Cascaded stochastic processes in optics

Processus stochastiques en cascade d'importance en optique

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abstract and key words

Thirty years ago, Bernard Picinbono and his colleagues carefully addressed an important problem : how an optical field is converted into a sequence of photoelectrons upon detection. Their choice of problem could not have been better, nor their timing more judicious. In a paper entitled "Photoelectron Shot Noise", published in the *Journal of Mathematical Physics* in 1970, when quantum optics was in its infancy, they obtained results that were to serve as an important building block in analyzing and generating many different forms of light. We present some variations on the theme of cascaded stochastic processes in optics.

Poisson process, shot noise, self-exciting process, doubly-stochastic process, compound Poisson process, doubly-Poisson, triggered optical emissions, multiply-Poisson, cascaded-Poisson, correlated-Poisson, nonclassical light, sub-Poisson light.

résumé et mots clés

Il y a trente ans, Bernard Picinbono et ses collègues ont traité rigoureusement un problème important : comment un champ optique est converti en une suite de photoélectrons après détection. Leur choix de ce problème ne pouvait pas être meilleur et a révélé un caractère pionnier. Dans un article intitulé « Photoelectron Shot Noise », publié dans le *Journal of Mathematical Physics* en 1970, alors que l'optique quantique n'était encore qu'à ses débuts, ils obtinrent des résultats qui constituèrent un important point de départ pour analyser et générer de nombreuses et diverses formes de lumières. Nous présentons des variations sur le thème des processus stochastiques en cascade, en optique.

Processus de Poisson, processus stochastique en cascade.

1. Introduction

Exactly thirty years ago, Bernard Picinbono, together with Chérif Bendjaballah and Jean Pouget, were putting the final touches on a manuscript they were preparing to submit to the *Journal of Mathematical Physics*. The work was accepted for publication by the Editor soon after receipt. The article appeared in print in July 1970, just a year after its original submission [1]. This classic paper has served as a cornerstone of statistical optics ever since.

In the late 1950s it was established that the statistical properties of a light source were manifested in the sequence of photoelectrons generated when the light impinged on a photocathode. In 1959, Leonard Mandel proposed that the link between the intensity fluctuations of the source and the sequence of time instants at which photoelectrons were produced was embodied in the doubly-stochastic (compound) Poisson process [2]. In the years that followed it became critical to carefully establish the validity of this relationship. Only then could the statistical properties of various sources of light be compared and contrasted. The 1960s and 1970s saw the invention of the laser and its development, and the focus was then on distinguishing laser light from other more conventional forms of light. In the 1980s and 1990s the focus shifted to the discrimination of nonclassical from classical light.

Throughout the course of these developments the photomultiplier tube (PMT) has been the instrument of choice for carrying out such measurements and indeed it remains so even today. The contribution by Picinbono and colleagues was exceptional because it recognized that the measurement instrument itself, the photomultiplier tube, introduces unavoidable excess fluctuations of its own in the course of detecting the light under study. The beauty of the photomultiplication process is that every photoelectron at the cathode undergoes a chain-reaction cascade which results in roughly a million more electrons being added to it by the time the current pulse traverses the dynodes and arrives at the anode. This millionfold amplification has the salutary effect of making each detected photon stand tall and contribute to the output current. But this comes at the cost of stochastic fluctuations associated with the cascade, noise known as gain fluctuations, which result in a variable sequence of current pulses at the anode.

These authors succeeded in providing a beautiful characterization of this additional noisiness so that proper inferences could be drawn about the statistical nature of the light itself. Though it had long been known that the central limit theorem provides an appropriate description for linearly filtered Poisson events (shot noise) under a broad variety of conditions [3], this seminal paper led to the recognition that such a description cannot be used for the detection of random light by a real photomultiplier tube.

The approach taken by Bernard Picinbono and his colleagues has suggested a whole host of generalizations. In this tribute, we devote particular attention to our own efforts toward establishing appropriate statistics and asymptotic forms for doubly- and multiply-Poisson processes, which are concatenations of homogeneous Poisson processes and shot noises. We begin with the most elemental of such conceptions : the homogeneous Poisson process, shot noise, and self-exciting point processes. We end with cascaded Poisson processes and shot noises.

