SQUEEZED AND ANTIBUNCHED LIGHT

New techniques that exploit the limits of the uncertainty principle promise nearly noise-free optical measurements and improved optical information transmission.

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As a direct result of the quantum nature of electromagnetic radiation, all forms of light exhibit inherent and unavoidable random fluctuations. Because these fluctuations reveal information about the nature of light and the underlying processes that generate it, they have received considerable attention since the time of Einstein.[1] But they are also a source of noise that limits the accuracy with which information can be transmitted by a beam of light. Although experiments often still struggle with instrumental noise, some are quickly approaching the measurement limits set by the quantum-statistical nature of light. With the help of new techniques to "squeeze" the uncertainty of light, researchers can now conduct experiments with greater precision than possible with laser light. The generation of various forms of squeezed light in recent years has raised hopes for its application to such diverse areas as gravity-wave detection based on optical interferometry and reduced-error lightwave communications.

Nonclassical light

To understand squeezed light, recall that the electromagnetic field associated with a single mode of radiation may be described by two independent components—its magnitude and phase or, alternatively, its cosine and sine quadratures. According to quantum mechanics, these components are represented by noncommuting Hilbert-space operators. The Heisenberg uncertainty principle therefore requires that the product of their uncertainties obey a fundamental lower bound. As a result, the two components cannot be simultaneously known with perfect precision, and the value of the electric field cannot be totally certain.

However, the uncertainty of *either* of the quadrature components may in principle be reduced without limit, rendering it noiseless. Of course, this reduction comes at the expense of an increase in the uncertainty of the other quadrature component. Similarly, the photon number associated with a mode may be known exactly, in which case the phase of that mode is totally uncertain. Light with a minimum uncertainty product, but with an unequal apportionment of fluctuations in the two quadratures, is said to be quadrature squeezed. Light whose photon-number fluctuations are smaller than those of the Poisson distribution is said to be photon-number squeezed. Such light is also termed "sub-Poisson" because the standard deviation associated with the number of photons is less than that for a Poisson distribution. The nomenclature indicates that some of the fluctuations are "squeezed" out of one component and into the other.

Ideal lasers emit coherent light, which has quadratures whose fluctuations are equal and satisfy the minimum product permitted by the uncertainty principle. The photon-number fluctuations of coherent light are governed by the Poisson distribution.

Squeezed light is one form of nonclassical light; it cannot be mathematically described as a superposition of coherent states with nonnegative weights. Another form of nonclassical light is "antibunched" light, in which the photon coincidence rate is reduced below its value for coherent light. Nonclassical light has attracted considerable interest[2–4] since the possibility of generating it was first suggested in the 1960s by Roy Glauber of Harvard University, H. Takahasi of the University of Tokyo, David Stoler, now at AT&T Bell Laboratories, and Horace Yuen,

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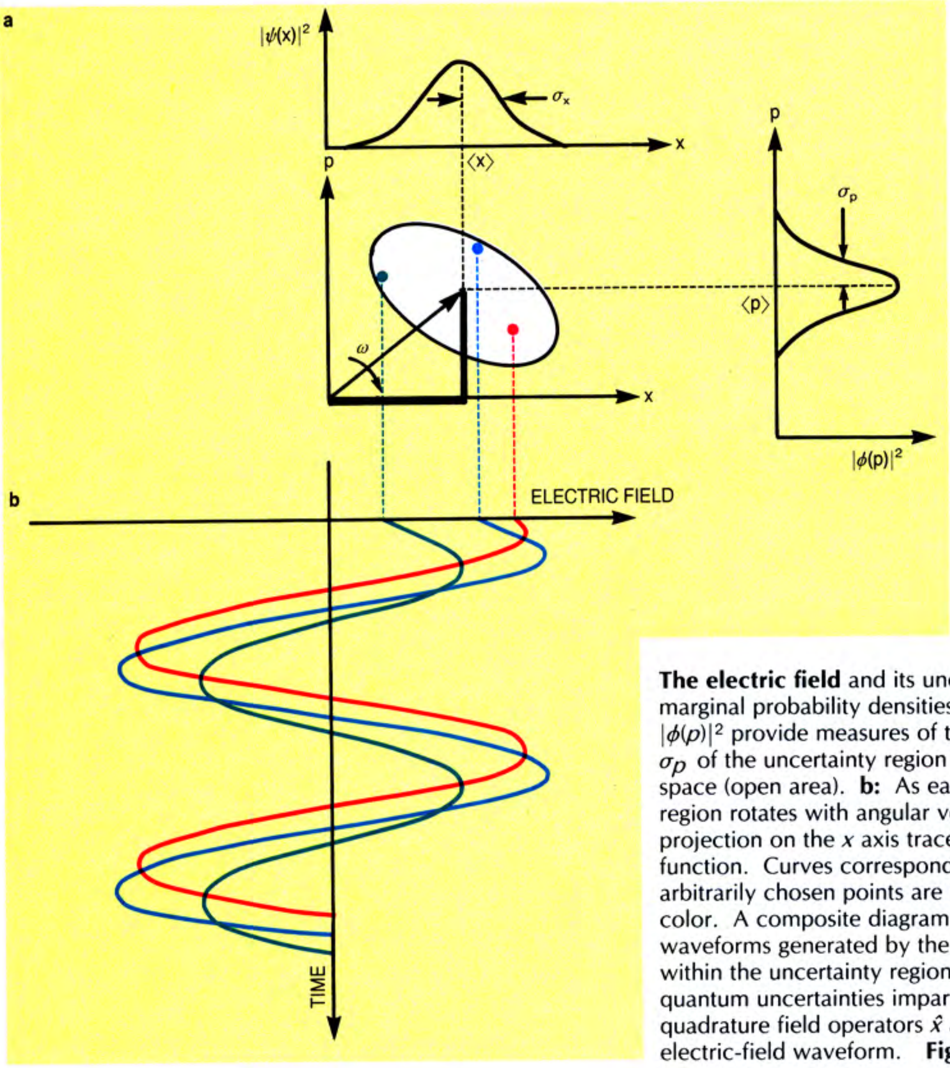


The electric field and its uncertainty. **a:** The marginal probability densities $|\psi(x)|^2$ and $|\phi(p)|^2$ provide measures of the widths σ_X and σ_P of the uncertainty region plotted in phase-space (open area). **b:** As each point in this region rotates with angular velocity ω its projection on the x axis traces out a sinusoidal function. Curves corresponding to three arbitrarily chosen points are illustrated in color. A composite diagram of all such waveforms generated by the set of points lying within the uncertainty region indicates the quantum uncertainties imparted by the quadrature field operators $\hat{x}$ and $\hat{p}$ to the electric-field waveform. **Figure 1**

