Energy Landscape Theory for Alzheimer’s Amyloid β-Peptide Fibril Elongation

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**ABSTRACT** Recent experiments on the kinetics of deposition and fibril elongation of the Alzheimer’s β-amyloid peptide on preexisting fibrils are analyzed. A mechanism is developed based on the dock-and-lock scheme recently proposed by Maggio and coworkers to organize their experimental observations of the kinetics of deposition of β-peptide on preexisting amyloid fibrils and deposits. Our mechanism includes channels for (1) a one-step prion-like direct deposition on fibrils of activated monomeric peptide in solution, and (2) a two-step deposition of unactivated peptide on fibrils and subsequent reorganization of the peptide–fibril complex. In this way, the mechanism and implied “energy landscape” unify a number of schemes proposed to describe the process of fibril elongation. This β-amyloid landscape mechanism (β-ALM) is found to be in good agreement with existing experimental data. A number of experimental tests of the mechanism are proposed. The mechanism leads to a clear definition of overall equilibrium or rate constants in terms of the energetics of the elementary underlying processes. Analysis of existing experimental data suggests that fibril elongation occurs through a two-step mechanism of nonspecific peptide absorption and reorganization. The mechanism predicts a turnover in the rate of fibril elongation as a function of temperature and denaturant concentration. Proteins 2001;42:217–229. © 2000 Wiley-Liss, Inc.

Key words: Alzheimer’s disease; Aβ-peptide; fibril; kinetics; reaction rates; denaturant; protein folding

**KINETICS OF FIBRILLOGENESIS AND ELONGATION**

The crucial role of amyloid peptide deposition as “a necessary but not sufficient factor for the pathogenesis” of Alzheimer’s disease has been persuasively argued. Experimental studies of fibril formation and elongation have led to a number of views of amyloidogenesis.2–4 One view suggests that unstructured monomers in solution cluster and form nuclei.2 Once the cluster reaches a critical size, the nucleus forms a fibril,5,6 which then grows to form full-length fibrils by the addition of monomers to the existing fibril ends.7 In another view, intermediate peptide “protofibrils” are formed5,9 and associate end-to-end or laterally to form fibrils.9,10 Subsequently, the amyloid peptide monomer/dimer may add directly to existing protofibrils and fibrils.11,12 This view may be augmented by the possibility that monomers associate to form micelles and that those micelles may convert to fibril nuclei upon reaching a critical size.2,5,6

It has been demonstrated that the process of elongation of existing fibrils through the process of monomer binding to fibril ends can be studied independently of the process of nucleation, micelle conversion, or association of protofibrils11 (Fig. 1). It is then possible to study the simple, first-order kinetics of fibril elongation9,12 where

\[
\frac{dN_f}{dt} = k_e [m]
\]

with \(N_f\) the number of peptide monomers in a fibril, \(k_e\) is the fibril elongation rate constant, and \([m]\) is the concentration of peptide monomer. By assuming a mechanism of “bimolecular association” of peptide monomer diffusion to the fibril end and activated reorganization of the peptide/fibril, further analysis of the elongation rate constant \(k_e\) led to estimates of the separate energy and entropy of activation based on an approximate reaction/diffusion rate constant

\[
k_e = D\sigma \exp(-G^*/RT)
\]

where \(\sigma\) is the diameter and \(D\) is the diffusion coefficient for the peptide monomer.9,12 The Gibbs free energy of activation can be written in terms of the separate energy, volume, and entropy of activation as

\[
G^* = E^* + PV^* - TS^* = H^* - TS^*
\]

where the experiments are performed at constant temperature and pressure.

The dimension \(\sigma\) was taken to be on the order of 10 Å, which is roughly the diameter of the monomeric peptide. The diffusion constant for the monomer was taken from experiment to be 1.6 × 10⁻⁷ cm²/s.12 With a knowledge of \(k_e\) and \(D\), experimental analysis assigned a value of \(TS^* = 16\) kcal/mol. When combined with a value of \(H^* = 23\) kcal/mol derived from a van’t Hoff analysis, the result is an overall free energy of activation of only \(G^* = 7\) kcal/mol at 300 K. The significant and positive entropy of activation \(S^* = 0.053\) kcal/(mol K) was taken to indicate that the
peptide/fibril undergoes an unfolding transition in reaching the transition state from a collapsed, unactivated monomer state.12

In demonstrating that fibril elongation occurs by a process of monomer diffusion to the fibril end and subsequent reorganization of the aggregate, these impressive experiments raise questions about the elementary molecular kinetic events of fibril elongation. What is the mechanism of monomer/fibril association? What is the structure of the initial deposit? What is the nature of the reorganization from an initially formed deposit to a well-formed fibril?

A set of key experiments recently performed by Maggio and coworkers13 partially answer these questions. These investigators explored the kinetics of formation of the “locked” or irreversibly well-formed fibril. A solution of radioactively labeled monomeric peptide in solution was allowed to “dock” onto existing deposits of unlabeled peptide for a loading time $t_L$. The deposits were then washed and the off rate for the labeled peptide to leave the deposit was measured. Two well-separated time scales for desorption were observed. The amount of peptide that was found to be “locked” to the deposit such that it would not desorb was found to be a sensitive function of the loading time $t_L$. Maggio and colleagues sketched a dock-and-lock scheme in which monomeric peptide diffuses to the fibril end, loads on the deposit, and then undergoes a conformational reorganization to the “locked” state. This scenario is similar to that proposed by Teplow and coworkers.9,12 However, through their carefully designed experiments, Maggio and coworkers have been able to analyze the kinetics of the association and reorganization steps separately by introducing what they call a “transition state” intermediate of the peptide between the reactant solution state and the irreversibly locked product fibril state. The term “transition state,” as used by Maggio and coworkers, refers to a metastable intermediate, rather than the typically unstable activated transition state that appears in the transition state theory of activated processes.

