Estimating a Search and Matching Model with Sticky Price and Staggered Wage Negotiation

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Abstract

This paper estimates a search and matching model of the aggregate labor market with sticky price and staggered wage negotiation. It starts with a partial equilibrium search and matching model and expands into a general equilibrium model with sticky price and staggered wage. I study the quantitative implications of the model. The results show that (1) the price stickiness and staggered wage are quantitatively important for the search and matching model of the aggregate labor market; (2) a relatively high outside alternative of the workers is needed to match the data; and (3) workers have relatively lower bargaining power than firms, which contrasts with the assumption in the calibration literature that workers and firms share equally the surplus generated from their employment relationship.

Keywords: Labor market, search and matching, sticky price, staggered wage negotiation.

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1 Introduction

The Mortensen-Pissarides (MP) search and matching model has become the driving force for labor market issues in macroeconomics. By focusing on the search and matching aspect, that is, workers searching for jobs, firms searching for workers, and both sides being matched with each other, the model assumes that search on the labor market is frictional and provides a description of employment flow in an economy. Moreover, the search and matching model is tractable and convenient to be integrated into standard macroeconomic models as an alternative to the perfectly competitive Walrasian labor market model. Merz (1995) and Andolfatto (1996) were among the first to integrate this search and matching framework into a general equilibrium model. They have illustrated its relative success in explaining cyclical behavior in wages and employment fluctuations.

However, Shimer (2005) pointed out that these models fail to account for the observed business cycle frequency fluctuations in unemployment and job vacancies, given shocks of a plausible magnitude. These variables are at least 10 times more volatile in U.S. data than in the MP model. The possible reason is that under standard parametrization, there is a linear relationship between wages and the labor market tightness. Any improvement in labor market conditions will be immediately transferred to wage, thus reducing the incentives for the firms to post vacancies. Researchers have put great efforts to remedy this "puzzle". To date, most of the literature is theoretical and based on calibration, for example, Hagedorn and Manovskii (2008) proposed a new calibration strategy to the standard search and matching model to identify the value of unemployment benefit and the bargaining power. Only recently, some researchers have started applying more formal estimation methods to study the quantitative implications of the search and matching framework, such examples include Gertler, Sala, and Trigari (2008) and Lubik (2009).

In addition, micro level data and everyday experience suggest that prices are sticky and wages adjust infrequently in a staggered manner. A growing body of literature, surveyed by Taylor (1999), shows how sticky prices can be fruitfully incorporated into macroeconomics models. Gertler, Sala, and Trigari (2008) also argued that the wages from the new wage negotiations are sticky in relation to previously bargained wages. By assuming sticky wages in a search and matching framework, the model can generate sizable volatility based on the assumption that the new hires are paid at the going wage, a positive shock can lead to more profits for firms from hiring and production, which encourages firms to post vacancies. Meanwhile, as in the framework of Pissarides (2009), Haefke, Sonntag, and van Rens (2009) and Rudanko (2009), the sticky wages should have no effect on real aggregates.

The purpose of this paper is to estimate a search and matching model of the aggregate labor market with sticky price and staggered wage negotiation. Applying the Bayesian methodology, we study the comprehensive quantitative implications of the entire search and matching model and the role of sticky price and staggered wage negotiation.

The first main finding is that sticky prices and staggered wages are quantitatively important for the search and matching model of the aggregate labor market. Our model matches the business cycle statistics fairly better than the model without price and wage rigidities as in Lubik (2009). Second, different from Shimer (2005) and Trigari (2009), a relatively high outside alternative of the worker is needed to match the data, which gives support to the considerably high outside option argument in Hagedorn and Manovskii (2008). Third, in most calibration literature, worker's share in the Nash bargaining is set to 0.5, meaning that workers and firms equally share the surplus generated from their employment relationship, however our estimation results indicate that the workers have relatively lower bargaining power than the firms.

This paper proceeds as follows. In the next section, we describe a simple search and matching model with sticky price and staggered wage negotiation, followed by a discussion in Section 3 of the data used and empirical strategy. In section 4, we present and discuss the baseline estimation results. Section 5 investigates the model sensitivity and compares the important statistics from data and different models. Section 6 concludes.

2 Model

This paper is based on a partial equilibrium search and matching model and expands the model to a general equilibrium model with sticky prices and staggered wages. We first describe the optimization problems of the households and firms, followed by a discussion of the price and wage determination.

The Household

Our economy is populated by a continuum of households of measure unity. In addition households equally share income and risk among all family members. The utility of a representative household is defined by

$$E\sum_{j=t}^{\infty}\beta^{j-t}\left[\frac{C_j^{1-\sigma}-1}{1-\sigma}-\chi_j n_j\right], \tag{1}$$

where C denotes aggregate consumption and $n \in [0,1]$ is the portion of employed household members. $\beta \in (0,1)$ is the discount factor, $\sigma \geq 0$ is coefficient of the relative risk aversion, and χ_t is an exogenous stochastic process, which gives the disutility of labor at t. Household members either supply labor services or search for a job. Since the employment is determined by the search and matching process, households cannot control it. The representative household's budget constraint is

$$C_t + T_t = w_t n_t + (1-n_t)b + \Pi_t, \tag{2}$$

where b is the unemployment benefit financed by a lump-sum tax T_t. w_t is the wage and Π_t is the dividend that the household receives as the owner of the firms. Since the employment status are determined by the search and matching process, first-order condition simply gives the Euler equation:

$$C_t^{-\sigma} = \lambda_t, \tag{3}$$

where λ_t is the Lagrange multiplier on the budget constraint.

