Optimal Admission Control of Secondary Users in Preemptive Cognitive Radio Networks

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***Abstract*—We study optimal admission control of secondary users (SUs) in cognitive radio (CR) networks in presence of preemption. In this model, when a primary user (PU) arrives to the system and finds all the channels busy, it preempts an SU unless all the users in the system are PUs. We apply admission control on the SUs only. Using dynamic programming (DP), we find the optimal admission control policy that maximizes the long-run average profit. As our main contribution, we show that the optimal admission control of the SUs depends only on the total number of users in the system (i.e. it does not depend on the number of PUs and SUs in the system individually) and is of threshold type. Therefore, although the system is modeled as a two-dimensional Markov chain, our findings allow simple and efficient computation of the optimal control policy.**

I. Introduction

Commercially available wireless spectrum has become drastically scarce because of the increasing use of wireless devices, such as smartphones and tablets. In conventional spectrum management, resources are reserved primarily to licensed users. However, latest studies show that the spectrum allocated to license holders is often underutilized in space and time [1].

Cognitive radio (CR) technologies enable smart use of the spectrum through opportunistic spectrum hand-off and secondary market usage to improve spectrum utilization. In CR systems, there are two classes of users: primary users (PU) which have permanent license to access the spectrum and secondary users (SU) which are temporarily allowed into the system whenever the system is underutilized. In general, the PUs are long term contract users and SUs lease excess spectrum when the system allows them [2]. A service provider (SP) serving both PUs and SUs must maximize its revenue by attracting the greatest number of SUs while the performance perceived by PUs should not be affected by the admission of SUs.

In order to emphasize the importance of PUs, we consider the PUs as the *higher priority users* whereas the SUs are the *lower priority users*. We use a mechanism that allows SUs in the system when there is capacity and aborts the service of an SU whenever a PU needs service. For instance, FCC Block D at 700 MHz employs such a termination model where public safety services are the PUs and commercial services are the SUs [3]. For every evicted SU, the SP has to pay a certain amount of punishment. With every successfully serviced SU, the SP gains a certain amount of reward. This ensures that PUs have access to spectrum without being burdened by SUs and SUs have access to excess spectrum when available [2]. We study the optimal admission control policy of SUs that maximizes the average profit.

We formulate the problem as a two-dimensional (2D) Markov decision process (MDP) optimization problem. Our main contribution is to show that the optimal admission control policy is of threshold type. Similar to the well-studied admission control problem where both PUs and SUs are blocked when the system is full [4], [5], the optimal threshold for our system depends only on the total number of SUs and PUs in the system. In turn, we can avoid the computational complexity introduced by dynamic programming (DP) while determining the optimal admission control policy.

This conclusion is non-trivial despite the extensive literature on optimality of threshold admission policies for *non-preemptive* loss systems (see, for example, [4] for a seminal paper on this issue). Conclusions concerning non-preemptive systems do not immediately extend to preemptive systems because in a non-preemptive system users cannot be evicted once they are admitted, and in turn, due to identical holding time statistics, users are indistinguishable once they are in the system. In contrast, a preemptive system

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entails an asymmetrical situation since admission of a PU when the system is full depends on the presence of SUs in the system at the time of its arrival. Hence it is not clear from the outset whether analysis of optimal profit requires consideration of a multi-dimensional model, at the expense of considerable computational complexity.

Interestingly, although optimal admission control policies have the same form for non-preemptive and preemptive systems with memoryless call durations, differences emerge when this latter condition is relaxed: On the one hand, it is known that for a two-class non-preemptive system with general call length distribution, the optimal occupancy-based policy, i.e. the policy that depends only on the number of users of *each* class, is of threshold type [6]. On the other hand, we show here by way of an example that for preemptive systems threshold type policies are not necessarily optimal even within occupancy-based policies under general call durations.

The rest of the paper is organized as follows: Section II presents the related work on admission control and preemption. We describe our system model in Section III. The model analysis and characterization of admission control, and performance analysis is given in Section IV. Furthermore, some numerical results are included in Section V. We discuss general call length distributions in Section VI and Section VII concludes the paper.

II. Related Work

The related work can be classified in two categories: dynamic control of queueing systems and preemption.