The experimental and interpretive work of these groups has begun to define a reaction mechanism for the process of fibril elongation in terms of elementary molecular processes. In this study, we present a detailed mechanism of fibril elongation that builds on that work. Our kinetic model is used to construct a schematic energy landscape with loosely defined reaction coordinates and transitions states for peptide/fibril association and reorganization. A steady-state kinetic analysis of the proposed mechanism is combined with the experimental data of Maggio and coworkers and is used to assign values to rate constants for the assumed elementary processes of adsorption/desorption and reorganization/deorganization of the amyloid peptide and fibril. A variety of experiments measuring the rate of fibril elongation for mutant and modified forms of the amyloid peptide are interpreted using the proposed energy landscape theory. The result is a kinetic model consistent with existing data that provide a molecular level interpretation of the process of fibril elongation.

**ENERGY LANDSCAPE MECHANISM FOR THE KINETICS AND THERMODYNAMICS OF FIBRIL ELONGATION**

An important first step in developing a mechanism for fibril elongation is to isolate the reversible and potentially rate limiting steps in the essentially irreversible process of fibril elongation. Figure 2 shows a schematic snapshot of the process of fibril elongation including monomeric $m$ peptide in solution, an amorphous deposit of “$\beta$” peptide, the reaction interface where the amorphous deposit meets the well-formed fibril, and the $\beta$ fibril itself. This picture is meant to capture the essence of the dynamics of fibril elongation such as that studied in the experiments of Maggio and coworkers.13 The dynamics of fibril elongation at lower concentrations is expected to be similar with regard to the reaction interface but may result in less “$\beta$” peptide being formed at steady state. At higher concentrations, the loading of layers of “$\beta$” peptide is expected to raise the barrier for reorganization of “$\beta$” to $\beta$ peptide at the reaction interface decreasing $k_r$.

In this simple model, we will assume that the origin of the essentially irreversible formation of fibril is that the peptide adds to existing fibril and then is buried as other peptide is added and the fibril continues to grow. Therefore, while the peptide at the reaction interface shows a reversible reorganization and “deorganization” with rate constants $k_r$ and $k_{d,t}$, respectively, once the peptide is buried within the fibril the rate of deorganization $k_{d,t}$ is effectively zero. Similarly, in the amorphous deposit the initial adsorption and desorption steps, with associated rate constants $k_a$ and $k_{d,t}$, respectively, are reversible. However, peptide within the $\beta$ peptide deposit is essentially trapped making $k_d$ effectively zero for those molecules that are not at the interface with the solution. Finally, within the amorphous deposit, “$\beta$” peptide that is not at the reaction interface in contact with existing fibril has a rate of reorganization to $\beta$ peptide $k_r$ that is effectively zero.

The overall picture is one of adsorption/desorption of monomers on the existing deposit and reorganization/deorganization of peptide at the reaction interface. Our proposed mechanism focuses on the dynamics of the association of monomers with the fibril end and the subsequent reorganization step that would be expected to
represent a steady-state condition at low peptide concentrations.

A simple set of elementary kinetic steps that capture the essential peptide dynamics can be written as follows. The peptide may encounter the fibril end ($P$) in an unreactive conformation $m$. In this case, the peptide may deposit itself to the fibril surface to form a poorly annealed extended fibril ("$b$"). That will occur with a rate $k_a$ for peptide adsorption. Subsequently, the peptide may desorb before the transition from the poorly annealed conformation to the well annealed fibril formation ($b$) occurs. That will occur with a rate $k_d$ for peptide desorption.

\[
m + P \xrightarrow{k_a} "b" \xrightarrow{k_d} m \]

After deposition, the peptide, as part of a poorly formed fibril deposit "$b$," may undergo reorganization and reach the reactive state leading to a secure deposit $\beta$ before the peptide monomers desorb. The peptide/fibril reorganization occurs with a rate constant $k_r$ as

\[
"b" \xrightarrow{k_r} \beta
\]

where $k_d$ is the rate of peptide deorganization. These steps represent a two-step mechanism for fibril formation as deposition of unactivated peptide and subsequent reorganization. A partially structured collapsed coil state encounters the fibril end through diffusion. The peptide deposits itself on the fibril end resulting in a loosely formed deposit. The peptide/fibril deposit then undergoes reorganization to accommodate the peptide in a deposited (product) state. The reorganization step may involve conformational changes in either the peptide or the fibril end, or both. The activation energy for the fibril elongation is associated with peptide/fibril reorganization. Again, at higher concentrations, the loading of layers of "$b$" peptide is expected to raise the barrier for reorganization of "$b$" to $\beta$ peptide, making $k_r$ a decreasing function of time.

It is also possible for the peptide to encounter one of a number of activated transition state conformations $m^*$

\[
m \xrightarrow{k_1} m^* \xrightarrow{k_2} m
\]

If the peptide encounters the precursor fibril end ($P$) in such a state the deposition is immediately reactive

\[
m^* + P \rightarrow \beta
\]

with a rate constant $k_a$ for peptide adsorption. This is a one-step mechanism of fibril formation as fast deposition of "activated" peptide. A partially structured collapsed coil state of the peptide encounters the fibril end by diffusion. The fibril can be considered frozen, as no fibril end reorganization is required for peptide deposition. The peptide will deposit on the fibril end if, when the peptide encounters the fibril end through diffusion, the peptide is in one of a set of reactive coil conformations. The activation
energy is associated with the peptide monomer reorganization in solution from a set of unreactive configurations to one of a set of transition state configurations.

