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ULTRAFAST GENERATION OF TWO-PHOTON ENTANGLED STATES USING TWO NONLINEAR CRYSTALS

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Quantum interference experiments with two-photon states generated by spontaneous parametric downconversion of an ultra-short pump pulse impinging on two nonlinear optical crystals are investigated. In particular, we analyze the influence of the frequency and wave-vector distributions on the visibility of the fourth-order polarization interference pattern and its connection with the polarization entanglement.

Photon pairs generated in spontaneous parametric downconversion (SPDC) have recently been employed in a number of interesting applications, such as quantum cryptography, teleportation, computation, metrology, and imaging. As such, considerable attention has been devoted to the design of improved sources of downconverted light. A number of different experimental configurations have been used to generate various types of quantum states, including those entangled in frequency, wave-vector, and polarization. We have recently demonstrated the utility of a model which considers entanglement in frequency and wave vector concurrently,[1] and extended this model to generation media with inhomogeneous distributions of nonlinearity.[2] In this work we present an extension of the latter model to the case of ultrafast pump pulses, and discuss the effect on fourth-order quantum interference patterns.

Let us consider the two-photon state generated in SPDC with an arbitrary pump-pulse profile and an arbitrary longitudinal distribution of nonlinearity. The general form of a two-photon state at the output of the generation medium is

$$|\Psi\rangle = \int d\mathbf{k}_o \, d\mathbf{k}_e \; \Phi(\mathbf{k}_o, \mathbf{k}_e) \; \hat{a}_o^\dagger(\mathbf{k}_o)\hat{a}_e^\dagger(\mathbf{k}_e)|0\rangle, \tag{1}$$

where $\mathbf{k}_{o,e}$ are three-dimensional wave vectors for the signal (ordinarily polarized) and idler (extraordinarily polarized) fields, respectively.

Within the limits of first-order time-dependent perturbation theory, the state function is found[1] to be $\Phi(\mathbf{k}_o, \mathbf{k}_e) = \mathcal{E}_p(\mathbf{k}_o + \mathbf{k}_e)\,\tilde{\chi}[\Delta(\mathbf{k}_o, \mathbf{k}_e)]$, where

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$\mathcal{E}_p(\mathbf{k})$ is the complex-amplitude profile of the pump field, $\tilde{\chi}[\Delta(\mathbf{k}_o,\mathbf{k}_e)] = \int dz\, \chi(z)\, \exp[i\Delta(\mathbf{k}_o,\mathbf{k}_e)]$ is the Fourier transform of the second-order nonlinearity distribution $\chi(z)$ along the longitudinal axis, and $\Delta(\mathbf{k}_o,\mathbf{k}_e) = \mathbf{k}_p - \mathbf{k}_o - \mathbf{k}_e$ is the wave vector mismatch function inside the generation medium. Note that the state function $\Phi(\mathbf{k}_o,\mathbf{k}_e)$ cannot be written as the product of two functions $\Phi_o(\mathbf{k}_o)\Phi_e(\mathbf{k}_e)$, and thus the state is said to be entangled in three-dimensional wave vector, or concurrently entangled in frequency and transverse wave vector.

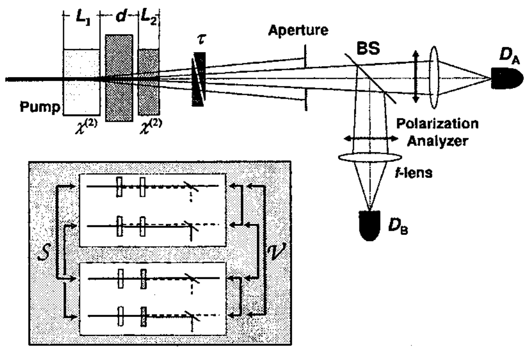

Figure 1. Schematic of a typical experimental arrangement. Collinear type-II SPDC is generated in two bulk crystals separated by a birefringent linear medium. The interferometer consists of a relative time delay τ introduced between signal and idler photons, a non-polarizing beam splitter, and two polarization analyzers. The inset at lower left illustrates the four relevant probability amplitudes for photon pairs from the first and second crystal to be transmitted and reflected from the beam splitter, and how these amplitudes enter into the expressions $\mathcal{S}$ and $\mathcal{V}$ of Eqs. (4) and (5).

