\Phi_i^p : B_i^p \to V_i$$

such that T_i^p is isomorphic to the graph $\left\{\left(\widehat{b}_i(\tau_i), \widehat{v}_i(\tau_i)\right) : \tau_i \in T_i^p\right\}$ of Φ_i^p.

By the revelation principle, no loss of generality is implied by assuming that the seller employs an incentive compatible and individually rational "direct revelation" auction game or mechanism $\langle q_i : T \to [0,1], m_i : T \to [0,1]\rangle_{i \in N}$ in which each bidder i is asked to report its type $\tau_i \in T_i$, and then to participate in a lottery in which it pays an amount $m_i(t)$, and wins the object with probability $q_i(t)$.

A mechanism $\langle q_i, m_i\rangle_{i \in N}$ is incentive-compatible (IC) if every type $\tau_i \in T_i$ of every bidder $i \in N$ maximizes its expected payoff by truthfully reporting its type, or

$$\int_{T_{-i}} \left(q_i(\tau_i, \tilde{\tau}_{-i}) \hat{v}_i(\tau_i) - m_i(\tau_i, \tilde{\tau}_{-i})\right) d\hat{b}_i(\tau_i)(\tilde{\tau}_{-i}) \geq \int_{T_{-i}} \left(q_i(\tau_i', \tilde{\tau}_{-i}) \hat{v}_i(\tau_i) - m_i(\tau_i', \tilde{\tau}_{-i})\right) d\hat{b}_i(\tau_i)(\tilde{\tau}_{-i}) \quad \text{(IC)}$$

for every $\tau_i' \in T_i$.

A mechanism $\langle q_i, m_i\rangle_{i \in N}$ is individually-rational if every type $\tau_i \in T_i$ of every bidder $i \in N$ prefers to participate in the mechanism than to opt out, or

$$\int_{T_{-i}} \left(q_i(\tau_i, \tilde{\tau}_{-i}) \hat{v}_i(\tau_i) - m_i(\tau_i, \tilde{\tau}_{-i})\right) d\hat{b}_i(\tau_i)(\tilde{\tau}_{-i}) \geq 0. \quad \text{(IR)}$$

Definition 2. A prior p permits the *full-surplus-extraction* from a set $K \subseteq N$ of bidders if there exists an incentive compatible and individually rational mechanism $\langle q_i, m_i\rangle_{i \in N}$ that generates an expected payment to the seller that is equal to the full surplus generated by the bidders in K –

$$\sum_{i \in K} \int_T m_i(\tau)\, dp(\tau) = \int_T \max_{i \in K} \{\hat{v}_i(\tau_i)\}\, dp(\tau).$$

A prior that permits the full-surplus-extraction from the set K of bidders is called a full-surplus-extraction (FSE) prior for K.

[8] If there were two distinct types $\tau_i \neq \tau_i'$ with the same preferences and beliefs in the private values universal type space, then the beliefs-preserving mappings of type spaces into the universal space would not have been unique, thus contradicting the definition of the universal type space.

Remark 1 (FSE implies that IR must be binding). Fix a prior p. In order to extract the full surplus from the bidders in $K \subseteq N$ in the environment described by p, in p-almost every state of the world $\tau \in T$, the seller must sell the object to the bidder in K who has the highest willingness to pay for it at τ at an expected price that is equal to this bidder's valuation of the object. The individual rationality constraint implies that the seller cannot sell the object to any bidder in K at τ for a higher price, and selling the object for a lower price implies a failure to extract the full surplus that is generated by the bidders in K. It therefore follows that the individual rationality constraint must be binding for any type $\tau_j \in T_j$ of a bidder in K that wins the object with a positive probability under a mechanism $\langle q_i, m_i \rangle_{i \in N}$ that extracts the full surplus of the bidders in K.

Remark 2 (FSE from different sets of players). The fact that a prior may permit the extraction of the full surplus from the set N of bidders does not imply that it is possible to extract the full surplus of each single bidder. For example, if there are two bidders and it is commonly known that bidder 1's willingness to pay for the object is strictly lower than that of player 2's, then it is possible to extract the full surplus of the set of bidders $\{1, 2\}$ if and only if it is possible to extract the full surplus of bidder 2 alone. However, if it is possible to extract the full surplus of each single bidder, then it is also possible to extract the full surplus of the set N of bidders.

Next, we show that BDP is necessary for full-surplus-extraction. Specifically, we show that if a prior p permits the extraction of bidder i's full surplus p-almost surely, then p is a BDP prior for player i.

Proposition 1. A prior p that is a FSE prior for bidder i is a BDP prior for bidder i.

2.4 Genericity

A natural definition of genericity is that of full Lebesgue measure. Unfortunately, there is no direct analog for Lebesgue measure in infinite dimensional spaces. Unlike the Lebesgue measure in a finite-dimensional Euclidian space $\mathbb{R}^k$, which is spread uniformly across the space, in infinite-dimensional spaces there does not exist any (sigma-additive) translation invariant measure. For example, in an infinite-dimensional separable Banach space, any open ball of radius $r > 0$ contains an infinite sequence of disjoint open balls of radius $\frac{r}{4}$, so if a translation-invariant measure were to assign a positive measure to these balls, then the r-ball would be assigned an infinite measure for any $r > 0$.[9] Therefore, probabilities or measures in infinite-dimensional spaces are not satisfactory devices for determining whether events are "typical" or not.

[9] Furthermore, confining attention to full-support *quasi-invariant* measures, which preserve null-sets under translations (such as the Gaussian measures on the Euclidean spaces), is unhelpful either. Under fairly general conditions, if there does not exist a non-trivial full support invariant measure on an infinite-dimensional space, then neither does there exist such a quasi-invariant measure (see, e.g., Yamasaki, 1985).

Recently a general notion of largeness, which coincides with full Lebesgue measure in finite-dimensional spaces, has been proposed. An event E in a finite-dimensional Euclidian space $\mathbb{R}^k$ has Lebesgue measure zero if and only if there exists a positive measure μ on $\mathbb{R}^k$ such that E and all its translations $\{x+y : x \in E\}$, $y \in \mathbb{R}^k$, have μ-measure zero. Christensen (1974) and Hunt, Sauer, and York (1992) have relied on this observation and defined a Borel subset of a complete metric topological vector space to be *shy* if there exists a positive measure μ on the space such that the set and all its translations have μ-measure zero. They called the complement of a shy set *prevalent*. They showed that shy sets satisfy all the properties one would expect "small" or "negligible" events to satisfy. Namely, a subset of a shy set is shy, every translation of a shy set is shy, a countable union of shy sets is shy, and no open set is shy.

Anderson and Zame (2001) have adapted Christensen (1974) and Hunt et al.'s (1992) definition to the case in which the relevant parameter set is a convex subset C of a topological vector space X. Since we are interested in determining the genericity of the set of FSE priors relative to the convex set of priors on the universal type space, this is the definition which is appropriate for our purpose.

It turns out that for our analysis it is not necessary to rely on Anderson and Zame's general definition of shyness, but rather on a simpler and stronger notion called "finite shyness." Let λ_H denote the Lebesgue measure on a finite-dimensional subspace $H \subseteq X$.

Definition 3. (Anderson and Zame, 2001) A universally measurable[10] subset $E \subseteq C$ is *finitely shy* in $C \subseteq X$ if there exists a finite-dimensional subspace $H \subseteq X$ such that $\lambda_H(C+p) > 0$ for some $p \in X$ and $\lambda_H(E+x) = 0$ for every $x \in X$. An arbitrary subset $F \subseteq X$ is finitely shy in C if it is contained in a finitely shy universally measurable set.

Anderson and Zame (2001) show that if a set E is finitely shy in C then it is also shy in C. A subset $Y \subseteq C$ is said to be prevalent in C if its complement $C \backslash Y$ is shy in C.[11]

Example 1. The set of everywhere differentiable concave functions is finitely shy in the cone of all concave functions (though it is of second-category)[12] – see Anderson and Zame (2001).

Example 2. Both the subspaces of purely atomic measures and purely non-atomic measures are finitely shy in the space of all measures on a Euclidian space (with any topology in which these subspaces are Borel, or more generally, universally measurable) – see Stinchcombe

[10] A subset $E \subseteq X$ is universally measurable if it is measurable with respect to the completion of every regular Borel probability measure on X.