The Labor Market

The MP job search and matching model assumes that the search on the labor market is frictional. These frictions are represented by a Cobb-Douglas matching function. Let u_t denote job seekers and v_t denote vacancies in the economy and $\theta_t \equiv v_t/u_t$ denote the labor market tightness. The flow of matches is given by a constant return to scale function $m(u_t, v_t) \equiv \mu_t u_t^{\xi} v_t^{1-\xi}$, where $\xi \in (0,1)$ is the match elasticity of the unemployed, and μ_t is an exogenous stochastic process that affects the efficiency of the matching process. We define the employment as $n_t = 1 - u_t$ and assume that the new matches become productive after one period. Both old and new matches are destroyed at a constant separation rate $\rho \in (0,1)$. So, the employment evolves according to

$$n_t = (1-\rho)[n_{t-1} + v_{t-1} q(\theta_{t-1})], \tag{4}$$

where $q(\theta_t) \equiv m(u_t, v_t)/v_t$, representing the probability of filling a vacancy.

The Firm

Monopolistically competitive firms produce differentiated products. The firm's output y_t is produced with labor being the only input, i.e.

$$y_t = A_t n_t^{\alpha}, \tag{5}$$

where A_t is an aggregate technology shock and $\alpha \in [0, 1]$. The firm's output is demanded by households with a preference for variety that results in a demand function:

$$y_t = \left(\frac{p_t}{P_t}\right)^{-\epsilon} Y_t, \tag{6}$$

where y_t is the firm production, p_t is the price set by the firm, P_t is the aggregate price, ϵ is the demand elasticity, and Y_t is the aggregate production. In addition, we assume that vacancy posting is subject to cost $\frac{\kappa}{\psi} v_t^{\psi}$, in which $\kappa > 0$ and $\psi > 0$. Following Rotemberg (2008), we introduce the nominal rigidity in terms of quadratic price adjustment costs:

$$\frac{\vartheta}{2}\left(\frac{p_t}{p_{t-1}} - \pi\right)^2 Y_t, \tag{7}$$

where ϑ controls the price stickiness in the economy and π is the steady state inflation rate associated with the final good. The firm chooses its optimal price p_t, the number of workers, n_t, the number of vacancies, v_t, to be posted by maximizing the profit function

$$E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[(\frac{p_t}{P_t})^{1-\epsilon} Y_t - w_t n_t - \frac{\kappa}{\psi} v_t^{\psi} - \frac{\vartheta}{2}\left(\frac{p_t}{p_{t-1}} - \pi\right)^2 Y_t \right], \tag{8}$$

subject to the employment flow equation (4), the firm's production equation (5), and the demand equation (6).

The first-order conditions are

$$p_t \quad : \quad 1 - \vartheta(\pi_t - \pi)\pi_t + E_t \beta_{t+1} \left[\vartheta(\pi_{t+1} - \pi)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = (1 - \varphi_t)\epsilon, \tag{9}$$

$$n_t \quad : \quad \tau_t = \alpha \frac{Y_t}{n_t} \varphi_t - w_t + (1 - \rho) E_t \beta_{t+1} \tau_{t+1}, \tag{10}$$

$$v_t \quad : \quad \kappa v_t^{\psi - 1} = (1 - \rho) q(\theta_t) E_t \beta_{t+1} \tau_{t+1}, \tag{11}$$

where $\beta_{t+1} = \beta \cdot \frac{\lambda_{t+1}}{\lambda_t}$ is the stochastic discount factor and φ_t is the Lagrange multiplier on the firm's production function (5) and represents the real marginal cost. τ_t is the Lagrange multiplier on the employment flow function (4) and represents the marginal value of a job at t. First-order conditions (10) and (11) imply that

$$\frac{\kappa v_t^{\psi - 1}}{q(\theta_t)} = (1 - \rho) E_t \beta_{t+1} \left[\alpha \frac{Y_{t+1}}{n_{t+1}} \varphi_{t+1} - w_{t+1} + \frac{\kappa v_{t+1}^{\psi - 1}}{q(\theta_{t+1})} \right]. \tag{12}$$

The left-hand side of this equation gives the firm's cost of hiring one more worker and the right-hand side is the firm's profit from creating a new job, taking into consideration the probability of separation.

