Doppler-independent selective heterodyne radiometry for
detection of remote species

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A multiple-frequency heterodyne radiometer is proposed for the detection of
radiation from known species moving with unknown velocities. Expressions for
the signal-to-noise ratio and the minimum detectable total power are obtained
for sinusoidal signals and for Gaussian signals with both Gaussian and
Lorentzian spectra. In distinction to the conventional system, knowledge of
absolute line rest frequencies and a stable, tunable local oscillator are not
required. The system may find use in the detection of certain remote species
such as interstellar molecules and pollutants. A number of potential applications
are examined.

I. INTRODUCTION

The radiation from known remote species, such as extra-
terrestrial molecules and smokestack effluents, is generally
shifted as well as broadened in frequency when detected at
a receiving station. Shifts in the center frequency can be
attributed to a number of effects, including Doppler shift
arising from the mass motion of a group of molecules and
red shift arising from emission in the presence of a strong
gravitational field. The magnitude of the Doppler shift is
proportional to velocity and can be quite large, leading to
an uncertainty in the appropriate frequency at which to
search for a weak signal. This problem is magnified at high
frequencies since Doppler shift is also proportional to
frequency.

In this paper we propose a technique for partially eliminat-
ing the effects of Doppler shift in detecting remotely radiat-
ing objects. It is useful where a pair (or pairs) of emission
lines exist with a definite and well-known frequency separa-
tion, such as those produced by two transitions of a given
molecular species or by a given transition of two isotopes of
that species. If the two radiated frequencies are close to
each other, they are Doppler shifted by essentially the same
amount, so that the effects of Doppler shift can be made to
nearly cancel in the difference frequency.\textsuperscript{1,3} By employing
two signal frequencies instead of one, an effective modulation
of the source is achieved so that the bandwidth of the
receiver can be narrowed about the difference frequency, in
a manner similar to that accomplished by using a radiometer.
But whereas the modulation occurs at the detector in the
classical radiometer (which is therefore nonspecific), the
modulation frequency in the system described here is
directly related to the remote species being detected.
Furthermore, the system can be coupled with a classical
Dicke radiometer\textsuperscript{4-7} to provide improved performance where
warranted. The technique is expected to be most useful in
the infrared and optical where Doppler shifts are large;
conventional heterodyne radiometry and spectroscopy have
recently begun to find use at these frequencies.\textsuperscript{8-9}

II. CONFIGURATION FOR TWO RECEIVED
FREQUENCIES

The simplest example of this kind of detection system,
useful in the acquisition and tracking of radar and com-
communications signals, has been discussed elsewhere.\textsuperscript{1,3} In
Fig. 1, we show a block diagram for the three-frequency
selective heterodyne radiometer. The remotely radiating
source emits two waves at frequencies $f_1$ and $f_2$ whose rest
difference frequency $f_{\Delta} = |f_1 - f_2|$ is known to high accuracy.
The waves experience a Doppler shift arising from the mass
motion (Doppler feature) of the source. (They are also
broadened due to the constituent particle velocity distribu-
tion.) Thus, a wave whose center frequency is $f_1$ is detected
at the receiving station with a frequency $f'_1$. Assuming that
the velocity of the cloud is much smaller than the speed of
light $c$, the nonrelativistic Doppler formula provides

$$f' = f \pm v_1/\lambda,$$

where $v_1$ is the radial component of the overall velocity
vector $v$. The frequency difference between the two received
waves $f'_1$ is therefore given by

$$f'_1 = |f'_1 - f_1| = f_1 \pm (v_1/c) f \equiv f_1' \Delta f\,$$

thus, the radiated and received difference frequencies are
independent of Doppler shift to good approximation when
$v_1/c \ll 1$.