[11] In their definition, Anderson and Zame required the convex subset $C \subseteq X$ to be completely metrizable, but as they mention in a footnote, the definition makes sense even without this requirement, which is needed only for establishing some enhanced properties of shyness and prevalence (e.g., that if E is prevalent in F and F is prevalent in G then E is prevalent in G). This additional requirement is not needed for establishing the basic desiderata that a subset of a shy set is shy, that every translation of a shy set is shy, that a countable union of shy sets is shy, and that no open set is shy.

[12] See Remark 3 at the end of this subsection

(2001). The subspace of non-atomic measures is of second category in the space of all measures with the topology of weak convergence, but when the topology is strengthened to that of the total variation norm, the complement of this subspace (the measures with some atoms) becomes open and dense.

The fact that no invariant measure exists on infinite-dimensional vector spaces prevents the notion of prevalence from satisfying *all* the properties that full Lebesgue measure satisfies in finite-dimensional spaces. For example, there could be a subset E of an infinite-dimensional space $Y \times Z$ such that the sections $\{z \in Z : (y, z) \in E\}$ are shy in Z for every $y \in Y$, while E itself is not shy in $Y \times Z$ (Anderson and Zame 2001, example 4). This, of course, is impossible if $Y \times Z$ is finite dimensional.

The sets that we identify as non-generic in our analysis are small also in a stronger and more straightforward sense. Each such set is contained in a *proper face* of the convex body (of priors), and moreover the face in which they are contained has an infinite co-dimension. That is, it "misses" infinitely many dimensions of that convex body.

Definition 4. (Rockafellar, 1970) A convex subset F of a convex set C is called a face if whenever $f \in F$ is a convex combination of $x, y \in C$, then $x, y \in F$.

Definition 5. A face F of C is called a proper face when F is a proper subset of C. The co-dimension of F in C is the dimension of the minimal subspace Y of X such that C is contained in the subspace spanned by F and Y.

Under fairly general conditions, it can be shown that a proper face of a convex set is finitely shy in the set. A proper face of a convex set is also *null*. In a recent paper, Perry and Reny (2003) define a set A to be null if it is a countable union of finitely shy sets $\{A_n\}_{n=1}^{\infty}$, where for each n there exists a *one-dimensional* subspace $H_n = \{\alpha x_n : \alpha \in \mathbb{R}\}$ such that $\lambda_{H_n}(A_n + x) = 0$ for every $x \in X$.

Lemma 1. Let C be a convex subset of the topological vector space X. Suppose that X is endowed with a topology that satisfies the following property: for every $c \neq c' \in C$ and $A \subseteq \mathbb{R}$ the one-dimensional set

$$\{\alpha (c - c') : \alpha \in A\}$$

is a Borel subset of X if and only if A is a Borel subset of $\mathbb{R}$. Let F be a Borel set which is a proper face of C. Then F is finitely shy in C and null.

Remark 3 (Alternative notions of genericity). There are two distinct concepts concerning the pervasiveness of a property or a set. One concept concerns directly the size of the set. In Euclidian spaces, this is captured by the set having full Lebesgue measure, and its complement having Lebesgue measure zero. A property that obtains in a set of full Lebesgue measure is called "generic." Inter alia, a set of full Lebesgue measure is dense in the ambient Euclidian space. The second concept concerns the persistence or robustness of the property which describes the set. This concept is captured by the requirement that the set be open.

A hybrid concept combines the previous two to require that a set be both open and dense to be considered "large." This hybrid topological notion does not coincide with largeness in the sense of size, because there are open and dense subsets of a Euclidian space with arbitrarily small Lebesgue measure. This topological notion is sometimes stretched further, so that sets of the second category, namely, sets that contain a countable intersection of open and dense sets, are also considered "large."

The fact that, as mentioned above, there is no analog for Lebesgue measure in infinite dimensional spaces, has led some researchers to use the notion of second category as a notion of largeness in infinite-dimensional spaces. Sets of the second category do indeed satisfy some desiderata that every notion of pervasiveness should satisfy: Sets of the first category, which are the complements of sets of the second category are (1) closed under countable unions, (2) a subset of a first-category set is of first-category, (3) a translation of a first-category set is of first-category, and (4) no open set is of first-category. However, these desiderata are only necessary, not sufficient, properties for largeness in the sense of size.[13] Indeed, there are subsets of the Euclidian space that are of second-category but have Lebesgue measure zero.[14]

2.5 FSE Priors are Non-Generic

In this section we show that the set of FSE priors, denoted $\mathcal{F}$, is a proper face of the set of priors on the private values universal type space, $\mathcal{P}$, and that $\mathcal{F}$ has an infinite co-dimension in $\mathcal{P}$. In particular, $\mathcal{F}$ is finitely shy in $\mathcal{P}$ by lemma 1.

Positive multiples of priors in $\mathcal{P}$ constitute a convex cone of (positive) measures. Taking the differences of pairs of such measures yields the vector space of signed measures that are generated by $\mathcal{P}$, denoted $\mathcal{M}$. We assume that the topological vector space $\mathcal{M}$ is endowed with a topology that satisfies the following two properties: (1) the mappings

$$\begin{aligned} (p, p') &\rightarrow p + p' \\ (\alpha, p) &\rightarrow \alpha p \end{aligned}$$

are continuous in $p, p' \in \mathcal{P}$ and $\alpha \in \mathbb{R}$; and (2) a subset $A \subseteq \mathbb{R}$ is Borel if and only if for every pair of priors $p, p' \in \mathcal{P}$ the one-dimensional set of weighted averages

$$\{\alpha p + (1 - \alpha) p' : \alpha \in A\} \tag{2}$$

is a Borel subset of $\mathcal{M}$. These two properties are satisfied by a large variety of topologies on $\mathcal{M}$, including the topology of weak convergence and the topology of the total variation norm, but not by extremely strong topologies such as the totally disconnected topology in

[13]For Example, Mas-Colell (1985, p. 318) writes: "Although the topological notion of genericity discriminates a bit more than merely asserting the density of a given property, it has to be thought of as much less sharp than the measure theoretic concept available in the finite-dimensional case."

[14]For example, let $r_1, r_2, \ldots$ be an enumeration of the rationals in R, and let $A_k = \bigcup_{n=1}^{\infty} \left(r_n - \frac{1}{k2^{n+1}}, r_n + \frac{1}{k2^{n+1}} \right)$. Then A_k is open and dense but has Lebesgue measure $\frac{1}{k}$, and $\bigcap_{k=1}^{\infty} A_k$ is of second category but has zero Lebesgue measure.

which every subset of $\mathcal{M}$ is open. The result below applies to any metric topology on $\mathcal{M}$ which satisfies the two properties above and that is also at least as strong as the topology of weak convergence.[15]

Theorem 1. The set $\mathcal{B}$ of BDP priors for bidder i is a proper face of the convex set $\mathcal{P}$ of priors on the private values universal type space. Moreover, $\mathcal{B}$ has an infinite co-dimension in $\mathcal{P}$.

Corollary 1. The set $\mathcal{B}$ of BDP priors is finitely shy and null in the set of common priors $\mathcal{P}$.

Corollary 2. The set $\mathcal{F}$ of Full-Surplus-Extraction priors for player i is contained in a proper face with infinite co-dimension within the set of common priors $\mathcal{P}$. In particular, $\mathcal{F}$ is finitely shy and null in $\mathcal{P}$.

Remark 4 (Priors with a finite support). Inspection of the argument presented in this section reveals that it also implies that the set $\mathcal{F}_f$ of FSE priors with a finite support (finite-support priors) for player i is a proper face of infinite co-dimension (and hence finitely shy and null) in the space of all finite-support private values priors $\mathcal{P}_f$ on the universal space T. The proofs apply verbatim.

2.6 Approximate Full Surplus Extraction and Robust Surplus Extraction

Until now we have addressed the issue of full surplus extraction. It remains an open question whether or not it is generically possible to approximate full surplus extraction. We conjecture, but have been so far unable to prove, that for every $\varepsilon > 0$, both the set of priors in which it is possible to extract at least $1-\varepsilon$ of the available surplus, and the set of priors in which it is impossible to extract at least $1-\varepsilon$ of the available surplus, are not small in the sense that neither of them is shy.[16]

A conceptually distinct question concerns the robustness of the extent of surplus extraction. Suppose that for a given prior p the principal has designed an optimal mechanism μ_p that extracts as much surplus as possible. If it turns out that the principal has mis-specified the prior slightly, would the mechanism μ_p extract nearly as much of the surplus as could be extracted with the correct prior? We conjecture that the answer to this question is negative, and furthermore that the extent of surplus extraction by a fixed optimal mechanism

[15] In particular, if a topology of strategic proximity as discussed in the introduction belongs to this range of topologies, then our result implies that the shy subset of FSE priors cannot be open in that topology. Hence, only the set of NFSE priors could be both open and dense in such a topology.