Wage Bargaining

Frictions in the labor market create a prospective mutual surplus between firm-worker matches. This surplus equals the value added of the match compared to the payoff of both parties in the labor market. We assume the economic surplus is split between workers and firms by maximizing the bargaining function from Lubik (2009):

$$SP_t = \left(\frac{1}{\lambda_t} \cdot \frac{\partial \mathcal{W}_t(n_t)}{\partial n_t} \right)^{\eta} \left(\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t} \right)^{1-\eta}, \tag{13}$$

where $\eta \in [0,1]$ is workers' bargaining power, λ_t is the marginal utility as defined in (3), $\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}$ is the marginal value of a job to the household, given by

$$\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t} = \lambda_t w_t - \lambda_t b - \chi_t + \beta E_t \frac{\partial \mathcal{W}_{t+1}(n_{t+1})}{\partial n_{t+1}} \cdot \frac{\partial n_{t+1}}{\partial n_t}, \tag{14}$$

where $\frac{\partial n_{t+1}}{\partial n_t} = (1-\rho)[1-\theta_t q(\theta_t)]$ from (4). $\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t} = \tau_t$ is the marginal value of a job to the firm. So, the bargaining solution is given by

$$(1-\eta)\left[\frac{1}{\lambda_t} \cdot \frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}\right] = \eta \left(\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}\right). \tag{15}$$

Substituting (9), (10), (11), and (14) into (15), and solving for w_t, we get

$$W_t^* = w_t = \eta(\alpha \frac{y_t}{n_t} \varphi_t + \kappa v_t^{\psi-1} \theta_t) + (1-\eta)(b + \chi_t C_t^{\sigma}). \tag{16}$$

where W_t^* represents the optimal wage after bargaining. From this representation of the bargaining wage, we can see that it is a linear combination of the firm's surplus from hiring and the value of worker's outside alternative from unemployment, where worker's outside alternative is defined as $b + \chi_t C_t^{\sigma}$, which includes both unemployment benefit b and the consumption utility of leisure $\chi_t C_t^{\sigma}$.

Staggered Wages

Following Erceg, Henderson, and Levin (2000), each household supplies specialized labor $n_t(j)$, which is combined according to

$$n_t = \left[\int_0^1 n_t(j)^{\frac{\epsilon_t^w - 1}{\epsilon_t^w}} dj\right]^{\frac{\epsilon_t^w}{\epsilon_t^w - 1}}, \tag{17}$$

by a representative labor aggregator (or “employment agency”), where $\epsilon_t^w > 0$ is an exogenous stochastic process. The profit maximization of the labor aggregator implies the labor service demand is given by

$$n_t(j) = \left[\frac{w_t(j)}{W_t}\right]^{-\epsilon_t^w} n_t, \tag{18}$$

where the aggregate wage index is

$$W_t = \left[\int_0^1 w(j)^{1-\epsilon_t^w} dj\right]^{1/(1-\epsilon_t^w)}. \tag{19}$$

Different from Lubik (2009), which assumes there is no wage rigidity in his paper, we follow Sala, Söderström, and Trigari (2010) and assume that in each period, a fraction of $1-\theta_w$ of workers are able to renegotiate their wages. In addition, θ_w of workers who are not able to renegotiate their wages and receive

$$w_t(j) = w_{t-1}(j)\pi_{t-1}^{\gamma_w}\pi^{1-\gamma_w}, \tag{20}$$

where $\gamma_w \in [0,1]$ is the wage indexation parameter. If the indexation parameter $\gamma_w = 0$, $w_t(j)$ indexes fully to steady-state inflation, π; if $\gamma_w = 1$, $w_t(j)$ indexes fully to lagged inflation, π_{t-1}. In the presence of staggered wages, the aggregate wage index evolves as

$$W_t = \left[(1-\theta_w)(W_t^*)^{1/(\epsilon_t^w-1)} + \theta_w(W_{t-1}\pi_{t-1}^{\gamma_w}\pi^{1-\gamma_w})^{1/(\epsilon_t^w-1)}\right]^{\epsilon_t^w-1}. \tag{21}$$

Market Clearing

To close the model, the resource constraint implies that

$$Y_t = (C_t + \frac{\kappa}{\psi}v_t^{\psi})M_t, \tag{22}$$

where M_t is an exogenous stochastic process, indicating the possible measuring error.

Model Summary

The complete model consists of 9 endogenous variables determined by 9 equations: the flow of employment function (4), the production function (5), the optimal pricing function (9), the optimal vacancy setting function (12), the optimal bargaining wage setting function (16), the stagger wage function (21), the resource constraint (22), and identity equations $\theta_t = \frac{v_t}{u_t}$ and $n_t = 1 - u_t$. In addition, there are 6 shocks[1]: to the aggregate technology A_t , to the matching efficiency μ_t, to the Lagrangian parameter with respect to equation (5) φ_t, to the disutility of labor χ_t, to the wage markup ϵ_t^w, and to the output measurement, M_t. The complete model is summarized in Appendix A and the log-linearized system is summarized in Appendix B.

3 Data and Estimation

Data

We will estimate the log-linearized version of the model using quarterly U.S. data from 1964Q1 to 2004Q4: u_t, v_t, C_t, Y_t, W_t, and π_t.

Bayesian Estimation

Recently, Bayesian estimation methods are widely used in estimating DSGE model such as in An and Schorfheide (2007) and Smets and Wouters (2007). Lubik (2009) also applied Bayesian estimation methods to search and matching model of the aggregate labor market. We are going to apply Bayesian estimation method to the search and matching model above with sticky price and staggered wage. The state-space form of the log-linearized model is

[1]There are 5 shocks in log-linearized system since wage markup ϵ_t^w is fixed at 0 and it does not show up in log-linearized model when the steady state inflation is set at 1.

characterized by the state equation

$$X_t = A(\theta)X_{t-1} + B(\theta)\zeta_{1,t}, \tag{23}$$

where X_t is a vector of endogenous variables, $\zeta_{1,t}$ is a vector of innovations, and θ is a vector of parameters; and the measurement equation is

$$S_t = C(\theta)X_t + D(\theta)\zeta_{2,t}, \tag{24}$$

where $\zeta_{2,t}$ is a vector of measurement errors and S_t is a vector of observable variables, that is,

$$S_t = 100[\Delta \log Y_t, \Delta \log u_t, \Delta \log v_t, \Delta \log C_t, \Delta \log W_t]. \tag{25}$$

$\zeta_{1,t}$ and $\zeta_{2,t}$ are mutually independent.