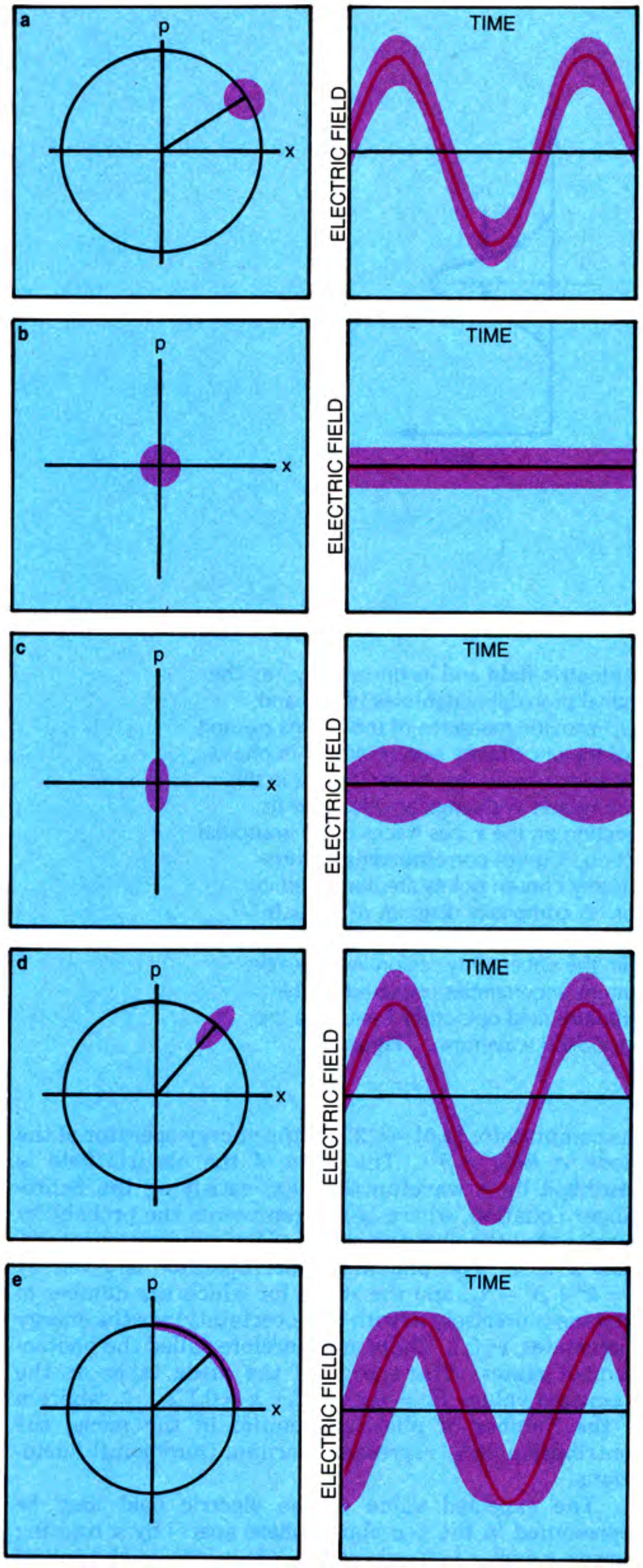
now at Northwestern University. Antibunched light was the first form of nonclassical light to be produced in the laboratory, in a pioneering resonance fluorescence experiment carried out in 1977 by Leonard Mandel of the University of Rochester and his students H. Jeffrey Kimble (now at Caltech) and Mario Dagenais (now at the University of Maryland).[5] Yet squeezed light was not generated until 1985, although there is no fundamental restriction on the degree to which noise may be reduced in a given component. Quadrature squeezing is difficult to achieve because nonlinear optical interactions are required to impart the phase-space asymmetry characteristic of it. And photon anticorrelations are needed to produce photon-number squeezing. Furthermore, squeezing is fragile; once produced, it is readily diluted by the ever-present random loss of photons and by the contamination of (unsqueezed) background photons.

Mathematical representation

The electromagnetic field in an optical cavity has the same mathematical description as a set of quantum-mechanical harmonic oscillators. The electric-field operator associated with a mode of angular frequency ω at a given position is $\hat{\mathscr{E}}(t) = \mathscr{E}_0[\hat{x}\cos(\omega t) + \hat{p}\sin(\omega t)]$, where the carets denote Hilbert-space operators and $\mathscr{E}_0$ is a constant. The quadrature field operators $\hat{x}$ and $\hat{p}$ are analogous to the position and momentum operators of a simple mechanical harmonic oscillator appropriately normalized such that the commutator $[\hat{x},\hat{p}] = i/2$ and the energy operator of the mode is $\hbar\omega(\hat{x}^2 + \hat{p}^2)$. The state of the electric field is described by a wavefunction $\psi(x)$ satisfying the Schrödinger equation, where $|\psi(x)|^2$ represents the probability density that the observed value of the quadrature component $\hat{x}$ is x. The photon-number operator is given by $\hat{n} = \hat{x}^2 + \hat{p}^2 - ½$, and the states for which the number of photons is precisely n (with no uncertainty) are the energy eigenstates, $\psi_n(x)$. These are therefore called the photon-number states. The energy of the mode takes on the quantized values $E_n = \hbar\omega(n + ½)$, $n = 0,1,2,\ldots$, where n is the number of photons (quanta) in the mode; the contribution of ½ represents vacuum (zero-point) fluctuations.

The expected value of the electric field may be represented in the x–p plane (phase space) by a rotating phasor $\alpha\exp(-i\omega t)$, where $\alpha = \langle\hat{x}\rangle + i\langle\hat{p}\rangle$ is the initial value of the phasor at $t = 0$. The projection of the phasor along the direction of the x axis is the mean electric field. Because uncertainties are present, however, the expected value of the field alone does not provide a complete picture of the light. For many states of light the mean field vanishes and the energy is totally contained in its random fluctuations.

The simplest measures of uncertainty or fluctuations are the standard deviations σ_x, σ_p, and σ_n. These quantities depend on the state $\psi(x)$ and its Fourier transform $\phi(p)$, so that the Heisenberg uncertainty rela-



Electric-field uncertainties for coherent and squeezed light. Uncertainty regions (left) for various types of light are shown together with the time dependences of the corresponding mean electric fields (red curves) and their uncertainties (shaded). **a:** The coherent state exhibits a circular uncertainty region centered about the phasor. Points (x,p) in the uncertainty circle trace out an electric field with an uncertainty that is independent of time. **b:** The uncertainty region for the vacuum state is a circle centered at the origin. **c:** The squeezed-vacuum state has an elliptical uncertainty region. **d:** The quadrature-squeezed coherent state has an elliptical uncertainty region centered about the phasor. As with the squeezed-vacuum state the electric field shows a periodic reduction and enhancement of its uncertainty. If the minor axis of the ellipse were oriented along the phasor, the state would also be photon-number squeezed. **e:** A sufficiently narrow crescent-shaped uncertainty region generates a photon-number-squeezed state with an electric field that shows substantial uncertainty at all times. **Figure 2**

tion $\sigma_x \sigma_p \geqslant \frac{1}{4}$ is obeyed. The quadrature components of the electric field therefore cannot be simultaneously specified with unlimited accuracy; states of minimum uncertainty obey the equality $\sigma_x \sigma_p = \frac{1}{4}$. Because the electric field comprises both quadrature components, light without uncertainty does not exist.

The uncertainties associated with the quadrature components are illustrated schematically in the x–p plane by means of an uncertainty region of dimensions σ_x and σ_p in the x and p directions, as shown in figure 1. This region is centered about the point $(\langle\hat{x}\rangle,\langle\hat{p}\rangle)$, and rotates with angular frequency ω. Diagrams of this type were first used by Carlton Caves, now at the University of Southern California.

