ADAPTIVE FUZZY C-MEANS ALGORITHM WITH SPATIAL INFORMATION FOR IMAGE SEGMENTATION

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ABSTRACT

This paper considers the problem of partitioning noisy images into different regions by fuzzy clustering approach. Based on two fuzzy c-means (FCM) algorithms (FCM$_{S1}$ and FCM$_{S2}$), we propose four adaptive algorithms (FCM$_{S11}$, FCM$_{S12}$, FCM$_{S21}$ and FCM$_{S22}$) which utilize the high correlation of image pixels to increase the algorithms’ robustness to noise. Unlike existing algorithms, our algorithms are free of parameter selection. They can automatically determine the balance between using the standard FCM cost function to segment image and using the local spatial information to eliminate the affect of noise. As a result, these adaptive algorithms are very effective in segmenting noisy images. The experiments demonstrate the effectiveness of our algorithms.

Index Terms— Image segmentation, FCM, Clustering

1. INTRODUCTION

Image segmentation is one of the most important and classical problems in computer vision. Its goal is to partition images into different regions and each region has similar characteristics such as color, intensity or texture, etc. Image segmentation algorithms can be grouped into five categories: thresholding, edge detection, region splitting and merging, graph-based segmentation and clustering [1]. In this paper, we propose four image segmentation algorithms using the clustering approach.

Clustering is one method of grouping objects or patterns into different clusters such that the objects in the same cluster are more similar to each other than objects in other clusters. Two clustering schemes are usually used in this process: hard clustering and fuzzy clustering. In hard clustering, the probability of one object belonging to one cluster is zero or one, while in fuzzy clustering, the probability of one object belonging to one cluster is between zero and one. We cluster one object into one cluster if the probability of this object belonging to that cluster is the highest among all the clusters. In image segmentation, fuzzy clustering can usually give much better results than hard clustering. Among all the fuzzy clustering methods, fuzzy c-means (FCM) is one of the most powerful techniques [2]. There are some existing algorithms in the literature using FCM to do image segmentation [3, 4, 5, 6, 7, 8, 9, 10, 11]. Because of space limitations, we refer the readers to the references for more details. Here we only talk about the algorithms proposed in [8] and [9]. In [8], one algorithm called FCM$_S$ is proposed to allow the labelling of one pixel’s neighborhood to influence the labelling of that pixel, thus utilizing the spatial information of images. One disadvantage of this algorithm is that the neighborhood labelling is computed at each step of the algorithm, which is very computationally expensive. To overcome this problem, the authors in [9] propose FCM$_{S1}$ and FCM$_{S2}$, which replace the every neighborhood pixel term by the neighboring mean and the neighboring median, respectively. Since the mean and the median can be computed in advance, these two new algorithms can decrease the computation time of FCM$_S$ significantly.

In FCM$_S$, FCM$_{S1}$ and FCM$_{S2}$, we need to choose a parameter to control the weight put on the neighboring term. When there is lots of noise in the image, we hope that this weight can be large in order to use one pixel’s neighborhood information to help labelling that pixel. We usually select this parameter by experience and prior knowledge about the image. Once it is chosen, the weight put on the neighboring term is fixed for every pixel in the image. But there may be some cases that some regions in the image are very noisy while other regions are not. In these cases, it is expected that more weights could be put on the neighborhood of the noisy regions while less weight is put on other regions, thus making the algorithm adaptive. This paper proposes four such adaptive algorithms called FCM$_{S11}$, FCM$_{S12}$, FCM$_{S21}$ and FCM$_{S22}$. These algorithms are free of weight selection. They can automatically determine the weight put on the neighborhood term of each pixel based on the noisiness of every pixel in the image. To achieve this, we make the weight depend on the difference between the pixel value and the mean (median) pixel value of its neighborhood, or the variance of the pixel values in the neighborhood. If one pixel is very noisy, then this difference and the variance will be large and more weights will be put on the neighboring term of that pixel.

