Optical Detection of Laser or Scattered Radiation Transmitted Through the Turbulent Atmosphere

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Photoelectron counting distributions are obtained for radiation, consisting of a coherent and a chaotic component, after passage through the turbulent atmosphere. Independent additive background radiation is also taken into account. The counting distributions broaden markedly, and their peak occurs at decreasing count numbers for increasing turbulence. Plots of the distributions are presented for various levels of turbulence and various values of the coherent-to-chaotic average irradiance ratio. The results are related to the continuous photocurrent probability density.

Introduction

In recent work,[1] we have examined the counting statistics obtained for stochastic radiation caused to scintillate by passage through a random medium. The result was applied specifically to the transmission of amplitude-stabilized radiation, with and without independent additive background, and of chaotic radiation through the log-normal turbulent atmosphere.

In this paper, we investigate the detection of amplitude-stabilized radiation together with interfering narrow band gaussian noise, after passage through such an atmosphere. Again we allow for independent, additive background at the detector. This type of combined radiation approximates the output of a laser[2–6] more closely than does the amplitude-stabilized source alone, which we considered previously. For a source with a coherent-to-chaotic (or signal-to-noise) ratio greater than 5, the model considered here compares well with the description of the laser as a van der Pol nonlinear oscillator.[5] Although the Risken irradiance distribution associated with that model provides a better approximation through the region of laser threshold, it is more difficult to deal with analytically, and unnecessarily so for most experimental systems where the SNR is almost always considerably greater than 5.

It must be kept in mind, however, that the distributions described here will be valid only for detector counting intervals short in comparison with the radiation coherence time. The model considered here is also suitable for describing the photocounting detection of scattered radiation, where a coherent and a chaotic component both occur, and when the interference beats fall within the detector bandwidth. Detection of a laser illuminated satellite is an example of such an application, provided that the illuminating irradiance at the satellite does not fluctuate.

Since the factorial moments of the counting distribution are proportional to the direct moments of the irradiance, the results of this paper also contain the ordinary moments of the continuous photocurrent probability distribution. Thus, we have a means of relating the results of our model to already existing continuous photocurrent experiments,[7] as well as providing new predictions for the expected photoelectron counting distribution.

Theory

We have shown previously[1] that the method of steepest descent applied to the integral for the modulated counting statistics[1,8] yields a photoelectron counting distribution expressible very closely as

$$p(n,\sigma,N) = \frac{p_0(n,M)\exp[-\frac{1}{2}\sigma^2 q_1{}^2(n,M)]}{[1 - \sigma^2 q_2(n,M)]^{\frac{1}{2}}}. \tag{1}$$

This result is valid for any reasonable single-peaked distribution $p_0(n,N)$ that would be observed in the absence of the atmosphere. Here σ is the standard deviation of the logarithmic irradiance, N is the over-all mean of the counting distribution, and the quantities q_m are given by

$$q_m(n,N) = \partial^m \ln p_0(n,N)/\partial(\ln N)^m. \tag{2}$$

The parameter M must be determined implicitly for each count number n from the stationarity condition

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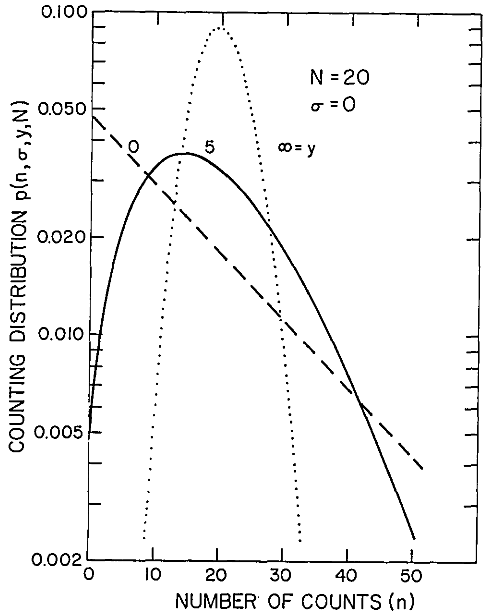

Fig. 1. Photoelectron counting distributions for linear superposition of amplitude-stabilized and chaotic radiation after transmission through a quiescent atmosphere ($\sigma = 0$). The average coherent-to-chaotic intensity ratio is indicated by y; the mean count is $N = 20$.

$$\ln M = \ln N - \tfrac{1}{2}\sigma^2 + \sigma^2 q_1(n,M). \quad (3)$$

Thus, the counting distribution is obtainable by first deriving $q_1(n,N)$ and $q_2(n,N)$ from Eq. (2), then calculating an M for each n from Eq. (3), for use in Eq. (1).