The two radiation fields $f'_1$ and $f'_2$ are mixed in a hetero-
dyne detector with a strong, coherent, and polarized local
oscillator (LO) signal at frequency $f_s$, yielding two electrical
beat signals at $|f'_1 - f_s|$ and $|f'_2 - f_s|$, along with a dc
component which is blocked. The third signal at $|f'_1 - f_s|$,\n
arising without benefit of the LO, is weak and may be
neglected. The ac output of the heterodyne mixer must then
be broadband coupled, through a filter of bandwidth $\Delta f$,
to a nonlinear device. The value chosen for $\Delta f$ should be as
small as possible in order to maximize the signal-to-noise
ratio (SNR), but must encompass the (somewhat unknown)
difference frequencies generated in the mixer. The nonlinear
device, which must also have a response over $\Delta f$, then generates a component at the frequency 

$$f' = |f'_1 - f'_2|.$$  

(3)

Since the output of the nonlinear device is essentially independent of the Doppler shift as well as the LO frequencies, variations in these quantities have little effect on the system output. In many instances, therefore, the necessity for a stable and tunable LO may be eliminated. The reception in this system has the additional advantage of being angle-independent, in the sense that the ordinary Doppler shift is proportional to the radial velocity $v_1$ and, therefore, is generally a function of angle.

A narrow-band filter centered at $f - f'_2$, and of bandwidth $B$, placed after the nonlinear device, achieves a low noise bandwidth. Thus, amplifiers and other detection apparatus can process electrical signals at (usually) moderate frequencies, which provides ease of matching as well as good receiver noise figure. This, in turn, decreases the LO power necessary for optimum coherent detection.\(^{10-12}\) Only the heterodyne mixer and the nonlinear device need have high-frequency response in many instances. For clarity, amplifiers have been eliminated from the block diagram. If warranted, the output of the narrow bandpass filter may be fed into a standard Dicke radiometer (dashed box in Fig. 1) consisting of a (third) detector, a phase-sensitive (synchronous) detector, and an integrator with time constant $\tau$. (Although we specify that this detector is square-law in Fig. 1, its characteristic is not critical and, in fact, a linear detector will often provide the cleanest signal.) The modulation may be obtained from a chopper as indicated. This technique can sometimes provide improvement in the SNR and has been coupled with a conventional ir heterodyne radiometer in a number of instances.\(^{13-15}\) It may also be advantageous to use a balanced mixer in this configuration.\(^7\)

The SNR at the output $(o)$ of the three-frequency nonlinear heterodyne system (SNR)$_o$ is given by [see Eqs. (45) and (77) of Ref. 2, Part 1]

$$(\text{SNR})_o = \alpha (\text{SNR})^2/[1 + 2(\text{SNR})],$$

(4)

assuming the use of an LO which produces no excess noise, a bandpass filter close to low-pass $(\Delta f \rightarrow f)$, and a square-law nonlinear device (for which it is easy to carry out the calculation). Here $(\text{SNR})$ represents the SNR at the input $(i)$ to the square-law device (see Fig. 1) which will generally be $\ll 1$. For the particular cases examined previously,\(^{1,2}\) the constant $\alpha$ is bounded by $1 < \alpha < 2$ for LO frequencies such that $f_L < f'_1$, $f'_2$ or $f_L > f'_1$, $f'_2$. For LO frequencies between the two signal frequencies, however, the output of the square-law device at $|f'_1 - f'_2|$ arises from the sum-frequency rather than the difference-frequency term and therefore falls in a region of lower noise. This, then, is the most desirable configuration from an SNR point of view, and allows us to choose $\alpha \approx 2$. The degenerate case where $f_L$ is precisely midway between the received signal frequencies should be avoided.\(^{16}\) The quantity $\kappa$ appearing in Eq. (4) is discussed in the next section.

III. n RECEIVED FREQUENCIES AND THE FACTOR $\kappa$

For the system involving two signal frequencies considered previously,\(^{1,2}\) the quantity $\kappa$ has been shown to depend on the magnitude and on the statistical and spectral nature of the received radiation, as well as on the widths of the two bandpass filters [see Eqs. (40), (176), and (89) of Ref. 2, Part 1]. Inasmuch as the radiation from remote molecular species may contain multiple frequencies, we consider operation of the system in the more general case when $n \geq 2$ lines with equal frequency spacing are passed through the broad bandpass filter and detected. A number of cases are of interest: sinusoidal signals $(s)$, independent Gaussian signals with Gaussian spectra $(g)$, and independent Gaussian signals with Lorentzian spectra $(l)$. The Gaussian signal case is considered in detail since radiation from astronomical sources
is generally Gaussian. The effect of multiple lines is included in the parameter \( \kappa \) by generalizing the previously obtained expressions. For the cases considered above, this quantity can be written as

\[
\kappa \approx \frac{f_n}{B} \sum_{j=1}^{n-1} A_j A_{j+1},
\]