Numerical methods are introduced to find the posterior for inference and for computing integration of functions of parameter. We can apply the Markov Chain Monte Carlo (MCMC) method to produce a Markov chain (or a sample of the posterior distribution). One efficient algorithm to generate the Markov chain is the Metropolis-Hastings algorithm and this paper focuses on the Random Walk Metropolis-Hastings (RWMH) algorithm.

Prior

The priors are reported in Table 1. The choice of priors for the Bayesian estimation is based on the typical values used in calibration studies. We assign share parameters a Beta distribution with support on the unit interval. We also use Gamma distributions for real-valued parameters and uninformative priors for labor market parameter. The details are as follows.

We set the discount factor, β, at 0.99 and the labor elasticity, α, at 0.67 as usual. Wage markup ϵ_t^w is fixed at 0 and it does not show up in log-linearized model when the steady state

Definitions	Parameter	Density	Mean	Std. Dev.
Discount factor	β	Fixed	0.99	-
Labor elasticity	α	Fixed	0.67	-
Wage markup	ϵ^w	Fixed	0	-
Unemployment benefit	b	Fixed	0.40	-
Relative risk aversion	σ	Gamma	1.00	0.10
Separation rate	ρ	Beta	0.10	0.02
Scaling factor on job matching	μ	Gamma	0.60	0.10
Match elasticity	ξ	Beta	0.70	0.15
Demand elasticity	ϵ	Gamma	10.00	2.00
Scaling factor on price adjustment	ϑ	Gamma	105.00	20.00
Scaling factor on vacancy creation	κ	Gamma	0.05	0.01
Elasticity of vacancy creation	ψ	Gamma	1.00	0.20
Mean of disutility of labor	χ	Gamma	0.70	0.20
Worker bargaining power	η	Uniform	0.50	0.25
Wage stickiness	θ_w	Beta	0.75	0.10
Indexation to past inflation	γ_w	Beta	0.75	0.10
AR-coefficients of shocks	ρ_i	Beta	0.90	0.05
Std of shocks	σ_i	Inverse Gamma	0.01	1.00
Std of log measurement error	σ_M	Inverse Gamma	0.01	1.00

Table 1: Prior of structural parameters

inflation is set at 1. The unemployment benefit b is set at 0.40, which is the same as in Shimer (2005) and lower than usual values discussed in other literature. For instance, Gertler, Sala, and Trigari (2008) find $b = 0.98$ in their framework. The reason for us to take $b = 0.40$ is that in this paper the overall outside alternative of the worker is $b + \chi_t C_t^{\sigma}$ instead of b, even if b is a small number, $b + \chi_t C_t^{\sigma}$ could be relatively large. So, we fix b and assign the mean of the disutility of labor, χ, a wide Gamma prior with the mean at 0.70 and a standard deviation of 0.20.

We choose a relatively narrow prior for the intertemporal substitution elasticity, σ, with mean 1 and a standard deviation of 0.1 as in Lubik (2009). According to the analysis in Shimer (2005), jobs last for about 2.5 years on average, which suggests a quarterly separation rate $\rho = 0.1$. So we set the mean exogenous separation rate at 0.1 and a standard deviation of 0.02. In order to match the observed job-finding rate of 0.7 per quarter (Shimer 2005), we set a prior mean of 0.6 for the scaling factor on job matching, μ, and mean of 0.7 for the match elasticity, ξ. We follow Trigari (2009) and set the demand elasticity, ϵ, at mean 10 with a standard deviation of 2. Based on Krause and Lubik (2007), the scaling factor on price adjustment is set to $\vartheta = 105$ with a standard deviation of 20. The scale parameter in the vacancy cost function is set to $\kappa = 0.05$ with a standard deviation of 0.01 as in Rotemberg (2008). The elasticity of vacancy creation ψ is set to 1 with a standard deviation of 0.50, because in most papers, vacancy creation costs are linear, i.e. $\psi = 1$.

The parameter η represents the bargaining power of the workers and calibration studies use a wide range of values. Since we are interested in how much information on η is from the data, we have chosen an uninformative prior for this parameter on the unit interval. We follow Sala, Söderström, and Trigari (2010) and set both wage stickiness, θ_w, and indexation to past inflation, γ_w, at mean 0.75 with a standard deviation of 0.1.

The log of measurement error process is a white noise with mean 0 and the prior for its standard deviation is an inverse-gamma distribution with mean 0.01 and standard deviation 1. The rest of the exogenous stochastic processes are described by AR(1) processes with a prior mean on the autoregressive parameters of 0.9 and the innovations as having inverse-gamma distributions with typical standard deviations.