The rest of this paper is organized as follows. We introduce our algorithms in Section 2. Section 3 presents the comparison of our algorithms with some existing algorithms and
demonstrates the effectiveness of our algorithms. We discuss conclusions in Section 4

2. ADAPTIVE FCM

In this section, we first introduce FCM_S1 and FCM_S2 proposed in [9]. Then we present our algorithms based on these two algorithms.

2.1. FCM_S1 and FCM_S2

Consider the problem of clustering a dataset \( \{x_k\}_{k=1}^{N} \subset \mathbb{R}^d \) into \( c \) clusters \( (2 \leq c < N) \), in FCM_S1, the authors construct a cost function as follows.

\[
J_m = \sum_{k=1}^{N} \sum_{i=1}^{c} u_{ik}^m \|x_k - v_i\|^2 + \alpha \sum_{k=1}^{N} \sum_{i=1}^{c} u_{ik}^m \|\bar{x}_k - v_i\|^2
\]

where \( \|\cdot\| \) denotes the standard Euclidean norm, \( v_i \) is the prototype of the center of cluster \( i \), \( u_{ik} \) is the degree of membership of \( x_k \) in cluster \( i \), \( m \) is a weighting exponent on each fuzzy membership, \( N_k \) is the number of neighbors of \( x_k \), \( \bar{x}_k \) is the average of the neighboring pixels inside a window centered at \( x_k \), and \( \alpha \) is the weight put on the neighboring term. It indicates the effect of neighboring pixels on the clustering of \( x_k \). To minimize this cost function, we can calculate the partial derivative of \( J_m \) with respect to \( u_{ik} \) and \( v_i \) and set them to be zero to get the necessary conditions the local minima has to satisfy [2]. They are as follows.

\[ u_i = \frac{\sum_{k=1}^{N} u_{ik}^m (x_k + \alpha \bar{x}_k)}{(1 + \alpha) \sum_{k=1}^{N} u_{ik}^m} \]
\[ u_{ik} = \frac{\sum_{j=1}^{N} (d_{jk}^2 + \alpha \|x_k - v_i\|^2)^{-1/(m-1)}}{\sum_{j=1}^{N} (d_{jk}^2 + \alpha \|x_k - v_j\|^2)^{-1/(m-1)}} \]

where \( d_{jk} = \|x_k - v_i\| \). Based on these equations, an iterative procedure can be used to get the local minima of \( J_m \) and minimize the cost function. The authors in [9] also propose another algorithm called FCM_S2. In FCM_S2, it uses the median of the neighboring pixels instead of the average.

\[ \text{FCM_S12: } \alpha_k = \text{Var}(x_r) = \frac{1}{N_k-1} \sum_{r=1}^{N_k} (x_r - \bar{x}_k)^2, \]

where \( \text{Var}(x_r) \) is the unbiased sample variance of the neighbors of \( x_k \). Note that \( \bar{x}_k \) and \( \text{Var}(x_r) \) can be calculated in advance and stored in the memory, thus decrease the computation time of the algorithm.

Similarly, based on FCM_S2, we replace the mean by the median of the neighboring pixels instead of the average. When a pixel is noisy, its distance to the average of its neighbors will be large and the variance of its neighbors will also be large. Thus more weights will be put on the neighboring term in the cost function. Therefore, this modification increases the robustness of the algorithms to noise. Next we give a necessary condition \( v_i \) and \( u_{ik} \) have to satisfy in order to minimize the modified cost function.

**Theorem 1.** Assume \( u_{ik}^* \) and \( v_i^* \) are local or global minimas to minimize the modified cost function, then they have to satisfy the following conditions:

\[ u_{ik}^* = \frac{\sum_{k=1}^{N} u_{ik}^m (x_k + \alpha_k \bar{x}_k)}{\sum_{k=1}^{N} (1 + \alpha_k) u_{ik}^m} \]
\[ u_{ik}^* = \frac{(d_{ik}^2 + \alpha_k \|x_k - v_i^*\|^2)^{-1/(m-1)}}{\sum_{j=1}^{N} (d_{jk}^2 + \alpha_k \|x_k - v_j^*\|^2)^{-1/(m-1)}} \]

**Proof.** Because of space limitations, we omit the proof and put it online at http://people.bu.edu/xlan/misc.html.

By this theorem, we can use an iterative procedure to get \( v_i^* \) and \( u_{ik}^* \). First, initialize \( u_{ik} \), then update \( v_i \), then update \( u_{ik} \), repeat this process until it converges.