Admission control of queueing systems has been widely studied. Earlier work includes various seminal papers such as Miller [4] and Ramjee et al. [5], which consider a multi-class and multi-server queueing system that uses admission control to maximize the expected average return. Recent studies include the work of Gans and Savin [7]. They characterize the optimal pricing policies in a system that consists of two types of users. None of the mentioned work considers preemption.

One of the earliest works on preemption is the work of Helly [8], which proposes two approaches on the control of two-class traffic with different priorities and limited number of servers. Garay and Gopal [9] use preemption as a control mechanism in networks and outline the optimal preemption decision for greater revenue. Xu and Shanthikumar [10] present some results on optimal admission control by using duality of two systems one of which employs preemption. Brouns and van der Wal [11] study a single server queue that consists of two different classes of users. They model the system as a 2D MDP and investigate the optimal termination and admission controls that maximize the expected revenue. Brouns [12] extends these results to the multi-server case, where there are only queueing costs. In contrary to our system, their system does not have preemption cost per user. Thereafter, Zhao et al. [13] investigate the effects of preemption and admission control mechanisms on the long-run average return in a two-class loss network. They state that preemption is optimal only when a link is full.

Thereafter, Zhao et al. [14] utilize preemption to provide differentiated services to various classes in a parallel loss network. They compute the preemption probability and the preemption rate for each class analytically but they do not consider any type of admission control.

Ulukus et al. [15] have the closest model to our system. Similar to [13], they prove that preemption is optimal only when the system is full and they characterize the optimal termination and admission policies as state dependent threshold policies. The difference with our system is that they consider a general case where classes have different service rates. Hence, they cannot deduce a threshold that depends on the total number of users in the system.

III. Model Description

The model we consider is a two-class $M/M/C/C$ queue. There are C identical channels. We consider two classes of users: the PUs and the SUs. We assume that all users request the same amount of bandwidth corresponding to a single channel. Arrivals of each class are independent Poisson processes with rate $\lambda_1 > 0$ for PUs and $\lambda_2 > 0$ for SUs.

We introduce the concept of *preemption*, which is a widely used priority queueing discipline when there is competition for limited available resources. In a preemptive system, there are several classes of users with different priorities. A user enters service even if another user with lower priority is already in service. In our system, PUs have preemptive priority over SUs. Thus, an SU can be removed from the system if all channels are busy upon a PU arrival. When an SU is preempted, it is withdrawn from the system permanently.

The call durations are independent and exponentially distributed with mean μ^{-1} unless terminated prematurely. With Poisson arrivals, this model is

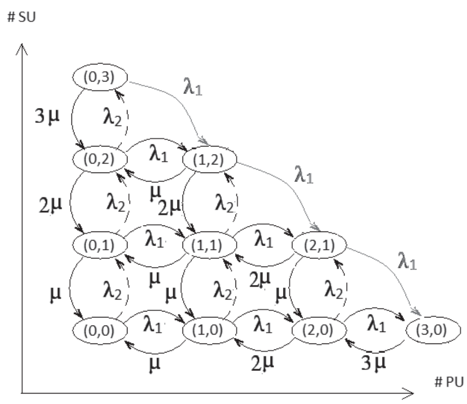


Figure 1: State diagram of 2D Markov chain for $C = 3$. Depending on the admission control, the dashed transitions with rate λ_2 upon SU arrivals may exist or not.

consistent with the observations obtained from real data traces [16]. Knowing that the PU and SU call lengths are exponentially distributed and the service of an SU may be interrupted at any time instant, the system behaves as a 2D continuous-time MDP. An example state transition diagram for 2D MDP with $C = 3$ is given in Fig. 1. The system description is as follows:

States: The state of the system is in the form of a tuple (x, y) where $x \geq 0$ represents the number of PUs in the system and $y \geq 0$ is the number of SUs in the system.

Decisions: The only decision control is admission control on the SUs. Upon an SU arrival, the system either accepts or rejects the user. PUs are always admitted if there are less than C PUs in the system.

Rewards and punishments: $r > 0$ is the reward collected from an SU per unit time. Then, $R = r/\mu$ is the average reward per SU, which is collected after an SU leaves the system with successful completion of service. Rejecting SUs and PUs upon arrival is free of charge. The system preempts an SU whenever a PU arrives and finds all the channels busy. Employing preemption is optimal only when all C channels are occupied [13], [15]. For every preempted SU, the SP pays a punishment $K > 0$.