2. Poisson Processes

The homogeneous Poisson point process, which is illustrated in figure 1 (top), is the simplest of all point processes [4]. It is characterized by a single quantity, its rate μ, which is constant. Its distinguishing feature is that it is memoryless; the occurrence times and numbers of events before an arbitrary time have no bearing on the subsequent occurrence times and numbers of events. Because of its simple properties it forms a suitable building block for more complex point processes that appear to resemble it little, if at all. In optics, a light source of constant intensity, such as an ideal amplitude-stabilized laser, leads to photoelectron statistics characterized by the homogeneous Poisson process.

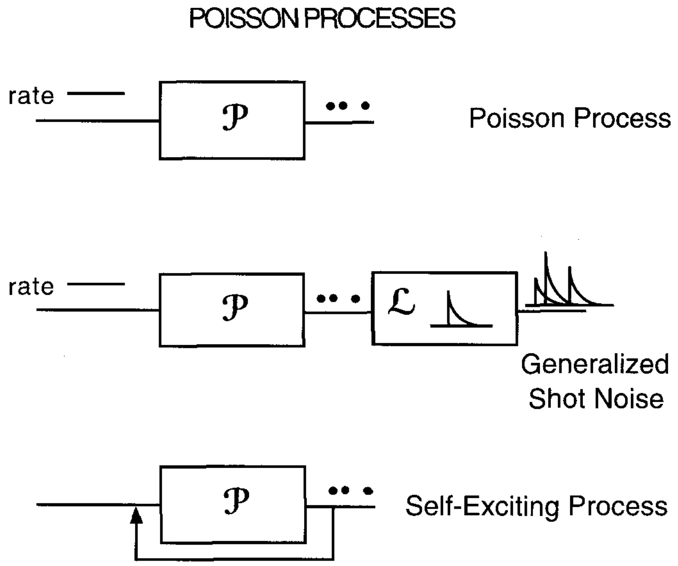

Figure 1. – ***Top*** **: Poisson generator $\mathcal{P}$ generates a Poisson point process with constant rate μ.** ***Middle*** **: A random linear filter $\mathcal{L}$ following $\mathcal{P}$ leads to generalized shot noise.** ***Bottom*** **: The evolution of a self-exciting point process depends on the occurrence times of past events. An arbitrary point process, with variability greater or less than that of the Poisson, can be cast in the form of a self-exciting process.**

2.1. Shot Noise

A Poisson process of rate μ passed through a deterministic linear filter with impulse response function $h(t)$ gives rise to shot noise. Campbell obtained values for the mean and variance of this process in 1909 and used it to characterize the emission of light [5]. The process was extensively studied, and named *shot effect*, by Schottky in 1918 [6]. When the impulse response function is of finite duration τ_p and the emissions are dense ($\mu\tau_p \gg 1$), the shot-noise amplitude distribution approaches Gaussian statistics [1, 3, 7].

Generalized shot noise arises when a Poisson process is passed through a linear filter whose impulse response is a random function chosen from a common distribution, as illustrated in figure 1 (middle). It has been shown [1, 8] that an ensemble of stochastic impulse response functions has an equivalent deterministic impulse response function that is suitable for calculating the first-order statistics of the shot-noise process.

When the impulse response function assumes the form of a decaying power law, its characteristic time can become arbitrarily large or small. Such fractal (power-law) shot noise can then violate the conditions of the central limit theorem whereupon the amplitude distribution does not approach Gaussian form for any value of the Poisson rate μ. The behavior of fractal shot noise, and its generalized cousin, have been extensively studied for a variety of parameters of the process [9]. For certain parameters the power spectral density exhibits $1/f$-type behavior over a substantial range of frequencies, so that the process serves as a source of $1/f^{\alpha}$ shot noise for α in the range $0 < \alpha < 2$. For other parameters the amplitude probability density function is a Lévy-stable random variable with an order parameter less than unity. This process then behaves as a fractal shot noise that fails to converge to a Gaussian amplitude distribution in the asymptotic limit as the driving rate increases. In the domain of optics fractal shot noise provides a suitable model for describing the intensity statistics of Čerenkov radiation arising from a random stream of charged particles.