For the radiation source of interest here, a linear superposition of an amplitude-stabilized beam and gaussian noise radiation, observed with a detector of adequate bandwidth, the undisturbed counting distribution is given by[2-6]

$$p_0(n,y,N) = (1 + y)N^nH^{n+1}\exp(-yNH)L_n(-y[1 + y]H), \quad (4)$$

where

$$H = (N + 1 + y)^{-1}, \quad (5)$$

N is the over-all mean count, y is the ratio of average coherent and chaotic irradiances, and $L_n(x)$ is the Laguerre polynomial. For use in the stationarity condition, Eq. (3), the first logarithmic derivative is

$$q_1(n,N) = [n(1 + y) - N]H - y(1 + y)NH^2[1 - L_n'(x)/L_n(x)], \quad (6)$$

where

$$x = -y(1 + y)H. \quad (7)$$

For a given source SNR y, turbulence parameter σ, and ultimate mean count N at the detector, Eq. (3) yields $M(n)$ when this function $q_1(n,M)$ is introduced into the implicit equation, with M replacing N in Eqs. (6) and (7). The second logarithmic derivative, $q_2(n,M)$, is then needed in Eq. (1); this is given by

$$q_2(n,N) = -(n + 1)(1 + y)NH^2 - y(1 + y)(1 + y - N)NH^3A - y^2(1 + y)^2N^2H^4B, \quad (8)$$

where

$$A = 1 - L_n'(x)/L_n(x) \quad (9)$$

and

$$B = [L_n'(x)/L_n(x)]^2 - L_n''(x)/L_n(x), \quad (10)$$

with χ as in Eq. (7). Note that q_1 and q_2, as well as p_0, are now functions of y.

The normalized mth order moment of the irradiance, which is directly related to the factorial moments of the counting distribution, is

$$\langle I^m\rangle/\langle I\rangle^m = [m!L_m(-y)/(1 + y)^m]\exp[\tfrac{1}{2}\sigma^2m(m - 1)]. \quad (11)$$

The exponential factor is characteristic of the atmosphere, while the factor containing the Laguerre polynomial is characteristic of the source considered here. This relationship is useful in comparisons with data from continuous photocurrent density measurements.

Discussion and Conclusions

In Fig. 1, we show the expected counting distribution [Eq. (1)] for $\sigma = 0$, i.e., a quiescent atmosphere, with three values of the coherent-to-chaotic ratio, $y = \infty$, 5, 0, and an over-all mean count chosen at $N = 20$. For $y = \infty$ the distribution becomes the usual Poisson while for $y = 0$ it reduces to the expected Bose-Einstein. For an intermediate value $y = 5$ it is given by Eq. (4) and differs considerably from both limiting

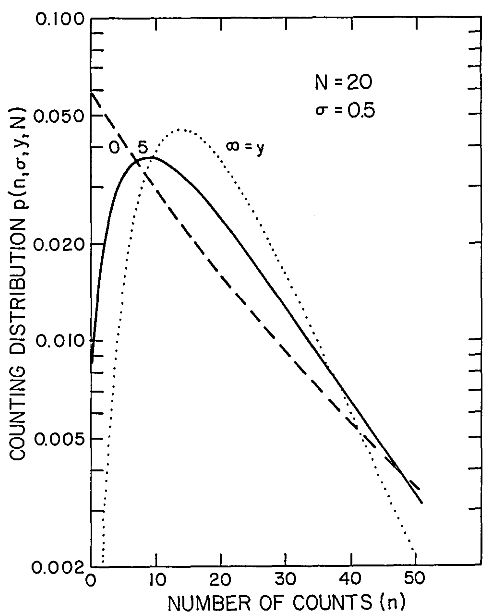

Fig. 2. Photoelectron counting distributions for linear superposition of amplitude-stabilized and chaotic radiation after transmission through a lightly turbulent atmosphere ($\sigma = 0.5$). The average coherent-to-chaotic intensity ratio is indicated by y; the mean count is $N = 20$.

