Relation between Radiation Statistics and Two-Quantum Photocurrent Spectra*

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Two methods of investigating the statistical properties of radiation fields in single beams have been found useful, particularly to obtain information about laser sources. One is the measurement of photocounting statistics[1]; the other is the analysis of the more easily obtained photocurrent spectrum.[2] The information most readily extracted from these measurements, such as moments of the counting distribution or bandwidths, is ultimately related to certain statistical parameters of the fluctuations in the radiation field. Correlation properties, such as photon-bunching effects, can be revealed by detailed processing of ordinary (single-quantum) photocounting measurements, but can be made more apparent by using double-quantum detectors.[3–5] The relation between two-quantum photocounting statistics and the distribution of irradiance fluctuations has been analyzed in an earlier paper.[6] In this paper, we complement that analysis with a determination of the kind of information about the stochastic properties of the radiation field that can be obtained from the two-quantum photocurrent spectrum.

The results generalize those of a similar analysis made for the single-quantum detector by Freed and Haus.[2] In both cases, the photocurrent spectrum is related to a moment of the two-time joint-probability density of the radiation, but the one-quantum spectrum corresponds to a second moment of that distribution, whereas the two-quantum spectrum relates to a particular fourth-order moment. Thus, from a single measurement, the two-quantum photocurrent spectrum provides a determination of a fourth-order correlation function of the incident radiation.

What is referred to in the spectral measurement is the excess photocurrent noise spectrum, above the shot-noise level. In the case of single-photon detection, that excess noise is directly related to the spectral noise power of the radiation.[2] The proper generalization to double-quantum detection (or to any order) is obtained by recognizing that the relevant statistics measured by the excess-noise spectrum are those of the probability of emission of a photoelectron in the detector. For the one-quantum detector, this is indeed proportional to the irradiance $I(t)$, but for the two-quantum detector, the emission probability $w(t)$ is proportional to the square of the fluctuating irradiance, $I^2(t)$.

For sufficiently narrow photocurrent pulses, the generalized relation between the photocurrent spectrum $\Phi_i(\omega)$ and the irradiance is given by

$$\Phi_i(\omega)=Aei_0[\Gamma+\Phi_w(\omega)/\langle w\rangle], \tag{1}$$

where $w(t)$ is the probability per unit time of photoemission and $\Phi_w(\omega)$ is its spectrum. The constant term represents the enhanced shot-noise level, with i_0 the average anode current, e the electron charge, A the photomultiplier gain, and Γ the enhancement factor due to secondary emission. For a single-photon detector, $w(t)=\alpha I(t)$ while for a two-photon detector,[4] $w(t)=\beta I^2(t)$, with α, β the respective quantum efficiencies.

The relation to the radiation statistics is obtained by noting that the spectrum is the Fourier transform of the autocorrelation function. Thus, in the single-photon case,

$$\Phi_w(\omega)=\alpha^2\mathfrak{F}\langle I(t)I(t+\tau)\rangle, \tag{2}$$

where $\mathfrak{F}$ denotes the Fourier transform. For the double-photon detector,

$$\Phi_w(\omega)=\beta^2\mathfrak{F}\langle I^2(t)I^2(t+\tau)\rangle, \tag{3}$$

which is a particular fourth-order correlation function of the radiation field. For either detector, when the irradiation is in the form of an attenuated classical field[7] that is stationary and ergodic, the photocurrent spectrum can be related to the two-time joint-probability density function $p(I_1,I_2,\tau)$. Here, I_1, I_2 are the irradiances at two times t_1 and $t_2=t_1+\tau$. The spectra are the transforms of two different moments of this joint density function. For one-quantum detection,

$$\Phi_w(\omega)=\alpha^2\mathfrak{F}\int_0^\infty\int_0^\infty p(I_1,I_2,\tau)I_1I_2dI_1dI_2, \tag{4}$$

whereas for a two-quantum detector,

$$\Phi_w(\omega)=\beta^2\mathfrak{F}\int_0^\infty\int_0^\infty p(I_1,I_2,\tau)I_1^2I_2^2dI_1dI_2. \tag{5}$$

Thus, the excess photocurrent noise spectrum is related to these second- and fourth-order moments of the joint intensity distribution for the single- and double-quantum detectors.

In general, the photocurrent spectra obtained with the two types of detector provide different information about the radiation statistics, through moments of different order of the underlying joint-probability density function. Information about this function is of significance in distinguishing radiation with different higher-order statistical properties, even if the intensity distribution $p(I)$ of two sources be the same.

In the special case of a bivariate gaussian joint-density function, however, the one- and two-quantum photocurrent spectra present the same correlation properties, in different form. The relation between the two is then

$$\langle I^2(t)I^2(t+\tau)\rangle=2\langle I(t)I(t+\tau)\rangle^2+\langle I^2\rangle^2-2\langle I\rangle^4. \tag{6}$$

Comparison of the current spectra from the two types of detector can thus confirm gaussian statistics or, more generally, provide completely distinct information on photon correlations.

The photocurrent spectrum can also be related to the second factorial moment of the two-quantum photocounting distribution discussed previously.[6] The generalized relation, which is similar to the one obtained by Freed and Haus for the single-quantum case,[2] is expressible as

$$\langle n(n-1)\rangle=\int_{-\infty}^\infty \Phi_w(\omega)\left[\frac{\sin(\omega T/2)}{(\omega T/2)}\right]^2\frac{d\omega}{2\pi}, \tag{7}$$

where n is the number of counts obtained during a time T of observation. It should be noted that this relation can, in principle, be inverted to yield the spectrum from the photocounting statistics, but only if the second factorial moment is known for all time T.

The correlation information obtainable from experiments such as those considered here is similar to that provided by a Hanbury Brown–Twiss experiment using two double-quantum detectors, a configuration which has been described by Teich and Wolga.[8] Coincidence experiments, such as those performed by Davidson and Mandel[9] give similar correlation information, but to lower order. Double-quantum photocurrent-spectrum measurements therefore give information which complements that obtained from other experiments, such as those of Freed and Haus, and Davidson and Mandel.

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