[16] For the case of public good provision, Neeman (2004) describes an example where if beliefs do not determine preferences, then the probability that the public good can be provided decreases to zero with the number of players while efficiency requires that the public good be provided with probability 1. It therefore follows that in such a setting the total surplus that can be extracted from the players converges to zero at the same time that the total surplus that could be generated by the players remains uniformly bounded away from zero.

is generally discontinuous in the prior. That is, we conjecture that arbitrarily close to any prior p, there exists another prior p' such that the mechanism μ_p extracts a much smaller portion of the surplus than the portion of surplus that μ_p extracts under p.[17]

Finally, it is also interesting to know how the portion of the extractable surplus varies with the prior p, when the mechanism is allowed to vary optimally with the prior. Since the set of FSE priors is dense (even though, as we showed, it is non-generic), the portion of the extractable surplus is discontinuous at any NFSE prior. The percentage of the surplus that can possibly be extracted as a function of the prior is thus discontinuous "almost everywhere" (i.e., on a prevalent set of priors). We conjecture that the extractable surplus may nevertheless be continuous at the non-generic set of environments described by the FSE priors themselves.

2.7 The Relationship to Crémer and McLean's Results

Crémer and McLean (1988) showed that within the set of models with a *fixed* finite number of types $n_i \geq 2$ for each player i (or equivalently, within the set of priors that are supported on a *fixed* finite number of types $n_i \geq 2$ for each player i), the set of priors that permit full-surplus extraction from any bidder is generic. Our argument cannot be phrased in this more limited setting, because the set of priors that are supported on a fixed finite number of types *is not convex*. For example, the mixture of the common priors that are represented by the two matrices

	$\tau_2 = (v_2, b_2)$	$\tilde{\tau}_2 = \left(\tilde{v}_2, \tilde{b}_2\right)$
$\tau_1 = (v_1, b_1)$	a	b
$\tilde{\tau}_1 = \left(\tilde{v}_1, \tilde{b}_1\right)$	c	d

	$\tau'_2 = (v_2, b'_2)$	$\tilde{\tau}'_2 = \left(\tilde{v}_2, \tilde{b}'_2\right)$
$\tau'_1 = (v_1, b'_1)$	a'	b'
$\tilde{\tau}'_1 = \left(\tilde{v}_1, \tilde{b}'_1\right)$	c'	d'

[17] Consider the sequence of common priors

$$p_n = \left(\begin{array}{|c|c|c|} \hline & v=1 & v=2-\frac{1}{n} \\ \hline v=1 & \frac{1}{3} & \frac{1}{6} \\ \hline v=2-\frac{1}{n} & \frac{1}{6} & \frac{1}{3} \\ \hline \end{array} \right).$$

For every n, the mechanism that extracts full surplus for the prior

$$p = \left(\begin{array}{|c|c|c|} \hline & v=1 & v=2 \\ \hline v=1 & \frac{1}{3} & \frac{1}{6} \\ \hline v=2 & \frac{1}{6} & \frac{1}{3} \\ \hline \end{array} \right)$$

fails to extract more than $\frac{1}{3}$ for any element of the sequence because it excludes bidders with WTP $2 - \frac{1}{n}$ in spite of the fact that total surplus on the sequence converges to $\frac{5}{3}$.

Weinstein and Yildiz (2004) present a related result. They show that unless a game is dominance-solvable, every equilibrium is highly sensitive to higher-order beliefs. Note that with the topology of weak convergence, a player's type *is* continuous in its beliefs of any given order.

(where $a+b+c+d=a'+b'+c'+d'=1$, but either $ab' \neq a'b$ or $cd' \neq c'd$) is *not* the prior that is represented by the matrix

	$\tau_2'' = (v_2, b_2'')$	$\tilde{\tau}_2'' = \left(\tilde{v}_2, \tilde{b}_2''\right)$
$\tau_1'' = (v_1, b_1'')$	$\frac{1}{2}(a+a')$	$\frac{1}{2}(b+b')$
$\tilde{\tau}_1'' = \left(\tilde{v}_1, \tilde{b}_1''\right)$	$\frac{1}{2}(c+c')$	$\frac{1}{2}(c+c')$

but rather the following prior

	$\tau_2 = (v_2, b_2)$	$\tilde{\tau}_2 = \left(\tilde{v}_2, \tilde{b}_2\right)$	$\tau_2' = (v_2, b_2')$	$\tilde{\tau}_2' = \left(\tilde{v}_2, \tilde{b}_2'\right)$
$\tau_1 = (v_1, b_1)$	$\frac{1}{2}a$	$\frac{1}{2}b$	0	0
$\tilde{\tau}_1 = \left(\tilde{v}_1, \tilde{b}_1\right)$	$\frac{1}{2}c$	$\frac{1}{2}d$	0	0
$\tau_1' = (v_1, b_1')$	0	0	$\frac{1}{2}a'$	$\frac{1}{2}b'$
$\tilde{\tau}_1' = \left(\tilde{v}_1, \tilde{b}_1'\right)$	0	0	$\frac{1}{2}c'$	$\frac{1}{2}d'$

which is supported on 8 rather than 4 states.

In particular, the mixture of the common priors that are represented by the two matrices

	$v_2 = 0$	$\tilde{v}_2 = 1$
$v_1 = 0$	$\frac{1}{2}$	0
$\tilde{v}_1 = 1$	0	$\frac{1}{2}$

	$v_2 = 0$	$\tilde{v}_2 = 1$
$v_1 = 0$	0	$\frac{1}{2}$
$\tilde{v}_1 = 1$	$\frac{1}{2}$	0

is not

	$v_2 = 0$	$\tilde{v}_2 = 1$
$v_1 = 0$	$\frac{1}{4}$	$\frac{1}{4}$
$\tilde{v}_1 = 1$	$\frac{1}{4}$	$\frac{1}{4}$

(3)

but rather

	$v_2 = 0$	$\tilde{v}_2 = 1$	$v_2 = 0$	$\tilde{v}_2 = 1$
$v_1 = 0$	$\frac{1}{4}$	0	0	0
$\tilde{v}_1 = 1$	0	$\frac{1}{4}$	0	0
$v_1 = 0$	0	0	0	$\frac{1}{4}$
$\tilde{v}_1 = 1$	0	0	$\frac{1}{4}$	0

(4)

(Notably, the average (4) preserves the fact that it is common knowledge that bidders know each other's type, while (3) does not.)

3 Implementation with Interdependent Valuations

In this section, we demonstrate how the results obtained in the previous section for an auction problem with private values can be generalized to any mechanism design problem with interdependent types. Specifically, we ask whether a given decision rule, which is a mapping from players' types into outcomes is generically implementable. Whenever possible, we rely on the notation used in the previous section.

Let $N = \{1, ..., n\}$ be a finite set of players, and $\mathcal{X}$ a measurable set of outcomes. The players' preferences over outcomes depend on the state of nature $k \in K$. The space of states of nature K is our *basic space of uncertainty.* It is assumed to be a complete, separable, metric space, that is endowed with its Borel σ-field. When the state of nature is $k \in K$, the outcome $x \in \mathcal{X}$ prevails, and player i receives a monetary transfer m_i, her payoff is given by

$$u_i(x, k) + m_i$$

where

$$u_i : \mathcal{X} \times K \to \mathbb{R}$$

is a Borel measurable utility function. The players are assumed to be expected payoff maximizers.

3.1 Type Spaces

For every player $i \in N$, the set of player i's types Θ_i is assumed to be a complete, separable, metric space. Every type $\theta_i \in \Theta_i$ is associated with a probability measure on the space of states of nature K and the other players' types $\Theta_{-i} = \prod_{j \neq i} \Theta_j$. The space of probability measures $\Delta(K \times \Theta_{-i})$ is endowed with the topology of weak convergence. With a slight abuse of notation, we say that $\Theta_i \subseteq \Delta(K \times \Theta_{-i})$.

This formulation, which implies that the uncertainty of $\theta_i \in \Theta_i$ is about $K \times \Theta_{-i}$ but not about Θ_i, captures the idea that each type has a sufficiently developed introspective ability to determine its own belief.