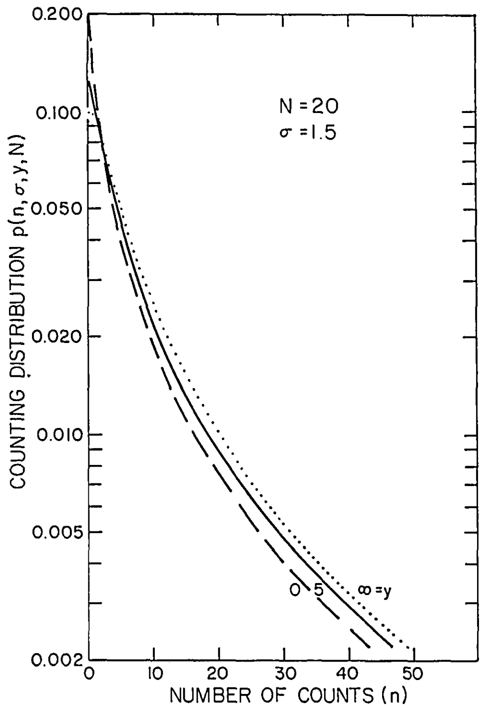

Fig. 3. Photoelectron counting distributions for linear superposition of amplitude-stabilized and chaotic radiation after transmission through a strongly turbulent atmosphere ($\sigma = 1.5$). The average coherent-to-chaotic intensity ratio is indicated by y; the mean count is $N = 20$.

cases in that it exhibits a relatively broad peak and has a long tail with a falloff intermediate between those of the limiting curves. While the value $y = 5$ may be realistic for scattered radiation, it is not for the usual laser where $y \gg 5$. In Fig. 2, we let $\sigma = 0.5$, representing a lightly turbulent atmosphere, and show the results for the same values of y. The atmosphere has its characteristic broadening effect in all three cases with peaks shifting to lower counts. In Fig. 3 we set $\sigma = 1.5$, which is its saturation value and represents the strongly turbulent case. This model gives results which are always intermediate between the limiting cases $y = \infty$ (amplitude-stabilized source) and $y = 0$ (chaotic source). But as σ increases, as pointed out previously,[1] the violent atmospheric fluctuations overshadow the statistics of the radiation source so that the counting distribution becomes relatively independent of y.

All the previous results are valid only for short counting intervals T (much smaller than both the source coherence time τ_c and the atmospheric fluctuation time τ_a) and for a receiver area smaller than the coherence area. As we noted previously, for $\tau_c \ll T \ll \tau_a$, fluctuations due to the source are averaged out and the results are identical to those for the stable source ($y = \infty$), while in the opposite limit $\tau_a \ll T \ll \tau_c$, only fluctuations due to the source are observed and atmospheric modulation need not be considered ($\sigma = 0$). Similarly for $T \gg \tau_c, \tau_a$, no fluctuations are resolved by the detector, and a simple Poisson distribution results.

If we now consider an additional source of noise in the form of independent, additive, noninterfering background radiation, the counting statistics are given by a convolution summation, as discussed previously.[1] For individual signal and noise counting distributions p_S and p_N, the over-all counting distribution is then

$$p_{S+N}(n,\sigma,y,z,N) = \sum_{m=0}^{n} p_S(m,\sigma,y,Nz/[1+z])p_N(n-m,N/[1+z]). \quad (12)$$

Here, z is the ratio of the average signal level at the detector, consisting of both coherent and chaotic contributions, to the average additive independent noise level resulting from incoherent background radiation. Usually p_N will be Poisson, and dark current can also be included directly.[1] In Fig. 4, we present the counting distributions resulting from a source with a coherent-to-chaotic ratio $y = 5$ passing through a lightly turbulent atmosphere of $\sigma = 0.5$. The background radiation is assumed to contribute a Poisson distribution, and the over-all mean count is $N = 20$. Curves are shown for various values of the detector signal-to-noise ratio, i.e. $z = 0, 0.25, 1, 4, \infty$.

It is clear that for $z = 0$, the distribution consists only of additive noise and is therefore Poisson. For $z = \infty$, it reverts to the distribution $p(n,\sigma,y,N)$ that was discussed above in the absence of additive noise. Curves for intermediate values of z show the transition between the two extremes. It is observed that as z

