Proofs

**Lemma 1** When qualities are independently distributed, every rationalizable Bayesian strategy is weakly increasing.

**Proof:** Consider two possible types of player 1, \( q_L < q_H \). Suppose that \( q_L \) puts positive probability weight on conceding at time \( t_L \), and \( q_H \) puts positive weight on conceding at time \( t_H \). Let \( \Gamma \) be player 1’s (perceived) distribution function of \( t_2 \), the time when player 2 concedes (if player 1 does not previously do so). Let \( E[q|t] = E[q_2|t_2 = t] \) be player 1’s expected value of player 2’s quality \( q_2 \) given that player 2 concedes at \( t \). (This conditional expectation does not depend on player 1’s own type by assumption of independence.) We now state player 1’s incentive compatibility constraints. If type \( q_L \) is willing to concede at \( t_L \), we have:

\[
W_{q_L} \int_0^{t_L} e^{-rt} d\Gamma(t) + L e^{-r t_L} \int_{t_L}^{\infty} E[q|t] d\Gamma(t) \geq W_{q_L} \int_0^{t_H} e^{-rt} d\Gamma(t) + L e^{-r t_H} \int_{t_H}^{\infty} E[q|t] d\Gamma(t).
\]

The same argument leads to an incentive compatibility constraint for type \( q_H \) (replace \( q_L \) with \( q_H \) and reverse the inequality in the previous expression). Adding these two inequalities yields:

\[
\int_0^{t_L} e^{-rt} d\Gamma(t) \leq \int_0^{t_H} e^{-rt} d\Gamma(t)
\]

Thus, if \( t_L > t_H \), there is zero concession probability for player 2 between \( t_H \) and \( t_L \). But then no type of player 1 should concede at \( t_L \) (it would be better to concede at or just after \( t_H \)), contradicting our assumption that \( q_L \) can optimally concede at that time. Thus, we conclude that \( t_L \leq t_H \). \( \square \)

**Proposition 1** When qualities are independently and continuously distributed, the symmetric equilibrium of an uninterrupted war of attrition selects the better proposal.

**Proof:** Suppose there is an atom in the distribution of concession times at \( T \), and let \( \bar{q} (q) \) be the supremum (infimum) of the set of types who can optimally concede at that time. For a given strategy, a player’s expected payoff is continuous in its type. So type \( \bar{q} \) can also optimally concede at \( T \).
If $\bar{q} < \infty$, then by waiting until just after $T$, type $\bar{q}$ would (strictly) increase its own probability of winning without reducing the expected quality of the system that emerges. This contradicts the statement that type $\bar{q}$ can optimally concede at $T$. If $\bar{q} = \infty$, all types $q \in (\bar{q}, \infty)$ concede at $T$ (by Lemma 1) and receive an expected payoff $\frac{1}{2}(Wq + LG(q))$. This is also a contradiction, as types $q > \frac{LG(q)}{W}$ could do strictly better by waiting an instant to get $Wq$ for sure.

Now suppose there is a gap in the distribution of concession times between $t_L$ and $t_H$. This also leads to a contradiction: the lowest type that can optimally concede at $t_H$ would do better to concede just after $t_L$. Thus, if quality is continuously distributed, the symmetric equilibrium strategies in an uninterrupted war of attrition must be one-to-one. The better system wins because concession times are strictly increasing in $q$. □

**Lemma 3** Suppose the equilibrium concession strategies of proponents who expect a random choice at $T$ remain unchanged (i.e., as if $T = \infty$). Then, if either $f(q)$ or $1 - F(q)$ is log-concave, there is a unique time $T^* \geq 0$ such that a neutral player prefers screening via the war of attrition at all $t < T^*$ and immediate random choice thereafter.

**Proof:** There are several cases to consider. First, if $G(q_{\min}) = \mu < (1 + v)q_{\min}$ then (7) implies that a neutral player prefers immediate random choice at $t = 0$. Because log-concavity implies that $G(q) - (1 + v)q$ is monotone decreasing, the neutral player will also prefer a random intervention at all $t > 0$; so $T^* = 0$.

If $\mu > (1 + v)q_{\min}$ then screening is initially worthwhile. Suppose the support of $F(\cdot)$ is $[q_{\min}, \infty)$. Log concavity implies that $G(q) - q$ is bounded above by $\mu - q_{\min}$. Because $vq$ grows without bound as $q \to \infty$, continuity and the intermediate value theorem imply the existence of a unique $q^*$ such that $G(q^*) - q^* = vq^*$. For all $t > t(q^*)$ the neutral player prefers random intervention.

If the support of $F(\cdot)$ is $[q_{\min}, \bar{q}]$, then $G(q)$ must converge to $\bar{q}$ as $q$ approaches the upper bound. Because $v\bar{q} > 0$, continuity and the intermediate value theorem imply a unique $q^* < \bar{q}$. □

**Proposition 3** The following strategies are a perfect Bayesian equilibrium: Sponsor-types $q < q^*$ use the concession strategy derived in Section 2.1. Types $q > q^*$ never concede. The neutral player waits until $T^*$ before making a random choice (and would also intervene at all times $t > T^*$).