The precise shape of the uncertainty region is difficult to define because $\hat{x}$ and $\hat{p}$ cannot be precisely determined simultaneously. The joint probability density of x and p is therefore not meaningful. Nevertheless, a formal measure of the uncertainty distribution can be provided by the Wigner distribution function[6]

$$W(x,p) = \int_{-\infty}^{\infty} \psi^*(x+y)\psi(x-y)e^{ipy}dy$$

This function is defined such that its projections on the x and p axes are the marginal probability densities $|\psi(x)|^2$ and $|\phi(p)|^2$, respectively, with rms widths σ_x and σ_p. The photon-number uncertainty is related to the spread of the uncertainty region in the radial direction since $\hat{n} = \hat{x}^2 + \hat{p}^2 - \frac{1}{2}$. The angle that the uncertainty region subtends is a measure of the phase uncertainty.

Graphical representation

In figure 2 we show the uncertainty regions and the corresponding time dependences of the electric field for various forms of light. The idealized noiseless limit of classical light would be a point of dimension zero in the phase-space plot which traces out a perfectly sinusoidal electric field with no uncertainty. Figure 2a represents the quantum state that most closely resembles noiseless classical light—the coherent state. It is a minimum-uncertainty state described by the Gaussian wavefunction $\psi(x) \propto \exp(-i\langle p\rangle x)\exp[-(x-\langle x\rangle)^2]$, with equal quadrature uncertainties given by $\sigma_x = \sigma_p = \frac{1}{2}$. The Wigner distribution function is then given by the expression $W(x,p) \propto \exp\{-2[(x-\langle x\rangle)^2 + (p-\langle p\rangle)^2]\}$, so that the uncertainty region is concentrated within a circle of radius $\frac{1}{2}$ centered about the phasor $\alpha = \langle x\rangle + i\langle p\rangle$. For $|\alpha| \gg 1$, the uncertainty region is simultaneously confined about the point α, to the maximum extent permitted by the Heisenberg uncertainty principle. The photon-number probability distribution $P(n)$ is then Poisson with mean $\langle n\rangle = |\alpha|^2$ and variance $\sigma_n{}^2 = \langle n\rangle$, so that $\sigma_n = \langle n\rangle^{1/2}$. For $|\alpha| \gg 1$, the angle σ_θ subtended by the uncertainty region is approximately $\frac{1}{2}/|\alpha| = \frac{1}{2}/\langle n\rangle^{1/2}$ so that $\sigma_n \sigma_\theta = \frac{1}{2}$.

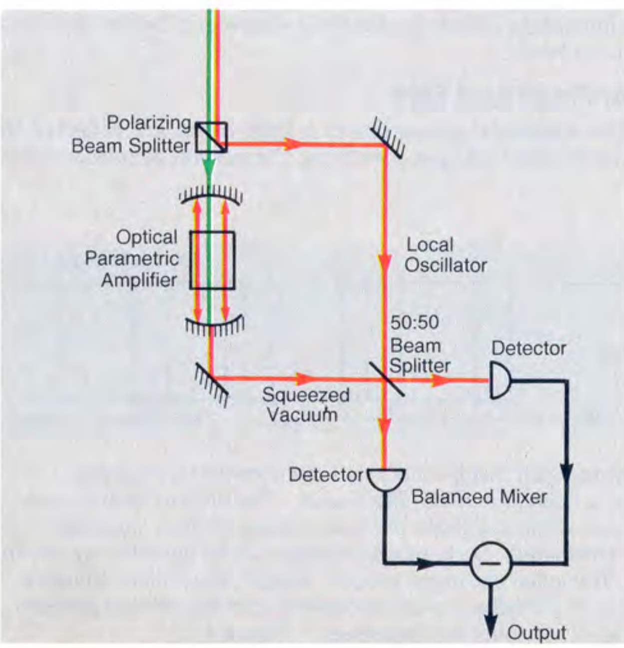
In the limit when $\alpha = 0$, the coherent state becomes the vacuum state, which is the same as the $n = 0$ photon-number state, $\psi_0(x) \propto \exp(-x^2)$. The vacuum-state Wigner distribution function, $W_0(x,p) \propto \exp[-2(x^2+p^2)]$, is illustrated in figure 2b. It has $n=0$, $\langle n\rangle = 0$, $\sigma_n = 0$ and $\sigma_\theta = \infty$. Although its mean photon number is 0, the vacuum state has zero-point energy $\frac{1}{2}\hbar\omega$, so that it exhibits residual fluctuations in x and p. Although the vacuum is devoid of photons, it is noisy!

The uncertainty region for the coherent state is circular, but for other states, it can assume other shapes, provided that the area is conserved as the minimum uncertainty product is approached. For minimum-uncertainty states, squeezing the uncertainty along one axis stretches it along the conjugate axis. A state whose x quadrature is squeezed has a probability density whose width σ_x is less than $\frac{1}{2}$. If, for example, the vacuum state $\psi_0(x) \propto \exp(-x^2)$ were somehow scaled in the x direction by a factor $s>1$, the wavefunction would become $\psi_s(x) \propto \exp(-s^2x^2)$. In that case, the probability density $|\psi_s(x)|^2$ would be a Gaussian function whose width had been squeezed from $\sigma_x = \frac{1}{2}$ to $\sigma_x = 1/2s < \frac{1}{2}$. The wavefunction $\phi(p) = \exp(-p^2/s^2)$ is also Gaussian, but the width of $|\phi(p)|^2$ is correspondingly stretched to a width $\sigma_p = s/2 > \frac{1}{2}$. The product $\sigma_x\sigma_p$ maintains its minimum value of $\frac{1}{4}$, so that the minimum-uncertainty property is maintained. The result is the squeezed-vacuum state. The Wigner distribution function that is associated with this state is given by $W_s(x,p) \propto \exp[-2(s^2x^2 + p^2/s^2)]$, demonstrating that the uncertainty circle is squeezed into an ellipse as illustrated in figure 2c. The mean photon number $\langle n\rangle$ is $(s-1/s)^2/4$, so that this state no longer truly represents a vacuum. The variance $\sigma_n{}^2 = 2(\langle n\rangle + \langle n\rangle^2)$ is twice that of the Bose–Einstein distribution; as a result the photon-number distribution is noisier than that for chaotic light.

The coherent state wavefunction, given by the expression $\psi(x) \propto \exp(-\mathrm{i}x\langle p\rangle)\exp[-(x-\langle x\rangle)^2]$, can be similarly transformed into a quadrature-squeezed state by using the transformation $(x,p) \to (sx, p/s)$ with $s>1$, yielding $\psi_s(x) \propto \exp(-\mathrm{i}x\langle p\rangle)\exp[-s^2(x-\langle x\rangle)^2]$. The corresponding Wigner distribution function transforms to $W_s(x,p) \propto \exp\{-2[s^2(x-\langle x\rangle)^2 + (p-\langle p\rangle)^2/s^2]\}$, which has an uncertainty region that is concentrated in an ellipse, as shown in figure 2d. The squeezed-coherent state can exhibit either super-Poisson ($\sigma_n > \langle n\rangle^{1/2}$) or sub-Poisson ($\sigma_n < \langle n\rangle^{1/2}$) photon-number statistics, depending on the angle φ that the phasor $\alpha = \langle x\rangle + \mathrm{i}\langle p\rangle$ makes with the minor axis of the ellipse. When $\varphi = 0$ (in-phase quadrature squeezing), the minor axis of the ellipse aligns with the phasor α and lends only a small uncertainty to the radial direction. The photon-number distribution is then sub-Poisson and the state is photon-number squeezed as well as quadrature squeezed. In contrast, when $\varphi = \pi/2$ (shown in figure 2d), the major axis of the ellipse aligns with the phasor α. This lends a large uncertainty to the radial direction, thereby giving rise to a super-Poisson photon-number distribution.