We now consider the propagation of the downconverted light generated using the crystal configuration described above through an arbitrary linear optical system to a pair of single-photon detectors, as illustrated in Fig. 1. The probability amplitude for detecting the photon pair at the space-time coordinates $x_{\rm A} = (\mathbf{x}_{\rm A}, t_{\rm A})$ and $x_{\rm B} = (\mathbf{x}_{\rm B}, t_{\rm B})$ is given by

$$\mathcal{A}(x_{\rm A}, x_{\rm B}) = \int d\mathbf{k}_o\, d\mathbf{k}_e\; \Phi(\mathbf{k}_o,\mathbf{k}_e)\; \Big[\mathcal{H}_A(\mathbf{k}_o)\; \mathcal{H}_B(\mathbf{k}_e)\; e^{-i(\omega_o t_A + \omega_e t_B)} + \\ + \mathcal{H}_A(\mathbf{k}_e)\; \mathcal{H}_B(\mathbf{k}_o)\; e^{-i(\omega_e t_A + \omega_o t_B)}\Big], \quad (2)$$

where the transfer function $\mathcal{H}_{A,B}(\mathbf{k})$ describes the propagation of a mode $\mathbf{k}$ through the optical system from the output plane of the generation medium to detection planes $\mathbf{x}_{\rm A}$ and $\mathbf{x}_{\rm B}$, respectively.

Consider SPDC from two crystals of equal thickness L, dispersion coefficients $D = 1/u_o - 1/u_e$ and $\Lambda = 1/u_p - (1/u_o + 1/u_e)/2$ (here $u_{p,o,e}$ are the

group velocities for the pump, signal, and idler fields inside each crystal), separated by a birefringent linear medium of thickness d. Let the birefringence of the medium impose a relative temporal delay τ_d between the signal and idler photons generated in the first crystal. When pairs from such a source are allowed to propagate through the interferometer of Fig. 1, the coincidence rate for slow bulk detectors (which integrate over space and time) can be written within the paraxial and quasi-monochromatic field approximations as

$$\mathcal{R}_c(\tau) = \mathcal{R}_\infty \{1 + \mathcal{S}(\tau) + v_{pol}\; \mathcal{V}(\tau)\}, \quad (3)$$

where τ is the relative optical-path delay between photons, the constant factor $\mathcal{R}_\infty$ is the "shoulder rate", and the polarization analyzer projection factor $v_{\rm pol} = 2\mu_{\rm Ao,Be}\,\mu_{\rm Bo,Ae}/[\mu^2_{\rm Ao,Be} + \mu^2_{\rm Bo,Ae}]$ with $\mu_{ij,lm}$ $(i,l = {\rm A,B}$ and $j,m = {\rm e,o})$ is the projection of the (i,l)-th photon polarization onto the (j,m)-th analyzer polarization. The interference functions $\mathcal{S}$ and $\mathcal{V}$ are given by

$$\mathcal{S} = 2\epsilon \frac{\rho}{1+\rho^2} \int d\xi\; \Pi_{0,1}(\xi)\; \Pi_{0,1}\left[\xi - \left(1 + \frac{\tau_d}{DL}\right)\right] \cdot \\ \cdot\, \mathcal{G}^{(0)}\left(\xi; \frac{\tau}{DL}\right)\, \mathcal{C}_p\left[\frac{\Lambda L}{2}\left(1 + \frac{\tau_d}{DL}\right)\right]\, \cos(\Delta' d) \quad (4)$$

