dark. No attempts were made to optimize the junction behavior.

The conditions affecting the structural and electrical characteristics of the films and the p-n junctions are being investigated together with the possibility of extending this technique to other materials which do not lend themselves to standard methods of chemical doping.

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3 The substrates employed were sodium free barium-aluminum boro-silicate glass (Corning No. 7059) and Eagle-Picher high-purity CdS, grade A.

THREE-FREQUENCY HETERODYNE SYSTEM FOR ACQUISITION AND TRACKING OF RADAR AND COMMUNICATIONS SIGNALS

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The operation of a three-frequency heterodyne system for communications and radar use is discussed. The technique provides important advantages over the conventional heterodyne detector, and is applicable in the infrared, optical, and microwave. The signal-to-noise ratio and minimum detectable power for the specific case of an optical or infrared system are calculated and compared with values for the standard configuration. A practical CO$_2$ laser radar has been examined as a particular example of this mode of operation.

A heterodyne system$^1$ which transmits two signals of a small, but well-known difference frequency $\Delta \nu$, is proposed.$^2$ Aside from the mixer, the receiver includes a nonlinear element such as a square-law device, and provides its output signal at a frequency very close to $\Delta \nu$ regardless of the Doppler shift of the transmitted signals. This obviates the need for high-frequency electronics, thus improving impedance matching, system noise figure, and range of operation over the conventional case. The usual frequency scanning of either the detector or the local oscillator (LO) is eliminated, as is the necessity for a stable LO. Furthermore, it allows targets to be continuously observed with Doppler shifts of considerably greater magnitude and range than presently possible.

Although the system requires a two-frequency transmitter, it is in some ways simpler than systems currently in use. If the radiation power of the received signal ($P_r$) is large and/or the expected Doppler frequency range of the signal ($\Delta f$) is small (i.e., $P_r/\Delta f \gg h\nu/\eta$ in the visible or infrared), the signal-to-noise ratio (SNR) for the system is approximately $\eta P_r/6kT\nu B$. Here, $\eta$ is the quantum efficiency of the detector, $h\nu$ is the photon energy, and $B$ is the bandwidth of the final filter centered about $\Delta \nu$ (see Fig. 1). Under certain circumstances $B$ may be made very small, and the SNR may be greater than that obtained with a conventional system. Consideration of the opposite limit yields a minimum detectable power which is larger than that obtained by use of the standard technique by a factor of approximately $(3\Delta f/B)^{1/2}$. Thus, the greater the knowledge of Doppler shifts to be encountered, the greater the achievable SNR and the lower the minimum detectable power. The system can, of course, be switched from the three-frequency mode to the ordinary heterodyne mode of operation (with or without AFC) if Doppler information is required.

In Fig. 1(a), we present a block diagram for a radar version of the system. A transmitter emits two waves of frequencies $\nu_1$ and $\nu_2$ whose difference $\Delta \nu$ is known to high accuracy. The waves are Doppler shifted by the moving target such that a wave of frequency $\nu$ returns with a frequency $\nu'$ given by the standard formula

$$\nu' = \nu(1 \pm 2v_{\parallel}/c), \quad (1)$$

where $v_{\parallel}$ is the radial velocity of the target and $c$ is the speed of light. After scattering from the target, therefore, the new frequency difference between the two waves $\Delta \nu'$ is given by

$$\Delta \nu' = \nu_1' - \nu_2' = \Delta \nu + (2v_{\parallel}/c)\Delta \nu. \quad (2)$$

Aside from this frequency shift resulting from the Doppler effect, there is a frequency broadening of each wave associated with the scattering by a moving target in a typical radar configuration.$^3$ For a rotating target, this broadening is of order $4\nu_0/\lambda$,
where \( \nu \) is the "radius" of the target, \( \omega _1 \) is its component of angular velocity perpendicular to the beam direction, and \( \lambda \) is the wavelength.\(^5\) For any practical system, as will be shown shortly, the difference \( \Delta \nu' - \Delta \nu \) is much smaller than the broadening effects and may usually be neglected. In the following considerations, therefore, we take \( \Delta \nu' - \Delta \nu = 0 \). The power-spectral-density \( S(\nu) \) for the transmitted and for the return signals from the target are shown in Fig. 1(b). The broadening of the signals is apparent.