This model can be interpreted graphically in terms of a dynamics on the amyloid peptide “energy landscape” depicted in Figure 3. The x-axis is a measure of the free energy of the monomer fibril system. The y-axis is a measure of the separation between the peptide monomer and the existing fibril. The x-axis is a coordinate that measures the conformation of the peptide monomer/fibril as it undergoes a transition between the collapsed coil conformation and the β form favored in the fibril. As shown on the landscape, the collapsed coil structure (m) is the predominant thermodynamic form in solution. As the peptide approaches the fibril end along the peptide–fibril separation coordinate it can remain as a collapsed coil and adhere to the fibril end. That structure is the amorphous or poorly formed peptide–fibril deposit (“β”). A slow conformational change of the peptide structure then occurs along the peptide reorganizational coordinate as the peptide reorganizes to the β structure that is energetically favored in the fibril. The well-formed fibril is a thermodynamically stable “global minimum” on the landscape. These steps make up the proposed β-amylloid landscape mechanism (βALM).

Steady-State Rate of Fibril Elongation at Low Peptide Concentrations

Given these two reaction channels, we can examine the overall kinetics. In the general case, the integrated rate laws for the concentration [m], ["β"] and [β] can be determined. Once a steady state of fibril growth is established, the concentration of activated monomer [m*] and poorly annealed monomer added to fibril ["β"] should be constant in time. The result for the steady-state rate of fibril growth is then given by

\[
\begin{align*}
\frac{d[β]}{dt} &= k_d[P][m]\left(\frac{k_r}{k_r + k_d} + \frac{k_1}{k_1 + k_d + k_1[P]}\right) - \frac{k_d[β]}{k_r + k_d} \\
&= \zeta_m[m] - \zeta_β[β] = -\frac{d[m]}{dt}.
\end{align*}
\]

In the steady-state regime, this model can be solved for the integrated rate of increase of [β] in time as

\[
[β](t) = [β](0)e^{-\zeta_βt} + \frac{\zeta_m}{\zeta_m + \zeta_β}([m](0) - [β](0))\left[1 - e^{-\zeta_βt}\right].
\]  (5)

In the special case in which the initial concentration [β](0) = 0, k_d is small, making ζ_β ≪ ζ_m and k_1/(k_1 + k_d[P]) is small compared with k_r/(k_r + k_d), we find

\[
[β](t) = [m](0)\left[1 - \exp\left(-\frac{k_r}{k_r + k_d}[P]\right)\left(\frac{k_r}{k_r + k_d}\right)t\right].
\]  (6)

The details of each reaction channel and the conditions under which each channel may predominate are described in the following section. For the purposes of the discussion that follows, we focus on the initial steps in β-fibril elongation and assume that the rate of “deorganization” is small and can be ignored. No such assumption is made in the general form of the βALM mechanism. For each channel, we derive a specific form for the compound elongation rate constant k_e in terms of the elementary rate constants defined by our mechanism.

Fast Equilibrium Formation of Activated m*

Assume that there is a rapid formation of activated monomer in its equilibrium concentration. The rate of deactivation of peptide monomer is assumed to be greater than the rate of adsorption of monomer on precursor fibrils or that k_{-1} ≫ k_r[P]. In that case the rate law is

\[
\frac{d[β]}{dt} = k_d[P][m]\left(\frac{k_r}{k_r + k_d} + \frac{k_1}{k_1 + k_d + k_1[P]}\right)
\]

(7)

Competing pathways for the formation of the well-formed extended fibril β are (1) direct deposition of activated peptide monomer m*, and (2) deposition of unactivated peptide and subsequent reorganization of peptide/fibril. From the definition of the elongation rate constant k_e per deposit or fibril (in units of L·mol^{-1}·s^{-1}) we define

\[
\frac{dN_f}{dt} = k_e[m] = \frac{1}{[P]} \frac{d[β]}{dt}
\]

(8)

and find that

\[
k_e = k_d\left(\frac{k_r}{k_r + k_d} + \frac{k_1}{k_1 + k_d + k_1[P]}\right)
\]

(9)

Fast Equilibrium with Small k_1/k_{-1} ≪ k_r/k_d

Assume that there is an equilibrium population of activated peptide monomers (m*) characterized by the equilibrium constant [m*]/[m] = k_r/k_{-1}. The ratio of the reorganization rate constant to the desorption rate constant exceeds the equilibrium constant or k_r/k_d ≫ k_1/k_{-1}. In such a case, the fibril elongation occurs rapidly as a process of deposition of unactivated peptide and subsequent reorganization depicted in Figure 4. The corresponding rate law is

\[
\frac{d[β]}{dt} = k_e\frac{k_r}{k_r + k_d}[P][m]
\]

(10)

The elongation rate constant is given by

\[
k_e = k_d\frac{k_r}{k_r + k_d}
\]

(11)

In the case of k_d ≫ k_r, the rate is determined by the magnitude of the equilibrium constant for adsorption k_r/k_d and the rate of reorganization of adsorbed peptide. An Arrhenius analysis of the rate of elongation would contain contributions from both the activation energy for peptide/fibril reorganization, as well as the equilibrium free energy of adsorption/ desorption

\[
G^* = G^*_a + \Delta G^\text{eq}
\]

(12)
In the opposite case of \( k_r \gg k_d \), the rate of elongation will reduce to \( k_e = k_a k_1 / k_{-1} \) and the Arrhenius analysis will determine the barrier for the association process \( G^*_a \).