4 Results

Posterior Estimates of the Parameters

Table 2 reports the estimation results and 90 percent coverage intervals. Some parameter estimates stand out. First, the posterior mean of η is 0.08 with a 90 percent coverage interval

$[0.07, 0.09]$, which shifts away from the prior. This implies that workers can only claim a smaller portion of the surplus than the firms, therefore the incentive for the firms to create vacancies is quite strong. This result is close to the values of 0.05 in Hagedorn and Manovskii (2008) and 0.03 in Lubik (2009). With low bargaining power for workers, the output, employment and wage respond marginally, while the job creation increases. The argument is that a positive shock, such as an increasing in productivity, will lead to a higher profit. Firms get a large share of the profit and have a strong incentive to post vacancies. However, because of low bargaining power of workers, wage will not change much, which gives little incentive for workers to work more hours and produce more products. This is important for the volatility of employment fluctuations, which has been discussed by Cooley and Quadrini (1999).

Parameter	Prior Mean	Posterior Mean	Posterior 90 Percent Interval
ρ	0.10	0.10	[0.09,0.12]
ξ	0.70	0.65	[0.60,0.70]
ϵ	10.00	7.00	[5.14,9.37]
ϑ	105.00	120.83	[107.85,134.32]
ψ	1.00	1.48	[1.18,1.69]
σ	1.00	1.00	[0.89,1.10]
κ	0.05	0.04	[0.03,0.05]
θ_w	0.75	0.46	[0.40,0.51]
γ_w	0.75	0.59	[0.43,0.72]
η	0.50	0.08	[0.07,0.09]
χ	0.80	0.49	[0.39,0.62]

Table 2: Posterior estimates: baseline model

Second, recent literature pays enough attention to the unemployment benefit or the outside option of the worker. The discussion in Lubik (2009) points out that the generic parameter, b, is not structural per se, but rather a reduced-form coefficient that captures only a part of the outside alternative of the worker. Its value changes with other components of the outside alternative. In order to get a full picture of this outside alternative, we compute $b + \chi C^{\sigma}$

at the posterior mean and find its value 0.84 with a 90 percent coverage interval $[0.72, 0.96]$, which indicates that a relatively high outside alternative of the worker is needed to match the data and gives support to the considerably high outside option argument in Hagedorn and Manovskii (2008).

Third, the posterior distributions of the price adjustment coefficient ϑ, the wage stickiness coefficient θ_w, and the indexation to past inflation coefficient γ_w indicate the presence of sticky prices and staggered wages. Further discussions and a brief sensitive analysis will be shown in the next section.

In most papers, vacancy creation costs are linear, i.e. $\psi = 1$. Rotemberg (2008) uses a low value as $\psi = 0.2$. The estimation results show that the vacancy creation cost function is not linear and the elasticity posterior mean is 1.48. A suggested explanation in Lubik (2009) is that this high value could balance potentially "excessive" vacancy created by the high bargaining power of firms.

Estimation results of other parameters are in line with the values from calibration studies. The posterior mean of the job separation rate ρ is 0.10 with a 90 percent coverage interval $[0.09, 0.12]$, which is exactly the suggested value of 0.1 in Shimer (2005). The posterior distributions of demand elasticity ϵ is slightly lower than the prior mean and scaling factor on price adjustment ϑ is a little higher than the prior mean. The posterior distribution of the scaling factor on vacancy creation κ is basically covered by its prior distribution. The autoregressive coefficients of the shocks (not reported) are all around 0.90, indicating that the model generates enough internal propagation to capture the substantial persistence in the data.

Variance Decomposition

To investigate the most important driving forces of the business cycle as seen through the model, we compute the variance decompositions. The selected results are reported in Table

	Matching	Technology	Marginal Cost	Labor	Output Measure
u	13.24	23.79	61.71	0.00	1.26
v	0.57	27.2	71.03	0.00	1.20
W	0.76	22.25	11.47	46.76	18.76
Y	2.77	84.07	12.90	0	0.26

Table 3: Variance decomposition: benchmark model (in percent)

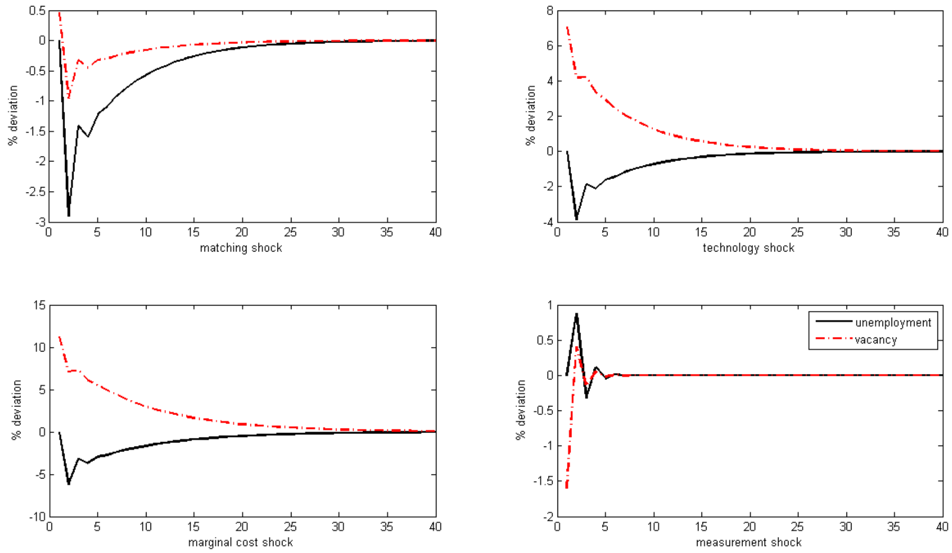
3. It shows that in the estimated model, unemployment dynamics are mainly driven by marginal cost shock and technology shock. The marginal cost shock also operates through the job creation as it affects the expected value of a job. So, the vacancies and wage dynamics are controlled by this shock, especially the dynamics of vacancies. The output variances are explained almost exclusively by the technology shock.

