

Secondary Pricing of Spectrum in Cellular CDMA Networks

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Abstract— We study secondary pricing of spectrum in wireless cellular networks employing CDMA at the physical layer. We consider a primary license holder who aims to lease its spectrum within a certain geographic subregion of its own network. Such a transaction has two contrasting economic implications for the seller: On the one hand the seller obtains a revenue due to the exercised price, or rent, of the region. On the other hand, the seller incurs a cost due to (i) reduced spatial coverage of its network and (ii) possible interference from the leased region into the retained portion of its network. We formulate an optimization problem with the objective of profit maximization, and characterize its solutions based on a reduced load approximation that can be shown to be asymptotically exact. The form of optimal prices suggests charging the buyer per admitted call, in proportion with the interference it generates. The charged amount balances the corresponding loss of revenue incurred by the seller due to the influence of an admitted call. We numerically argue that this pricing approach yields better profit compared to some other simplistic techniques.

I. INTRODUCTION

Legacy regulatory frameworks of cellular wireless communications grant limited property rights to license holders of spectrum: a license holder can only provide a specific service and cannot resale any part of its license. Economists have long argued against such rigid regulation [1], whose inefficiency has recently gained wider recognition and led to global regulatory effort centered around more flexible frameworks that allow secondary trading of spectrum among license holders. The Secondary Markets Initiative [3] of the Federal Communications Commission (FCC), for example, permits leasing of spectrum licenses subject to approval by FCC. Similar regulatory efforts are also underway in the EU [9].

In this paper we focus on secondary pricing of spectrum in wireless cellular networks. We consider a primary license holder who aims to lease its spectrum within a certain geographic subregion of its own network. Such a transaction has two contrasting economic implications for the seller: On the one hand the seller obtains a revenue due to the exercised price, or rent, of the region. On the other hand, the seller incurs a cost due to (i) reduced spatial coverage of its network and (ii) possible interference from the leased region into the retained portion of its network. We formulate optimal pricing as an optimization problem with the objective of profit maximization.

While the pricing problem can in principle be considered

within the framework of monopolistic markets in classical microeconomic theory [11], complexity of network-wide consequences of interference presents a major hurdle in obtaining explicit solutions. For example, a call in progress leads to a temporal reduction in utilization in its immediate neighborhood, which may in turn help accommodate more calls in the second-tier cells around it. In view of such knock-on effects determining the marginal cost of traffic in a given area appears involved. A seemingly appealing solution to this issue might be to eliminate interference by isolating the activity in the two subregions by way of guardbands [10]. A guardband, however, is an unutilized resource whose cost needs to be internalized either by the seller or by the buyer involved in the transaction. The situation leads to an inevitable loss of efficiency in the transaction, which may in fact be significant. The attendant inefficiency in turn limits the granularity and thereby liquidity of a secondary spectrum market.

Here we pursue optimal secondary price of spectrum without resorting to conservative methods to eliminate interference. In particular, the form of optimal price suggests charging the buyer per admitted call that generates interference for the seller. The charged amount is shown to depend on the extent of generated interference, namely, it balances the corresponding loss of revenue incurred by the seller due to the influence of an admitted call. This effort entails convenient analytical techniques that avoid the alluded difficulties associated with network-wide effects of interference at the expense of reasonable loss of modeling accuracy. Towards that end we adopt reduced load approximations that have found application in classical telephony. We show that the profit resulting from such prices may significantly exceed those of less sophisticated pricing techniques that ignore or eliminate interference.

Although pricing of communication networks is a well-studied topic, the setting considered here is specific to secondary wireless markets and, to the best of the authors' knowledge, it has not been considered before. In related work, [4] pursues interference based pricing in a single cell via adaptive optimization techniques, and [7], [8] adopt a performance oriented viewpoint in considering dynamic spectrum access within a cell. Main contributions of the present paper are

1. Global consideration of network: We consider general network topologies rather than a single cell. Rather than lumping any portion of the network into an approximate module, the paper accounts for sophisticated dependence

between cells due to generated interference.

2. Characterization of optimal price: We characterize the form of optimal prices under a general framework. Optimal prices are shown to have an interpretation that offers insight on dominant factors that determine the value of spectrum under spatial interactions.