**Fast Equilibrium with Large \( k_1/k_{-1} \gg k_r/k_d \)**

Assume that the ratio of the reorganization rate constant to the desorption rate constant is large compared to the equilibrium constant or \( k_r/k_d \ll k_1/k_{-1} \). In such a case, the fibril elongation occurs rapidly from the direct deposition of activated peptide as depicted in Figure 5. The corresponding rate law is

\[
\frac{d[\beta]}{dt} = k_a k_1 k_{-1} [P] [m]
\]

The elongation rate constant is

\[
k_e = k_a k_1 / k_{-1}
\]

An Arrhenius analysis of the elongation rate will have a leading contribution from the activation energy for adsorption but will be complicated by contributions from monomer activation equilibrium and peptide reorganization

\[
G^* = G^*_a + \Delta G_{eq}^{mon}
\]

**No Significant Formation of Activated \( m^* \)**

Assume that the activated monomer is rapidly converted to well-formed elongated fibril and

\[
\frac{d[\beta]}{dt} = k_{1}[m]
\]

When it is also true that the pathway of direct “nucleation” dominates that of deactivated association and reorganization, which might be the case when the rate of reorganization of adsorbed peptide is slow and \( k_1 \gg k_a [P](1 + k_d/k_r) \), we find

\[
\frac{d[\beta]}{dt} = k_{1}[m]
\]

The rate constant for elongation then becomes

\[
k_e = k_1 [P]^{-1}
\]

Note that this rate is inversely proportional to the concentration of precursor amyloid deposits \([P]\). When the concentration of precursor deposits is high, an increase in precursor concentration may lead to a slowing of the rate of elongation, as those deposits are competing for scarce monomers.

In this case, the rate is proportional to the rate of activation of the amyloid peptide monomer in solution. In an Arrhenius analysis of the rate constant, the activation energy \( G^* \) would correspond to the process of conformational reorganization of the solvated peptide. An example would be the opening of the peptide from a tight collapsed coil state.

This simple kinetic scheme serves to demonstrate that the rate constant for fibril elongation may have contributions from a variety of kinetic processes. The take home message is that any Arrhenius analysis must be carefully done to include the contributions from secondary equilibria as well as peptide reorganization on the fibril surface and in solution. Only in the case of direct deposit of activated monomer from solution can we expect to have a “clean” Arrhenius analysis of an activation or equilibrium free energy difference for an isolated elementary process assumed in previous studies.\(^9,12\)

In the next section, we determine the magnitude of the rate constants for peptide reorganization, deorganization and desorption through a fit to experimental data for the rate of peptide deposition measured by Maggio and coworkers.\(^13\)
loading time

existing deposits and allowed to age on the deposit for a radioactively labeled amyloid peptide was “loaded” onto deposition on existing amyloid deposits. A solution of experiments that explore the rate of amyloid peptide fibril the deposit was washed as a function of the fraction of peptide that remained on the deposit after the quantity of peptide deposited in a given time, and (2) the elongation rate (with a slope $1.89 \times 10^{-3}$).

where $V$ is the volume of the reaction solution taken to be $1 \times 10^{-4}$ L (J.E. Maggio, private communication). The fit to the initial rise provides a slope of $0.0159 \times 10^{-7}$ L/s. Given that the solution concentration of monomeric peptide was 100 pM = $1 \times 10^{-10}$ M, we find that $k_a[P]=1.59 \times 10^{-7}$ L/s.

Fig. 6. Absolute quantity of amyloid peptide deposited from solution on synthetic amyloid fibrils is shown as a function of time. The experimental data of Maggio and coworkers $^{13}$ is shown with the theoretical fit to the initial rise (with a slope $1.59 \times 10^{-2}$ fmol/min) and long-time steady-state elongation rate (with a slope $1.89 \times 10^{-3}$ fmol/min).
deposition, initially \( A(\tau_L) \), is converted to trapped \( \beta \) peptide, initially \( B(\tau_L) \).

In the analysis of their experimental data, Maggio and coworkers identify a “fast” component of peptide that leaves the deposit relatively quickly and a “slow” component of peptide that leaves the deposit slowly or not at all. The “fast” component is essentially the loosely held “\( \beta \)” peptide of the amorphous deposit with an initial fraction \( A(\tau_L) \). However, no distinction was made regarding the peptide at the reaction interface and the peptide trapped in the deposit that we assign to initial fractions \( B(\tau_L) \) and \( C(\tau_L) \), respectively. Both fractions were said to constitute the “slow” component. Our analysis makes it clear that the fraction \( B(\tau_L) \) of peptide at the reaction interface is independent of the loading time and the conversion of “fast” to “slow” components is a conversion of \( A(\tau_L) \) to \( C(\tau_L) \) that occurs as the reaction interface moves outward in the process of fibril elongation.

We find that a best fit is obtained with \( k_d = 6.7 \times 10^{-3} \) min\(^{-1}\) and \( k_{d'} = 8.5 \times 10^{-4} \) min\(^{-1}\) and the coefficients showing in Table I.

These values of the rates for dissociation and deorganization can be compared with the time constants for the biexponential fits of Maggio and coworkers, using a functional form \( Y = A \exp(-k_d t) + (1 - A) \exp(-k_{d'} t) + C/(1 + C) \) and values of \( t_{1/2} = \ln(2)/k_1 = 10.4 \pm 2.1 \) min or \( k_1 = 6.6 \times 10^{-2} \) min\(^{-1}\) and “the half time for the second (slower) dissociation process was at least 100-fold longer”\(^{13}\).

**Fitting the Rate of Reorganization \( k_r \)**

The reorganization of the deposited peptide is the rate-limiting step in the conversion of the “\( \beta \)” to \( \beta \) peptide form. Figure 8 shows the relative fractions of deposited peptide as a function of the loading time \( \tau_L \). We expect that the rate of interconversion of the fraction \( A(\tau_L) \) to \( C(\tau_L) \) of “\( \beta \)” to \( \beta \) will vary as the sum of the rate of reorganization and deorganization \( k_r + k_{d'} \).