Meanwhile, wage dynamics are mainly driven by the labor shocks with technology shock contributing a considerably small portion of the wage dynamics. It is not surprising that the labor shock matters most for wage dynamics since it directly affects wage through the outside alternative of the worker. In addition, it appears directly in the wage equation (16).

Impulse Response Analysis

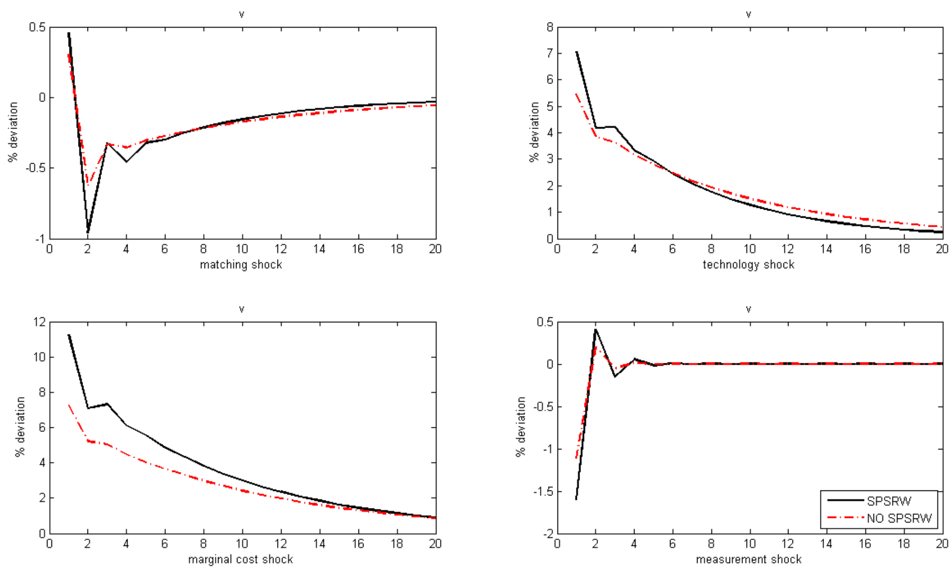
Consider first the dynamic effects of different shocks on vacancies and unemployment. The impulse response functions are depicted in Figure 1. Since the variance decomposition shows that the labor shock has no effect on both vacancies and unemployment, we do not graph the impulse response function with respect to this shock. With one unit increase in the efficiency of the matching process, unemployment decreases and vacancy posting increases immediately.

Figure 1: Impulse response of unemployment and vacancies



After the first period, the unemployment keeps decreasing and the vacancy posting also decreases. This is because a positive shock to matching efficiency creates employment and fills vacancies. In the longer term, both unemployment and vacancy posting impulse responses vanish to zero. With one unit of technology shock or marginal cost shock, the unemployment rate decreases. The first reason is that with the stagger real wage setting, a larger effective share of the surplus accruing to firms increases their incentive to create vacancies in response to the shocks, meanwhile, the wage is rigid and does not change a lot immediately after shocks in our baseline model, which makes posting vacancies very attractive to firms but less attractive for workers to fill these job offers. The second reason is that the introduction of price stickiness creates a more gradual adjustment within the labor market because of more persistent consumption and output under price rigidity. Both interpretations explain the considerably smaller changes in unemployment than the changes in vacancies after the shock. In

Figure 2: Compare impulse responses of vacancies under two models



Note:

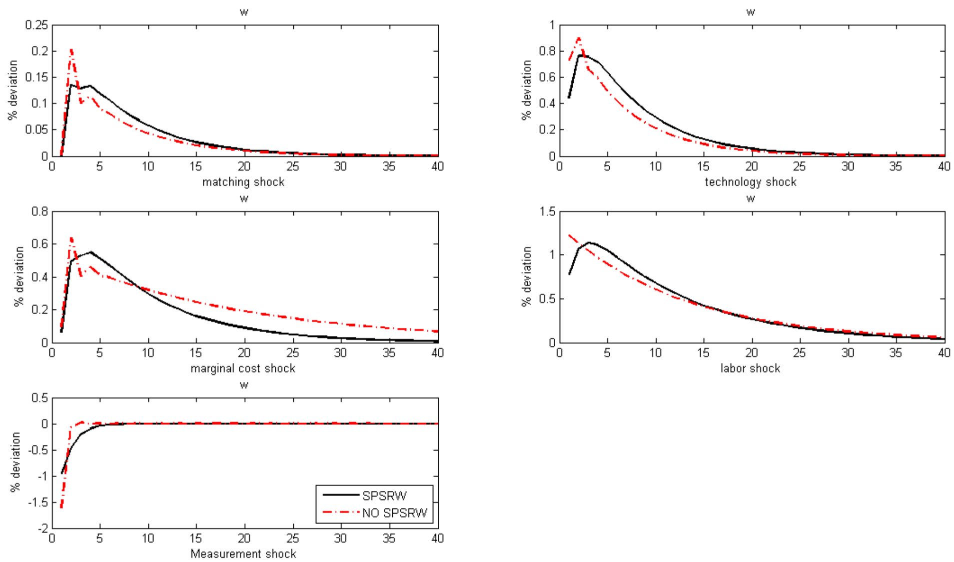
SPSRW: baseline model with Sticky Price and Stagger Real Wage.