2.2. Self-Exciting Processes

An arbitrary regular point process can be cast in the form of a Poisson process with a rate controlled by past events, as illustrated in figure 1 (bottom). In its most general form, the future evolution of such a self-exciting point process depends on the occurrence times of past events as well as on their total number [4].

A special but useful case occurs when the process has limited memory; in particular, the interevent intervals of a homogeneous self-exciting Poisson process with a memory that reaches back exactly one event form a sequence of statistically independent random variables (a renewal process) [4]. The dead-time-modified Poisson process, a renewal process, is of particular interest in optics [10]. The circuitry at the output of a photodetector, such as a photomultiplier tube operated in the photon-counting mode, generally exhibits a fixed period of time τ_d following the registration of an event, during which it is incapable of registering another event, i.e. it is dead. Whether the dead time assumes nonparalyzable or paralyzable form [11], its presence serves to substantially regularize the photoelectron point process. This, in turn, results in a photoelectron counting distribution whose normalized variance (count variance divided by count mean) can reach substantially below unity, a value at which it is fixed for a homogeneous Poisson process [11]. Such counting statistics are therefore called *sub-Poisson*.

3. Doubly-Stochastic Poisson Processes

The doubly-stochastic Poisson process has a rate that takes on a stochastic nature of its own. This process was first examined by Cox [12], who provided an example of its use in textile technology. The designation *doubly-stochastic* was introduced to emphasize that two kinds of randomness are operative : randomness associated with the point process itself and an independent randomness associated with its rate, as illustrated in figure 2 (top). The doubly-stochastic Poisson process, often called the compound Poisson process, has become the basis for understanding photoelectron statistics of all orders that result from the detection of classical light of all forms [13]. The rate of the point process is the squared magnitude of the complex field. Particular attention was devoted early on to unraveling the photoelectron statistics for thermal light, which is characterized by a circularly symmetric complex Gaussian field [1, 13, 14, 15], and for interfering superpositions of thermal and amplitude-stabilized (ideal-laser) light [1, 13, 16]. Bernard Picinbono, together with M. Rousseau, considered the statistical properties of a generalized class of Gaussian optical fields, including pseudo-Gaussian fields and Gaussian fields with real amplitude [17, 18]. Interestingly, the fluctuations of the real-amplitude Gaussian field turn out to be greater than those of the thermal field.

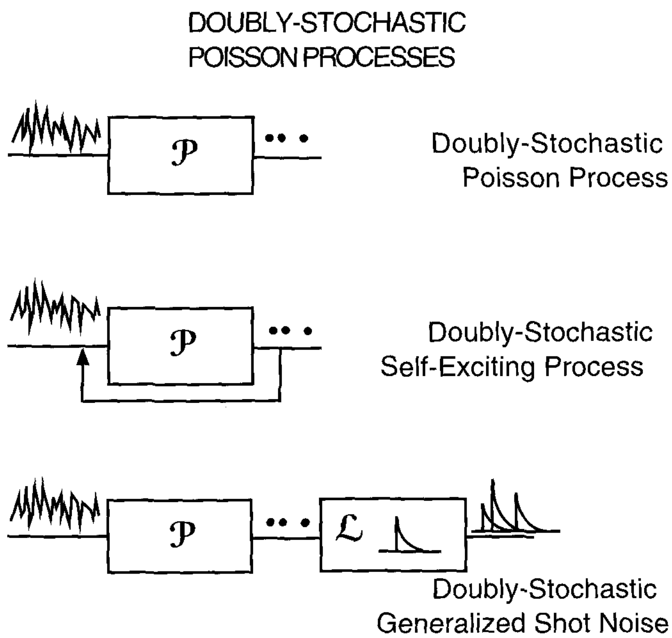

Figure 2. – *Top* : A Poisson point process $\mathcal{P}$ whose rate is a stochastic process in its own right, is termed a doubly-stochastic Poisson point process. *Middle* : A self-exciting point process whose rate is a stochastic process. *Bottom* : A random linear filter $\mathcal{L}$ following a doubly-stochastic Poisson process leads to doubly-stochastic generalized shot noise, first analyzed in detail by Bernard Picinbono and colleagues in 1970 [1].