Although its electric-field uncertainty is always large, as illustrated in figure 2e, a sufficiently narrow crescent-shaped uncertainty region results in a photon-number squeezed state. As the radial extent of the crescent is decreased, its angular extent increases until it becomes a ring. This limit corresponds to the photon-number state $\psi_n(x)$, for which $\sigma_n = 0$. The phase is then totally random and the quadrature uncertainties are symmetrical and large.

As figures 2c and 2d show, the unequal uncertainties in the two quadrature components of the squeezed-vacuum state and the squeezed coherent state are manifested in



A squeezed-vacuum state generated by a phase-sensitive process. The scheme shown here involves degenerate parametric downconversion in a crystal of $MgO{:}LiNbO_3$. A Nd^{3+}:YAG laser (not shown) provides both the local oscillator at 1.06 μm (red) and the parametric amplifier pump at 0.53 μm (green). The pump beam is derived from a portion of the laser light by passage through a frequency-doubling crystal (not shown). The nonlinear process in the cavity reduces the noise in one quadrature of the vacuum entering the cavity, and a squeezed vacuum state at 1.06 μm emerges. The squeezed-vacuum state is detected by a balanced homodyne detection scheme in which it is combined with the local oscillator at a beam splitter before dual detection. Differencing the detector outputs reveals the squeezed vacuum signal. (Adapted from ref. 7.)
Figure 3

the electric field by periodic occurrences of a large uncertainty followed, one quarter-cycle later, by a small uncertainty.

Quadrature-squeezed light

The generation of quadrature-squeezed light essentially involves the differential scaling of the two quadratures, so that it must be produced by some phase-sensitive process. It can be achieved by mixing a wave with its conjugate in a nonlinear optical material. In 1979, Yuen and Jeffrey Shapiro, working at MIT, suggested that a process known as four-wave mixing might produce squeezed light, and indeed it does. This technique relies on the use of a nonlinear medium to couple four lightwaves. Two of the waves are strong pump waves, and the other two are weak waves. As a result of the coupling mechanism, one of the weak waves becomes a conjugate of the other. When the two weak waves are combined at a beam splitter the real and imaginary parts are scaled by different factors so that an asymmetry is introduced in the uncertainty region.

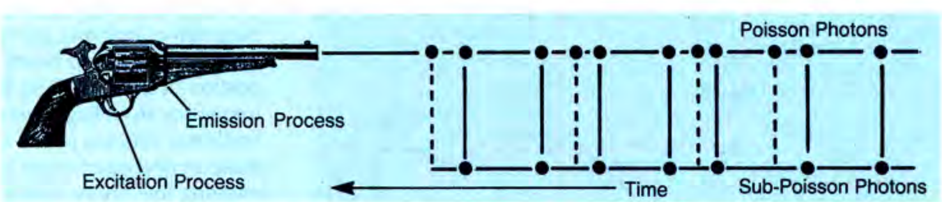
In 1985 Richart Slusher, Bernard Yurke and their colleagues at AT&T Bell Labs used a form of four-wave mixing to generate quadrature-squeezed light. At about the same time, several other groups, using three- and four-wave mixing and other nonlinear optical processes, also demonstrated quadrature squeezing. (See PHYSICS TODAY, March 1986, page 17.) The strongest effect published to date (about 60 % noise reduction) was achieved by Kimble and his collaborators working at the University of Texas at Austin.[7] (See PHYSICS TODAY, March 1987, page 20.) This group used a three-wave mixing technique involving parametric downconversion in which an incoming photon of angular frequency ω is split into two photons, each of angular frequency of $\omega/2$. The three-wave parametric amplification process leads to phase-sensitive gain, and the cavity in which the nonlinear crystal is placed increases the effect by multiple passes. (See figure 3.)

A phase-sensitive process is required not only to generate quadrature-squeezed light but also to detect it. Noise reduction will be achieved if the electric field is measured only at those times when its uncertainty is small. Direct detection is generally not suitable for detecting quadrature-squeezed light because it is sensitive to the photon number, which involves equal contributions from both the quiet and the noisy quadrature components. One technique for sensing squeezed light makes use of homodyne detection, in which the light is mixed with coherent light (from a laser local oscillator) at a beam splitter; this superposition is then detected with a photon counter or photodiode. If the phase of the local oscillator is appropriately selected, the superposition emerging from the beam splitter is photon-number squeezed, which gives rise to a sub-Poisson photocount at a detector (or a photocurrent below the shot-noise level at a photodiode).

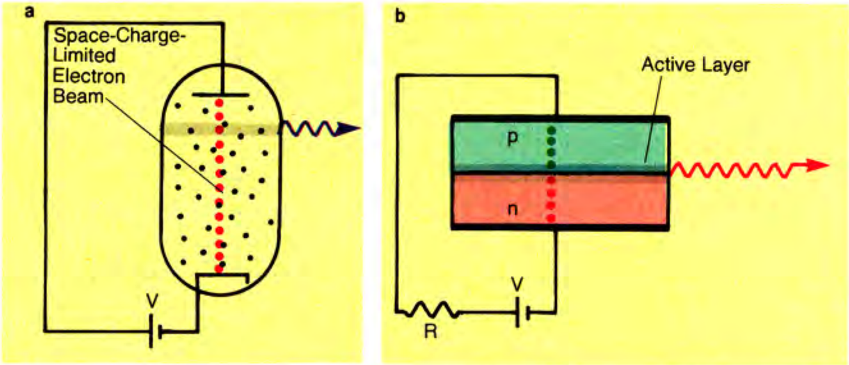
In 1983 Yuen and Vincent Chan (MIT Lincoln Laboratory) proposed a balanced homodyne detection scheme (balanced mixer) that has come to be commonly used today. As diagrammed in the lower portion of figure 3, the squeezed-vacuum signal and a coherent local oscillator beam impinge on a 50:50 beam splitter. The light at both output ports is detected, so no energy is lost. Because of the phase shift at the beam splitter, the contributions of the squeezed vacuum at the output ports differ by 180°. The balanced-detector output is the difference of the photocurrents after detection. This differencing operation suppresses all of the fluctuations in the local oscillator, and serves to extract the quadrature of the signal with reduced fluctuations. Therefore, as the phase of the local oscillator is varied, the detector will alternately detect fluctuations above and below the shot-noise level.

Antibunched light

The statistical properties of a light beam are reflected in the random process governing the arrival of photons at a



Hypothetical photon gun that is randomly fired generates a stream of photons that have a Poisson number distribution. The stream becomes sub-Poisson (bottom row of dots) if those photons arriving within a specified "dead" time are eliminated. Such anticorrelations can be imparted by any of three processes: The initial excitation process (trigger); the emission process (firing mechanism); or a feedback process derived from the emitted photons that controls either of the other two processes. **Figure 4**



Photon-number-squeezed light can be generated by means of excitation control using both of the devices shown below. **a:** The Franck–Hertz apparatus contains mercury vapor (black dots). In the shaded region the electrons (red dots) have attained sufficient kinetic energy to excite the mercury atoms by means of inelastic collisions. Because of space charge associated with Coulomb repulsion, the electrons have a sub-Poisson distribution, and so too do the 253.7-nm photons (violet) emitted when the atoms relax back to the ground state. **b:** A similar scheme in an InGaAsP/InP distributed-feedback semiconductor injection laser. By virtue of the external resistor in series with the battery, the current flowing through the device is constant, that is, the pump fluctuations are suppressed, so that the electrons (red dots) and holes (green dots) have sub-Poisson distributions. Their recombination in the active region (shaded) generates 1.56-μm photons (red) with a sub-Poisson distribution. The optical feedback inherent in the laser structure enhances the effect. **Figure 5**

detector. Light for which the photons arrive at perfectly regular time intervals has lower photon-number fluctuations than light whose photons arrive at statistically independent times. In the former case the number of photons counted in any time T is fixed, so that $\sigma_n = 0$, whereas in the latter case the number has a Poisson distribution, so that ${\sigma_n}^2 = <n>$ for all T. In general, however, the mean and variance of the number of counts are functions of the counting time T, so that light may be sub-Poisson for one counting time and super-Poisson for another.

