Statistics of Entangled-Photon Coincidences in Parametric Downconversion[a]

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INTRODUCTION

We present experimental counting distributions and interevent-time histograms for photon detections from spontaneous parametric downconversion,[1] both marginally and as coincidences. The experiments were conducted with a lithium-iodate downconverter pumped by 413-nm krypton-ion laser light. The data are consistent with Poisson statistics; a model leading to this result is presented.

EXPERIMENT

A block diagram of the experimental setup is shown in FIGURE 1. Light from a single-mode Kr^+-ion laser, operated at 413 nm and attenuated to 0.75 mW, is directed by a lens onto an l = 10-mm-long 42.8°-cut lithium iodate ($LiIO_3$) nonlinear optical crystal, oriented for type-I (ooe) phase matching. Unconverted pump photons pass straight through the crystal and enter a beam dump. Downconverted photons emerge at angles to the pump beam determined by energy- and phase-matching, with degenerate photons emerging symmetrically in a cone of full angle ~ 15°. By using apertures for the downconverted beams of about 2-mm diameter, we selected out desired degenerate photon pairs[2] with center wavelengths of ~826 nm. The entangled photon pairs were directed to two passively quenched avalanche-photodiode (APD) counting modules. A pulse counter and a time-interval counter recorded events from the detectors. Coincidences were generated by passing the sequence of standardized pulses from the two detectors through a 10-ns AND gate. The coincidence events were also fed to the pulse and time-interval counters for statistical analysis.

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In the upper portion of FIGURE 2, we present the marginal photon-counting distribution $p(n, T)$ obtained from one of the twin beams (detector A) using a counting time of $T = 1$ ms (solid dots). Using a total of $N = 50{,}004$ samples, the mean number of photocounts was found to be $\bar{n} = 20.238$ and the Fano factor was $F = \mathrm{Var}(n)/\bar{n} = 0.974$.

The solid curve in the upper portion of FIGURE 2 is a Poisson distribution with the same mean as the data. The standard deviation of the Fano factor estimate for a Poisson distribution based on N samples is known to be $\approx\sqrt{2/N}$ so that, for $N = 50{,}004$, F is expected to lie between 0.994 and 1.006. The Poisson distribution fits the data quite well, but the observed value of F ($= 0.974$) is too low.

The discrepancy is readily resolved by making use of the nonparalyzable dead-time-modified Poisson (DTMP) counting distribution.[3,4] This distribution is, in fact, appropriate because the electronic pulses produced by the APD-module circuitry incapacitate the counting system for a duration $\tau_d \approx 1$ μs following the registration of a photon. The theoretical expression for the DTMP Fano factor is[5]

$$F \approx (1 - \bar{n}\tau_d/T)^2 \leq 1. \tag{1}$$

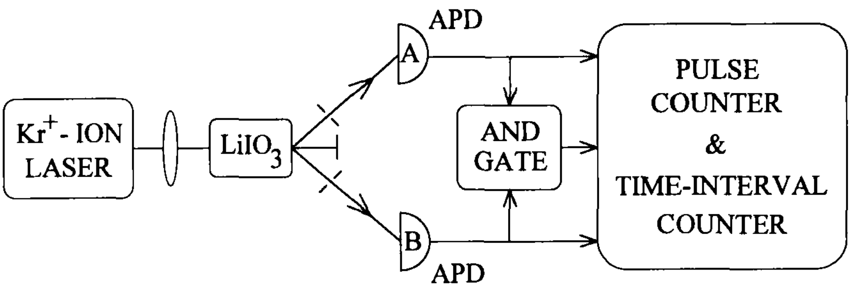

FIGURE 1. Block diagram of the experimental arrangement. The pulse-counter electronics produces a pulse of finite duration that acts as a dead time τ_d.

Using the observed value for F, we obtain $\tau_d \approx 0.64$ μs, which is in reasonable accord with the APD-module pulse duration. Photon-counting distributions were obtained using counting times T ranging from 5×10^{-7} to 2×10^{-3} s, and the DTMP counting distribution was found to describe properly the data in all cases.

The marginal interevent-time probability density function $p(t)$ from detector A is shown in the lower portion of FIGURE 2 (solid dots). The histogram bins are 1 μs in duration and the number of samples is $N = 100{,}000$. The data are well fit by a straight line that, on this semilogarithmic plot, represents an exponential distribution with the same mean as the data. The only exception is the initial data point, which lies below the exponential curve, and this is a result of dead time. The observed mean interevent time of $\bar{t} = 50.336$ μs accords well with the expected value of

$$\bar{t} \approx \tau_d + T/\bar{n} \tag{2}$$

using the parameters determined from the counting distribution.