The technical focus of this paper is on networks that employ CDMA as spectrum access mechanism, where a call uses the whole spectrum but can be sustained under some interference. Narrowband networks, in which a channel cannot be utilized simultaneously in neighboring cells, generally appear harder to analyze due to combinatorial consequences of interference. While the techniques presented in this paper apply to certain narrowband topologies and channel assignment policies, a general treatment of such networks is not pursued here.

The paper is organized as follows. Sections II and III describe respectively the teletraffic and economic aspects of the considered network model. Optimal pricing is formulated as a profit maximization problem in Section IV. The reduced load approximation employed in approximating the objective function is provided in Section V and the resulting optimal prices are given in Section VI. Numerical solutions on some hexagonal lattice networks are provided in Section VII and are compared with less sophisticated pricing techniques. The paper concludes with final remarks in Section VIII.

II. NETWORK MODEL

In this section we introduce the operational model of the generic cellular network considered in this paper. We represent a wireless cellular network with a weighted graph $G = (N, W)$ where N and W refer respectively to nodes and positive edge weights. Each node $i \in N$ in the graph represents a cell. For each pair i, j of cells the associated weight $w_{ij} \in W$ is a measure of electromagnetic interference between the cells due to their geographic proximity. Self-loops are allowed, in fact it will consistently be the case that $w_{ii} > 0$. The example of Figure 1 illustrates the graphical representation of a hexagonal lattice model.

We consider the network under circuit-switched operation and refer to each communication session as a call. Let n_i be the number of calls in progress at each cell i and let \mathbf{n} denote the cell loads ($n_i : i \in N$). A call is subject to interference from other calls in the same cell, as well as from calls in other cells in proportion with the associated weights. We shall assume that a call can be sustained only if it experiences small enough interference. A cell, however, may receive unbounded interference if does not accommodate a call. A network load \mathbf{n} is thus feasible if for all cells j and certain constants $\kappa_j > 0$

$$\sum_{i \in N} n_i w_{ij} \leq \kappa_j \quad \text{whenever } n_j > 0. \quad (1)$$

Network models based on similar constraints have been considered in earlier works on cellular wireless CDMA networks. See, for example, [2] for an in-depth discussion of this model and specification of model parameters w_{ij} and κ_j in terms of

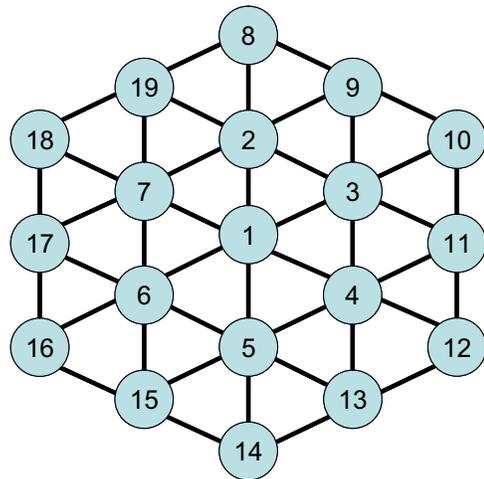


Fig. 1. The network graph of a hexagonal lattice topology with 19 cells.

physical layer parameters. In this paper we shall assume that the model parameters satisfy the following mild condition:

Assumption 1: For all $i, j \in N$ the parameters w_{ij} and κ_j are rational numbers. Hence, without loss of generality in the feasibility condition (1), these parameters are further taken as integers.

Calls arrive at each i cell according to a Poisson process of rate $\nu_i \geq 0$. Arrival processes for different cells are mutually independent. Each call has a holding time that is exponentially distributed with unit mean, independently of the history prior to its arrival. An incoming call is accepted if and only if its inclusion in the network conserves the feasibility condition (1) and the call is blocked otherwise. We denote the vector of call arrival rates by $\nu = (\nu_i : i \in N)$ and define $B_i(\nu)$ as the associated probability of call blocking on cell i .