3.1. Doubly-Stochastic Self-Exciting Processes

The effects of dead time on a doubly-stochastic Poisson point process, illustrated in figure 2 (middle), are substantial and dramatic. Though the complexity of the calculations quickly escalates, expressions for the dead-time-modified count mean and variance have been obtained for thermal light [19] and for shot-noise light [20], a doubly-Poisson form of light that will be discussed in some detail subsequently.

3.2. Doubly-Stochastic Shot Noise

The calculation of the statistical properties of doubly-stochastic generalized shot noise, illustrated in figure 2 (bottom), was, of course, carried out in the seminal paper by Picinbono and colleagues [1]. To incorporate the effects of gain fluctuations inherent in the photomultiplication process, they modeled the photomultiplier-tube anode-current pulses as a sequence of independent nonstationary brief random impulse-response functions drawn from a single probability distribution. They established that in the limit of dense photoemissions, $\mu\tau_p \gg 1$ where μ is the rate of photoelectron arrivals and τ_p is their effective duration at the anode, the asymptotic photocurrent resulting from such a filtered doubly-stochastic Poisson point process in general fails to converge to Gaussian form. They therefore concluded that the central-limit theorem is not applicable in this circumstance. They likened this behavior to the result of finding the limit of a random number of independent random variables, and cited Robbins' 1948 study [1].

Among the specific results they derived in this classic paper are the single- and two-fold anode current distributions when thermal light is incident at the faceplate of a photomultiplier tube. The resulting generalized shot noise was found to be characterized by an asymptotic probability distribution proportional to the zeroth-order modified Bessel function of the second kind, K_0. In recent years a number of important studies have elaborated on the emergence of the K_0 distribution in just this context [21]. The same distribution emerges repeatedly in optics; it is useful for characterizing the field and intensity fluctuations of scattered light, as well as light that has been transmitted through a random medium such as the turbulent atmosphere [22].

4. Doubly-Poisson Processes

The designation *doubly-Poisson* indicates the participation of a *pair* of Poisson processes. The simplest of the doubly-Poisson processes, illustrated in figure 3 (top), concatenates a homogeneous Poisson process, a deterministic linear filter, and a second Poisson process. Since the output of the linear filter is shot noise, this construct is given the appelation *shot-noise-driven Poisson process* [23]. It is a special kind of doubly-stochastic Poisson process as can be understood by comparing figure 3 (top) with figure 2 (top). A representative example of the applicability of this process is provided by cathodoluminescence. Such light is generated when a Poisson stream of electrons (the first Poisson) impinges on a phosphor whereupon it splays out, over a small range of times of duration $\sim \tau_p$ (the linear filter), a random