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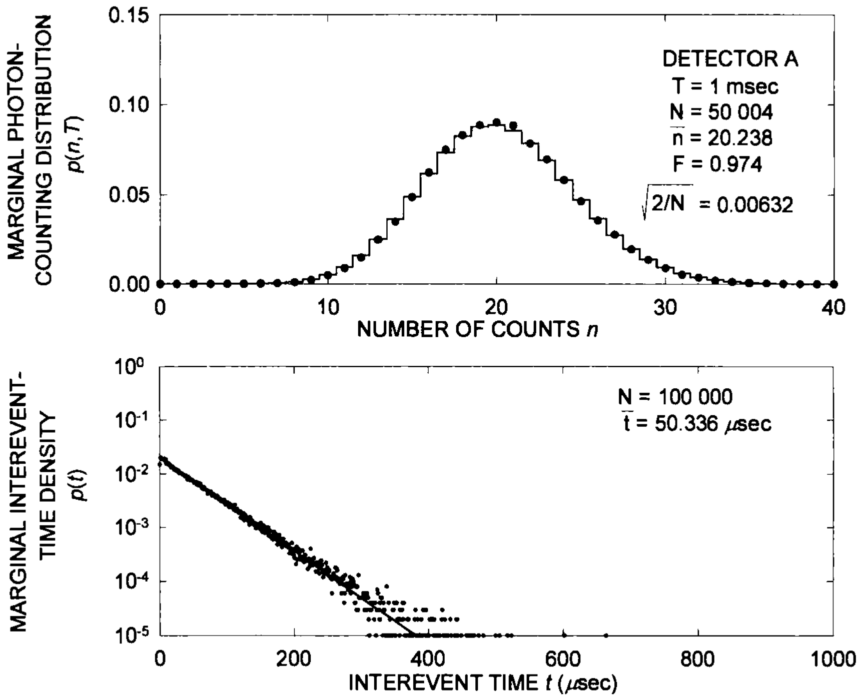

FIGURE 2. Marginal photon-counting distribution $p(n, T)$ versus number of counts n at the output of detector A (upper panel, solid dots): the solid curve is the Poisson distribution with the same mean as the data. Marginal interevent-time probability density function $p(t)$ versus interevent time t at the output of detector A (lower panel, solid dots): the solid curve is the exponential distribution with the same mean as the data. Both results are consistent with a photon stream at the input to the detector that obeys a Poisson point process, observed by a detector with dead time.

We conclude that the photocount occurrences observed at the output of detector A are well modeled by a dead-time-modified Poisson point process. This is consistent with a photon stream at the input to the detector described by a Poisson point process.

The marginal photon-counting distribution and interevent-time probability density function for detector B are shown in FIGURE 3, again using a counting time of $T = 1$ ms (solid dots). In this case, the mean number of photocounts was $\bar{n} = 9.501$, about half that in detector A (probably as a result of the different quantum efficiencies of the two detectors). The solid curve in the upper portion of FIGURE 3 represents a Poisson distribution with the same mean as the data. Again, it fits well, but the observation that $F = 0.983 < 1$ suggests the use of the DTMP counting distribution. Using this value for F in equation 1 leads to $\tau_d \approx 0.88$ μs, which is again reasonable. The observed mean interevent time $\bar{t} = 102.32$ μs from detector B also accords well with the value expected from equation 2. Thus, the photon stream impinging on detector B also appears to be describable by a Poisson point process.

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The statistics of the coincidences are displayed in FIGURE 4. Using a counting time of $T = 2$ ms (solid dots), the coincidence-counting distribution is shown in the upper panel. Using a total of $N = 25{,}002$ samples, the mean number of coincidences was $\bar{n} = 3.206$ and the Fano factor was $F = 0.988$. The solid curve is a Poisson distribution with the same mean as the data. Similar results were obtained using counting times T ranging from 5×10^{-7} to 2×10^{-3} s. We can therefore also invoke the DTMP counting distribution to describe the coincidence data. Using the observed value for F, we infer that $\tau_d \approx 3.8$ μs, which is somewhat greater than the values obtained for the rates of singles. The reason for the increased dead time in the coincidence case is unclear. Except for the initial point, the coincidence interevent-time probability density function is also well fit by an exponential distribution. The mean interevent time $\bar{t} \approx 623.77$ μs is in accord with equation 2. Thus, the coincidences are also well modeled as a DTMP, which is consistent with the photon pairs arriving at the detectors as a joint Poisson point process.