and

$$\mathcal{V}(\tau) = \frac{1}{1+\rho^2} \int d\xi\; \Pi_{0,1}(\xi)\; \Pi_{0,1}\left(2\frac{\tau}{DL} - \xi\right) \cdot \\ \cdot\, \mathcal{G}^{(1)}\left(\xi; \frac{\tau}{DL}\right)\, \mathcal{C}_p\left[\Lambda L\left(\xi - \frac{\tau}{DL}\right)\right] + \\ + \frac{\rho^2}{1+\rho^2} \int d\xi\; \Pi_{0,1}(\xi)\; \Pi_{0,1}\left[2\frac{\tau}{DL} - 2\left(1 + \frac{\tau_d}{DL}\right) - \xi\right] \cdot \\ \cdot\, \mathcal{G}^{(2)}\left(\xi; \frac{\tau}{DL}\right)\, \mathcal{C}_p\left[\Lambda L\left(\xi - \frac{\tau}{DL} + \left(1 + \frac{\tau_d}{DL}\right)\right)\right] + \\ + 2\epsilon \frac{\rho}{1+\rho^2} \int d\xi\; \Pi_{0,1}(\xi)\; \Pi_{0,1}\left[2\frac{\tau}{DL} - \left(1 + \frac{\tau_d}{DL}\right) - \xi\right] \cdot \\ \cdot\, \mathcal{G}^{(12)}\left(\xi; \frac{\tau}{DL}\right)\, \mathcal{C}_p\left[\Lambda L\left(\xi - \frac{\tau}{DL} + \frac{1}{2}\left(1 + \frac{\tau_d}{DL}\right)\right)\right] \cos(\Delta' d), \quad (5)$$

where ρ is the acceptance angle of the interferometer, Δ' is the wave vector mismatch in the inter-crystal medium, $\mathcal{C}_p(t)$ is the temporal autocorrelation of the pump field and the spatiotemporal interference functions $\mathcal{G}^{(j)}(\xi; \frac{\tau}{DL})$ are defined in a previous work.[2]

This equation predicts two deviations from the familiar single-mode theory of interferometry with SPDC. The presence of transverse wave vectors leads to spatial distinguishability between signal and idler photons at the detector plane, and an attendant loss of interference visibility. Similarly, pumping with ultrafast pulses leads to spectral distinguishability between photons, which also leads to reduced visibility.

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Figure 2 shows, for a quasi-monochromatic cw pump field, a plot of the visibility function $\mathcal{V}$ where the delay is fixed at the point of maximum interference DL, as a function of the system aperture size r (determining the acceptance angle ρ) and the distance d between crystals where the linear medium separating the crystals is taken to be air. In the limit of very thin crystals, small apertures, and small separation distances, the photons are spatially indistinguishable and Eq. (3) reduces to the conventional single-mode theory.

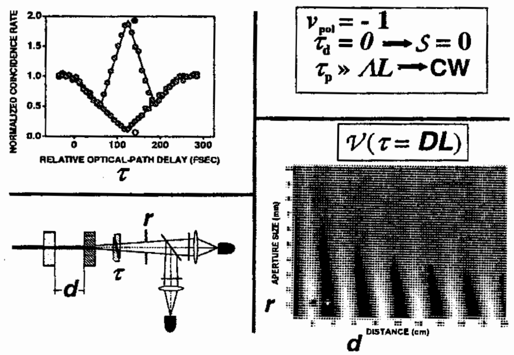

Figure 2. Quantum interference patterns obtained with a cw pump field and two 0.5-mm-long BBO crystals with parallel optical axes separated by an air gap. The lower right inset shows the visibility function $\mathcal{V}(\tau = DL)$ as a function of crystal separation d and aperture diameter r. Note the periodicity in d and decay in r. The upper left inset is a reproduction of experimental data from a previous work, with a fixed acceptance angle $\rho = 3.3$ mrad.

We have reported a model of quantum interference patterns from degenerate collinear type-II SPDC with an arbitrary longitudinal distribution of nonlinearity and an arbitrary pump profile. The model treats a state concurrently entangled in frequency and transverse wave vector. In the case of two crystals separated by an air gap, the structure of the observed quantum interference pattern is seen to depend on the optical path length between the two crystals, the acceptance angle of the optical system, and the bandwidth of the pump.

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2. G. Di Giuseppe, M. Atatüre, M. D. Shaw, A. V. Sergienko, B. E. A. Saleh, and M. C. Teich Phys. Rev. A **66**, 013801 (2002).