**Proof:** To begin, note that Lemma 3 implies $Wq^* = LG(q^*)$, so the last type to concede before a random intervention must be indifferent between immediate victory and concession. We need to show that two kinds of deviations are unrewarding for the players. First, in the candidate equilibrium, a sponsor with type $q > q^*$ will wait until its rival concedes or the random choice is made; might this player deviate by conceding early, before $T^*$? Second, a sponsor with type $q < q^*$ is meant to concede before the random choice at $T^*$; might it deviate by waiting till $T^*$ and having a chance of winning?
First, by Lemma 3, at $T^*$ a sponsor with $q > q^*$ prefers random choice to concession. Since the type $q^*$ sponsor at least weakly prefers concession at $T^*$ to concession at any $t < T^*$, Lemma 3 implies that the same is true of any higher type. Thus, types $q > q^*$ do not have an incentive to deviate from equilibrium play. We must also check that they would not deviate from the prescribed out-of-equilibrium play. That is, if at $t > T^*$ there has still been no random choice, but they expect one imminently, would a high-type try to concede? No: by Lemma 3, they would prefer an immediate random choice.\footnote{This would break down if the sponsors believe that types $q \in (q^*, Q)$ would have already conceded by now (after $T^*$), so that concession looks better (relative to random choice) because the opponent’s expected quality is now $G(Q) > G(q^*)$. But this is not an issue, as we are specifying that each believes that the other is playing “never concede after $T^*$” and there can be no evidence to falsify that belief: the only available evidence would be concession, after which the question doesn’t arise.}

Second, a player with $q < q^*$ could, by waiting till $T^*$, obtain a strictly positive chance of winning. Couldn’t that be a tempting deviation? No—by Lemma 3, for a low quality sponsor, concession just before $T^*$ dominates random choice at $t \geq T^*$: the expected benefits of choosing a rival system exceed those of being the winner.

Finally, consider the neutral player’s random intervention at $T^*$. By the definition of $T^*$, this player prefers to allow screening at all $t < T^*$ but wants to intervene then. Because the out-of-equilibrium expectations are that no vendors will concede at any $t > T^*$, if no actual concession has happened by such a time, the same calculation says that the neutral player wants to intervene now.

Of course, if a concession has happened, there is no incentive question to resolve. □

**Lemma 4:** When proponents expect a random intervention at $T < \infty$, their symmetric equilibrium concession strategies have thresholds $q \leq \overline{q}$, such that types $q < \overline{q}$ concede according to $t(q)$; types $q \in (\underline{q}, \overline{q})$ concede at $T$; and types $q > \overline{q}$ wait for the random intervention.

**Proof:** Suppose $\underline{q}$ is the supremum of the set of types that concede strictly before $T$. As $t(q)$ is weakly increasing, types $q < \underline{q}$ will also concede before $T$. And because the intervention at $T$ has no impact on the marginal costs or benefits of delay for types below $\underline{q}$, they will concede according to their first-order conditions. To see that $\overline{q}$ is finite, note that $t(q) \to \infty$ as $q$ approaches $\infty$ (or any finite upper bound). Moreover, because $F(\cdot)$ and the payoffs are continuous in $q$, type $\overline{q}$ will be indifferent between conceding at $t(q)$ and $T$.

The remaining types $q > \overline{q}$ have two options; they can concede at $T$ — and win with probability $\frac{1}{2}$ if the other player also concedes at that time — or “wait” (an instant) for the random intervention. Lemma 1 says that if $\overline{q}$ is the supremum in the set of types that can optimally concede at $T$, then any proposals between $\underline{q}$ and $\overline{q}$ must also concede at that time. Types greater than $\overline{q}$ wait for the random choice by construction. □

**Proposition 4:** If the marginal benefit of unobserved ex ante R&D is constant in $q$, a predetermined
standard setter has the strongest incentive to innovate, followed by firms facing an immediate random choice. Firms that anticipate a random intervention at \( T^* > 0 \) have weaker incentives than under immediate random choice, but stronger than under an uninterrupted war of attrition: \( I^{PS} = W > I^{RC} = \frac{1}{2}W ≥ I^{RI} ≥ I^{WOA} \).

**Proof:** Because \( I^{WOA} \equiv E[u'(q)] \), equation (10) implies that

\[
I^{WOA} = W \int_{q_{\text{min}}}^{\infty} \int_{q_{\text{min}}}^{x} \delta(y) \, dF(y) \, dF(x) \leq W \int_{q_{\text{min}}}^{\infty} \int_{q_{\text{min}}}^{x} dF(y) \, dF(x) = \frac{W}{2} = I^{RC}
\]

and the inequality is strict for \( v > 0 \), because \( \delta(y) < 1 \) for all \( y > q_{\text{min}} \) (by Proposition 1).