Discounting: By discounting at a rate $\alpha \geq 0$, we ensure that the rewards and costs at time t are scaled by a factor of $exp(-\alpha t)$. Namely, the reward gained in the present is more valuable than the reward gained in the future [17].

Uniformization: The process we have is a continuous-time Markov chain. We can develop the discrete-time equivalent of this system using a uniformization technique [18]. Without loss of generality, we set the maximum possible rate out of a state to 1 (i.e. $\lambda_1 + \lambda_2 + C\mu + \alpha = 1$). Hence, a PU (SU) arrival occurs with probability $\lambda_1(\lambda_2)$, a PU (SU) departure occurs with probability $x\mu$ $(y\mu)$, the process terminates with probability α and the system stays at the same state with probability $(1 - \lambda_1 - \lambda_2 - x\mu - y\mu - \alpha) = (C - x - y)\mu$.

Criterion: The objective is to maximize the total expected long-run average profit gained from SUs.

IV. Model Analysis and Characterization of Admission Control

In this section, we formulate the average profit rate of the system and determine a method to maximize it through the optimal admission control of SUs. We define S as the state space for all states given that there are C channels, that is:

$$S = \{(x, y) \mid x + y \leq C, \; \forall x, y \geq 0\}.$$

Let $S_1 \subset S$ be the space of preemptive states. Formally:

$$S_1 = \{(x, y) \mid x + y = C, \; y \; \geq 1\}.$$

We define $\pi_p(x, y)$ as the steady state probability that the system is in state (x, y) under policy p. In this scheme, J_p the average profit rate under policy p is as follows:

$$J_p = R \sum_{(x,y)\in S} y\mu\pi_p(x, y) - K\lambda_1 \sum_{(x,y)\in S_1} \pi_p(x, y). \quad (1)$$

The first term in Eq. (1) corresponds to the total average revenue rate collected from SUs and the second term is the total average cost due to the preempted SUs. Their difference yields the total average profit rate.

In order to find the optimal admission control policy p^* of SUs that maximizes the average profit rate J_p in Eq. (1) and yields J^*, the problem can be formulated as a *stochastic dynamic programming (DP) problem* [17].

A. Dynamic Programming Formulation

In this section, we formulate the problem and provide a solution using DP for finite horizon. The time index n is the observation points left until the end of the horizon. $V_n(x, y)$ is the maximal expected discounted profit for the system in state (x, y) at time period $n \geq 0$. The corresponding DP equations are as follows:

For $n = 0$:
$V_0(x, y) = 0 \quad \text{for} \quad x, y \geq 0$
For $n \geq 1$**:**
• $x + y < C$

$$\begin{aligned} V_n(x, y) = {} & \lambda_1 V_{n-1}(x+1, y) \\ & + \lambda_2 \max\{V_{n-1}(x, y), V_{n-1}(x, y+1)\} \\ & + x\mu V_{n-1}(x-1, y) \\ & + y\mu[V_{n-1}(x, y-1) + R] \\ & + (C - x - y)\mu V_{n-1}(x, y) \end{aligned} \tag{2}$$

• $x = C$ and $y = 0$

$$\begin{aligned} V_n(x, y) = {} & \lambda_1 V_{n-1}(x, y) \\ & + \lambda_2 V_{n-1}(x, y) \\ & + C\mu V_{n-1}(x-1, y) \end{aligned}$$

• $x \neq C$ and $x + y = C$

$$\begin{aligned} V_n(x, y) = {} & \lambda_1[V_{n-1}(x+1, y-1) - K] \\ & + \lambda_2 V_{n-1}(x, y) \\ & + x\mu V_{n-1}(x-1, y) \\ & + y\mu[V_{n-1}(x, y-1) + R]. \end{aligned}$$

We set $V_n(-1, y) = V_n(0, y)$ and $V_n(x, -1) = V_n(x, 0)$ when required.