THEORY

The theory developed by Yurke and Potasek[6] for the photon statistics of parametrically downconverted light is based on the assumption of a classical pump

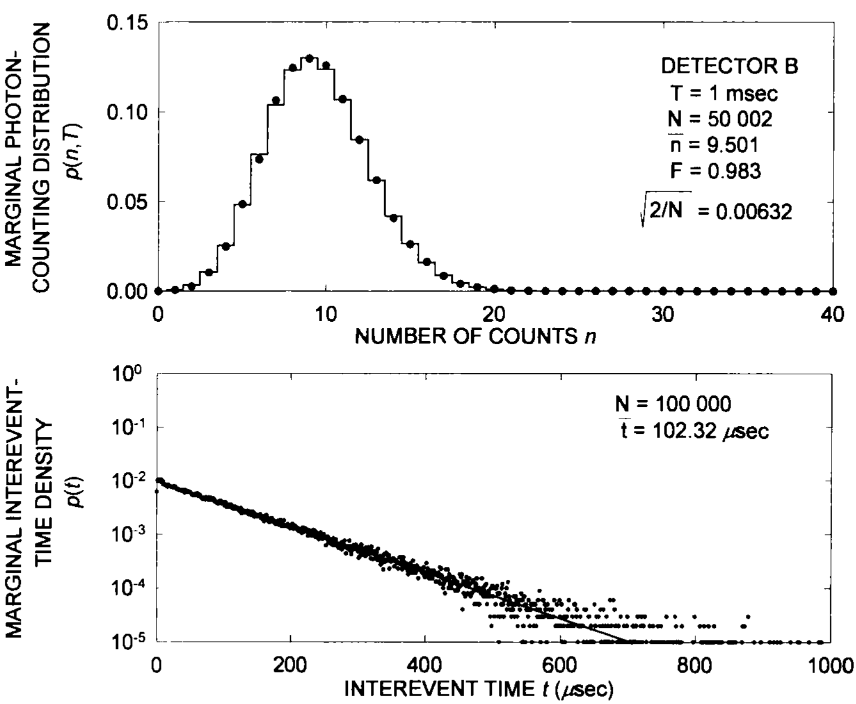

FIGURE 3. Same as FIGURE 2, but at the output of detector B.

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applied for a time interval between 0 and t and of single signal and idler modes of a cavity that are initially in the vacuum state. Writing the Hamiltonian for the process, they showed that the final (pure) state is one for which the marginal photon-counting distributions for each of these modes constitute the Bose-Einstein distribution, and the numbers of photons in the two modes are always the same. In the framework of this model, the mean number of photons is a monotonically increasing function of time because the emitted photons continue to build up in the cavity.