No SPSRW: baseline model without Sticky Price and Stagger Real Wage.

Figure 2, we compare the impulse response functions of vacancies for the baseline model and for the baseline model without price and wage rigidity. It is clear that the initial responses of the vacancies are higher under the baseline model than the model without price and wage rigidities, which coincides with our previous argument that firms are more willing to post jobs when there is rigidity. Meanwhile, we can see the impulse responses of vacancy posting from the model without price and wage rigidity are relatively close to the impulse responses of vacancy posting in the baseline model. The reason is that both models apply the Nash wage bargaining and the estimations from both models give low bargaining power to workers, which leads to sizable changes in vacancy posting in both models.

The impulse response functions for wage are depicted in Figure 3. Based on the baseline model, which includes the stagger real wage, all shocks produce hump-shaped wage responses. With a positive shock to the efficiency of the matching process, workers, who are seeking for a job, are more likely to find a job. Meanwhile firms need to pay more to attract workers, which leads to the increase in real wage. With a positive shock to the technology, the real wage is procyclical and increases in the boom. The positive response of wage to one unit shock in marginal cost indicates a positive relationship between these two and this relationship is mainly determined in the optimal bargaining wage setting process. However, the wage rigidity makes the dynamics of wage change relatively small immediately after the shock, but gradually increase in the next few periods. With a positive shock to leisure, workers will dislike work more and require higher return from working, which explains the increase in wage. Moreover, comparing with the model without real wage rigidity, the introduction of the stagger real wage reduces the volatility of the real wage and strongly increases the incentive for firms to hire workers. The effect of measurement shock on wage is immediate and does not last long in both models.

Figure 3: Impulse response of wage with respect to different shocks



Note:

SPSRW: baseline model with Sticky Price and Stagger Real Wage.

No SPSRW: baseline model without Sticky Price and Stagger Real Wage.

5 Model Sensitivity and Summary Statistics

Model Sensitivity

The posterior mean of the scaling factor on price adjustment ϑ is 107.82 with a 90 percent coverage region of $[120.83, 134.32]$, which is slightly higher than Krause and Lubik (2007). To test the sensitivity of the model with respect to the value of this factor, we set ϑ at 1 and then estimate the model. This experiment shows that the marginal likelihood drops from -419.01 in the baseline model to -497.99 in this model almost without price rigidity. The introduction of price stickiness creates a more gradual adjustment within the labor market because of the interaction of prices with labor market variables.

The posterior mean of wage stickiness θ_w is 0.46 with a 90 percent coverage region of $[0.40, 0.51]$. Therefore, this wage stickiness coefficient is different from 0, meaning that about a half of the workers can renegotiate their wages each period. The posterior mean of the indexation to past inflation γ_w is 0.59 with a 90 percent coverage region of $[0.43, 0.72]$. Therefore, the price changes index not only to the lagged inflation, but also to the steady-state inflation. An experiment of setting both θ_w and γ_w at 0 shows that the marginal likelihood drops from -419.01 in the baseline model to -508.67 in the model with θ_w and γ_w fixed. Both experiments on price rigidity and stagger wage negotiating parameters indicate that the introduction of the price stickiness and staggered wage is important for overall fit of the model.

Business Cycle Statistics

Shimer (2005) pointed out that MP models fail to account for the observed business cycle frequency fluctuations in unemployment and job vacancies, given shocks of a plausible magnitude. These variables are at least 10 times more volatile in U.S. data than in the MP model. In addition unemployment is countercyclical whereas vacancies are procyclical or the two variables demonstrate a Beveridge curve. To show how well our estimated model matches unconditional second moments in the data, we compute various statistics both from data and from simulation of the estimated model with parameters set at their posterior means. The statistics are listed in Table 4.

Second Moments	Data	Baseline Model	Baseline Model Without Price and Wage Rigidities
$\sigma(y)$	1.57	1.54	1.68
$\sigma(u)/\sigma(y)$	7.34	6.47	6.38
$\sigma(v)/\sigma(y)$	9.23	9.53	7.93
$\rho(u,v)$	-0.87	-0.44	-0.38

Table 4: Summary Statistics

The model matches these statistics reasonably well. The relative standard deviation of output, vacancies and unemployment are close to the data. The estimated model is less successful in capturing the high negative correlation between unemployment and vacancies in the data, the so-called Beveridge curve.

6 Conclusion

We estimate a search and matching model with sticky price and staggered wage negotiation on aggregate data using Bayesian method. The structural estimation of the full model allows us to assess the viability of the model as a plausible description of labor market dynamics, taking into account all moments of the data and not just selected covariates.