Note that there are three different equations in the DP formulation. The first DP equation is for the case when there are idle channels in the system for the use of PUs and SUs. The second equation is for the case when all channels are used by PUs and there are no SUs in the system. Lastly, the third equation is for the preemption case, i.e. all channels are busy and there exists at least one SU in the system that can be preempted by a PU arrival.

B. Characteristics of the Optimal Policy

In this subsection, we characterize the optimal admission control policy of SUs.

Theorem 1. *The optimal admission control policy p^* of SUs is of threshold type and it depends only on the total number of SUs and PUs in the system. Thus, there exists an optimal threshold T^* such that if an SU arrival finds the system in state (x, y) and if $x + y < T^*$, the SU is accepted. Otherwise, the SU is rejected.*

We first prove certain monotonicity properties of the system in the following lemmas. Note that full proofs can be found in [19].

The following lemma sets a lower bound on the value of an additional SU call. The proof technique is similar to the proof of Lemma 1 in [15], in which the authors study non-identical service rates.

Lemma 1. *For all (x, y) with $x + y + 1 \leq C$ and $\forall n \geq 0$:*

$$V_n(x, y+1) - V_n(x, y) \geq -K. \tag{3}$$

Our proof techniques in the next two lemmas follow similar lines as [11], [12]. However, our proofs are simpler as we avoid the use of sample path arguments. They are proven by using an induction argument on n.

Lemma 2 declares that the value of an additional SU call is non-increasing in the number of PUs for fixed number of SUs in the system.

Lemma 2. *For all (x, y) with $x + y + 2 \leq C$ and $\forall n \geq 0$:*

$$V_n(x, y+1) - V_n(x, y) \geq V_n(x+1, y+1) - V_n(x+1, y). \tag{4}$$

Lemma 3 states that values of an additional SU at two neighbor states on the same diagonal, say $(x + 1, y)$ and $(x, y + 1)$, are equal. Consequently, all the states with the same total number of users have the same reward upon accepting an SU under the optimal policy. Hence, the optimal admission decisions on SUs of these states are also identical.

Lemma 3. *For all (x, y) with $x + y + 2 \leq C$ and $\forall n \geq 0$:*

$$\begin{aligned} & V_n(x+1, y+1) - V_n(x+1, y) \\ & = V_n(x, y+2) - V_n(x, y+1). \end{aligned} \tag{5}$$

Adding Eq. (4) to Eq. (5), we obtain Eq. (6). The next corollary states that the profit function is concave in the number of SUs.

Corollary 1. *For all (x, y) with $x + y + 2 \leq C$ and $\forall n \geq 0$:*

$$V_n(x, y+1) - V_n(x, y) \geq V_n(x, y+2) - V_n(x, y+1). \tag{6}$$

Theorem 1 can be inferred from Lemma 3 and Corollary 1. For the states with the same total number of users to have the same admission control decision, the value of an additional accepted SU must be identical. Lemma 3 states that the rewards gained by accepting an additional SU to the states with the same total number of users are equal. Therefore, Lemma 3 implies that the states with the same total

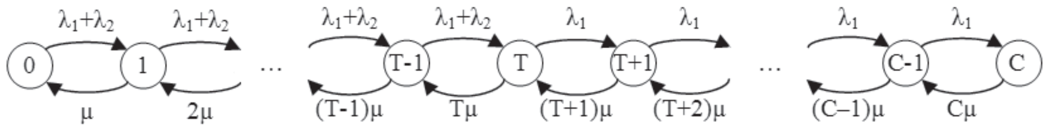


Figure 2: State diagram of the 1D Markov chain used in determination of the optimal threshold.

number of users have the same optimal admission control.

Furthermore, the optimal admission control policy of SUs is of threshold type by Corollary 1. In the DP equations, Eq. (2) designates the admission control decision on the SUs. The optimal decision d^* in state (x, y) at time period n is as follows:

$$d^* = \begin{cases} accept \quad if \;\; V_{n-1}(x, y+1) - V_{n-1}(x, y) \geq 0 \\ reject \quad otherwise. \end{cases}$$

Assume x is fixed. For small values of y, it is optimal to accept SUs until the optimal threshold value T^* is reached. Then, for larger values of y such that $x + y \geq T^*$, we always reject SUs as the profit function is concave in y.