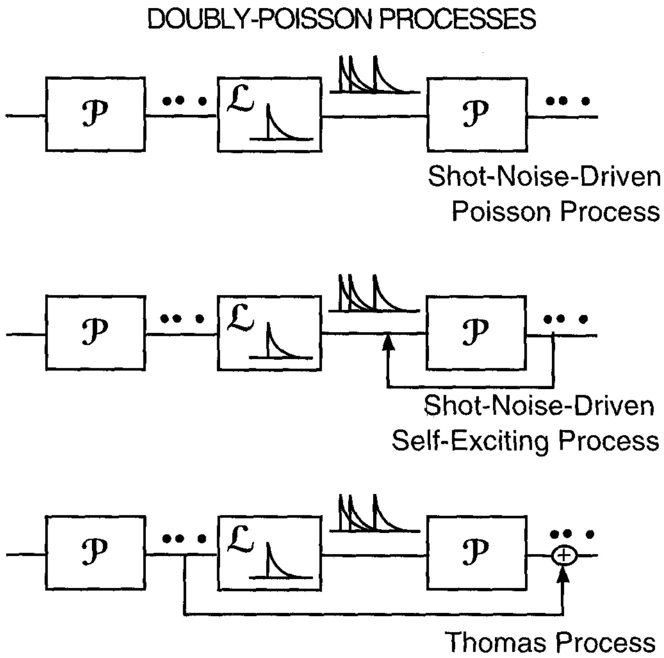

Figure 3. – *Top* : A Poisson point process $\mathcal{P}$ (right) whose stochastic rate is shot noise is termed a shot-noise-driven Poisson process. *Middle* : A self-exciting process driven by a shot-noise rate is, by analogy, referred to as a shot-noise-driven self-exciting process. *Bottom* : The Thomas process emerges when the initial shot events not only excite linear-filter responses but are also carried forward to the output; it is a variation on the shot-noise-driven Poisson process.

cluster of photons (the second Poisson). The statistical properties of cathodoluminescence photons are well-described by the shot-noise-driven Poisson process [23].

A number of variations on this construct have been set forth. A nonstationary version of the shot-noise-driven Poisson process has been developed [24], as has a fractal shot-noise version [25]. Moreover, general analysis, synthesis, and estimation techniques have been developed for such fractal-rate point processes [26].

Unfortunately, expressions for the photocounting distributions associated with shot-noise light are rather complex. This is because they depend on a number of features of the process : the means of the two Poisson distributions, the spectrum of the light, and the detector counting time. It turns out, however, that a simple two-parameter distribution, the Neyman type-A [27], provides a remarkably good approximation to the photocounting statistics of shot-noise light with arbitrary spectral properties and arbitrary counting times [28]. This distribution therefore plays the role for shot-noise light that the negative-binomial distribution plays for thermal light [29]. An accurate method for computing the tails of the Neyman type-A distribution has been developed [30]. Conditions under which it converges in distribution to the fixed-multiplicative Poisson and to the Gaussian have been established [31].

In the context of optics, shot-noise light provides a suitable description for the statistical properties of light generated by a number of mechanisms, including cathodoluminescence as described above [23]; Čerenkov radiation from a random stream of charged particles [25, 31]; betaluminescence photons generated by high-energy electrons at the glass faceplate of a photomultiplier tube [32]; beta- and radioluminescence noise in star-scanner detection systems operating in ionizing-radiation environments of space [20, 31]; and bending-magnet light produced at the Brookhaven National Laboratory vacuum-ultraviolet electron storage ring [33]. In the context of visual psychophysics, the Neyman type-A distribution is also useful for understanding how a brief flash of Poisson photons at the retina is transformed into a sequence of neural events at higher centers in the human visual system [34].

In certain cases, such as when detector dead time is present, superior agreement with experiment is obtained by replacing the second Poisson process in figure 3 (top) with a self-exciting process, as illustrated in figure 3 (middle). This representation is also a special case of figure 2 (middle) since shot noise is a special stochastic rate. Analytical results have been obtained for the count mean and variance [19, 20], and for the interevent-interval distribution [35], of shot-noise light in the presence of dead time. Two optics experiments for which the shot-noise-driven self-exciting process provides an excellent description are betaluminescence in transparent materials [20] and the interspike-interval histogram recorded from a cat retinal ganglion cell in darkness [35].

Another variation, the Thomas process [36] is illustrated in figure 3 (bottom). This point process is a modified version of the shot-noise-driven Poisson process in which primary events are carried forward. In the limit of long counting times, it yields the Thomas counting distribution [36, 37], whence its name. Much like the Neyman type-A distribution, the Thomas also converges in distribution to the fixed-multiplicative Poisson and to the Gaussian in certain limits [31].

5. Triggered Optical Emissions

Many processes associated with the generation of light, classical and nonclassical alike, take place via the triggering of optical emissions by point excitation events [38, 39].

The shot-noise rate function considered in the previous section comprised a superposition of brief intensity flashes generated, for example, by luminescence emissions. This construct is satisfactory when interference is absent so that intensities may be added. A more general approach to shot-noise light considers the superposition of brief nonstationary optical-*field* wavepackets which may interfere with each other. The shot-noise intensity rate function is then obtained as the absolute square of the superposed analytic signal, as shown in figure 4 (top). The statistical nature of the emission times introduces fluctuations manifested in the relative contributions of different emissions at a given observation time. This results in the introduction of an additional particle-like contribution to the normalized second-order correlation function of the light, and a concomitant increase in the photocount variance [38].