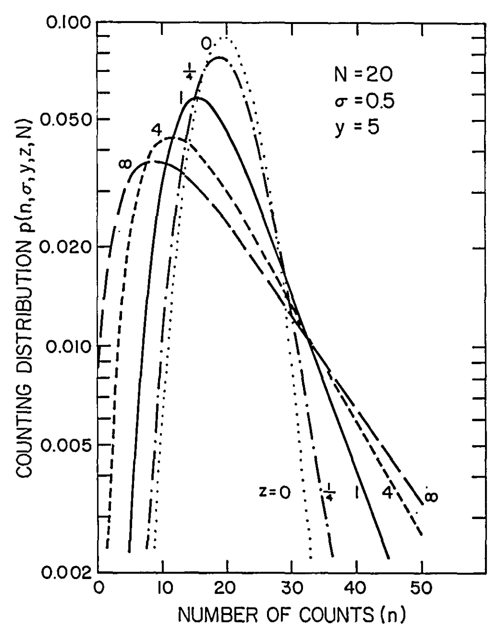

Fig. 4. Photoelectron counting distributions for linear superposition of amplitude-stabilized and chaotic radiation after transmission through a lightly turbulent atmosphere ($\sigma = 0.5$), combined with independent, noninterfering, Poisson-distributed noise counts. The combined mean count is $N = 20$, and the source SNR is $y = 5$ for each detector SNR (z) shown.

increases from zero, the distribution gradually broadens while its peak shifts toward lower count numbers. The shape of the curve for $z = 4$ is quite similar to that at $z = \infty$, indicating that additive independent noise at the detector may not be significant even at moderate signal-to-noise ratios z. More general situations, in which the additive noise is not Poisson, can be treated in the same way, except that p_N is then the appropriate distribution. The counting statistics presented here contain the two cases considered previously[1] as limiting cases, i.e., amplitude stabilized radiation ($y = \infty$) and chaotic radiation ($y = 0$).

As indicated earlier, the factorial moments of the counting distribution are effectively the direct moments of the irradiance [Eq. (11)] and, therefore, of the photocurrent in a continuous-current experiment. Thus, we can take into account the effect of fluctuations of the incoming radiation on the photocurrent probability distribution, which should be strictly log-normal only for the amplitude-stabilized case. As may be seen from Figs. 1–4 for a laser operated very near to threshold or for a hypothetical scattering source with $y = 5$, the stochastic nature of the radiation source will yield a current density that differs considerably from log-normal. In principle, the entire photocurrent probability distribution can be obtained from a knowledge of all its moments, but this is not practical because of the strong dependence on extremely rare events. Rather, the experimentally obtained direct moments for continuous photocurrent experiments should be compared with the theoretically predicted moments presented in Eq. (11), provided that the time resolution of the two experiments is comparable.

As a final note, we point out that Eq. (4) (with $\sigma = 0$) is also the photoelectron-counting distribution that results from the Rice-Nakagami irradiance distribution, and as such provides the appropriate counting statistics for an ideal amplitude-stabilized radiation source coupled with the *Rayleigh* model for atmospheric fluctuations. In this case, the factor y relates to the level of turbulence; the quiescent atmosphere is represented by $y = \infty$ which is a Poisson distribution.

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Optics for the Airborne Observer. By L. Larmore and F. F. Hall, Jr. Research Communication 135, Douglas Advanced Research Laboratories, Huntington Beach, California, July 1970. 30 pp.

We do not ordinarily review technical reports in this journal, but this one deserves special treatment. Everyone who works in atmospheric optics has long been aware of the splendid book by M. Minnaert, *The Nature of Light and Colour in the Open Air* (Dover, New York, 1954), in which Minnaert describes the many remarkable optical phenomena of the atmosphere that can happen around us almost any day but that generally pass unnoticed simply because we do not look: mock suns, mirages, interesting shadows, twilight phenomena, and so on. We remember a few years ago sitting in a New Mexico desert one evening with John Strong, another atmospheric optics man of the Minnaert type: "Look," he said, "there goes the shadow of the earth," and sure enough, there it was, sweeping across the sky; something one never sees near the hazy glow of a big city. In this report, Lewis Larmore and Freeman Hall, both frequent authors in this journal, repeat the Minnaert treatment for the phenomena easily observed by the airplane traveler: contrails and cloud phenomena, heiligenschein, glories, subsuns, strange polarization effects, and so on. All of these phenomena are well known to a few experts and are thoroughly described in the strictly technical literature (usually in German); what makes this report special is that the authors briefly describe and explain when and where to look for these phenomena, and it includes photographs of many of them. There are about thirty good sharp photographs in full color, individually tipped into the report. Such a report is expensive, and this was understandably produced in a limited edition of about one hundred copies: available only to the lucky few who write in first, but we understand that the SPIE intends to reproduce this article in their journal, with the figures in color.* We only wish our own page budget (and budget for color) were not in such straitened circumstances that we must needs send you elsewhere for such a mickle treat.

* Vol. 9, p. 87 (1971).

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