In the experiment, what is measured is the reorganization of the newly deposited peptide. Peptide that was part of the deposit before the loading began is not labeled and not counted in the statistics. The first step is to populate the fibril with labeled peptide. Once a site is covered with labeled peptide, there is a chance that the amorphously deposited peptide “\( \beta \)” will convert to the well-formed \( \beta \). That occurs with a probability

\[
\text{Probability of reorganization} = e^{-k_r + k_{d'} \tau_L} \tag{21}
\]

\( ^{13} \)Fitted values of coefficients \( A, B, \) and \( C \) as a function of the loading time \( \tau_L \).

<table>
<thead>
<tr>
<th>Load time (min)</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>68.2</td>
<td>6.7</td>
<td>25.1</td>
</tr>
<tr>
<td>30</td>
<td>53.0</td>
<td>11.7</td>
<td>35.3</td>
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<tr>
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<td>21.4</td>
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<td>27.9</td>
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<td>9.8</td>
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</tr>
<tr>
<td>720</td>
<td>0.3</td>
<td>73.9</td>
<td>25.8</td>
</tr>
</tbody>
</table>

We expect that \( k_r \gg k_{d'} \), so that the observed rate of propagation of the reaction interface can be taken to be the rate of reorganization \( k_r \). This allows us to fit the fraction of \( \beta \) (fast) and “\( \beta \)” (slow) peptide, using the approximation

\[
A(\tau_L) = A(0) e^{-k_r \tau_L} \tag{22}
\]

and

\[
C(\tau_L) = C(0) [1 - e^{-k_r \tau_L}] \tag{23}
\]

These general fitting functions provide us with an estimate of the rate of reorganization as well as an estimate of the diffusional association rate. The fit based on the value \( k_r = 7.3 \times 10^{-3} \) min\(^{-1}\) is shown in Figure 8.

While the single exponential fit captures the essential features of the loading time dependence of the relative fractions of deposited peptide, a more accurate fit can be obtained using a biexponential or stretched exponential time-dependent model. For example, using a biexponential time dependence, the relative fractions of \( A, B, \) and \( C \) peptide are fitted to \( A(\tau_L) = 35.7 e^{(-2.9 \times 10^{-2} \text{ min}^{-1} \tau_L)} + 39.1 e^{(-3.9 \times 10^{-3} \text{ min}^{-1} \tau_L)} \), \( B(\tau_L) = 29.2 \) and \( C(\tau_L) = 100 - A(\tau_L) - B(\tau_L) \) also shown in Figure 8. The stretched exponential fit of the form \( A(\tau_L) = 82.0 e^{(-0.068 \text{ min}^{-1} \tau_L^{0.59})} \), where \( \gamma = 0.59 \) is similar in appearance. The goodness of the biexponential or the stretched exponential fits to \( A(\tau_L) \) may be taken to indicate the presence of a distribution of barriers for the conversion of “\( \beta \)” to \( \beta \)-peptide.

Using the fits to the relative fractions of peptides we can compute the absolute quantities of peptide in the “\( \beta \)” phase within the deposit (A), the “\( \beta \)” phase at the reaction interface (B), and the \( \beta \) phase of the well-formed fibril away from the interface (C). The results are shown in Figure 9. Initially, there is an increase in the concentration of peptide in the amorphous deposit and at the
reaction interface. The rate of increase is determined by the rate of peptide diffusion to the fibril ends. Subsequently, there is an increase in the quantity of peptide in the fibril—the “irreversibly” formed fibril. The rate of increase is determined by the rate of peptide reorganization at the reaction interface. The sum total of the three components is the total quantity of peptide added to the fibril as a function of time (Fig. 6).

Another test of this mechanism is to predict the rate of increase of $\beta$-fibril shown in Figure 6 by directly solving the differential rate laws of the $\beta$-amyloid landscape mechanism proposed here for $\beta(t)$. The results based on the rate constants $k_{aP}$, $k_{d}$, $k_{d'}$ and time-independent $k_\alpha = 0.0073 \text{ min}^{-1}$ are shown in the upper panel of Figure 10. Overall, the general features of the increase in deposited peptide are captured by the proposed $\beta$-amyloid landscape mechanism ($\beta$ALM). While the values of the elementary rate constants should be somewhat independent of the concentration of monomeric peptide in solution, the kinetics of peptide deposition is more complex at high peptide concentrations (Fig. 2). At higher concentrations, it is expected that the rate of reorganization of deposited peptide will depend on the surrounding deposit with the rate of reorganization decreasing as the steady-state thickness of the deposit is increased. In the lower panel of Figure 10, the kinetics resulting from the use of a time dependent $k_{aP}(t) = [0.0185 \exp(-k_{aP}[P]t) + 0.0069] \text{ min}^{-1}$ where $k_{aP}[P] = 0.00556 \text{ min}^{-1}$ is shown. Note that for the rate of decrease in the rate we have used the rate of peptide association from solution. The fit is in excellent agreement with the experimentally measured rate of increase in the deposited peptide.13

In the next section, we suggest forms for the absolute rate constants for bimolecular reaction/diffusion and unimolecular conformational reorganization that appear in the elementary steps of the energy landscape mechanism.

**ABSOLUTE REACTION RATE CONSTANTS FOR REACTION/DIFFUSION AND UNIMOLECULAR CONFORMATIONAL TRANSITION**

The rate constants that appear in the proposed two channel mechanism can be modeled as a “bimolecular” reaction/diffusion of peptide and deposit and “unimolecular” reorganization of the peptide and peptide/deposit. Specific forms of the elementary rate constants are developed in this discussion.