Our results so far hold for all $n \geq 0$. We can extend these α-discounted finite horizon results to the infinite horizon by taking $n \to \infty$, i.e. $V(x, y) = \lim_{n\to\infty} V_n(x, y)$. We refer to [20] for the conditions that make this extension possible. In addition, the properties of the optimal admission control policy hold for the long-run average return case since the control space and state space are finite. The average return case corresponds to $\alpha \to 0$ [15]. Thus, Theorem 1 holds for both the infinite horizon discounted return and average return formulations.

C. Performance Analysis

In this section, we show that the performance of the algorithm that determines the optimal admission control policy significantly improves once we know that the optimal admission control policy depends only on the total number of users in the system and is of threshold type. Without the knowledge of the characteristics of the optimal admission control policy, we have to solve the DP equations of the 2D Markov chain to determine the optimal decision at each state. We may use the *policy iteration* technique [17] to determine the optimal admission control policy which theoretically requires $O(2^{C^2})$ iterations to converge for our case. Hence, the running time significantly increases as the number of states increases.

On the other hand, once we characterize the optimal admission control policy, we can use linear search to find the optimal threshold T^*. The computational complexity of the algorithm that finds the optimal threshold T^* outperforms that of the DP algorithm. In order to achieve this, we derive the average profit rate of a system at state (x, y) in terms of the total number of users $N = x + y$ and threshold T.

Although our system cannot be reduced to a one-dimensional (1D) Markov chain, we can utilize a birth-death 1D MDP with $C + 1$ states to determine the optimal threshold. The states of this Markov chain are the total occupancy levels N.

Consider the Markov chain given in Fig. 2. We define $\pi_T(N)$ as the steady state probability that the system is in state N with threshold T. We define S' the entire state space as shown in Fig. 2:

$$S' = \{N \in \mathbb{N} \,|\, 0 \leq N \leq C\}.$$

Let $S'_1 \subset S'$ be the sub-space of states where SUs are rejected:

$$S'_1 = \{N \in \mathbb{N} \,|\, T \leq N \leq C\}.$$

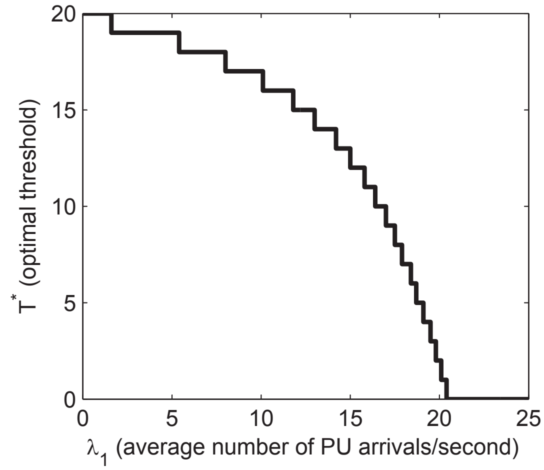
To compute J_T, the average profit rate of a system with threshold T, we use the following formula:

$$J_T = R\lambda_2\Bigg(1 - \sum_{N\in S'_1} \pi_T(N)\Bigg) - (K + R)\lambda_1\Bigg(\pi_T(C) - E(\lambda_1/\mu, C)\Bigg), \qquad (7)$$

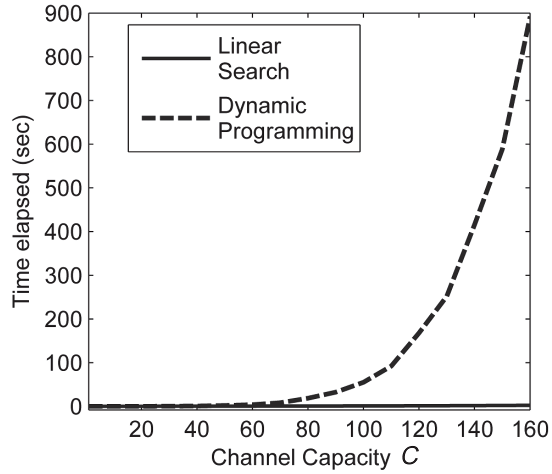
where $E(\lambda_1/\mu, C)$ is the blocking probability of PUs in the absence of SU arrivals. This quantity corresponds to the well-known *Erlang-B* formula:

$$E(\lambda_1/\mu, C) = \frac{\frac{(\lambda_1/\mu)^C}{C!}}{\sum_{n=0}^{C} \frac{(\lambda_1/\mu)^n}{n!}}.$$