(5a)

and

\[
\kappa \approx \frac{f_n}{\sqrt{8\gamma}} \left( \sum_{j=1}^{n-1} \gamma P_j \right)^{-1} \left( \frac{2\Phi(B/\sqrt{8\gamma}) - 1}{(B/\sqrt{8\gamma})} \right),
\]

(5b)

Here, \( A_j \) represents the amplitude of the \( j \)th line in the sinusoidal case, \( P_j \) represents the peak value of the Gaussian spectral distribution, and \( \gamma \) is its standard deviation, whereas \( D_i \) and \( \Gamma \) represent the height and width of the Lorentzian spectrum, respectively. The quantity \( \Phi \) is the error function. It has been assumed for simplicity that all spectral widths are identical, i.e., \( \gamma = \gamma_{i+1} = \gamma \) and \( \Gamma_i = \Gamma_{i+1} = \Gamma \); similar but more complex expressions are obtained when this is not the case.

Inasmuch as the quantities in large parentheses in Eqs. (5b) and (5c) above are of order unity for \( B \leq \gamma (\Gamma) \), it is the larger of \( B \) and \( \gamma (\Gamma) \) which limits \( \kappa \) and therefore the SNR in the Gaussian signal case. In particular, for the Gaussian spectrum case with \( \beta = \sqrt{8\gamma} \), \((2\Phi(1) - 1) = 0.68\) whereas for the Lorentzian spectrum case with \( B = 4(\sqrt{2} - 1) \Gamma \approx 1.66\Gamma \), \(\pi^2 \tan^{-1} 1 = \frac{\pi}{4} \). Thus the SNRs for the Gaussian and Lorentzian cases are reduced below that for the sinewave case (data-function spectrum), for the same bandwidth \( B \). This is understood to arise from the fact that some signal is being excluded in the Gaussian and Lorentzian cases in comparison with the delta-function case, but the noise is approximately the same. For fixed \( \gamma (\Gamma) \), the best SNR for the Gaussian and Lorentzian cases is obtained as \( B \rightarrow 0 \), since the noise decreases faster than the signal, as \( B \) decreases, in the approximation \( \Gamma_i \rightarrow 0 \). Of course, \( B \) cannot be decreased below the Doppler shift of the frequency difference \( |f' - f_0| = (\eta_\lambda/c)f_0 \) which is unknown but can generally be estimated. For \( B \gg \gamma (\Gamma) \), essentially all of the signal is included, and the results reduce to those obtained in the sinewave case. If possible, therefore, lines should be chosen for which the (Doppler) width and the Doppler shift are minimized, i.e., the lines should be narrow and closely spaced in frequency.

In the case where all such lines are of equal spacing, power, and width \( (A_j = A_{j+1}; P_j = P_{j+1}; \gamma_j = \gamma_{j+1} = \gamma; \Gamma_j = \Gamma_{j+1} = \Gamma) \), the large square brackets in Eq. (5) can be replaced by

\[
[-1/n^2, n = 2, 3, 4, \ldots].
\]

(6)

For fixed input radiation power, the best operation is clearly achieved for \( n = 2 \) (so that \( [-1, 1] = \frac{1}{2} \)), since additional lines increase the (signal-by-noise contribution to the total noise) more than they do the signal. When increased radiation power becomes available by virtue of the additional lines, however (e.g., the detection of more than one Doppler feature), \( n > 2 \) can be advantageous.