The emissions may be deterministic or stochastic, as indicated in figure 4 (top). For coherent or thermal wavepacket emissions at Poisson trigger times, interference between the randomly delayed emissions produces additional wave-like noise. In the limit when the emissions overlap strongly, $\mu\tau_p \gg 1$, the field exhibits the correlation properties of thermal light, whatever the statistics of the individual emissions. This is a consequence of the central limit theorem. In the opposite limit, when emissions seldom overlap, the light intensity is describable by a superposition of intensities as considered in the previous section, and the photocounts show enhanced particle-like noise which exhibits its largest value when the counting time is long. The photocounts then obey the Neyman

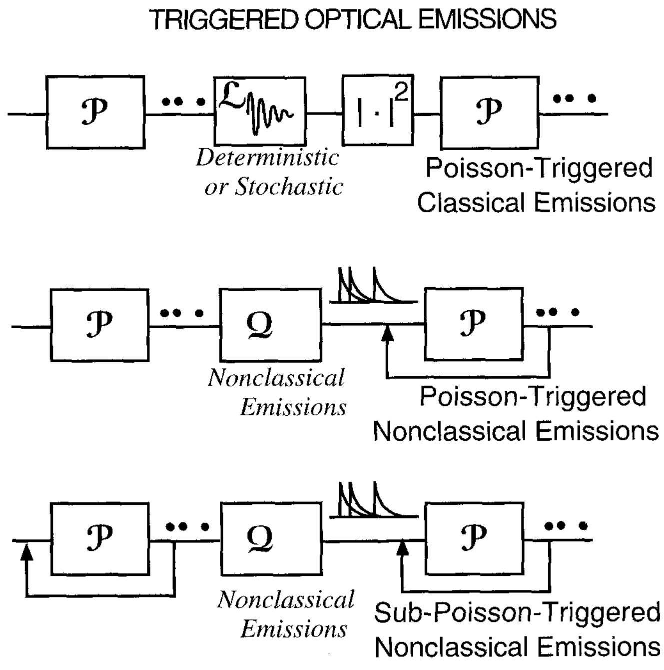

Figure 4. – ***Top*** **: A homogeneous Poisson point process $\mathcal{P}$ (left) followed by a deterministic (stochastic) linear filter $\mathcal{L}$ generates ordinary (generalized) shot noise. In this case the shot noise represents the optical *field*, rather than the optical *intensity* as in figure 3, so it must be squared before serving as the rate for the second Poisson process ($\mathcal{P}$ at right). This sequence serves as a model for Poisson-triggered classical emissions and gives rise to photon statistics that are noisier than those of the homogeneous Poisson process.** ***Middle*** **: Poisson events can, in the alternative, trigger nonclassical photon emissions $\mathcal{Q}$ (e.g., single photons), modeled by a self-exciting process. This construct cannot generate stationary nonclassical light however.** ***Bottom*** **: The generation of unconditionally sub-Poisson (photon-number-squeezed) light requires a concatenation in which sub-Poisson events trigger sub-Poisson emissions, both of which are represented by self-exciting processes.**

type-A and generalized Polya-Aeppli distributions, for coherent and thermal emissions respectively [38].

For nonclassical emissions, neither the intensity representation nor the Poisson-photon generation process embodied in figure 4 (top) is applicable [39]. Rather, these must be replaced by a quantum photon-generation process Q represented by a self-exciting process, as shown in figure 4 (middle). This admits the possibility of triggering single photons, or sub-Poisson clusters of photons. However, even when the individual emissions comprise number states, the Poisson trigger times result in the reduction or elimination of their nonclassical character. Indeed, when the emissions overlap strongly the asymptotic behavior of the field is that of thermal light, just as if the individual emissions were classical [38]. This perspective provides a physical underpinning for the ubiquity of Gaussian light which, as Picinbono and Rousseau [18] have shown, takes a variety of forms and can be generated in various ways. Detecting thermal light with a real photomultiplier tube yields anode current fluctuations associated with the K_0 distribution, in accordance with the results of Picinbono, Bendjaballah, and Pouget [1].