The contribution of this paper is that it answers three questions. (i) is the price stickiness or the staggered wage plausible, (ii) what is the proper value of unemployment benefit, and (iii) what is the worker's power in Nash wage bargaining. To answer the first question, we have studied the marginal likelihood of the baseline model and the models without price stickiness and staggered wage. The results shed light on the importance of sticky prices and staggered wages as it allows firms to benefit from increasing employment in a boom. To answer the second question, we analyze the composition of the outside alternative of worker in this particular model and compute its mean and 90 percent coverage interval based on the posterior distribution, which indicates that a relatively high outside alternative of the worker is needed to match the data and gives support to the considerably high outside option argument in Hagedorn and Manovskii (2008). To answer the third question, we assign an uninformative prior for the bargaining power parameter on the unit interval and get a low bargaining power for workers from the data. The findings in this paper are broadly consistent with recent literature and would support continued use of the search and matching framework with sticky price and staggered wage negotiation to analyze aggregate labor market issues.

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Appendix

A. General Equilibrium

We can derive a system of equations for 9 variables

1. The flow of employment function

$$n_t = (1-\rho)[n_{t-1} + v_{t-1}q(\theta_{t-1})].$$

2. The production function

$$Y_t = A_t n_t^{\alpha}.$$

3. The optimal pricing function

$$1 - \vartheta(\pi_t - \pi)\pi_t + E_t\beta_{t+1}\left[\vartheta(\pi_{t+1} - \pi)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = (1-\varphi_t)\epsilon.$$

4. The optimal vacancy setting function

$$\frac{\kappa v_t^{\psi-1}}{q(\theta_t)} = (1-\rho)E_t\beta_{t+1}\left[\alpha\frac{Y_{t+1}}{n_{t+1}}\varphi_{t+1} - w_{t+1} + \frac{\kappa v_{t+1}^{\psi-1}}{q(\theta_{t+1})}\right].$$

5. The optimal bargaining wage setting function

$$W_t^* = \eta(\alpha\frac{Y_t}{n_t}\varphi_t + \kappa v_t^{\psi-1}\theta_t) + (1-\eta)(b + \chi_t C_t^{\sigma}).$$

6. The stagger wage function

$$W_t = \left[(1-\theta_w)(W_t^*)^{1/(\epsilon_t^w - 1)} + \theta_w(W_{t-1}\pi_{t-1}^{\gamma_w}\pi^{1-\gamma_w})^{1/(\epsilon_t^w - 1)}\right]^{\epsilon_t^w - 1}.$$

7. The resource constraint

$$Y_t = (C_t + \frac{\kappa}{\psi} v_t^{\psi}) M_t.$$

8. The job market tightness identity

$$\theta_t = \frac{v_t}{u_t}.$$

9. The employment identity

$$n_t = 1 - u_t.$$

B. Log-linearized System

The log-linearized system is summarized as follows:

1. The flow of employment function

$$\hat{n}_t = (1-\rho)[\hat{n}_{t-1} + \frac{v\mu\theta^{-\xi}}{n}(\hat{v}_{t-1} + \hat{\mu}_{t-1} - \xi\hat{\theta}_{t-1})].$$

2. The production function

$$\hat{Y}_t = \hat{A}_t + \alpha\hat{n}_t.$$

3. The optimal pricing function

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\epsilon - 1}{\vartheta}\hat{\varphi}_t.$$

4. The optimal vacancy setting function

$$\begin{aligned}(\psi - 1)\hat{v}_t + \xi\hat{\theta}_t - \hat{\mu}_t \quad &= \quad \sigma(\hat{C}_t - \hat{C}_{t-1}) \\ &\quad +\beta(1-\rho)\frac{\mu\theta^{-\xi}}{\kappa v^{\psi-1}} E_t[\alpha\frac{Y}{n}\varphi(\hat{Y}_{t+1} - \hat{n}_{t+1} + \hat{\varphi}_{t+1})]\end{aligned}$$

$$+\beta(1-\rho)E_t[(\psi-1)\hat{v}_{t+1}+\xi\hat{\theta}_{t+1}+\hat{\mu}_{t+1}]$$

5. The optimal bargaining wage setting function

$$\begin{aligned}\hat{W}_t^* &= \frac{1}{W}\left\{\eta\alpha\frac{Y}{n}\varphi(\hat{Y}_t-\hat{n}_t+\hat{\varphi}_t)+\eta\kappa v^{\psi-1}\theta[(\psi-1)\hat{v}_t+\hat{\theta}_t]\right\}\\ &\quad +\frac{1}{W}\left\{(1-\eta)\chi C^{\sigma}(\sigma\hat{C}_t+\hat{\chi}_t)\right\}.\end{aligned}$$

6. The stagger wage function

$$\hat{W}_t=(1-\theta_w)\hat{W}_t^*+\theta_w(\gamma_w\hat{\pi}_{t-1}+\hat{W}_{t-1}).$$

7. The resource constraint

$$\hat{Y}_t=\frac{C}{Y}\hat{C}_t+\frac{\kappa}{Y}v^{\psi}\hat{v}_t+\hat{M}_t.$$

8. The job market tightness identity

$$\hat{\theta}_t=\hat{v}_t-\hat{u}_t.$$

9. The employment identity

$$\hat{n}_t=-\frac{u}{n}\hat{u}_t.$$