Type θ_i's *belief type* $\widehat{b}_i(\theta_i) \in \Delta(\Theta_{-i})$ is the marginal $\theta_{i|\Theta_{-i}}$ of the probability measure θ_i on the other players' types Θ_{-i}. Type θ_i's *preference type* $\widehat{v}_i(\theta_i)$ is any version of the expected payoff functions

$$U_i\left(\tilde{x}; \theta_i, \tilde{\theta}_{-i}\right) : \mathcal{X} \times \Theta_{-i} \to \mathbb{R}, \quad \theta_i \in \Theta_i$$

that satisfies

$$\int_{\Theta_{-i}} U_i\left(\delta\left(\theta_i, \tilde{\theta}_{-i}\right); \theta_i, \tilde{\theta}_{-i}\right) d\theta_{i|\Theta_{-i}} = \int_{K \times \Theta_{-i}} u_i\left(\delta\left(\theta_i, \tilde{\theta}_{-i}\right), \tilde{\kappa}\right) d\theta_i$$

for every measurable *decision rule* $\delta : \Theta \to \mathcal{X}$.

The type space is the product $\Theta \equiv \prod_{i \in N} \Theta_i$ of the players' type sets. Each $\theta \in \Theta$ is called a state of the world.

The private-values setting of the previous section is a particular case of the formulation described in this section. If a type θ_i is a product probability

$$\theta_i = \theta_{i|K} \times \theta_{i|\Theta_{-i}}$$

then the expected payoff functions $U_i\left(\tilde{x}; \theta_i, \tilde{\theta}_{-i}\right)$ are independent of $\tilde{\theta}_{-i}$, and can therefore be denoted $U_i(\tilde{x}; \theta_i)$.

In the setting of the single object private values auction considered in the previous section, outcomes $x = (x_1, ..., x_n)$ are given by vectors that describe the probability with which each bidder or player wins the object. Bidders' payoffs are linear in the probability with which they win the object and are independent of both other bidders' types and the probabilities with which other bidders win the object. Hence, if we let $\mathbf{1}_i$ denote the vector that has 1 in the i-th place and 0 everywhere else, then for every vector x, $U_i(x; \theta_i) = x_i \cdot U_i(\mathbf{1}_i; \theta_i)$. Moreover, because $U_i(\mathbf{1}_i; \theta_i)$ describes θ_i's utility when it wins the object for sure, which we denoted in the previous section by $\widehat{v}_i(\theta_i)$, every type's preferences can be completely described by its preference type $\widehat{v}_i(\theta_i)$.

To further illustrate the definition, consider now the case of a single object pure common value auction with two bidders. In this case $k \in K$ is the true value of the object. Suppose that bidder 1 knows the value k with certainty; that bidder 2 has no private information, only a belief which specifies the probabilities p_k of the potential values of k; and that all of this is common knowledge among the bidders. Then bidder 2 has a single type $\bar{\theta}_2$. Bidder 1's types have the form[18]

$$\theta_1^k = \delta_k \times \delta_{\bar{\theta}_2}, \quad k \in K,$$

respectively. Bidder 1's preference and belief types are therefore given by

$$\begin{aligned} \widehat{v}_1\left(\theta_1^k\right) &= U_1\left(x; \theta_1^k, \bar{\theta}_2\right) = x_1 k \\ \widehat{b}_1\left(\theta_1^k\right) &= \delta_{\bar{\theta}_2} \end{aligned}$$

The unique type $\bar{\theta}_2$ of bidder 2 is a probability measure over $K \times \Theta_1$, which assigns probability p_k to the combination $\left(k, \theta_1^k\right)$ for $k \in K$, and zero probability to any other combination in $K \times \Theta_1$. The preference and belief types of $\bar{\theta}_2$ are given by

$$\begin{aligned} \widehat{v}_2\left(\bar{\theta}_2\right)\left(x, \theta_1^k\right) &= U_2\left(x; \bar{\theta}_2, \theta_1^k\right) = x_2 k \\ \widehat{b}_2\left(\bar{\theta}_2\right)\left(\theta_1^k\right) &= p_k \end{aligned}$$

Thus bidder 2's preference type depends non-trivially on bidder 1's type θ_1^k.

3.2 The Universal Type Space

Given the basic space of uncertainty K and the set of players N, there exists a *universal type space*

$$T = \prod_{i \in N} T_i$$

into which every other type space T can be uniquely mapped in a beliefs-preserving way (Mertens and Zamir, 1985; Brandenburger and Dekel, 1993; Heifetz, 1993). That is, for every type space Θ there exists a unique set of measurable mappings[19] $(\mathbb{E}_i : \Theta_i \to T_i)_{i \in N}$ satisfying

$$\mathbb{E}_i(\theta_i)(A) = \theta_i\left(\mathbb{E}_{-i}^{-1}(A)\right)$$

[18] δ denotes the unit-mass probability measure.

[19] which are in fact also continuous.

for every measurable $A \subseteq K \times T_{-i}$, where

$$\mathbb{E}_{-i} : K \times \Theta_{-i} \to K \times T_{-i}$$

is defined by

$$\mathbb{E}_{-i}\left(k, (\theta_j)_{j \neq i}\right) = \left(k, (\mathbb{E}_j(\theta_j))_{j \neq i}\right)$$

It turns out that in the universal type space for player i, T_i is isomorphic with $\Delta(K \times T_{-i})$ (and not just with a subset of it) for every $i \in N$. We therefore refer to T_i and $\Delta(K \times T_{-i})$ interchangeably.

The private-values universal type space T^{PV} described in the previous section is a subset of the universal type space T that is presented here in the special case in which $K = \prod_{i \in N} V_i$. It is the subset of T in which it is commonly known that each player i's types $\tau_i \in T_i$, $i \in N$, have a product form as follows

$$\tau_i = \tau_{i|V_i} \times \tau_{i|V_{-i} \times T_{-i}}.$$

The definition of a common prior on the universal space is the same as in section 2.3 above. The space of environments of interest is the set of priors $\mathcal{P}$ on the universal type space T.

3.3 EDR Priors are Non-Generic

In section 2 we have considered private-values environments of a particular kind, in which the private preferences of a player could be represented by a one-dimensional valuation. All the results about the genericity of full surplus extraction obtained in Section 2 also obtain under the general (quasi-linear) setup defined at the beginning of this section. We now proceed to consider the implementation of general decision rules.

Definition 6. A prior p with support T_p permits the implementation of a decision rule $\delta : T_p \to X$ if there exists an incentive compatible and individually rational direct revelation mechanism $\left\langle \delta, (m_i)_{i \in N} \right\rangle$ where $m_i : T_p \to R$ denotes the payment to player i as a function of the players' types.[20] A prior that permits the implementation of every decision rule is called an EDR prior.

Definition 7. A prior p with support T_p permits the full extraction of the players surplus relative to a decision rule $\delta : T_p \to X$ if there exists an incentive compatible and individually

[20] A direct revelation mechanism $\left\langle \delta, (m_i)_{i \in N} \right\rangle$ is incentive compatible if

$$\int_{T_{-i}} \left(U_i\left(\delta\left(\tau_i, \tilde{\tau}_{-i}\right)\right) + m_i\left(\tau_i, \tilde{\tau}_{-i}\right)\right) d\hat{b}_i\left(\tau_i\right)\left(\tilde{\tau}_{-i}\right) \geq \int_{T_{-i}} \left(U_i\left(\delta\left(\tau_i', \tilde{\tau}_{-i}\right)\right) + m_i\left(\tau_i', \tilde{\tau}_{-i}\right)\right) d\hat{b}_i\left(\tau_i\right)\left(\tilde{\tau}_{-i}\right)$$

for every player $i \in N$, and player i's types τ_i, τ_i' in the support of p. It is individually-rational if

$$\int_{T_{-i}} \left(U_i\left(\delta\left(\tau_i, \tilde{\tau}_{-i}\right)\right) + m_i\left(\tau_i, \tilde{\tau}_{-i}\right)\right) d\hat{b}_i\left(\tau_i\right)\left(\tilde{\tau}_{-i}\right) \geq 0$$

for every player $i \in N$, and player i's types τ_i in the support of p.

rational direct revelation mechanism $\langle \delta, (m_i)_{i \in N} \rangle$ that implements δ with payment functions that leave each type of each player in T_p with zero surplus.[21]

Remark 5. As in the previous section, the revelation principle implies that in the two definitions above, no loss of generality is entailed by restricting attention to direct revelation mechanisms.