An important characteristic of this random process is the rate of photon coincidence $G^{(2)}(\tau)$ at two times separated by the interval τ. The normalized coincidence rate is defined by $g^{(2)}(\tau) = G^{(2)}(\tau)/\lambda^2$ for a stationary optical beam whose photons arrive at a rate of λ photons per second. The photons associated with coherent-state light arrive independently, so that the normalized coincidence rate $g^{(2)}(\tau)$ is 1 for all time delays τ. If $g^{(2)}(\tau) < 1$, pairs of photons delayed by the time τ are less likely to occur, so their occurrences are anticorrelated. Light for which the arrival of photon pairs is anticorrelated when τ is small is called antibunched light.[3] Two definitions of antibunching are commonly used: The value of $g^{(2)}(\tau)$ at $\tau = 0$ is less than unity, or the slope of $g^{(2)}(\tau)$ at $\tau = 0$ is positive.

The coherent state provides a boundary between antibunching and bunching. Whereas thermal light is bunched ($g^{(2)}(0) > 1$), coherent-state light is unbunched ($g^{(2)}(0) = 1$). Antibunching ($g^{(2)}(0) < 1$) indicates that a source of light is nonclassical because anticorrelation at $\tau = 0$ is inconsistent with viewing $G^{(2)}(\tau)$ as a correlation function of the optical intensity.

Photon-number-squeezed light

The connection betwen photon-number squeezing and antibunching is subtle.[8] For short counting times the two effects must accompany each other. For single-mode light, $g^{(2)}(\tau)$ is independent of τ, so that antibunching implies photon-number squeezing and sub-Poisson statistics. In general, however, the photon-number distribution may be sub-Poisson for short T and super-Poisson for long T, or vice versa.

Photon-number-squeezed light may be generated by mixing quadrature-squeezed light with a coherent local oscillator at a beam splitter, as discussed earlier. It is also possible to produce it directly by introducing anticorrelations into photon occurrences that are spaced closely in time. The former technique is used when access to the phase of the field is required, for example, in applications where information is to be conveyed by the phase. However, when information is to be conveyed directly by the photon number, the latter technique is simpler.

The direct generation of photon-number-squeezed light may be visualized in terms a sequence of photons randomly "shot" from the hypothetical gun shown in figure 4. As the photons emerge they have a Poisson number distribution. One may make the sequence more regular by deleting every photon that follows another by less than some specified interval of time. The presence of such anticorrelations results in a more predictable stream of photons that exhibits sub-Poisson behavior. Anticorrelations may be introduced by pulling the trigger at regular time intervals (excitation control), by restrictions imposed by the firing mechanism such as the time required for it to reset (emission control) or by using information about the photon-occurrence times to control future excitations or emissions (feedback control).

To understand excitation control, consider the light generated by a collection of atoms excited by inelastic collisions with a stream of electrons, as, for example, in the Franck–Hertz experiment. In terms of the photon gun shown in figure 4, the electrons are the excitation process and the atoms represent the emission process. In the Franck–Hertz experiment illustrated in figure 5a, Coulomb repulsion can render the electron stream space-charge limited, and this phenomenon imparts a regularity to the electron flow. As a result there is a regularity in the number of atoms they excite and hence in the number of photons the atoms spontaneously emit. Such excitation control therefore results in the emission of spontaneous fluorescence photons that are photon-number squeezed.[8] We used an apparatus very much like this in 1985 to produce the first unconditionally photon-number-squeezed light.[9]

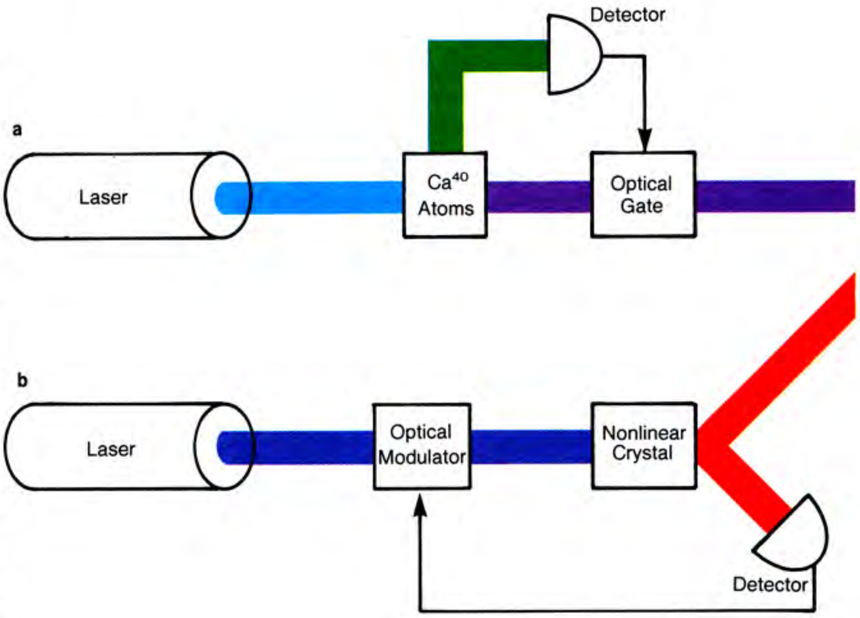
Unfortunately, the loss of photons resulting from imperfect photon generation, collection and detection diminishes the sub-Poisson behavior, and these losses account for the small amount of photon-number squeezing in our initial experiment. Although the photon-number uncertainty can in principle be reduced to zero, the effect is fragile (as is quadrature squeezing), so that loss and the presence of background photons must be assiduously avoided.