It is clear from the foregoing that the direct generation of stationary sub-Poisson (photon-number-squeezed) light cannot be accommodated using Poisson trigger times. We have developed a more general theory in which the trigger times are determined by a self-exciting process, as illustrated in figure 4 (bottom). In particular, results have been derived using trigger times that fluctuate in accordance with a stationary renewal point process (which can assume sub-Poisson form), and individual emissions that are coherent, thermal, or n-state in nature [39, 40]. Spatial effects are incorporated into the model by choosing the positions of the emissions to be independent and uniformly distributed over the source volume. The normalized second-order correlation function that emerges from this construct assumes the usual form for thermal light, but has two additional terms. The first of these is determined by the statistical nature of the individual emissions (it is positive for coherent and thermal, and zero for single-photon, emissions). The second term is governed by the statistics of the trigger process (it is positive for super-Poisson, zero for Poisson, and negative for sub-Poisson excitations). Both additional terms become small for light with a high degeneracy parameter (many total photons per emission lifetime), in which case the light is asymptotically Gaussian. In the opposite limit, when the degeneracy parameter is small (or the emissions are instantaneous), the correlation properties of the trigger process are directly transferred to the correlation properties of the photons. The first-order spatial-coherence properties of the field are identical to those of thermal light (the van Cittert-Zernike theorem is obeyed), although the second-order properties differ. The photon-counting distribution reflects the character of the correlation function. Thus sub-Poisson primary excitations, together with single-photon emissions, leads to sub-Poisson photon counts under appropriate conditions. Such nonclassical light may be made arbitrarily intense if interference effects are eliminated by detecting many spatial modes.

This theory is applicable to the Franck-Hertz experiment excited by a space-charge-limited electron beam. If the electron excitations are represented as a sub-Poisson renewal point process, and the photon emissions as single-photon states, the light generated should be antibunched and sub-Poisson. This indeed does turn out to be the case [41]. Ultraviolet (253.7-nm) sub-Poisson photons were generated in mercury vapor by inelastic collisions with a space-charge-limited electron beam. This first stationary and unconditionally photon-number-squeezed source, dating from 1985, was only weakly sub-Poisson. However the same excitation-control approach was successfully implemented by Yamamoto and colleagues to produce strongly sub-Poisson light from an InGaAsP distributed-feedback semiconductor laser driven by a suppressed-noise current source. This and other related techniques for generating nonclassical light have been summarized in a number of review articles [39, 42, 43, 44].

6. Multiply-Poisson Processes

The analysis of doubly-Poisson processes can be extended to multiply-Poisson processes. The multifold statistics of the events at the output of a series cascade of an arbitrary number of Poisson processes have been determined [45]. A linear filter following the output of each stage converts the pulsatile sequence of events into a stochastic rate function suitable for driving the next Poisson process, as illustrated in figure 5 (top). The greater the number of stages of the cascade, the longer the tail of the counting distribution.

If, instead, the cascade comprises Thomas processes, so that trigger events are carried forward as illustrated in figure 5 (middle), the result is the Poisson branching process [46]. This process characterizes electron multiplication at the dynodes of a photomultiplier tube. When the cascade ends in a linear filter, it provides a physical model for the random impulse response functions considered by Picinbono and colleagues in "Photoelectron Shot Noise" [1]. A useful limiting process of the Thomas cascade is obtained when the number of branching stages is infinite, while the average number of added events per event of the previous stage is infinitesimal. In particular, when the branching is instantaneous, the limit of continuous branching yields the the Yule-Furry process with an initial Poisson population [46].