The first term in Eq. (7) represents the total average revenue rate collected from SUs in each state an SU is accepted. The second term, on the other hand, is the average punishment rate due to preempted SUs. In order to find the sum of the steady state probabilities of the preemptive states, we subtract the blocking probability of PUs from the probability that all channels are busy. When we subtract the



(a)



(b)

Figure 3: (a) Optimal threshold for admission control of SUs versus average PU arrival rate; (b) Comparison of the running times of Linear Search and DP versus number of channels.

average punishment rate from the average revenue rate, we obtain the average profit rate of the system under threshold T.

Once we know the optimal policy is of threshold type, we use linear search on Eq. (7) to find the optimal threshold T^* that yields the maximum average profit rate J^*. The comparison of the performance of linear search and the performance of DP is given in Section V.

V. Numerical Results

In this section, we provide numerical results illustrating our theoretical analysis. First, we consider a 20 channel system with $\lambda_2 = 10$, $\mu = 1$, $K = 5$, $R = 1$ and λ_1 is a parameter. Fig. 3(a) demonstrates how the optimal threshold T^* varies as λ_1 increases.

Furthermore, we compare the running times of two different methods that give the optimal policy. First, we use linear search over all values of $0 \leq T \leq C$ to find the optimal admission control given it is of threshold type. Then, without taking the threshold property into account, we use DP to find the optimal admission decision at each state. The solution of DP also verifies the threshold type of policy, however, the running time drastically increases with increasing C. Fig. 3(b) demonstrates the comparison of running times of these methods with increasing number of channels. The experiments are performed on a Sony Vaio laptop with Intel Core i5 CPU at 2.53 GHz and 4 GB RAM. For instance, for $C = 160$, the running time of DP is 891.96 seconds. On the other hand, the running time of the linear search is only 2.06 seconds, which is about 430 times smaller than the running time of the DP algorithm.

VI. Applicability of Threshold Property to General Call Length Distributions

In the previous sections, we have examined a system with exponentially distributed call durations with mean μ^{-1}. In this section, we investigate whether the threshold structure holds for general call length distributions.

In earlier work [6], a non-preemptive loss system which consists of PUs and SUs and employs an occupancy-based policy is studied. The PUs and SUs have the same general call length distributions. The SP pays a punishment when a PU is blocked due to an SU. In [6], the optimal occupancy-based policy depends only on the total number of PUs and SUs in the system for general call length distributions. Through a counterexample, we show that for a preemptive system, the threshold property of the occupancy-based optimal admission control policy proven in Theorem 1 cannot be extended to general call length distributions.

Example 1. We assume that PU and SU calls have hypo-exponential distribution with two phases. In the first phase, the call length is exponentially distributed with mean $1/\mu_a$ and the call proceeds to the second phase where the call length is exponentially distributed with mean $1/\mu_b$. We determine the optimal occupancy-based admission control policy of a system with $C = 2$, $\lambda_1 = 0.5$, $\lambda_2 = 0.2$, $\mu_a = 1/3$, $\mu_b = 2/3$, $K = 9$ and $R = 3$. Note that the mean call duration of this system is the same as $\mu^{-1} = 1$ which is the value we used in our simulations in Section V. In the simulations, we observe that the optimal admission decision for state (0,1) is *reject* whereas it is *accept* for state (1,0). The optimal admission control decision at the given states had to be identical for the optimal admission control policy to depend only on the total occupancy. Hence, unlike the non-preemptive loss system, the threshold structure does not hold for general call length distributions for the preemptive system.

VII. Conclusion

We have studied a preemptive priority CR system with C channels and two user classes: PUs and SUs, where PUs have higher priority. We model the system as a 2D MDP and we analytically compute the optimal admission control of SUs which maximizes the profit. As our main contribution, we show that the optimal admission control policy depends only on the total number of users in the system and is of threshold type. The characterization of the optimal admission control policy of SUs reduces the computational complexity of the DP and allows computing the optimal admission control policy more efficiently.

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