To minimize the loss, a number of compact, Franck–Hertz-type devices with high collection efficiencies have been developed. The electrons supplied from a dc source, such as a battery, provide a convenient source of sub-Poisson excitations because of their intrinsic Coulomb repulsion (the principal source of noise is Johnson noise). A light-emitting diode, which ideally emits one photon per injected electron, can serve as the emitter. A solid-state analog of the space-charge-limited Franck–Hertz experiment is therefore provided by a simple LED driven by a constant-current source. Indeed, Paul Tapster, John Rarity and Julian Satchell of the Royal Signals and Radar Establishment in Malvern, England showed that this device emits photon-number-squeezed light.[10]

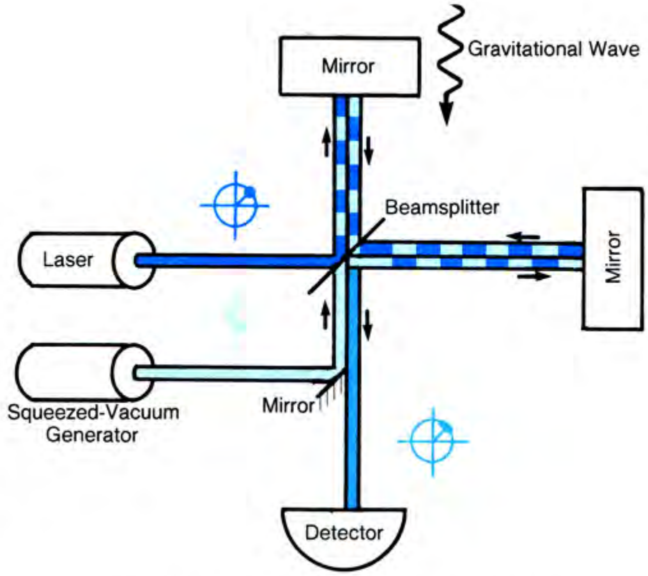
A significant advance was achieved by Yoshihisa Yamamoto and his colleagues at NTT Basic Research Labs in Tokyo, who in 1987 developed a semiconductor injection laser driven by a constant current source that produced photon-number-squeezed light.[11] This device behaves like a solid-state, stimulated-emission version of the space-charge-limited Franck–Hertz experiment, which is illustrated in figure 5b. Although Rarity, Tapster and Eric Jakeman have attained a greater degree of photon-number squeezing by the use of feedback control in a parametric downconversion device,[12] the injection laser has many advantages. It is compact and produces a large photon flux, and it has a broad spectral bandwidth and high efficiency. Other semiconductor device structures, employing solid-state space-charge-limited current flow and recombination photons, have also been proposed.[13]

Although excitation-control methods may hold the greatest promise for producing useful photon-number-squeezed sources, emission control can lead to photon-number squeezing. For example, dead time prohibits a second event from occurring within a fixed time following a given event. It therefore prevents the events from being arbitrarily close to each other and, as shown in figure 4, regularizes them. This reduces the uncertainty in the number of events registered in a fixed counting time T. A trigger or firing mechanism that requires time for resetting between consecutive shots, but is otherwise random, produces a sub-Poisson distribution.

Because isolated atoms subjected to Poisson excitations cannot emit photons during the time they are being reexcited, resonance fluorescence emissions are characterized by this description. In the earliest photon-number-



Feedback control for generating photon-number-squeezed light. **a:** In this simplified diagram the laser excites a beam of Ca^{40} atoms which then decay by the sequential emission of a pair of photons (one green and one violet). The electrical signal produced by the detection of a green photon is fed forward to operate an optical gate that selectively allows certain of the violet companion photons to pass. **b:** A scheme involving parametric downconversion used by a group at the Royal Signals and Radar Establishment to produce the strongest directly generated photon-number squeezing to date. The nonlinear crystal (KD*P) produces two correlated streams of photons, one of which provides a feedback signal to control the excitations by means of an optical modulator. **Figure 6**



Detecting gravity waves with a Michelson interferometer. Squeezed-vacuum light entering the output port reduces the noise and improves the sensitivity, as shown by the phase-space-uncertainty sketches. Superposed beams between the arms are schematically illustrated by alternating shades of blue. **Figure 7**

squeezing experiments carried out with resonance fluorescence radiation,[14] single atoms could not be isolated, so the photodetector had to be gated to assure operation with only a single active atom in the apparatus. The resulting light was therefore conditionally photon-number squeezed. Subsequent experiments, conducted in 1987 by Herbert Walther and his colleagues at the Max Planck Institute for Quantum Optics in Garching, were successful in trapping single ions and thus in producing unconditionally photon-number-squeezed resonance fluorescence radiation.

The control of the excitations or emissions may be derived from the emitted photons themselves by using feedback control, or photon control. If the arriving photons can be monitored without being destroyed, their arrival times can be used to modify subsequent excitations or emissions. Feedback control of this type can be carried out if the photons are observed by means of a quantum-nondemolition measurement, which allows an observable to be measured without perturbing it. Schemes for implementing quantum-nondemolition measurements have been suggested by Yamamoto and his colleagues, and have been implemented in optical fiber systems by Marc Levenson of IBM's Almaden Research Center together with Daniel Walls of the University of Auckland, and their colleagues.

Feedback control can also be achieved if twin photon streams are available, in which case one of the streams can be annihilated to create the control signal, while the clone stream survives. Configurations of this kind may be useful for generating photon-number-squeezed light with arbitrary photon-number statistics.[15] One suggested photon-feedback configuration makes use of cascaded atomic emissions,[16] as portrayed in figure 6a. A Poisson stream of laser-excited Ca^{40} atoms enters the apparatus. Each atom decays by the sequential emission of two photons—one green and one violet. The green photon is detected in a conventional manner to provide a feedback signal. This signal is used to selectively permit some of the violet companion photons to pass through an optical gate. Since the photons are always emitted in correlated pairs, only selected companions survive to produce a sub-Poisson photon stream at the output. (Correlated photon pairs from Ca^{40} were first used by Alain Aspect, now at the Ecole Normale Supérieure in Paris, and his colleagues for carrying out experiments demonstrating violations of the generalized Bell inequalities.)

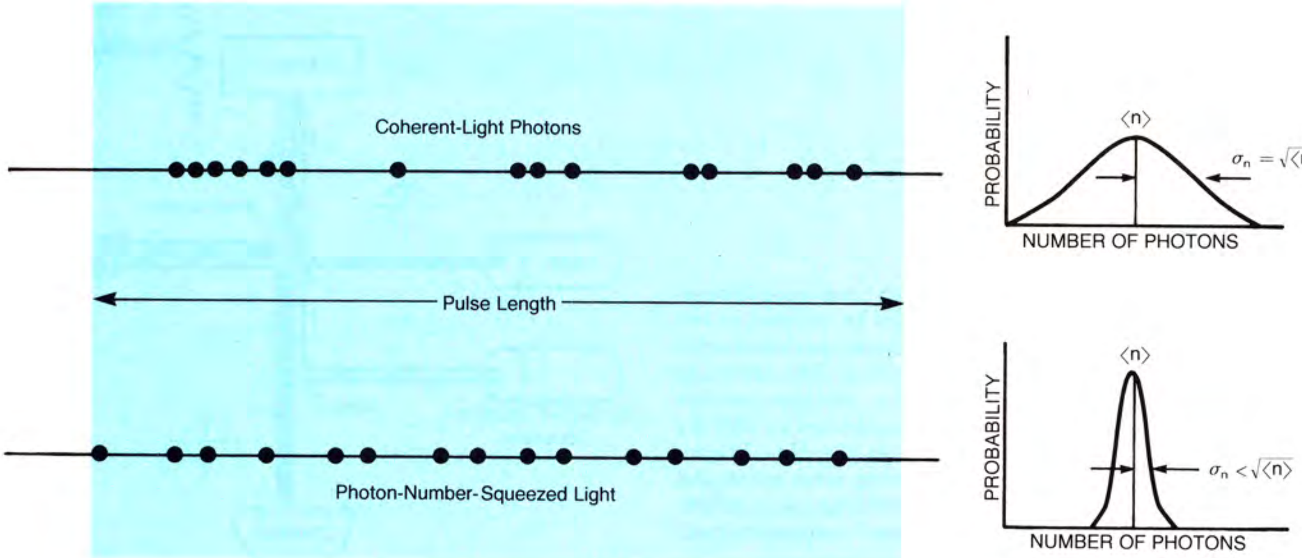
The same approach has been implemented by making use of a parametric downconversion experiment,[12] as shown in figure 6b. In this case, the feedback signal is used to control the excitations (pump) rather than one of the twin photon beams. This technique has produced the strongest directly generated photon-number squeezing to date (about 50% noise reduction).