**Peptide Reaction/Diffusion in Deposition from Solution**

The Smoluchowski rate theory provides an estimate of the equilibrium flux for the formation of contact between two species $P$ and $m$ of radii $\sigma_p$ and $\sigma_m$ diffusing in solution with a relative diffusion coefficient $D = D_P + D_m$. The rate constant for association is simply

$$k_{aP} = 4\pi D\sigma_\alpha$$

(24)

where $\sigma = \sigma_p + \sigma_m$ is the contact radius. These features are summarized in Figure 11. The parameter $\alpha$ is a measure of the probability that on contact the peptide deposits on the fibril. It has been shown that the monomer of the amyloid peptide congener in aqueous solution has a large hydrophobic surface area $A_{2P}$.16,17 If we take the peptide to have a total surface area $4\pi\sigma_m^2$, we expect that the probability that on approach the peptide will be
We can make a quick estimate of the number of fibril ends readily accessible to the diffusing monomers. Suppose that the association process has a rate constant is 

\[ k_a = 4\pi D \sigma \exp(-G^*/RT) \]

The exposed hydrophobic patch has been estimated to be on the order of 600–1000 Å² for the peptide of total solvent exposed surface area of roughly 3600 Å². \(^\text{15}\) That leads to a value of \( \alpha = 0.2 \). Using a value of \( D = 1.4 \times 10^{-6} \text{ cm}^2/\text{s} \), \(^\text{15}\) a reaction diameter of \( \sigma = (\sigma_p + \sigma_m) = 2 \times 10^{-7} \text{ cm} \) leads to an estimate of 

\[ k_a [P] = 4.2 \times 10^8 \exp(-G^*/RT) \text{ L/(mol s)} [P]. \]

Our estimate, then, is that

\[ \exp(-G^*/RT)[P] = 7.2 \times 10^{-9} \]

“Unimolecular” Peptide and Peptide/Fibril Conformational Transitions

The “reaction coordinate” for the reorganization of peptide/fibril may be defined in terms of the peptide reorganization alone, fibril reorganization alone, or a combination of peptide/fibril reorganization. The standard theory for describing such an event is the transition state theory. \(^\text{17–19}\)

The transition state theory estimate of the rate constant can be written

\[ k_{TST} = \nu \exp(-G^*/RT) \]

where the free energy of activation \( G^* \) determines the statistical probability of accessing the transition state conformation, and the prefactor is a measure of the equilibrium flux of activated reactants across the transition state.

Transition state theory assumes that every reactant that accesses the transition state will necessarily react and form product. It may be that the system dynamics will cause the activated reactant to undergo (1) inertial oscillation back to reactant or (2) collisions that return the activated state to a reactant state. In either case, the actual rate constant will be 

\[ k = \kappa k_{TST} \]

where \( 0 \leq \kappa \leq 1 \) is the transmission coefficient, which accounts for those dynamic recrossings. Dynamic recrossings only serve to lower the reaction rate making the \( TST \) rate constant an upper bound to the true reaction rate constant (see below).

A van’t Hoff analysis of the individual steps of association and reorganization is necessary to determine the contributions of activation enthalpy and entropy to the peptide/fibril conformational transition. In our proposed mechanism, the overall rate constant for fibril elongation is a composite of the elementary rate constants for peptide/fibril association and reorganization. A van’t Hoff analysis of the temperature dependence of the overall rate constant will not, in general, provide the activation enthalpy for the elementary molecular processes alone. However, a van’t Hoff analysis of the individual steps observed in the experiments of Maggio and coworkers \(^\text{13}\) will make such an assignment possible.

**COMPARISON WITH EXPERIMENT: EFFECT OF MUTATION AND CROSSLINKING ON FIBRIL ELONGATION**

The proposed energy landscape model of fibrillogenesis can be used to organize a variety of experimental observa-
tions of the rate of elongation and growth of amyloid fibrils for a variety of modified forms of the amyloid β peptide and a congener. In this discussion we focus on experiments on the Aβ(1–40) peptide and a truncated congener Aβ(10–35)—NH2, which has been shown to be a good model of the full Aβ peptide in plaque competence and deposition assays.16,21

**Wild-Type Congener Aβ(10–35)—NH2**

The WT peptide congener has been shown to exist in a loosely formed collapsed coil state in aqueous solution.21,22 The structure of the collapsed coil is characterized by a central hydrophobic cluster in the LVFFA (17–21) region. There is also a dominant turn in the VGSN (24–27) region, which is observed in both the aqueous solution structure15 and the TFE–water solution structure,22 also studied in dimethylsulfoxide (DMSO), which shows two short α-helical regions. Analysis of the exposed hydrophobic surface area of the collapsed coil structure14 shows that the peptide presents a large fraction of hydrophobic surface.

In the model above, the large hydrophobic surface increases the value of the equilibrium constant $k_a/k_d$ for the adsorption/desorption of the peptide monomer on the amyloid deposit. The collapsed coil structure is also expected to be a low-lying intermediate with a reduced free energy barrier to adsorption relative to the unstructured coil state (Fig. 3).

**Wild-Type Aβ(1–42) and Aβ(1–40) Peptides**

It has been shown that the Aβ(1–40) peptide deposits more slowly than the Aβ(1–42) peptide. This points to the importance of the C-terminus in affecting the formation of seeds by the amyloid peptide.23 The rate of deposition of the peptide is approximately a factor of two larger in the case of the Aβ(1–42) relative to the Aβ(1–40).

In the energy landscape mechanism, it is unclear whether the change in the C-terminus affects the rate of association $k_a$ through a change in the solution structure of the peptide or a change in the rate of reorganization $k_r$ of the deposited peptide, or both.

**Wild-Type D- and L-Aβ(1–40) Peptides**

Studies of the D- and L-stereoisomers of the Aβ peptide have shown stereospecificity in the fibril elongation process.26 It was shown that the L-Aβ(1–40) peptide deposited on L-Aβ(1–40) peptide templates with a first-order kinetic dependence on the concentration of peptide monomer in solution. It was also shown that D-Aβ(1–40) peptide would deposit onto the D-Aβ(1–40) peptide template according to a rate that was first order in the peptide concentration. However, no fibril elongation of L-Aβ(1–40) peptide on D-Aβ(1–40) peptide template, or D-Aβ(1–40) peptide on L-Aβ(1–40) peptide template, was observed. These experiments demonstrate that the elongation process is similar in both peptide enantiomers, as long as the depositing peptide and the existing template are of similar chirality. When the chiralities differ, the deposition does not occur.