Heterodyning with a local oscillator (LO) of frequency \( \nu _L \) then provides two electrical difference-frequency signals\(^6\)\(^7\) (centered at \( \nu _1 - \nu _L \) and \( \nu _2 - \nu _L \)), and a dc component which is blocked. The ac output of the detector must then be broadband coupled, through a filter of bandwidth \( \Delta f _c \), to a square-law device. The value chosen for \( \Delta f _c \) should be as small as possible in order to maximize the SNR, as will be seen later, but must encompass the difference frequencies generated in the mixer [see Fig. 1(b)]. The square-law device, which must have a response over \( \Delta f _c \), then generates a component at the frequency \( (\nu _1 - \nu _2) - (\nu _1 - \nu _L) \approx \Delta \nu \). Since the output of the square-law device is essentially independent of the Doppler\(^6\)\(^7\), as well as the LO frequencies, variations in these parameters have no effect on the system output. A narrowband filter centered near \( \Delta \nu \) and placed after the square-law device achieves a low noise bandwidth. Thus all amplifiers and other electronic detection equipment process narrowband signals at a low frequency, which provides ease of matching as well as good noise figure. This might, in turn, decrease the LO power necessary for optimum coherent detection.\(^5\) Only the heterodyne mixer and the square-law device need have high-frequency response and be broadband coupled. Similar results would, in fact, be expected with any nonlinear device (e.g., a half-wave linear, in place of the square-law element) so that there is a wide choice for designing a heterodyne-nonlinear detector combination, perhaps in a single package.

The power-spectral-density, and therefore, the SNR at the output of the square-law device, may be calculated by the "direct method" of Davenport and Root.\(^9\) Generalizing their treatment to two signals plus Gaussian noise (arising from the strong LO), we have obtained the power-spectral-density schematically shown in Fig. 1(b). The noise \times noise \((n \times n)\), plus the signal \times noise \((s \times n)\), contributions are shown as the solid line. In calculating these contributions, it has been assumed that \( \Delta f = \nu _n \), i.e., the first filter is lowpass rather than bandpass. The dotted lines represent signal \times signal \((s \times s)\) contributions which are not shown, except for the one of interest centered at \( \Delta \nu' = \Delta \nu \). Calculations have been performed for two cases: (a) when the signals \( \nu _1 \) and \( \nu _2 \) are sinusoidal and independent, and (b) when \( \nu _1 \) and \( \nu _2 \) are independent.
Gaussian random processes with individual power spectral densities which are also Gaussian. This appears to be a reasonable approximation to at least some experimental observations in the microwave, infrared, and visible.

We have calculated a bound on the over-all three-frequency heterodyne system SNR for the special case of sinusoidal infrared or optical returns from a (specular) target. If the received radiation power is equally divided between beams 1 and 2, and a lowpass filter follows the heterodyne mixer ($\Delta f \rightarrow f_B$), the SNR is bounded by:

$$\frac{\Delta f}{4B} \left[ \frac{\eta P_{\gamma}(h \nu \Delta f)^2}{1+2\eta P_{\gamma}/h \nu \Delta f} \right] \leq \text{SNR} \leq \frac{\Delta f}{2B} \left[ \frac{\eta P_{\gamma}/h \nu \Delta f)^2}{1+2\eta P_{\gamma}/h \nu \Delta f} \right].$$

(3)

The bound arises from different possible noise levels, and depends on the relative values of $\Delta \nu$ and $f_B$, as may be seen from Fig. 1(b). There are two limiting cases which may be examined. For strong signals and/or small uncertainties in Doppler frequency ($P_{\gamma}/\Delta f \gg h \nu/\eta$), Eq. (3) reduces to:

$$\eta P_{\gamma}/8hB \leq \text{SNR} \leq \frac{\eta P_{\gamma}}{4hB}.$$  

(4)

For this case, the SNR ($\approx \eta P_{\gamma}/8hB$) is degraded over the conventional heterodyne detector by $\sim 6$ db assuming equivalent output bandwidth $B$. However, the independence of the three-frequency system on the LO and Doppler frequencies might, in practice, allow $B$ to be reduced to a value well below that which could be used in the standard heterodyne configuration. Thus, a net increase in SNR might actually be possible.