We also consider the case in which \( n \) equal-power, equal-width lines are allowed through the broad bandpass filter, these not being equally-spaced, however, so that only one pair of lines contributes to the output signal. In this case, the large square brackets in Eq. (5) must be replaced by

\[
[-1/n^2, n = 2, 3, 4, \ldots].
\]

(7)

Performance in this case is degraded for \( n > 1 \) since the additional lines contribute only to the noise.

Finally, we consider the case in which only two lines \( (n = 2) \) of arbitrary width are received and detected. Defining \( \beta \) as the ratio of received power in these two lines, i.e.,

\[
\beta(\text{sinusoidal}) = A_2^2/A_1^2,
\]

\[
\beta(\text{Gaussian}) = \gamma_1 P_2 / \gamma_1 P_1,
\]

\[
\beta(\text{Lorentzian}) = D_2 / D_1,
\]

the expressions in large square brackets in Eq. (5) become

\[
[-1/2(1 + \beta)^{-2}, \beta = 1].
\]

(9)

which is again equal to \( \frac{1}{2} \) for equal-power received signals (\( \beta = 1 \)).

IV. SNR AND MDP FOR TWO GAUSSIAN SIGNALS

The expression for the SNR at the output of the three-frequency system (SNR), for two Gaussian signals with Gaussian spectra (standard deviation \( \gamma_1 \) and \( \gamma_2 \)) is obtained by using Eqs. (4), (5b), (8), (9), along with Eq. (76) of Ref. 2, Part 1. To good approximation, assuming \( (SBR)_r \ll 1 \), this is given by

\[
(SNR) \approx \frac{f_n}{(\gamma_1^2 + \gamma_2^2)} \left[ \frac{\beta}{(1 + \beta)^2} \right] \times \left( \frac{2\Phi(B/[2(\gamma_1^2 + \gamma_2^2)]) - 1}{B/[2(\gamma_1^2 + \gamma_2^2)]} \right)(SNR)^2.
\]

(10)

For quantum-noise limited detectors such as photoemitters and reverse-biased photodiodes operating in the ir and optical, assuming that the incident radiation and the coherent LO are polarized in the same plane, the input SNR to the nonlinear device is

\[
(SNR)_t = \eta P_s / h\nu f_n, \quad \nu \gg kT.
\]

(11)

Here \( \eta \) is the detector quantum efficiency, \( P_s \) is the total
received signal radiation power, $h \nu$ is the photon energy, and $kT$ is the thermal excitation energy ($k$ is Boltzmann's constant, and $T$ is the detector temperature). For photo-}
voltic and photocathode detectors, the input SNR is generally one-half that given in Eq. (11).11,12

Heterodyne detectors in the microwave and millimeter wave regions ($h \nu < kT$) include square-law mixers such as the crystal diode detector,16 the InSb photodiode detector,17 the Golay cell,18 the pyroelectric detector,18 the metal–oxide–metal diode, and the bolometer.5 The latter three types of detectors have also been used successfully in the middle infrared (at 10.6 $\mu$m).5–21 For this type of detector Johnson noise generally predominates, and the input SNR is given by23

$$\text{(SNR)}_i = P_e/kT_{\text{eff}}f_e.$$  

(12)

For simplicity, we have lumped a number of detector parameters and operating conditions into the receiver effective temperature $T_{\text{eff}}$. Of particular interest in the mm and far-infrared regions are the low-noise fast Schottky-barrier diodes recently used in a number of experiments for astronomical observations.19,24

Inserting Eq. (11) or Eq. (12) into Eq. (10), and letting ($\text{SNR})_i = 1$, we obtain a minimum detectable total power (MDP) at the output of the three-frequency system given by

$$\text{(MDP)} \approx \frac{h \nu}{kT_{\text{eff}}} \left[ 1 + \frac{\beta}{\sqrt{\theta}} \right]$$

$$\times \left( \frac{B_j/\left[ 2(\gamma_1^2 + \gamma_2^2) \right]}{2 \Phi(B_j/\left[ 2(\gamma_1^2 + \gamma_2^2) \right])} \right)^{1/4} f_0^4(\gamma_1^2 + \gamma_2^2)^{1/4}$$  

(13a)

for quantum-noise limited detection, and

$$\text{(MDP)} \approx kT_{\text{eff}} \left[ 1 + \frac{\beta}{\sqrt{\theta}} \right]$$

$$\times \left( \frac{B_j/\left[ 2(\gamma_1^2 + \gamma_2^2) \right]}{2 \Phi(B_j/\left[ 2(\gamma_1^2 + \gamma_2^2) \right])} \right)^{1/4} f_0^4(\gamma_1^2 + \gamma_2^2)^{1/4}$$  

(13b)

for Johnson-noise limited detection.