In the energy landscape theory proposed here, we expect that the association process and $k_a$ would be largely unaffected by the change in chirality of the peptide. This would be true if we assumed that the solution conformation of the peptide enantiomers were similar in size and exposure of hydrophobic surface. However, the reorganization process could be quite different if the chirality of the depositing peptide and existing template differed. In that case, the conformations required to make a well-formed deposit may not be accessible to the peptide rendering $k_r$ effectively equal to zero.

**Less Restrained Dutch Mutant Aβ(10–35)—NH2—E22Q**

Experimental analysis of the Dutch mutant of the WT amyloid peptide has shown the mutant to be significantly more active than the WT peptide with a twofold increase in the rate of fibril elongation and deposition competence.13 Experimentally determined H$_\alpha$ chemical shifts in the WT and Dutch mutant indicate that the structures of the monomeric peptides in solution are similar.13 The increased deposition rate observed for the Dutch mutant has been explained in terms of a more disordered solution state relative to the WT peptide.13 The looser structure is believed to lower the entropic barrier for “opening” of the peptide, which is necessary in the reorganization process.

In the energy landscape mechanism proposed in this discussion, we would expect to see in the E22Q mutant an enhanced value of $k_a$, as a result of the replacement of the charged glutamate residue with a polar glutamate residue lowering the desolvation barrier in adsorption. The greater flexibility in the peptide may also lead to a reduced barrier to reorganization and a larger $k_r$, producing a greater rate of fibril elongation.

**More Restrained Cyclic Crosslinked Aβ(10–35)—NH2—CycloH14K–E22 Peptide**

Experimentally measured H$_\alpha$ chemical shifts in the cyclic peptide indicate that the structure of the monomeric peptide is similar to the structure of the WT peptide.36 Nevertheless, the cyclic mutant peptide is found to be inactive in deposition. This has been interpreted as a demonstration that the peptide must be allowed to access an “open” or extended conformation in order to add to a well-formed amyloid deposit.

In the mechanism described above, we would expect that if the exposed hydrophobic surface area is similar in the cyclic peptide and the WT peptide, the cyclic peptide would adsorb on the fibril. However, the peptide would be unable to reorganize to access conformations consistent with a well-formed amyloid deposit. Therefore, we might expect the value of $k_a$ to be similar for the WT and cyclic peptides while the values of $k_r$ and $k_r$ would be essentially zero for the cyclic peptide. The cyclic peptide would be capable of adsorption on the existing amyloid deposit but would be easily washed off of the deposit as the peptide/fibril would not be able to reorganize to a conformation consistent with the well-formed β fibril.
**Disrupted Central Hydrophobic Cluster of Aβ(10–35)-NH₂-F19T**

NMR structural analysis of the F19T mutant of the amyloid peptide congener in aqueous solution indicates that there is a serious disruption of peptide structure in the central hydrophobic cluster (CHC) region of the mutant peptide. This disruption of the CHC is correlated with a diminished ability of the peptide to add to well-formed amyloid deposits.

In both the F19T mutant and the E22Q Dutch mutant the amyloid peptide monomer in solution is found to be less constrained in the coil state. However, in the case of the E22Q Dutch mutant and the WT peptide, the structure of the CHC is preserved.

In the mechanism proposed here, we would find that in the F19T mutant the ability of the peptide to adsorb would be significantly diminished due to a reduced hydrophobic surface area $A_H$ and adsorption rate constant $k_a$.

**DEPENDENCE OF THE ELONGATION RATE CONSTANT $K_e$ ON DENATURANT CONCENTRATION, TEMPERATURE, AND SOLVENT VISCOSITY**

The proposed mechanism makes predictions of a specific temperature dependence in the rate of fibril elongation.

**Turnover in Rate of Fibril Elongation With Increasing Denaturant Concentration**

Experiments have shown that the addition of denaturant can increase the rate of protein folding by reducing the time spent in misfolded intermediate states. The addition of denaturant is expected to impact two elementary steps in the energy landscape mechanism. The denaturant will favor a less structured collapsed coil state. This may decrease the diffusion constant for the peptide monomer. It is also expected that the less structured peptide will be able to reorganize more readily lowering the barrier to reorganization and increasing the rate constant for reorganization. If that is the dominant effect, the rate of reorganization and the addition of denaturant should increase the rate of fibril elongation.

At high enough concentrations of denaturant the structure of the collapsed coil state will be severely destabilized and the monomers will be predominantly unstructured coil states. This should lead to a decrease in the diffusion constant for the peptide monomer and a decrease in the rate of adsorption.

Overall, we expect that as the concentration of denaturant is increased there will be a “turnover” in the rate of elongation $k_e$. This effect is shown schematically in Figure 12.

In earlier experiments of Maggio and coworkers preliminary evidence for such a turnover was reported in the rate of amyloid peptide deposition as a function of urea denaturant concentration.

**Turnover in Rate of Fibril Elongation With Increasing Temperature**

At lower temperatures, experiments have found the collapsed coil states to be stable relative to largely random coil states. In the 5–35°C range, we expect that a temperature increase will increase the rates of association and reorganization due to an increase in the probability of being found in an activated state in the reaction/diffusion or reorganization process. Increasing temperature should also “loosen” the collapsed coil state and, at lower temperatures, increase $k_e$.

At higher temperatures the collapsed coil state will become destabilized relative to the random coil states. This will eventually deteriorate the ability of the peptide to associate with the amyloid deposit (an effect similar to that induced by high concentration of denaturant). The result should be a decrease in the rate of association and lead to an overall decrease in $k_e$ with increasing temperature.