In the opposite limit ($P_{\gamma}/\Delta f \ll h \nu/\eta$), with $\Delta f \gg B$, Eq. (3) reduces to:

$$\frac{(\eta P_{\gamma}/2h\nu)^2}{B \Delta f} \leq \text{SNR} \leq \frac{(\eta P_{\gamma}/\sqrt{3}h\nu)^2}{B \Delta f}.$$  

(5)

For convenience, we estimate this to be:

$$\approx \frac{\eta P_{\gamma}/\sqrt{3}h\nu}(B \Delta f)^{1/2},$$

which may be expressed as a minimum detectable power (SNR = 1) for the three-frequency mixing system $\eta P_{\gamma}(\text{min})$. Thus,

$$\frac{\eta P_{\gamma}(\text{min})}{(\sqrt{3}h\nu/\eta)(B \Delta f)^{1/2}},$$  

(6)

which is to be compared with the following quantity for the conventional configuration $P_{\gamma}(\text{min}) = (h\nu/\eta)B$, when the bandwidths $B$ are the same. The quantity $\eta P_{\gamma}(\text{min})$ approaches $P_{\gamma}(\text{min})$ only when $\Delta f \sim B$. Since the system provides its advantage for $\Delta f \gg B$, the minimum detectable power (MDP) will generally be increased (in dB) over the equivalent amount for the usual system by $5 \log(3\Delta f/B)$. Nevertheless, it must be kept in mind that the three-frequency scheme may allow $B$ to be considerably reduced in a practical system.

For the Gaussian signal case (both statistics and spectrum), the SNR results are very similar to those already presented, except for a multiplicative function containing the width and height of the spectrum, as well as the quantities $B$ and $f\nu$. For the special case of equal signals, with $B = \sqrt{B} \gamma$, and centered precisely about $\Delta \nu$ ($\gamma$ is the individual beam spectral width), the SNR need simply be multiplied by the quantity $[2\gamma(1 - 1) = 0.68$. This particular combination of parameters gives the simple result above in terms of the probability function $\phi$.

The SNR is, therefore, reduced below that obtained for the sinusoidal case (delta-function spectrum) for the same bandwidth $B$. This is understood to arise from the condition that some signal is being excluded in the Gaussian case as compared with the delta-function case, but the noise is the same. In fact, if $\Delta \nu$ is known to high accuracy, the best SNR for the Gaussian case is obtained in the limit as $B \rightarrow 0$, since the noise decreases faster than the signal, as $B$ decreases. In the limit of $\gamma \rightarrow 0$, maintaining constant power, the Gaussian results reduce to the delta-function results as expected.

As a practical example of this mode of operation, consider a CO$_2$ laser radar operating at 10.6 $\mu$m in the infrared [see Fig. 1(c)]. If we assume that we wish to acquire and track a 1-m-radius satellite with a rotation rate of 1 rpm, the expected bandwidth (resulting from rotation) of the radar return is of order $4\nu/\lambda \approx 40$ kHz. We, therefore, choose a difference frequency $\Delta \nu$ at a (convenient) value of 1 MHz, which eliminates spectrum overlap. If the satellite has a radial velocity $v = 10$ km/sec, the Doppler frequency $v_B$ is $\approx 2$ GHz, yielding a value $3\nu - 3\nu = 2v(\Delta \nu)/c = (\Delta \nu/c)v_B \approx 60$ Hz. This shift is very small indeed compared with general frequency modulations in an ordinary heterodyne system, justifying the assumption that $\Delta \nu - \Delta \nu \rightarrow 0$. Thus, assuming we have only an upper bound on the satellite velocity, i.e., its velocity may be anywhere in the range 0–10 km/sec, we choose $\Delta f \approx 2$ GHz and $B \sim 20$ kHz. The MDP for this system would, therefore, be $\sim 5(3\nu/h\nu)(B \Delta f)^{1/2}$ which is equivalent to the MDP obtained from a conventional setup with a bandwidth of approximately 10 MHz. If the velocity of $v_B$ is more confined, the MDP is correspondingly reduced. For strong returns, of course, the SNR will show an enhancement commensurate with the bandwidth $B$. Thus, the advantages of the three-frequency system may be secured with such a radar. Similar results would obtain at other frequencies; in the microwave, for example, $\Delta \nu$ may be made as small as tens of Hertz. In some cases, it may be possible to reduce clutter by the insertion of an extremely sharp notch filter at exactly $\Delta \nu$.