Applications

Because the capacity of light to carry information is limited by its random fluctuations, squeezed light is likely to find use in lightwave communications as well as in high-precision measurements ranging from gravitational-wave detection to spectroscopy to biology. In principle, one can circumvent the quantum mechanical uncertainty by putting the information on the quiet component of a squeezed light beam and using a detection scheme that is insensitive to the noisy component.

In a coherent communication system, for example, a laser local oscillator can lock the photodetector onto the quiet quadrature carrying the information, while the noisy quadrature is ignored.[17] Similarly, one can make a direct-detection communication system noise-free by using a fixed number of photons to represent a bit of information.[18] The noise is squeezed into the phase fluctuations, which are not registered by the photodetector for the system.

There is currently interest in a suggestion first made by Caves for using quadrature-squeezed light to measure the gravitational waves expected to emanate from a cataclysmic event such as a supernova explosion. Figure 7 shows a schematic representation of a Michelson interferometer suitable for this purpose. The light from a laser is divided at a beam splitter, then reflected by two mirrors at the ends of the interferometer arms, and finally recombined at the beam splitter and sent on to the detector. If a gravitational wave caused the mirrors to vibrate it would modulate the phase of the laser light reflected from it. The interference pattern at the detector would then be perturbed from its quiescent null operating point and the resulting intensity variations at the detector would register the gravitationally induced motion of the mirror. (See PHYSICS TODAY, February 1986, page 17.) In conventional operation without the benefit of squeezed light, the interferometer sensitivity is limited by the fluctuations of the vacuum light entering the output port, which leads to shot-noise-limited operation. To understand the importance of feeding squeezed vacuum light into this normally unused port, note that the interferometer, when operated in a balanced configuration, is formally equivalent to a balanced homodyne detector (compare figures 3 and 7). As in the homodyne detector, the fluctuations of the laser



Direct-detection lightwave communications could in principle be improved if pulses of photon-number-squeezed light were used to carry information. The photon occurrences in a light pulse (shaded region) are more regular for photon-number-squeezed than for coherent light. The reduced variability results in a reduced error rate in communication systems limited by photon noise. **Figure 8**

light cancel and the output noise depends only on the fluctuations that enter the output port, as Kimble and his coworkers have experimentally demonstrated.

We now turn to two specific examples where the use of photon-number-squeezed light might prove beneficial. In an idealized direct-detection digital lightwave communication system, errors (misses and false alarms) can be caused by noise from many sources, including photon noise intrinsic to the light source.[3] If photon noise is the limiting factor (which is rarely the case in the current state of our technology), the use of photon-number-squeezed light in place of coherent light could reduce this noise and thereby the probability of error. As shown schematically in figure 8, for a coherent source each pulse of light carrying one bit of information contains a Poisson number of photons so that the photon-number standard deviation is $\sigma_n = \langle n \rangle^{1/2}$. For photon-number-squeezed light, each pulse contains a sub-Poisson number of photons, so that σ_n is less than $\langle n \rangle^{1/2}$. In a simple binary on–off keying system whose only source of noise is photon-number fluctuations obeying the binomial distribution (with variance-to-mean ratio $F < 1$) rather than the Poisson, the mean number of photons per bit $\langle n' \rangle$ required to achieve an error probability of 10^{-9} decreases below its so-called coherent-light quantum limit of 10 as F decreases below unity. The "quantum limit" of a lightwave communication system should therefore more properly be designated as the "shot-noise limit."

Photon-number-squeezed light also could be used in visual science to clarify the functioning of ganglion cells in the mammalian retina. These cells transmit signals to higher visual centers in the brain via the optic nerve. In response to light, the ganglion cell generates a neural signal that takes the form of a time sequence of nearly identical electrical events. The statistical nature of this neural signal is generally assumed to be governed by two nonadditive elements of stochasticity: the incident photons (which are Poisson distributed in all experiments to date) and a randomness intrinsic to the cell itself.[19] If the statistical fluctuations of the photons were reduced by exciting the retina with photon-number-squeezed light, the essential nature of the randomness intrinsic to the cell could be isolated and unambiguously determined. The use of photon-number-squeezed light as a stimulus in visual psychophysics experiments could also help clarify the nature of seeing at threshold.[20]

* * *

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erally adduced in favor of "unwinding the program": Unavoidable energy dissipation occurs only where information is discarded, that is, when resetting the output register (or the measuring apparatus) to the standardized state; "after a long computation the final state of a reversible computer has many more balls [the information-bearing particles in the Fredkin–Toffoli billiard-ball model] whose state depends on the computation"[3]; and after unwinding the program, the input register can be "uncopied," without dissipation, against a copy of the input information preserved there for just this purpose. However, if the second argument were correct (even though every reversible computing step is a 1:1 imaging process!) this computation would reduce entropy (or generate negentropy), evidently by taking heat from the environment. The computer would get colder the longer it ran. During unwinding, this negentropy certainly would get lost again. But with Landauer's dissipationless copying process we can do much better by providing a resetting store at the output (with all bits in the 0 position). Copying this "information" into the output register after computation is completed involves precisely the same, presumedly dissipationless copying steps Landauer described in reference 3. Evidently, his copying process makes it possible to discard information without dissipating energy. (Unwinding the program then is just a waste of time and energy, and there is no need for extra hardware to store all the input information.)

In conclusion, the question arises of whether "reversible computation" is another of those "many episodes" Landauer so correctly describes in his Opinion column, where "the advocates are . . . carried away by their enthusiasm" and the skeptic will not be "invited to the conferences, which the proponents . . . dominate." In all generality, the skeptics are simply those who take the trouble to think a few steps further than the enthusiasts, who try to ignore everything that does not fit into their (frictionless) dreams.

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EGINHART BIEDERMANN

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LANDAUER REPLIES: My Opinion column provided a conservative assessment of some logic technology proposals and undoubtedly appears controversial to their respective advocates. Eginhart Biedermann uses this as an opportunity to return to a different and earlier debate already published in *Nature*. The earlier debate concerned a much more fundamental and conceptual question: What is the minimal energy dissipation requirement imposed on computation by the laws of physics? Biedermann takes issue with Charles Bennett's notion of reversible computation, first published in 1973. Bennett demonstrated that computation can be carried out with an energy dissipation *per step* that can be reduced to any desired extent, if we are willing to compute sufficiently slowly. This concept has been confirmed and elaborated by a great many subsequent investigators with different backgrounds and viewpoints, including the late Richard Feynman.[1] Bennett's work has been labeled "epoch-making."[2] I do not believe that reversible computation requires a detailed defense against *all* of Biedermann's critique, and cite below three recent items to lead to the citation trail.[3]

I do suggest that the reader of Biedermann's critique keep two items in mind. First of all, reversible computation as viewed by Bennett and by me is not *totally* without dissipation, as was claimed for one of the proposals that my Opinion item analyzed. Additionally, Biedermann states that for computation with a roughly predictable execution time, a limited total energy expenditure, say, $100kT$, is required. I do not consider the exact energy expenditure significant; the key point is that the expenditure is not proportional to the number of elementary logic functions carried out during a long computation. But why should we even require a computation to be characterized by a "predictable passage time"? Even for today's practical computers, which have a well-defined execution time per step, the number of successive logic steps required to carry out a program is, in general, not predictable. Finally, Biedermann ignores the fact that some reversible computer proposals, such as that of Konstantin Likharev using Josephson junction circuits,[4] are clocked just as actual current computers are.