Remark 6. The previous section was devoted to investigating the possibility of the extraction of the players' full surplus relative to an ex-post efficient allocation rule in the context of a single object private values auction.

Recently, Aoyagi (1998), d'Aspremont, Crémer, and Gérard-Varet (2003) and Kosenok and Severinov (2004) showed that in models with at least 3 players and a fixed finite number of types for each player that is larger than or equal to 2,[22] it is generically possible to implement every decision rule.[23] In contrast, we show below that finite-support EDR priors are BDP priors. Since the set $\mathcal{B}_f$ of finite-support BDP priors is non generic within the set $\mathcal{P}_f$ of *all* finite-support priors (the proofs of Theorem 1 and Corollary 1 apply verbatim), we conclude that the set $\mathcal{E}_f$ of finite-support EDR priors is non-generic within $\mathcal{P}_f$.

In order to establish this result, we impose the mild assumption that for every player i there exists an outcome x_0^i that if implemented, generates a payoff of 0 for player i regardless of player i's type. Allowing the players to "opt out" of the mechanism ensures the existence of such outcomes.

Proposition 2. A finite support EDR prior is a BDP prior.

Corollary 3. The set $\mathcal{E}_f$ of finite-support EDR priors is non-generic in the set of priors with finite support $\mathcal{P}_f$. The set $\mathcal{E}_f$ is contained in a proper face with infinite co-dimension in $\mathcal{P}_f$, and is therefore finitely shy and null.

4 Related Literature

Myerson (1981) showed that a seller in an auction may be able to exploit the presence of correlation among bidders to extract the bidders' full surplus. Crémer and McLean (1985,

[21] That is, the payment functions $(m_i)_{i \in N}$ are such that

$$\int_{T_{-i}} m_i(\tau_i, \tilde{\tau}_{-i})\, d\hat{b}_i(\tau_i)(\tilde{\tau}_{-i}) = -\int_{T_{-i}} U_i(\delta(\tau_i, \tilde{\tau}_{-i}))\, d\hat{b}_i(\tau_i)(\tilde{\tau}_{-i})$$

for every player $i \in N$, and player i's types τ_i in the support of p.

[22] Kosenok and Severinov (2004) require one player to have at least 3 types when there are exactly 3 players.

[23] d'Aspremont et al. (2004) do not require interim individual rationality but rather budget-balance. However, every budget balanced mechanism can be transformed into an individualy rational mechanism by adding a sufficiently large constant to each player's payment function m_i. Kosenok and Severinov (2004) do require both interim individual rationality and budget balance.

1988) showed that a monopolistic seller can generically extract the full surplus of risk neutral consumers and bidders, respectively, in models with a fixed, finite, number of types.[24] McAfee and Reny (1992) constructed a similar auction to the one described by Crémer and McLean that (approximately) extracts the full surplus of the bidders when the number of bidders' types is uncountably large, but did not explicitly address the issue of genericity. McAfee et al. (1989), Johnson et al. (1992), and Brusco (1998) have established related results in more specific contexts. For a general formulation of the full surplus extraction result, which allows for a continuum of multidimensional, mutually payoff relevant agents' types, see Johnson et al. (2002). Recently, Aoyagi (1998), d'Aspremont et al. (2003) and Kosenok and Severinov (2004) showed that it is generically possible to implement any decision rule in models with a fixed finite number of types.[25,26]

In the two previous sections we showed that the conclusions about the generic possibility of full surplus extraction and about the implementation of every decision rule are reversed when one considers the set of all models with *any* finite number of types. This still leaves open the question of whether or not a specific decision rule can be generically implemented. An answer to this question can be provided for the case of efficient decision rules.

Implementation of a decision rule requires that the mechanism designer be able to induce players to reveal both the preference and belief components of their types. A player's beliefs about other players' types can always be fully extracted at a cost by standard arguments (see e.g. d'Aspremont and Gérard-Varet, 1979). Thus a mechanism designer may generally face a trade-off between the cost and benefit of extracting a player's belief. However, if the decision rule to be implemented is efficient, then as shown by Bergemann and Morris (2003), the players' beliefs can be extracted at no cost. It therefore follows that it is possible to provide a precise characterization of whether a given efficient social choice function is (interim) implementable on arbitrary finite type spaces in terms of conditions that arise when looking at standard implementation in an environment with stochastically independent types (for details, see Bergemann and Morris, 2003). The gist of this characterization is that after a player's belief type has been costlessly extracted, then if the player's beliefs do not determine its preferences, the player has to be given some rent in order to induce it to reveal its payoff type truthfully, in a similar way to the rent players have to be given when their types are stochastically independent.

A number of authors have argued that the conditions that are imposed in order to obtain these full-rent-extraction results, while standard in many applications, are nevertheless very

[24] Crémer and McLean (1988, Appendix B) have indicated how some of their results can be generalized to allow for a continuum of types. See Parreiras (2004) for an alternative characterization of Crémer and McLean's results in terms of the principal's belief about the quality of the agents' information.

[25] What turns out to be the necessary and sufficient condition for implementation of every decision rule was first introduced by d'Aspremont and Gérard-Varet (1982). d'Aspremont et al. (2004) demonstrate that this condition (condition B) is strictly weaker than Aoyagi's (1998) strict regularity condition which have been shown to be sufficient for implementation of every decision rule. Kosenok and Severinov (2004) devise a stronger condition, Identifiability, which enables surplus extraction with a mechansim that also satisfies interim individual rationality.

[26] See Lopomo, Rigotti, and Shannon (2004) for an altogether different perspective on the subject of full surplus extraction.

strong. Crémer and McLean (1988) suggested that full rent extraction is not robust to the introduction of risk aversion or limited liability constraints, and emphasized the dependence of these results on the common prior assumption. Following their suggestion, Robert (1991) showed that for any given auction mechanism, when agents are risk averse or face limited liability constraints, the function that relates the common prior to the seller's profit and to total surplus (and hence also to the sum of information rents captured by the agents) is continuous in the prior. Since it is known that agents do obtain positive information rents in independent environments, Robert concluded that full information rent extraction also fails in "nearly independent" environments with risk averse agents or agents that face limited liability constraints. More recently, Laffont and Martimort (2000) have established the continuity of the mechanism's outcome function also for environments with risk-neutral agents who are not constrained by limited liability, but who may form collusive coalitions. Intuitively, the reason that full rent extraction fails under these circumstances is that the auction mechanisms that extract the full buyers' rent rely on lotteries whose variance increases to infinity at independence. Thus, in nearly independent environments, mechanisms that rely on such lotteries violate the buyers' limited liability or participation constraints. Because these lotteries also prescribe payments to and from agents that strongly depend on the actions of other agents, mechanisms that rely on such lotteries are highly susceptible to collusion among the agents, and fail in nearly independent environments where these payments are large.

5 Conclusion

This paper makes a contribution to the growing literature about robust mechanism design that has stemmed out of Robert Wilson's view that further progress in game theory depends "on successive reduction in the base of common knowledge required to conduct useful analyses of practical problems" (Wilson, 1987).[27] As shown by Neeman (2004), full-surplus extraction hinges on there being common belief that a player's belief determines, or predicts with certainty, the player's preferences. Once this assumption is relaxed, the full surplus of the players cannot be extracted. This paper presents a model in which it is shown that it is generically incorrect to assume that the mechanism designer and players maintain such common belief assumptions.

6 Appendix: Proofs

Proof of Proposition 1. Suppose that p is a FSE prior for bidder i. Let $\langle q_i, m_i \rangle_{i \in N}$ be an incentive compatible and individually rational mechanism that extracts the full surplus of bidder i. By Remark 1, bidder i must win the object with p-probability 1 under the

[27] See, e.g., Bergemann and Morris (2003), Chung and Ely (2004), Neeman (2003, 2004), Weinstein and Yildiz (2004) and the references therein.

mechanism $\langle q_i, m_i \rangle_{i \in N}$, and bidder i's individual rationality constraint must be binding with p-probability 1 under the mechanism $\langle q_i, m_i \rangle_{i \in N}$.