Random intervention is equivalent to an immediate random choice as of \( T^* \), and the incentive comparison is no different, as both proposals must be better than \( q^* \) for random intervention to make any difference in outcomes. Formally, we have

\[
I^{RI} - I^{WOA} = \int_{q^*}^{\infty} \left( \frac{1}{2}W\delta(q^*) - W \int_{q^*}^{x} \delta(y) \, dF(y) \right) \, dF(x)
\]

Because \( \delta(y) < \delta(q^*) \) for all \( y > q^* \), we can factor out \( \delta(q^*) \) and integrate the previous expression to show that

\[
I^{RI} - I^{WOA} > \frac{1}{2}W\delta(q^*)F(q^*)[1 - F(q^*)] > 0 \quad \square
\]

**Calculations**

All examples are based on the Pareto distribution: \( F(x) = 1 - x^{-(1+a)} \) for \( x \geq 1 \) and \( a > 0 \).

**Example 1:** Suppose quality has the Pareto distribution, and that the equilibrium concession strategies of proponents who expect a random choice at \( T \) remain unchanged (i.e., as if \( T = \infty \)). Then a neutral player makes an immediate random choice if \( av > 1 \) and allows an uninterrupted war of attrition if \( av < 1 \).

\[
G(x) = \frac{\int_{x}^{\infty} sf(s) \, ds}{1 - F(x)} = \frac{-(1 + a) \int_{x}^{\infty} s^{-(1+a)} \, ds}{x^{-(1+a)}} = \frac{-(1 + a)[s^{-a}]_{x}^{\infty}}{ax^{-(1+a)}} = \frac{(a + 1)x}{a}
\]

The results follows immediately from substituting \( G(x) \) into equation (7).

**Example 2** In the uninterrupted war of attrition with a Pareto distribution, \( I^{WOA} = E[u'(q)] \) is increasing in \( W \) if and only if \( k < \frac{a+2}{a(a+1)^2} \).

To find \( E[u'(q)] \), start with \( u'(q) \) from equation (10). From the calculations for Example 1, we know
\[ \mu = \frac{(a+1)}{a} \] and \( K(x) = \frac{(a+1)x^{-a}}{a} \). Thus, we have

\[ u'(q) = W \int_1^q \left[ \frac{K(s)}{\mu} \right]^v dF(s) = W(1 + a) \int_1^q s^{-a(v+1)-2} ds = \frac{W(1 + a) \left[ 1 - q^{-a(v+1)-1} \right]}{a(v + 1) + 1} \]

and taking expectations yields

\[ E[u'(q)] = \frac{W(1 + a)}{a(v + 1) + 1} \int_1^\infty \left[ 1 - s^{-a(v+1)-1} \right] (1 + a)s^{-a-2} ds \]

\[ = \frac{W(1 + a)}{a(v + 1) + 1} \left( 1 - \frac{(a + 1)}{a(v + 2) + 2} \right) = \frac{LW(a + 1)}{Wa + L(a + 2)} \]

where the final equality is found by substituting \( v = \frac{W-L}{L} \) and simplifying. In Example 2 we differentiate this expression with respect to \( w \) where (by assumption) \( \frac{dW}{dw} = 1 \) and \( \frac{dL}{dw} = -k \), and ask whether the result is greater than zero. This yields

\[ \frac{dI^{W/O}}{dw} = \frac{(a+1)(L^2(a+2) - kaW)}{(Wa + L(a + 2))^2} \]

and the numerator is positive if and only if \( k < \frac{L^2(a+2)}{Wa^2} = \frac{a+2}{a(v+1)^2} \).

**Example 3:** \( E[s'(q)] < E[u'(q)] \) if and only if \( v(v + 2)(a + 1) > 1 \): private incentives to improve quality are too high if there is strong vested interest.

Integrating equation (11) yields \( E[s'(q)] \) as a function of \( E[u'(q)] \). Thus, using \( E[u'(q)] \) from Example 2, we have

\[ E[u'(q)] < E[s'(q)] \iff \frac{L}{W} E[u'(q)] > (W + L)E[qf(q)v\delta(q)] \iff \frac{L(a + 1)}{a(v + 2) + 2} > (W + L)E[qf(q)v\delta(q)] \]

Further calculations show that

\[ E[qf(q)v\delta(q)] = v(1 + a)^2 \int_1^\infty s^{-a(2+v)-3} ds = \frac{v(1 + a)^2}{a(v + 2) + 2} \]

and substituting this expression into the previous inequality, yields the result stated in the text.

\[ E[u'(q)] < E[s'(q)] \iff \frac{L(a + 1)}{a(v + 2) + 2} > \frac{(W + L)v(1 + a)^2}{a(v + 2) + 2} \iff 1 > v(v + 2)(a + 1) \]