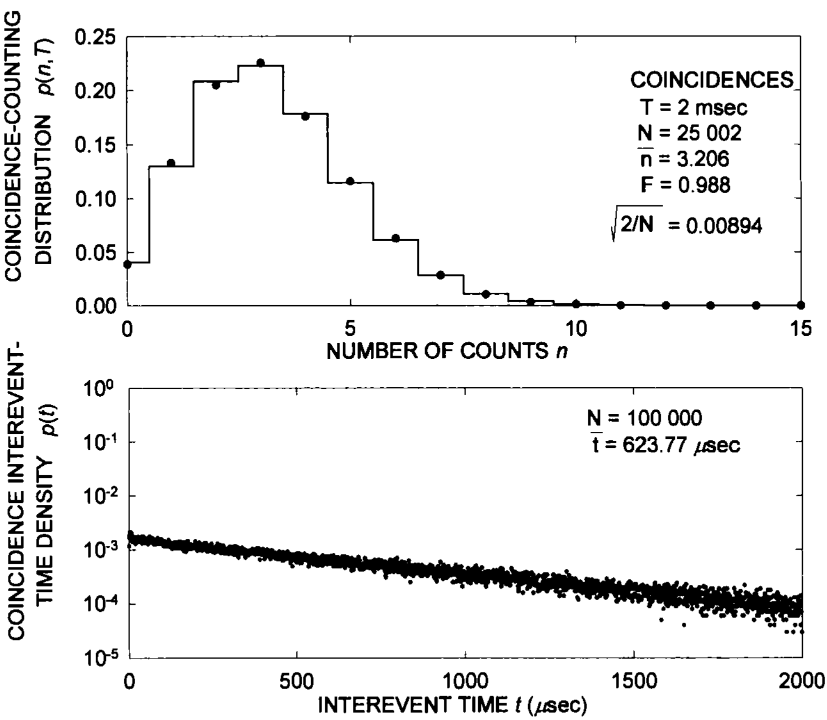

FIGURE 4. Coincidence-counting distribution $p(n, T)$ versus number of coincidences n at the output of the AND gate (upper panel, solid dots): the solid curve is the Poisson distribution with the same mean as the data. Coincidence interevent-time probability density function $p(t)$ versus interevent time t at the output of the AND gate (lower panel, solid dots): the solid curve is the exponential distribution with the same mean as the data. These results are consistent with the photon pairs arriving at the detectors as a joint Poisson point process.

These results can be adapted to a cavityless open system in which the pump travels continuously through the downconversion crystal and the emitted photons propagate away from the interaction region. We consider the continuous classical pump wave as divided into $N = T/\tau_r$ contiguous segments, each of which has a duration given by the transit time τ_r through the crystal. Because each transit-time segment may be described by the Yurke-Potasek cavity formulation with $t = \tau_r$, it will give rise to a Bose-Einstein-distributed number of photons in each of the downcon-

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verted beams. Random deletion, associated with loss and finite detector quantum efficiency, leaves the form of the Bose-Einstein distribution intact.[7] The total number of photons n generated in each beam, in the counting time T, is therefore a sum of N independent Bose-Einstein random variables. The result is the well-known negative-binomial distribution[8] with a mean $\bar{n} = \lambda T$ that is the sum of the individual means and with a variance given by

$$\mathrm{Var}(n) = \bar{n} + \left(\frac{\bar{n}^2}{N}\right) = \bar{n}(1 + \lambda\tau_r). \tag{3}$$

The quantity $\lambda\tau_r$ represents the mean number of photons emitted in a single transit time. In our experiments, $\lambda = \bar{n}/T \approx 20{,}000$ and $\tau_r = l/c \approx 30$ ps, so $\lambda\tau_r \approx 6 \times 10^{-7}$. The fact that $\lambda\tau_r$ is small means that $\mathrm{Var}(n) \approx \bar{n}$ in equation 3, as for Poisson statistics. Stated differently, the mean number of photons emitted into each downconversion beam in a transit time is sufficiently small such that the probability of generating more than one photon is negligible; thus, the overall sequence of photon emissions is consistent with a Poisson point process. If it is desired to observe the bunching accompanying Bose-Einstein emissions, $\lambda\tau_r$ would have to be increased significantly.

The results described above assume that the pump is classical; that is, it exhibits no fluctuations. It also applies if the pump exhibits phase fluctuations with coherence time τ_c, but is effectively sinusoidal during any transit time ($\tau_r \ll \tau_c$).

Using the foregoing results, we conclude that the coincidence events must arise from the intersection of two initially identical point processes that have been independently randomly deleted. Because the relevant point processes all maintain their form under random deletion,[7] the surviving pairs of events comprising the coincidences follow the same statistics as the marginals, albeit with lower mean. For the parameters of our experiment, joint Poisson coincidences ensue, as confirmed by our observations.

More comprehensive results for the statistical behavior of the downconverted light require the use of a multimode approach. Joobeur *et al.*[9] constructed a theory of downconversion based on an interaction Hamiltonian that couples a pump wave of finite spectral width with a nonlinear crystal of finite spatial width l. The downconverted light is not cross-spectrally pure. Marginal signal and idler spectral densities, denoted $S_{\theta_s}(\omega_s)$ and $S_{\theta_i}(\omega_i)$, respectively, have been calculated for different observation directions. These can be used to obtain a coherence volume V_c and a degrees-of-freedom parameter $M_V = V/V_c$ characterizing the marginal count variances.

The signal-idler cross-spectral density $\bar{S}_{\theta_s,\theta_i}(\omega_s)$ has also been obtained.[9] It is characterized by entanglement volumes V_s and V_i and associated spatiotemporal entanglement degrees of freedom M_s and M_i that determine the joint (and coincidence) statistics. One important result has already emerged from using this approach: the normalized coincidence rate R_{si} has been found to be suppressed substantially below unity unless M_s and M_i are both $\gg 1$.

All of this notwithstanding, for the range of parameters used in our experiments, the data are consistent with a simple model in which the pump may be regarded as a sequence of primary photons obeying Poisson statistics, each of which splits into a

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pair of downconverted photons that are created essentially simultaneously. Models of this type have been considered previously in other contexts.[10–12]

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