The quantities in large square brackets and in large parentheses in Eq. (13) are both typically of order unity. Since $f_0^4(\gamma_1^2 + \gamma_2^2)^{1/4} \approx (\gamma_{\text{max}}^{\text{max}}/\gamma_0)^4(\gamma_0^{1/4})$ for $\gamma > B$ and $\gamma_{\text{max}}^{\text{max}}(\gamma_0^{1/4})$ for $\gamma < B$, where $\gamma_{\text{max}}^{\text{max}}$ is the maximum expected velocity, the system provides increasing advantage at higher radiation frequencies $f$ (since the effective bandwidth $\sim f^4$) for fixed $\gamma$ and $(f_1 - f_2)$. Small linewidths and close spacing of the lines are also important. For certain choices of parameters, which are determined by the species which it is desired to detect, the SNR at the output of the three-frequency selective system will provide a sufficient confidence level for detection. For situations in which this is not the case, further improvement in the SNR could be obtained by using a multichannel receiver and/or a classical radiometer, as mentioned previously.

V. ASTRONOMICAL RADIATION FROM CN: AN EXAMPLE

As an example of the use of the system in the mm region, we calculate the MDP for astronomical radiation arising from the following $N = 1 \rightarrow 0$. $J = 1/2 \rightarrow 1/2$ hyperfine transitions of the CN radical: $F = \frac{3}{2} \rightarrow \frac{1}{2}$ ($f_1 = 113490.9 \pm 0.2$ MHz) and $F = \frac{3}{2} \rightarrow \frac{3}{2}$ ($f_2 = 113488.1 \pm 0.3$ MHz). Recent radiometric observations of this radiation made use of the simple and sharply-defined velocity structure of the Orion-A molecular cloud; a measurement of the $N = 1 \rightarrow 0$ line of $^{13}$CO provided the Doppler effect correction due to the cloud’s motion. Using Doppler-independent heterodyne radiometry, on the other hand, requires only a bound on the velocity range. A radial velocity within the (substantial) range $-200 \text{ km/sec} \leq v_i \leq 200 \text{ km/sec}$, for example, yields a Doppler shift uncertainty of $2 |v_i|/c \sim 151.2$ MHz. In this case, the detected frequencies would be bounded by $113415.3 \text{ MHz} \leq f_i \leq 113566.5 \text{ MHz}$ and $113412.5 \text{ MHz} \leq f_i \leq 113563.7 \text{ MHz}$. Choosing $f_i$ between the rest frequencies, e.g., at 113490.0 MHz, we obtain $|f_i - f_1| \leq 76.5$ MHz and $|f_i - f_2| \leq 77.5$ MHz, inducing us to choose $\Delta f = f_s = 78$ MHz. Depending on the actual velocity of the cloud, this might allow the beat signal of the LO with other hyperfine lines to be passed to the nonlinear device, which will not impair operation if these other lines are relatively weak. The narrowband filter is centered at the rest difference frequency $f_i = |f_i - f_2| = 2.8$ MHz, with a minimum bandwidth $B = 2 |v_i|/c \sim 1.87$ kHz. Since $B < 2(\gamma_1^2 + \gamma_2^2)^{1/4}$, the MDP is essentially determined by $f_0$ and $\gamma$ (which are $\approx 1.5$ MHz since $\gamma \leq \gamma_{\text{max}} \leq 1.5$ MHz). Inasmuch as $\beta = \gamma_{\text{max}} B/\gamma_1 P \approx 1/4$ for these lines,25 the MDP given in Eq. (13b) becomes $\text{MDP} \approx kT_{\text{eff}}/\left[ 2.12(1.11) \times (8.80) \times 10^9(1.45) \times 10^4 \right] \approx 8 T_{\text{eff}}/f_s$, with $B \approx 30$ MHz representing the effective bandwidth for the calculation. Using the conventional system with this uncertainty in Doppler shift, and assuming that a one-channel receiver is used, the MDP would be $kT_{\text{eff}}/\Delta f$ with $\Delta f \approx 78$ MHz, indicating that improvement is possible with the proposed system.