2.2. Adaptive FCM

As discussed above, FCM_S1 and FCM_S2 are parameter dependent. The weight \( \alpha \) is fixed for every pixel in the image. Based on FCM_S1, we propose two algorithms FCM_S11 and FCM_S12 which can automatically decide the weight \( \alpha \) based on the noisiness of every pixel in the image.

FCM_S11: \( \alpha_k = \|x_k - \bar{x}_k\| \),

FCM_S12: \( \alpha_k = \text{Var}(x_r) = \frac{1}{N_k-1} \sum_{r=1}^{N_k} (x_r - \bar{x}_k)^2, \)

where \( \text{Var}(x_r) \) is the unbiased sample variance of the neighbors of \( x_k \). Note that \( \bar{x}_k \) and \( \text{Var}(x_r) \) can be calculated in advance and stored in the memory, thus decrease the computation time of the algorithm.

3. EXPERIMENTS

In this section, we compare the performance of our algorithms with K-means, FCM, FCM_S1 and FCM_S2. Like [8], we choose the segmentation accuracy (SA) as the segmentation performance metric, where SA is defined as follows.

\[ \text{SA} = \frac{\text{Number of correctly clustered pixels}}{\text{Total number of pixels}} \]
Table 1. Segmentation accuracy (SA%) on an image (shown in 1(a)) corrupted by different noise.

<table>
<thead>
<tr>
<th></th>
<th>FCM_{S1} (\alpha = 1)</th>
<th>FCM_{S1} (\alpha = \frac{1}{15})</th>
<th>FCM_{S1} (\alpha = \frac{1}{30})</th>
<th>FCM_{S2} (\alpha = 1)</th>
<th>FCM_{S2} (\alpha = \frac{1}{15})</th>
<th>FCM_{S2} (\alpha = \frac{1}{30})</th>
<th>FCM_{S11}</th>
<th>FCM_{S12}</th>
<th>FCM_{S21}</th>
<th>FCM_{S22}</th>
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<tbody>
<tr>
<td>Gaussian 10%</td>
<td>84.6698</td>
<td>98.4486</td>
<td>98.4015</td>
<td>97.3113</td>
<td>97.3270</td>
<td>98.3543</td>
<td>98.3333</td>
<td>97.3795</td>
<td>97.3480</td>
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<tr>
<td>Gaussian 20%</td>
<td>76.5828</td>
<td>95.5451</td>
<td>95.9696</td>
<td>93.2914</td>
<td>93.3071</td>
<td>95.9958</td>
<td>96.0115</td>
<td>93.7159</td>
<td>93.5954</td>
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<tr>
<td>Gaussian 30%</td>
<td>72.6520</td>
<td>91.6929</td>
<td>92.5105</td>
<td>90.4459</td>
<td>90.6656</td>
<td>93.1132</td>
<td>93.2862</td>
<td>91.1583</td>
<td>91.0692</td>
<td></td>
</tr>
<tr>
<td>Gaussian Localvar 1</td>
<td>70.6709</td>
<td>88.8365</td>
<td>89.7484</td>
<td>85.0737</td>
<td>89.7432</td>
<td>89.7397</td>
<td>90.8595</td>
<td>90.2568</td>
<td>90.1677</td>
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<tr>
<td>Gaussian Localvar 2</td>
<td>69.8166</td>
<td>87.1174</td>
<td>87.9245</td>
<td>82.6520</td>
<td>88.6485</td>
<td>89.1771</td>
<td>89.4182</td>
<td>89.2715</td>
<td>89.0461</td>
<td></td>
</tr>
<tr>
<td>Gaussian Localvar 3</td>
<td>70.4088</td>
<td>88.4539</td>
<td>89.3082</td>
<td>78.6373</td>
<td>89.6488</td>
<td>90.2725</td>
<td>90.4245</td>
<td>90.1887</td>
<td>90.2044</td>
<td></td>
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<tr>
<td>Salt &amp; pepper 20%</td>
<td>90.2411</td>
<td>97.7883</td>
<td>97.9355</td>
<td>90.0105</td>
<td>99.2453</td>
<td>98.2075</td>
<td>98.4130</td>
<td>98.0294</td>
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<tr>
<td>Salt &amp; pepper 30%</td>
<td>85.0786</td>
<td>94.6698</td>
<td>95.4979</td>
<td>84.7694</td>
<td>99.1247</td>
<td>96.3732</td>
<td>96.2159</td>
<td>99.1247</td>
<td>99.1247</td>
<td></td>
</tr>
<tr>
<td>Speckle 20%</td>
<td>98.0031</td>
<td>99.4497</td>
<td>99.3973</td>
<td>98.3805</td>
<td>99.3239</td>
<td>98.5797</td>
<td>98.0294</td>
<td>97.5681</td>
<td></td>
<td></td>
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<tr>
<td>Speckle 30%</td>
<td>94.7170</td>
<td>93.3444</td>
<td>93.2662</td>
<td>93.9308</td>
<td>97.4319</td>
<td>96.1667</td>
<td>96.2687</td>
<td>97.1226</td>
<td>96.4151</td>
<td></td>
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</table>