Suppose that p is not a BDP prior for bidder i. It follows that there exist two disjoint measurable subsets of bidder i's types, $A_i, A'_i \subseteq T_i$, that each have a positive p-probability

$$p_{|T_i}(A_i) > 0, \quad p_{|T_i}(A'_i) > 0,$$

and the same range of beliefs

$$\hat{b}_i(A_i) = \hat{b}_i(A'_i) \subseteq \Delta(T_{-i}),$$

but different valuations. That is, if $\tau_i \in A_i$ and $\tau'_i \in A'_i$ are such that

$$\hat{b}_i(\tau'_i) = \hat{b}_i(\tau_i)$$

then

$$\hat{v}_i(\tau'_i) < \hat{v}_i(\tau_i).$$

In particular, for every type $\tau_i \in A_i$ there exists a type $\tau'_i \in A'_i$ such that $\hat{b}_i(\tau'_i) = \hat{b}_i(\tau_i)$ but $\hat{v}_i(\tau'_i) < \hat{v}_i(\tau_i)$. It follows that

$$\begin{aligned}
& \int_{T_{-i}} (q_i(\tau_i, \tilde{\tau}_{-i}) \hat{v}_i(\tau_i) - m_i(\tau_i, \tilde{\tau}_{-i})) \, d\hat{b}_i(\tau_i)(\tilde{\tau}_{-i}) \\
\geq & \int_{T_{-i}} (q_i(\tau'_i, \tilde{\tau}_{-i}) \hat{v}_i(\tau_i) - m_i(\tau'_i, \tilde{\tau}_{-i})) \, d\hat{b}_i(\tau_i)(\tilde{\tau}_{-i}) \\
= & \int_{T_{-i}} (q_i(\tau'_i, \tilde{\tau}_{-i}) \hat{v}_i(\tau_i) - m_i(\tau'_i, \tilde{\tau}_{-i})) \, d\hat{b}_i(\tau'_i)(\tilde{\tau}_{-i}) \\
> & \int_{T_{-i}} (q_i(\tau'_i, \tilde{\tau}_{-i}) \hat{v}_i(\tau'_i) - m_i(\tau'_i, \tilde{\tau}_{-i})) \, d\hat{b}_i(\tau'_i)(\tilde{\tau}_{-i}) \\
\geq & \; 0.
\end{aligned}$$

The first inequality follows from the (IC) constraint for type τ_i; the following equality follows from the fact that $\hat{b}_i(\tau'_i) = \hat{b}_i(\tau_i)$; the next strict inequality follows from the fact that $\hat{v}_i(\tau'_i) < \hat{v}_i(\tau_i)$ and that $q_i(\tau'_i, \tilde{\tau}_{-i}) = 1$ for p-almost every type $\tau'_i \in A'_i$; and the last inequality follows from the (IR) constraint for type τ'_i. It therefore follows that bidder i's individual rationality constraint is not binding for p-almost every type $\tau_i \in A_i$. A contradiction. ■

Proof of Lemma 1. Let $g \in C \setminus F$ and let $f \in F$. Consider the one-dimensional subspace of X

$$H = \{\alpha (g - f) : \alpha \in \mathbb{R}\}.$$

Let λ_H be Lebesgue measure on H. We have that $\alpha(g - f) + f = \alpha g + (1 - \alpha) f \in C$ if and only if $\alpha \in [0, 1]$ and hence $\lambda_H(C - f) = 1 > 0$. However, $\lambda_H(F + x) = 0$ for every $x \in X$. In fact $H \cap (F + x)$ is either empty or a singleton. Indeed, assume by contradiction that

$$\begin{aligned}
f_1 + x &= h_1 = \alpha_1 (g - f) \\
f_2 + x &= h_2 = \alpha_2 (g - f)
\end{aligned}$$

where $h_1, h_2 \in H,\ f_1, f_2 \in F$ and $\alpha_1 > \alpha_2$. Then

$$f_1 - f_2 = (\alpha_1 - \alpha_2)\, g - (\alpha_1 - \alpha_2)\, f,$$

or

$$\frac{1}{1 + (\alpha_1 - \alpha_2)} f_1 + \frac{(\alpha_1 - \alpha_2)}{1 + (\alpha_1 - \alpha_2)} f = \frac{1}{1 + (\alpha_1 - \alpha_2)} f_2 + \frac{(\alpha_1 - \alpha_2)}{1 + (\alpha_1 - \alpha_2)} g$$

where the left-hand side is a convex combination of $f_1, f \in F$, and hence in F, while the right-hand side is a convex combination of f_2 and g. Since F is a face, this implies that $f_2, g \in F$, contradicting the assumption that $g \in C \setminus F$. ■

Proof of Theorem 1. We first have to show that $\mathcal{B}$ is convex. Suppose that

$$f = \alpha f_1 + (1 - \alpha)\, f_2$$

where $f_1, f_2 \in \mathcal{B}$. We have to show that $f \in \mathcal{B}$.

Because f_k, $k = 1, 2$ are BDP priors for player i, there exist subsets $T^{f_k} \subset T$ such that

$$f_k\left(T^{f_k}\right) = 1,$$

the projection $T_i^{f_k}$ of T^{f_k} on T_i is the graph of a function

$$\Phi_i^{f_k} : B_i^{f_k} \to S_i$$

(where $B_i^{f_k}$ is the projection of $T_i^{f_k}$ on $\Delta\left(T_{-i}\right)$), and for every $\tau = (\ldots, \tau_j, \ldots) \in T^{f_k}$ and every player $j \neq i$, the marginal $\widehat{b}_j\left(\tau_j\right)_{|T_i}$ of τ_j on T_i assigns probability 1 to $T_i^{f_k}$. This means that if

$$T^{f_1} \cap T^{f_2} \neq \emptyset$$

then for every $\tau = (\ldots, \tau_j, \ldots) \in T^{f_1} \cap T^{f_2}$ and every player $j \neq i$, the marginal $\widehat{b}_j\left(\tau_j\right)_{|T_i}$ of τ_j on T_i assigns probability 1 to $T_i^{f_1} \cap T_i^{f_2}$. It then follows that the graphs of $\Phi_i^{f_1}$ and $\Phi_i^{f_2}$ coincide on $B_i^{f_1} \cap B_i^{f_2}$ almost surely according to both $f_{1|T_i}$ and $f_{2|T_i}$, because on $T^{f_1} \cap T^{f_2}$ each of the priors is an average of j's beliefs $\left\{\widehat{b}_j\left(\tau_j\right) : \tau = (\ldots, \tau_j, \ldots) \in T^{f_1} \cap T^{f_2}\right\}$. So if we define $\Phi_i : B_i^{f_1} \cup B_i^{f_2} \to S_i$ by

$$\Phi_i\left(b_i\right) = \begin{cases} \Phi_i^{f_1}\left(b_i\right) & b_i \in B_i^{f_1} \\ \Phi_i^{f_2}\left(b_i\right) & \text{otherwise} \end{cases}$$

then both $f_{1|T_i}$ and $f_{2|T_i}$ assign probability 1 to the graph of Φ_i.

This implies that the marginal of $f = \alpha f_1 + (1 - \alpha)\, f_2$ on T_i assigns measure zero to the complement of the graph of Φ_i, and hence f is a BDP prior.

Next, we have to show that $\mathcal{B}$ is a face of $\mathcal{P}$. Suppose that a BDP prior $f \in \mathcal{B}$ is a convex combination

$$f = \alpha p + (1 - \alpha)\, q$$

where $\alpha \in (0, 1)$ and $p, q \in \mathcal{P}$. We have to show that $p, q \in \mathcal{B}$.

We argue by contradiction. Suppose, without loss of generality, that $p \in \mathcal{P} \setminus \mathcal{B}$. It follows that for some bidder $i \in N$ there exist two disjoint measurable subsets of i's types, $A_i, A_i' \subseteq T_i$, that each have a positive p-probability

$$p_{|T_i}(A_i) > 0, \quad p_{|T_i}(A_i') > 0,$$

and the same range of beliefs

$$\hat{b}_i(A_i) = \hat{b}_i(A_i') \subseteq \Delta(T_{-i}),$$

but different valuations – if $\tau_i \in A_i$ and $\tau_i' \in A_i'$ are such that

$$\hat{b}_i(\tau_i') = \hat{b}_i(\tau_i)$$

then

$$\hat{v}_i(\tau_i') \neq \hat{v}_i(\tau_i).$$

It follows that also

$$\begin{aligned} f_{|T_i}(A_i) &= \alpha p_{|T_i}(A_i) + (1-\alpha) q_{|T_i}(A_i) > 0, \\ f_{|T_i}(A_i') &= \alpha p_{|T_i}(A_i') + (1-\alpha) q_{|T_i}(A_i') > 0, \end{aligned}$$

so f is not a BDP prior, in contradiction with our assumption.