For situations in which $B > \gamma$, a multichannel receiver using a bank of narrow bandwidth filters could be used in place of the narrow bandpass filter ($B$), compressing the number of channels below that required in the conventional system. In the infrared and optical, an unknown Doppler shift provides a greater range of uncertainty in the received frequencies than at longer wavelengths; this system should therefore be useful in detecting atomic and molecular radiation at these higher frequencies, particularly in those wavelength regions where atmospheric windows exist. For example, strong CN optical transitions from interstellar sources were first observed in 194026–37; one could attempt to definitively detect the presence of the CN R(2) line, which would provide an improved estimate for the cosmic black-body radiation at 1.32 mm.27 Particular attention might also be given to possible IR emission from CO, which exists in relatively high densities and with a very broad range of velocities in interstellar regions, as determined by its mm-wave emission.28–29 Clearly, the same considerations

apply to the detection of maser radiation from astronomical sources, and to the detection of remote pollutants.

VI. DISCUSSION

A selective heterodyne radiometer has been described for use in the detection of remote species such as pollutants and interstellar molecules. The system operates on the basis of the difference frequency between two radiated lines which, for closely spaced lines, is relatively insensitive to Doppler shift. This allows for the sensitive detection of known species moving at unknown velocities. The two frequencies may be obtained from individual transitions or from two isotopes of the same species. The system introduces little loss over the conventional heterodyne radiometer and has a number of specific advantages. In particular, it requires knowledge only of rest difference frequencies and not of line rest frequencies which are sometimes difficult to determine, and it requires neither a stabilized nor a tunable LO. Clearly it requires little knowledge of the source velocity and consequently is generally unsuitable for spectroscopy. Changes in the source velocity or direction do not alter system detectibility appreciably. This is particularly important in the infrared and optical where Doppler shifts are generally large.

The SNR and MDP at the output of the system have been obtained for a number of cases of interest, including sinusoidal signals and Gaussian signals with both Gaussian and Lorentzian spectra. A configuration involving multiple signal frequencies has also been considered. Other desirable operating conditions are as follows: (1) The LO frequency should be chosen to be nearly between the signal frequencies, (2) lines with minimum broadening (low γ) and minimum frequency separation (low B) are most desirable, (3) Δf should be minimized by bounding the expected Doppler shift as closely as possible, and (4) the strongest pair of lines consistent with the above conditions should be chosen.

The detection of CN radiation provided an example of the use of the technique in the mm region; an indication of the use of possible use at higher frequencies was provided. For the submillimeter region, it may be possible to use a combination Schottky-barrier-diode/harmonic mixer which would provide an output at low frequencies as long as the high-frequency beat signals are generated and mixed within the detector. LO harmonics are also readily generated in these devices so that harmonic-mixing selective heterodyne radiometry could be performed. Josephson junctions, which can sometimes be made to produce their own LO power, and metal–oxide–metal diodes could also be used. An IMPATT solid-state oscillator could conveniently be used as an LO in these regions since frequency stabilization, which is difficult to achieve in these devices, is not required. At higher frequencies, some fixed-line lasers could possibly be used since the LO frequency need not be tunable.

Disadvantages of the system include the lack of Doppler information, the difficulty of observing absorption lines and continuum radiation, and the added complexity. Use of a calibration load is also more complicated than in the conventional case. Finally, there will be an uncertainty that the detected difference-frequency signal can be properly identified, in analogy with the identification problem for the Doppler shifted signal in the conventional configuration. Thus, the system should be used for the application in which it is most effective: the search for a known emitting weak remote species with an unknown Doppler feature.

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