Our first test is on a synthetic image corrupted by different kinds of noise, see Figure 1(b). The original image is a 120 × 159 pixels image with two classes of gray level values set as 25 and 125 [13]. The experiment parameters are chosen as follows. For K-means, the only parameter is \( K = 2 \). For all the FCM-based algorithms, the fuzzy parameter is chosen as \( m = 2 \) and the error tolerance \( \epsilon = 10^{-5} \). For the neighbors, we choose a 5 × 5 pixels squared region. Note that the shape of the neighboring region does not matter too much and other shapes of neighboring region also apply. The comparison result is shown in Table 1. Because of space limitations, we omit the results for Kmeans and FCM. Their performance is much worse than the algorithms shown in Table 1 see Figure 1. The reason is that K-means and FCM do not use the spatial information of the image to help the clustering. Table 1 also shows that FCM_{S1} and FCM_{S2} are parameter dependent. FCM_{S1} is very effective in dealing with speckle noise while FCM_{S2} works much better for salt & pepper noise. Our algorithms FCM_{S21} and FCM_{S22} perform as well as FCM_{S2} for salt & pepper noise. For Gaussian noise, FCM_{S1} works well if the noise is small while FCM_{S12} works best if the noise is large. Also, if the noise has different variance (Gaussian Localvar), our algorithms perform much better than FCM_{S1} and FCM_{S2} since they are adaptive. They can adjust the weights put on the neighboring term based on the noisiness of the pixels. Figure 1 shows the segmented image by all these algorithms for the variance-locally-varying Gaussian noise. Figure 2 shows the segmentation result on a medical MRI image [14]. From this figure, we can see that our algorithms FCM_{S11}, FCM_{S12}, FCM_{S21} and FCM_{S22} are very effective in segmenting the upper noisy region of this MRI image.

4. CONCLUSIONS

In this paper, we propose four algorithms called FCM_{S11}, FCM_{S12}, FCM_{S21} and FCM_{S22} to do image segmentation. Our algorithms build on FCM_{S1} and FCM_{S2}. We utilize the high correlation between image pixels to help eliminate the affect of noise on clustering. Unlike FCM_{S1} and FCM_{S2}, our algorithms are free of weight selection. Based on the local neighborhood information, they can automatically determine how much weight to put on the neighborhood information to help the clustering. As a result, the clustering performance is improved. The experiments demonstrate the effectiveness of our algorithms.

5. REFERENCES

Fig. 1. Segmented result of a noisy image corrupted by locally-varying Gaussian noise (SA is shown in Row 5 of Table I).

Fig. 2. Partition a medical MRI image into 2 clusters.