Last, we have to show that the co-dimension of $\mathcal{B}$ in $\mathcal{P}$ is infinite. This follows from the fact that there are infinitely many (in fact, a continuum) of i.i.d. common priors which are not convex combinations of other priors, and the fact that i.i.d priors are not BDP.

To see this, consider two different bidders i, j, and two distinct valuations of each – $v_i \neq v_i' \in V_i$, $v_j \neq v_j' \in V_j$. Then there is a continuum of priors $p_{r,s}$ with $r, s \in (0,1)$, such that the support of each such prior consists of 4 type-pairs: With the prior $p_{r,s}$ that bidder i has the valuation v_i with probability r and the valuation v_i' with probability $1-r$; while independently of i's valuation, bidder j has the valuation v_j with probability s and the valuation v_j' with probability $1-s$. With this prior, with probability 1 each bidder has the same belief about the other's types irrespective of her own valuation, so $p_{r,s}$ is not a BDP prior.[28] Moreover, each prior $p_{r,s}$ is not a convex combination of other common priors in $\mathcal{P}$, because if $(r,s) \neq (r',s')$, then the priors $p_{r,s}$ and $p_{r',s'}$ on the universal space T have disjoint supports.[29] ■

Proof of Corollary 1. This follows from Theorem 1, lemma 1 and lemma 2 below. ■

Lemma 2. The set $\mathcal{B}$ of BDP priors for bidder i is a Borel subset of the space of priors $\mathcal{P}$.

Proof of Lemma 2. If the lemma obtains when $\mathcal{P}$ is equipped with the topology of weak convergence, it also obtains for any stronger metric topology. It is therefore enough to proceed assuming that $\mathcal{P}$ is equipped with the topology of weak convergence.

[28] If there are more bidders, the definition of $p_{r,s}$ could be extended by choosing some particular valuation for each of those extra bidders, and have $p_{r,s}$ assign probability 1 to that combination of valuations for each of the 4 combinations of valuations of i and j.

[29] Section 2.7 above contains an illustrative discussion of this point.

By definition, a prior $p \in \mathcal{P}$ is a BDP prior if and only if the marginal of p on $T_i = V_i \times \Delta(T_{-i})$ is concentrated on a measurable graph $\Phi_i^p : B_i^p \to V_i$. This is expressible by countably many conditions, in the following way.

Since V_i is separable, there is a countable collection $\{A_i^n\}_{n\geq 1}$ of subsets of V_i which is closed under complements and finite unions and generates the Borel sigma-field of V_i. Hence there are also countably many partitions $\{\Gamma_i^m\}_{m\geq 1}$ of V_i to finitely many disjoint subsets $\left\{A_i^{n_k^m}\right\}_{k=1}^{N_i^m} \subseteq \{A_i^n\}_{n\geq 1}$. Similarly, Since $\Delta(T_{-i})$ is separable, there exists a countable collection $\left\{Y_i^\ell\right\}_{\ell\geq 1}$ of subsets of $\Delta(T_{-i})$ which is closed under complements and finite unions and generates the Borel sigma-field of $\Delta(T_{-i})$. Hence, there are also countably many partitions $\{\Lambda_i^r\}_{r\geq 1}$ of $\Delta(T_{-i})$ to finitely many disjoint subsets in $\left\{Y_i^{\ell_k^r}\right\}_{k=1}^{L_i^r} \subseteq \left\{Y_i^\ell\right\}_{\ell\geq 1}$.

The marginal of p on $T_i = V_i \times \Delta(T_{-i})$ is concentrated on the graph of Φ_i^p if and only if for every partition $\Gamma_i^m = \left\{A_i^{n_k^m}\right\}_{k=1}^{N_i^m}$ of S_i

$$p\left(\bigcup_{k=1}^{N_i^m}\left(A_i^{n_k^m} \times \left(\Phi_i^p\right)^{-1}\left(A_i^{n_k^m}\right) \times T_{-i}\right)\right) = 1$$

Intuitively, as the partitions $(\Gamma_i^m)_{m\geq 1}$ of V_i get finer, the union of the rectangles $A_i^{n_k^m} \times \left(\Phi_i^p\right)^{-1}\left(A_i^{n_k^m}\right)$ approximates increasingly well the graph of Φ_i^p.

Now, for each partition $\Gamma_i^m = \left\{A_i^{n_k^m}\right\}_{k=1}^{N_i^m}$ of V_i, $\left\{\left(\Phi_i^p\right)^{-1}\left(A_i^{n_k^m}\right)\right\}_{k=1}^{N_i^m}$ is a partition of $\Delta(T_{-i})$, that can be approximated arbitrarily well (in terms of the probabilities assigned to the partition members by the marginal of p on $\Delta(T_{-i})$) by partitions in $\{\Lambda_i^r\}_{r\geq 1}$. Hence, the marginal of p on $T_i = V_i \times \Delta(T_{-i})$ is concentrated on a measurable graph from $\Delta(T_{-i})$ to V_i if and only if for every natural number $q \geq 1$ and for each partition $\Gamma_i^m = \left\{A_i^{n_k^m}\right\}_{k=1}^{N_i^m}$ of V_i there exists a partition $\Lambda_i^r = \left\{Y_i^{\ell_k^r}\right\}_{k=1}^{L_i^r}$ of $\Delta(T_{-i})$ with $L_i^r = N_i^m$ and

$$p\left(\bigcup_{k=1}^{N_i^m}\left(A_i^{n_k^m} \times Y_i^{\ell_k^r} \times T_{-i}\right)\right) \geq 1 - \frac{1}{q}$$

Formally, therefore, the set $\mathcal{F}$ of BDP priors is

$$\bigcap_{i\in N}\bigcap_{m\geq 1}\bigcap_{q\geq 1}\bigcup_{r\geq 1}\left\{p \in \mathcal{P} : p\left(\bigcup_{k=1}^{N_i^m}\left(A_i^{n_k^m} \times Y_i^{\ell_k^r} \times T_{-i}\right)\right) \geq 1 - \frac{1}{q}\right\}$$

which is a Borel subset of the space of priors $\mathcal{P}$. ■

Proof of Corollary 2. By proposition 1 we have $\mathcal{F} \subseteq \mathcal{B}$, so the corollary follows from Corollary 1 ■

Proof of Proposition 2. Suppose that p is not a BDP prior for player i. Then player i has two types $\tau_i, \tau_i' \in T_i$ that each have a positive p-probability, the same beliefs

$$\hat{b}_i(\tau_i) = \hat{b}_i(\tau_i') \equiv b_i \in \Delta(T_{-i}),$$

but different preference types

$$\hat{v}_i(\alpha_i') \neq \hat{v}_i(\alpha_i).$$

That is, there exist a profile of other players' types $\overline{\tau}_{-i} \in T_{-i}$ such that

$$b_i(\overline{\tau}_{-i}) > 0$$

and an outcome $\bar{x} \in \mathcal{X}$ such that (without loss of generality)

$$U_i(\bar{x}; \tau_i, \overline{\tau}_{-i}) > U_i(\bar{x}; \tau_i', \overline{\tau}_{-i}) \geq 0.$$

Define the decision rule

$$\delta : T \to \mathcal{X}$$

by

$$\delta(\tau) = \begin{cases} \bar{x} & \tau = (\tau_i', \overline{\tau}_{-i}) \\ x_0^i & \text{otherwise} \end{cases}$$

Consider any system of monetary transfers

$$m_i : T \to \mathbb{R}, \quad i \in N$$

and suppose that the mechanism $\left\langle \delta, (m_i)_{i \in N} \right\rangle$ is incentive compatible. In particular, for type τ_i'

$$\sum_{\tilde{\tau}_{-i} \in T_{-i}} \left(U_i(\delta(\tau_i', \tilde{\tau}_{-i}); \tau_i', \tilde{\tau}_{-i}) + m_i(\tau_i', \tilde{\tau}_{-i}) \right) b_i(\tilde{\tau}_{-i}) \geq$$

$$\sum_{\tilde{\tau}_{-i} \in T_{-i}} \left(U_i(\delta(\tau_i, \tilde{\tau}_{-i}); \tau_i', \tilde{\tau}_{-i}) + m_i(\tau_i, \tilde{\tau}_{-i}) \right) b_i(\tilde{\tau}_{-i})$$

or

$$U_i(\bar{x}; \tau_i', \tilde{\tau}_{-i}) b_i(\tilde{\tau}_{-i}) + \sum_{\tilde{\tau}_{-i} \in T_{-i}} m_i(\tau_i', \tilde{\tau}_{-i}) b_i(\tilde{\tau}_{-i}) \geq \sum_{\tilde{\tau}_{-i} \in T_{-i}} m_i(\tau_i, \tilde{\tau}_{-i}) b_i(\tilde{\tau}_{-i}).$$