Use of the three-frequency method for a communications system (in which the transmitter and receiver may be moving relative to each other) is similar to the radar already described, but with the signal sent through the channel to the mixer. In this case, only one of the carrier waves (say $\nu_1$) is modulated, and the output of the narrowband filter is fed into a demodulator. By modulating only one of the beams, the $S \times S$ component reaching the demodulator results from the convolution
of a delta-function (at \( \nu_2 \)) with the modulated signal (centered at \( \nu_2 \)), which is simply the original un-distorted spectral information ready for demodulation by a suitable device such as a mixer, an envelope detector, or a discriminator.

We have also considered the case of four-frequency mixing where the transmitter and receiver are identical. This would be convenient in some cases. But because of the very large signal component arising from the beating between the two strong LO beams, the \( s \times n \) term becomes very large. Thus, it turns out that the SNR is multiplied by a factor \( (P_T/P_{10})^2 \ll 1 \), where \( P_T \) and \( P_{10} \) are typical signal and LO powers, respectively. Thus, four-frequency mixing in this configuration does not appear to be a useful concept.

A more complete account of this theory, along with some general cases not presented here, will be published shortly. I am grateful to E. N. Protonotarios and to R. Yen for valuable discussions.


2Multiple-frequency cw microwave radar has been used previously for range measurements. The receiver configuration, as well as the object of the system, is different from that considered here, however. See Ref. 1 (M. I. Skolnik, p. 106).

3This is particularly easy to accomplish if the transmitter is a two-mode laser, since the modes tend to drift together, keeping \( \Delta \nu \) constant.

4Propagation through the atmosphere may also impose frequency broadening on the beams.


6A third signal at \( \nu_1' - \nu_2' \) is very weak and may be neglected.

7The quantum theory of heterodyne detection predicts that only difference, and not sum- and double-frequency components, occur for \( h\nu \gg kT \), where \( kT \) is the thermal excitation energy of the detector. See M. C. Teich, Appl. Phys. Letters 14, 201 (1969).

8*The reception in this system has the additional advantage of being angle-independent, in the sense that the Doppler shift is proportional to the radial velocity \( v_{||} \), and, therefore, is generally a function of angle.


11*This result applies for a photoemitter or a reverse-biased photodiode. For a photocductive or photovoltaic detector, the quantity \( \eta \) must be replaced by \( \eta/2 \). See Ref. 5.

12From Eq. (3), it may be seen that \( P_T(\min) \) is always greater than \( P_T(\min) \).


PASSIVE Q-SWITCHING OF A N₂O LASER USING ETHYLENE*

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Passive Q-switching of a N₂O laser operating on the \( P(19) \) line at 922.37 cm\(^{-1}\) has been achieved using ethylene as the saturable absorber. Use of a grating together with an iris eliminates spurious oscillations on the other laser lines.

Passive Q-switching of the CO₂ laser has been achieved using different gases among which are sulfur hexafluoride,\(^1\) vinyl chloride,\(^2\) boron trifluoride,\(^2\) but only rotating mirrors Q-switching\(^4\) of the N₂O laser has been reported to date. In this letter, we report the passive Q-switching of a N₂O laser oscillating on the \( P(19) \) line at 922.37 cm\(^{-1}\) by ethylene. The absorption coefficient of \( \text{C}_2\text{H}_4 \) at 922.37 cm\(^{-1}\) (10.84 \( \mu \)) has been measured to be 0.12 cm\(^{-1}\) Torr\(^{-1}\). At N₂O wavelengths other than 10.84 \( \mu \), ethylene is nearly transparent so that a laser cavity with a high Q in a narrow spectral band centered at 10.84 \( \mu \) is needed to suppress cw oscillations on lines other than \( P(19) \). A laser ful-

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