The occasional published dissent that, like Biedermann's, still considers reversible computation to be excessively optimistic is balanced on the other side by the proposal of Eiichi Goto and his colleagues asserting that the special precautions invoked in reversible computation are not needed.[5] Reversible computation uses logic functions at every step that are one-to-one and do not discard any information. Goto and his colleagues claim that this is not essential. In a fashion typical of critics of reversible computation, Biedermann analyzes the energy requirements of his own notion of minimally dissipative computation. Once again he tells us that static friction is essential, but does not tell us what is wrong with Likharev's scheme, which clearly avoids static friction.

In his final paragraph, Biedermann suggests that the exponents of reversible computation have been carried along by uncritical enthusiasm. Actually, reversible computation is a somewhat counterintuitive notion on first exposure, as demonstrated by Biedermann's repeated objections. Feynman, at a 1981 workshop, was the only one I have ever seen who caught on immediately. My own history was very different. When I first heard from Bennett about his evolving ideas, in 1971, I was totally skeptical. After all, this was a major departure from my own earlier publications. It took me six months to become convinced. It is now, however, almost two decades and many papers later, and it should no longer be that difficult!

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ROLF LANDAUER

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Wigner Distribution Malfunction

I am writing to point out some minor mathematical inconsistencies in the otherwise interesting and informative article on squeezed and antibunched light by Malvin C. Teich and

Bahaa E. A. Saleh (June, page 26).

The coherent state localized at $\langle x \rangle$ and $\langle p \rangle$ is properly described by the (normalized) wavefunction

$$\psi(x) = (2/\pi)^{1/4} \exp(2\mathrm{i}\langle p \rangle x) \times \exp[-(x - \langle x \rangle)^2]$$

When this wavefunction is inserted into the Wigner phase-space distribution function, defined as

$$W(x,p) = \frac{1}{\pi}\int \psi^*(x + \frac{1}{2}y)\,\psi(x - \frac{1}{2}y)\exp(2\mathrm{i}py)\,\mathrm{d}y$$

the result given by Teich and Saleh is obtained, namely

$$W(x,p) = (2/\pi)\exp[-2(x - \langle x \rangle)^2] \times \exp[-2(p - \langle p \rangle)^2]$$

It is easily shown that the above definition of $W(x,p)$ properly yields $|\psi(x)|^2$ when integrated with respect to p, and $|\varphi(p)|^2$ when integrated with respect to x. The "momentum" wavefunction corresponding to $\psi(x)$ is defined here by

$$\varphi(p) = \left(\frac{1}{\pi}\right)^{1/2}\int \exp(-2\mathrm{i}px)\,\psi(x)\,\mathrm{d}x$$

The extra factors of 2 that appear in the above formulas can be traced back to the commutation rule $[\hat{x},\hat{p}] = \mathrm{i}/2$, from which it follows that an appropriate representation of the "momentum" operator is $\hat{p} = (\mathrm{i}/2)\,\partial/\partial x$, and the wavefunction of a momentum eigenstate with momentum p is

$$\psi_p(x) = (1/\pi)^{1/2}\exp(2\mathrm{i}px)$$

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Teich and Saleh reply: The definition of the Wigner distribution function used in our article should indeed be modified, as John Philpott points out. The results presented in the article are not affected by this error, however.

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Angular Momentum Quantization Qualm

In his news story about "anyons" (November 1989, page 17) Anil Khurana apparently makes the *general* statement that angular momentum is not quantized in two spatial dimensions. In the *absence* of electromagnetic fields like flux lines, I find this hard to reconcile with the superposition principle and the probability interpretation of quantum mechanics. If one writes the wavefunction of a single spinless particle in polar coordinates ρ and φ, an arbitrary normalizable function $f(\rho)$ is an eigenfunction with angular momentum zero, while $f(\rho)\exp(\mathrm{i}m\varphi)$ has angular momentum $\hbar m$. If one considers a linear superposition of the two wavefunctions, the corresponding probability density is given by $2|f(\rho)|^2[1 + \cos(m\varphi)]$. As probabilities should be single-valued, the quantization of angular momentum follows without invoking the single-valued-ness of the wavefunction as the starting point. This argument holds in two as well in higher spatial dimensions. The reasoning given for the quantization of angular momentum in integer units for "normal" (non-fractional-statistic) particles shows that the description of anyons has to involve a superselection rule for states of different orbital angular momentum.

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'Doc' Draper Praised; A-Bomb Reappraised

It is unfortunate that Brian Reid (December 1989, page 101) was troubled by the fact that the National Academy of Engineering decided to name an award honoring engineers and technologists for "contributing to the advancement of human welfare and freedom" after Charles Stark Draper. It is even more unfortunate that Reid did not know "Doc" Draper.

The Charles Stark Draper Prize was established and endowed at the request of the Draper Laboratory because we think it a fitting tribute to Doc's memory and his contributions to engineering and technology. We intend that the prize will focus world attention on the important work of engineers in the same way that the Nobel Prize now focuses attention on accomplishments of scientists.

It is perhaps tragic that Reid does not recognize the contributions to "the advancement of human welfare and freedom" of technologically superior weapons developed to deter war. One of the important lessons of history is that the scourge of war is most likely to occur if free nations are *not* adequately prepared for it. We at Draper Laboratory are proud of our contributions to national defense and consider that work among the most noble in the engineering profession. So did Doc Draper.

It is also unfortunate that Reid apparently does not recognize how useful some engineering achievements initially developed for defense have been for society at large. Mechanical heart valves, silicon carbide ceramics, Mylar, flameproof epoxy paint, cordless tools, graphite composite materials, self-contained breathing apparatus, freeze-dried food, microwave technology, nuclear power, pacemakers, helicopters, electric analog computers and nuclear medicine are just some examples.

Ironically, Reid feels the Greek mathematician, physicist and inventor Archimedes would be a much worthier person for the academy to name a prize after. I say "ironically" because while Archimedes made original contributions in geometry and mathematics and founded the fields of statics, hydrostatics and mathematical physics, he also invented mechanical devices useful both in peace and in war and the defense of his society—just as Doc did.

In 214 BC, when Archimedes's native city of Syracuse was besieged by the Roman general Marcus Claudius Marcellus, the defense of the city was aided by military machines designed by Archimedes—including catapults, missile throwers and grappling hooks (*Encyclopedia Americana*, 1986). Legend has it Archimedes also devised concave mirrors that burned Roman ships by concentrating the Sun's rays on them.

Thus Archimedes made significant contributions to the advancement of human welfare and freedom, at least from the perspective of the Greeks, as Doc Draper did through his numerous engineering developments for his own nation. The achievements of both men had far-reaching effects on all aspects of their respective societies. I think Doc would be quite pleased with the parallel, and to be in such rich company.

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Contrary to Brian Reid, I feel that the citation "contributing to the advancement of human welfare and freedom" precisely describes the career of my late friend Charles Stark Draper.

Most of today's airline passengers are guided to their destinations by his Inertial Navigation System, which also took the Apollo astronauts to the Moon. As the NASA history reports, Charlie volunteered to operate it himself if the astronauts couldn't be taught to do so!

The last time we met—here in Sri