Parallel, rather than series, configurations of multiply-Poisson processes can also be constructed, as displayed in figure 5 (bottom left). (The doubly-Poisson form is illustrated.) In the context of optics, photon streams with precisely these properties are generated by optical spontaneous parametric downconversion. A pump laser beam emits photons in accordance with a Poisson

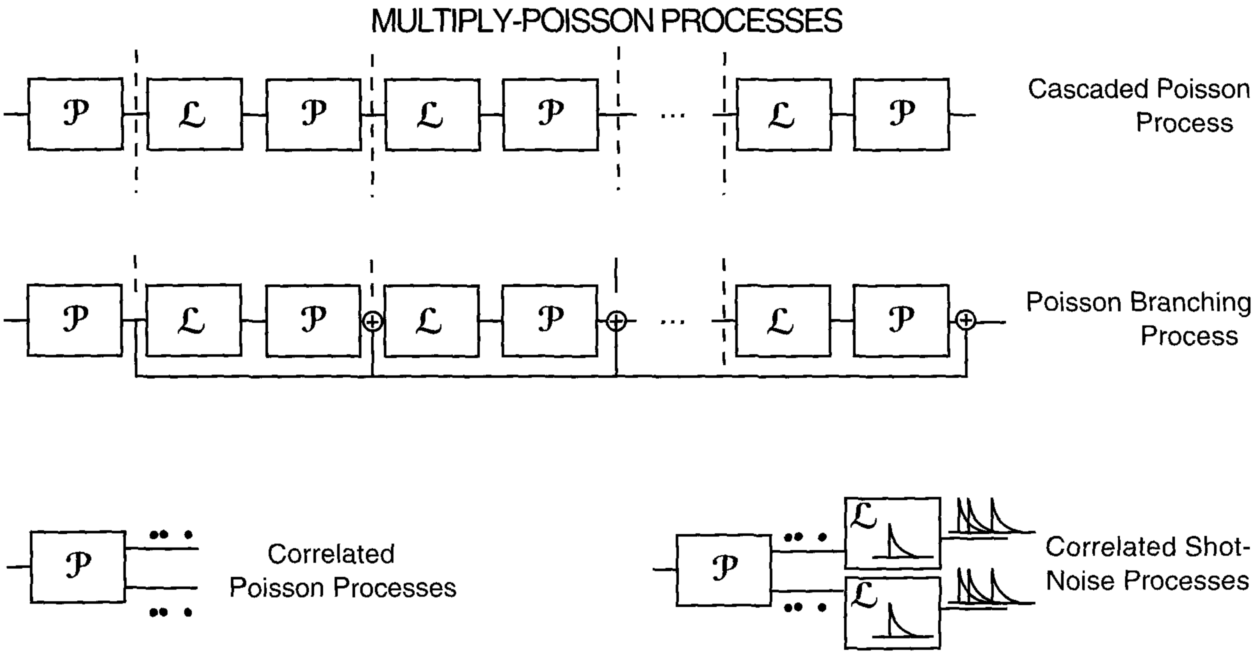

Figure 5. – ***Top*** **: Series cascade of Poisson processes linked by linear filters.** ***Middle*** **: Series cascade of Thomas processes. Because the trigger events are carried forward, the result is a Poisson branching process.** ***Bottom Left*** **: Parallel excitation of fully correlated Poisson processes such as occurs in spontaneous optical parametric downconversion.** ***Bottom Right*** **: Fully correlated parallel shot noises arising from the simultaneous detection of twin photon beams.**

process; each of these photons splits into an entanged pair in a nonlinear optical crystal such that energy and momentum are conserved. The resultant twin photon beams are marginally Poisson, but are fully correlated with each other [47]. Filtered versions of these photon streams, illustrated in figure 5 (bottom right), correspond to correlated shot-noise processes.

7. Conclusion

The Poisson point process and its variations describe many particle-like phenomena in statistical optics, including light emission by various luminescence sources, photoelectron generation, and secondary emission or impact ionization in electron devices [48]. We have described the cascading and concatenation of two or more Poisson processes with linear filters mediating the point processes. The outcome is a rich hierarchy of models whose statistical properties can always be determined explicitly, thanks to the inherent independence property of the initial Poisson point process. A cascade of filtered Poisson processes is in fact a *generalized shot noise* process with a random filter whose statistical characteristics are determined by secondary Poisson processes. Thus the generalized shot noise theme considered by Bernard Picinbono and his colleagues some thirty years ago is applicable to all such cascades.

Acknowledgments

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