Overall, with increasing temperature we expect to see an initial increase in the rate of amyloid deposition and fibril elongation followed by a “turnover” and decrease at high temperatures.

**Rate of Fibril Elongation Decreases With Increasing Solvent Viscosity**

For the energy landscape mechanism proposed, the rate constants for reaction/diffusion and reorganization can be parameterized through experiment and simulation to account for the various elementary reaction processes described in the two channel mechanism for amyloid fibril elongation. The rate constant $k_e$ is expected to be a strong function of the solvent viscosity. An increase in solvent viscosity $\eta$ will decrease the diffusion constant $D = RT/\gamma$ through an increase in the friction $\gamma = 6\pi n r_H$, where the peptide hydrodynamic radius is $r_H$. The rate of peptide
reorganization is expected to be a comparatively weak function of the solvent viscosity. Therefore, we expect the elongation rate \( k_e \) to be a monotonically decreasing function of the solvent viscosity.

**DISCUSSION OF PROTEIN FOLDING AND MISFOLDING AND THE RELATION TO \( \beta \) PEPTIDE DEPOSITION**

One view of amyloid peptide fibril elongation suggests that the solution phase of the amyloid peptide monomer is largely unstructured and samples a large number of disordered coil states on the time scale of fibril elongation. Fibril elongation follows a mechanism of peptide adsorption and subsequent peptide/fibril reorganization. Another view suggests that the conformation of the peptide monomer in solution is a crucial determinant of the rate of fibril elongation. The latter view has led to suggestions that theories of protein folding may elucidate aspects of the mechanism for fibril elongation.14,30

What can be learned from the analysis of amyloid peptide fibrillization as a protein folding problem? Modern theories of protein folding have been successful in organizing a significant volume of experimental data for folding stability analysis and kinetics. In the best case, they have not only provided a means of “organizing one’s thinking” about the problem, but a framework for predicting unobserved behavior and suggesting novel experiments as well.28,31–35 An obvious and suggestive isomorphism exists between the processes of protein folding and amyloid fibril elongation.

The kinetic process by which the protein arrives at the native state may follow one of two mechanisms. Both mechanisms follow a three-stage pathway of (1) collapse from a largely unstructured coil state to a compact state, (2) search through a set of compact states for a transition state, and (3) folding to the native state. The process is depicted for the case of the amyloid peptide in Figure 3, where it is imagined that the compact state is represented by a collapsed coil state and the “native” state of the protein corresponds to the peptide in a well formed amyloid deposit.

In the kinetic partitioning mechanism (KPM) of protein folding33 proposed by Thirumalai and coworkers,31 it is assumed that a certain fraction of proteins follows a fast folding process through a nucleation-collapse mechanism, while the remaining fraction folds to a non-native compact intermediate or misfolded state and subsequently overcomes an energetic barrier to “reopen” and fold to the native state. The energy landscape mechanism for amyloid peptide deposition proposed can be considered isomorphic with this KPM.

**“Fast Folding” and “Fast Deposition”**

In the kinetic nucleation–collapse process, the protein searches for a set of critical contacts and, upon forming that critical “nucleus,” folds rapidly to the native state. In the kinetics of amyloid peptide deposition, it has been proposed that an activated or “transition state” conformation of the \( \beta \) peptide monomer can be found that is rapidly and well deposited on an existing fibril. That process of “fast deposition” corresponds to the direct nucleation mechanism by which a protein may access its native state.

**Misfolded Intermediates and Poorly Formed Amyloid Deposits**

A fraction of proteins do not follow an efficient nucleation-collapse mechanism but instead initially misfold into a low energy but non-native compact state. These proteins undergo an activated transition to escape the misfolded state and refold to the native conformation. In the energy landscape mechanism for fibril elongation proposed, the process of deposition of peptide in a nonoptimal conformation to form a poorly ordered or “amorphous” deposit corresponds to such a misfolded but low-energy compact state. Subsequently, the peptide/fibril must undergo a reorganization reaction which corresponds, in this isomorphism, with the activated transition from misfolded intermediate to native state protein.

**SUMMARY**

The proposed energy landscape mechanism for amyloid peptide deposition and \( \beta \)-amyloid fibril growth rests on recent advances in the theory of protein folding and carefully devised laboratory experiments measuring the rate of amyloid fibril elongation. The mechanism incorporates several possible channels for peptide deposition, including (1) fast deposition from solution through an activation/nucleation event, and (2) deposition of peptide from solution onto existing fibrils followed by reorganization of the peptide/fibril deposit. As such, it unifies several views of \( \beta \) peptide deposition and fibril elongation.

The mechanism is consistent with a substantial body of experimental data for the rate of fibril elongation for WT \( \beta \) peptides, \( \beta \) peptide congeners, and mutant \( \beta \) peptide congeners. Moreover, it allows for the clear definitions of equilibrium and kinetic constants, in terms of the energetics of elementary processes of peptide absorption and reorganization, that should be valuable in the interpretation of experimental data for fibril reorganization. The existence of a turnover in the rate of peptide deposition as a function of denaturant concentration or temperature are predicted and may be tested experimentally.

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12. Kusumoto Y, Lomakin A, Teplow DB, Bendek GB. Temperature dependence of amyloid β-protein fibrillization. Proc Natl Acad Sci USA 1998;95:12277–12282. In that article, the diffusion constant is stated to be 1.6 × 10⁻⁷ cm²/s, whereas more recent estimates set it to be an order of magnitude larger 1.4 × 10⁻⁶ cm²/s. With that revised estimate of the diffusion constant, the revised activation energy is Q* = 5.7 kcal/mol. However, the central conclusion—that the activation entropy is positive—remains unchanged.