Therefore, if instead of truthfully reporting its type, τ_i reports it is type τ_i', then its expected payoff is

$$\begin{aligned}
& \sum_{\tilde{\tau}_{-i} \in T_{-i}} \left(U_i(\delta(\tau_i', \tilde{\tau}_{-i}); \tau_i, \tilde{\tau}_{-i}) + m_i(\tau_i', \tilde{\tau}_{-i}) \right) b_i(\tilde{\tau}_{-i}) \\
= \; & U_i(\bar{x}; \tau_i, \overline{\tau}_{-i}) b_i(\overline{\tau}_{-i}) + \sum_{\tilde{\tau}_{-i} \in T_{-i}} m_i(\tau_i', \tilde{\tau}_{-i}) b_i(\tilde{\tau}_{-i}) \\
> \; & U_i(\bar{x}; \tau_i', \overline{\tau}_{-i}) b_i(\overline{\tau}_{-i}) + \sum_{\tilde{\tau}_{-i} \in T_{-i}} m_i(\tau_i', \tilde{\tau}_{-i}) b_i(\tilde{\tau}_{-i}) \\
\geq \; & \sum_{\tilde{\tau}_{-i} \in T_{-i}} m_i(\tau_i, \tilde{\tau}_{-i}) b_i(\tilde{\tau}_{-i}) \\
= \; & \sum_{\tilde{\tau}_{-i} \in T_{-i}} \left(U_i(\delta(\tau_i, \tilde{\tau}_{-i}); \tau_i, \tilde{\tau}_{-i}) + m_i(\tau_i, \tilde{\tau}_{-i}) \right) b_i(\tilde{\tau}_{-i})
\end{aligned}$$

in contradiction to the presumed incentive compatibility of $\left\langle \delta, (m_i)_{i \in N} \right\rangle$. It follows that the decision rule δ cannot be implemented, so p is not an EDR prior. ■

References

Anderson, R. M. and W. R. Zame (2001) "Genericity with Infinitely Many Parameters," *Advances in Theoretical Economics* **1**, pp. 1-62.

Aoyagi, M. (1998) "Correlated types and Bayesian incentive compatible mechanisms with budget balance," *Journal of Economic Theory* 79, 142-151.

d'Aspremont, C., and L.-A. Gérard-Varet (1979) "Incentives and Incomplete Information," *Journal of Public Economics* 11, 25-45.

d'Aspremont, C., and L.-A. Gérard-Varet (1982) "Bayesian incentive compatible beliefs," *Journal of Mathematical Economics* 10, 83-103.

d'Aspremont, C., J. Crémer, and L.-A. Gérard-Varet (2003) "Correlation, Independence and Bayesian Incentives," *Social Choice and Welfare* **21**, 281-310.

d'Aspremont, C., J. Crémer, and L.-A. Gérard-Varet (2004), "Balanced Bayesian Mechanisms," *Journal of Economic Theory* **115**, 385-396.

Aumann, R. (1998) "Common priors: a reply to Gul," *Econometrica* **66**, 929-938.

Bergemann, D. and S. Morris (2003) "Robust mechanism design," mimeo, Yale University.

Brandenburger, A. and E. Dekel (1993) "Hierarchies of Beliefs and Common Knowledge," *Journal of Economic Theory*, **59**, 189- 198.

Brusco, S. (1998) "Unique implementation of the full surplus extraction outcome in auctions with correlated types," *Journal of Economic Theory*, **80**, 185-200.

Christensen, J. P. R. (1974) *Topology and Borel Structure*, North Holland Mathematical Studies Vol. 10, Amsterdam: North-Holland.

Chung, K. and J. C. Ely (2004) "Foundations of dominant strategy mechanisms," mimeo, Boston and Northwestern Universities.

Crémer, J. and R. McLean (1985) "Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent," *Econometrica* **53**, 345-361.

Crémer, J. and R. McLean (1988) "Full extraction of the surplus in Bayesian and dominant strategy auctions," *Econometrica* **56**, 1247-1257.

Gul, F. (1998) "A comment on Aumann's Bayesian view," *Econometrica* **66**, 923-927.

Harsanyi, J. (1967, 1968) "Games with incomplete information played by Bayesian players," *Management Science* **14**, 159-182, 320-334, 486-502.

Heifetz, A. (1993) "The Bayesian Formulation of Incomplete Information – the Non-Compact Case," *International Journal of Game Theory* **21**, 329-338.

Hunt, B. R., T. Sauer and J. A. Yorke (1992) "Prevalence: A Translation-Invariant 'Almost Every' on Infinite-Dimensional Spaces," *Bulletin (New Series) of the American Mathematical Society* **27**, pp. 217-238.

Johnson, S., J. Pratt, and R. Zeckhauser (1992) "Efficiency Despite Mutually Payoff-Relevant Private Information: The Finite Case," *Econometrica* **58**, 873-900.

Johnson, S., N. Miller, J. Pratt, and R. Zeckhauser, Efficient Design with Interdependent Valuations and an Informed Center," mimeo, Kennedy School of Government, Harvard University, 2002

Kajii, A. and S. Morris (1998) "Payoff Continuity in Incomplete Information Games," *Journal of Economic Theory* **82**, 267-276.

Kosenok, G. and S. Severinov (2004), "Individually Rational, Balanced-Budget Bayesian Mechanisms and the Informed Principal Problem," mimeo, Duke University.

Laffont, J.-J., and D. Martimort (2000) "Mechanism design with collusion and correlation," *Econometrica* **68**, 309-342.

Lopomo, G., L. Rigotti, and C. Shannon (2004) "Uncertainty in Mechanism Design," mimeo, Duke University.

Mas-Colell, A. (1985), "The Theory of General Economic Equilibrium – a Differentiable Approach," *Cambridge University Press.*

McAfee, P. R. and P. J. Reny (1992) "Correlated information and mechanism design," *Econometrica* **60**, 395-421.

McAfee, P. R., J. McMillan, and P. J. Reny (1989) "Extracting the surplus in the common-value auction," *Econometrica* **57**, 1451-1459.

Mertens, J. F., S. Sorin and S. Zamir (1994), "Repeated Games, part A: Background Material," CORE discussion paper 9420.

Mertens, J. F. and S. Zamir (1985) "Formulation of Bayesian analysis for games with incomplete information," *International Journal of Game Theory* **10**, 619-632.

Morris, S. (1995) "The common prior assumption in economic theory," *Economics and Philosophy* 11, 227-253.

Morris, S. (2001) "Typical Types," mimeo, Yale University.

Myerson, R. (1981) "Optimal Auction Design, *Mathematics of Operations Research* **6**, 58-73.

Neeman, Z. (2003) "The Effectiveness of the English Auction," *Games and Economic Behavior* 43 (2003), 214-238.

Neeman, Z. (2004) "The Relevance of Private Information in Mechanism Design," *Journal of Economic Theory* **117**(1), 55-77.

Parreiras, S. (2004) "Correlated Information, Mechanism Design, and Informational Rents," mimeo, University of North Carolina at Chapel Hill.

Perry, M. and P. Reny (2003) "Toward a Strategic Foundation for Rational Expectations Equilibrium," mimeo, University of Chicago.

Robert, J. (1991) "Continuity in auction design," *Journal of Economic Theory* **55**, 169-179.

Rockafellar, T. R. (1970) *Convex Analysis*, Princeton University Press.

Weinstein, J. and M. Yildiz (2004) "Finite-Order Implications of Any Equilibrium, mimeo, MIT.

Wilson, R. (1987) "Game-Theoretic Analyses of Trading Processes," in *Advances in Economic Theory: Fifth World Congress*, Ed. T. Bewley, Cambridge: Cambridge University Press.

Yamasaki, Y. (1985) *Measures on Infinite Dimensional Spaces,* World Scientific, Singapore.