The network provider generates unit revenue per admitted call in the network. We denote by $R(\nu)$ the long-term average rate of revenue generation of the provider per unit time, which is given by

$$R(\nu) = \sum_{i \in N} (1 - B_i(\nu)) \nu_i.$$

III. ECONOMIC MODEL

We consider pricing of a region, i.e. a given subset $L \subset N$ of cells, from the perspective of the network provider. Namely we seek optimal price, more precisely optimal rent, in leasing to another provider the license to provide service in region L . Henceforth we refer to the original provider of the network as the *seller* and to the potential provider in region L as the *buyer*.

A price for region L is a scalar which we denote by p . The unit of p is determined by the pricing philosophy adopted by the seller. For example if a flat price is employed then the unit of p is currency per unit time, whereas if a usage-based price is employed then p may be expressed in currency per Erlang.

It is assumed that the buyer reflects the transaction price p onto pricing of its own service, and thereby p would affect

the demand that the buyer receives in region L . Specifically, we denote by $\alpha_i(p)$ the call arrival rate of the buyer to cell $i \in L$ provided that the transaction is realized at price p . The demand statistics for the seller in the region $N - L$ after the sale remain unaltered. The overall network demand after a transaction at price p is denoted by $\lambda(p) = (\lambda_i(p) : i \in N)$ where

$$\lambda_i(p) = \begin{cases} \alpha_i(p) & \text{if } i \in L \\ \nu_i & \text{if } i \in N - L. \end{cases}$$

The seller has an expected rate of revenue over the term of a lease signed at price p . To keep the discussion general we succinctly denote this value by $F(p)$. For the special cases alluded earlier in this section the expected revenue rate $F(p)$ may take the following forms:

a) *Flat price*: A flat price would be taken if it does not exceed the valuation of the commodity by potential buyers. Suppose that the seller's a priori perception of the market value of the spectrum in region L can be represented by a random variable V . The seller's expected rate of revenue generation from a flat price p would then be given by

$$F(p) = pP(V > p).$$

b) *Price per demand*: The seller may price the spectrum per unit demand generated in the region L , in which case p refers to the revenue of the seller per call request in region L after the sale. The revenue rate $F(p)$ of the seller would then be

$$F(p) = \sum_{i \in L} \alpha_i(p)p. \quad (2)$$

c) *Price per honored demand*: Alternatively, the seller may choose to tax the interference that the buyer generates on its (retained) network. This abstract principle may be interpreted as imposing a tax p per *accepted* call in the region L , thereby entitling the seller to a certain share of the buyer's revenue. The revenue rate from the sale would then be given by

$$F(p) = \sum_{i \in L} (1 - B_i(\lambda(p)))\alpha_i(p)p. \quad (3)$$

IV. PROBLEM FORMULATION

For each nonnegative vector $\lambda = (\lambda_i : i \in N)$ of arrival rates let

$$Q(\lambda) = \sum_{i \in N-L} (1 - B_i(\lambda))\lambda_i. \quad (4)$$

Note that $Q(\lambda)$ is a network revenue due to the service provided over the region $N - L$, however it is affected by the demand in region L through the blocking probabilities $B_i(\lambda)$. In particular $Q(\lambda(p))$ is the after-sale revenue of the seller over the retained region $N - L$, provided that the transaction is realized at price p .

The cost incurred by the seller in leasing region L at price p is then given by

$$C(p) = R(\nu) - Q(\lambda(p)), \quad (5)$$

namely, by the reduction in rate of revenue generation from the service it provides to its end-users. The seller aims to choose

the price parameter p so as to maximize its profit; hence an optimal price for the seller solves:

$$\max_p (F(p) - C(p)). \quad (6)$$

In characterizing solutions of problem (6) we shall assume that the following technical condition holds:

Assumption 2: The functions F and α_i , $i \in L$, are differentiable.

The discussion of the next section establishes that the blocking probabilities $B_i(\cdot)$ are also differentiable. Hence, in light of Assumption 2, the profit $F(p) - C(p)$ is differentiable in p and a solution p^* to the seller's problem (6) satisfies

$$F'(p^*) = C'(p^*) = -\frac{d}{dp}Q(\lambda(p))|_{p=p^*}. \quad (7)$$

In this paper we seek insight on the nature of optimal prices by focusing on characterizing solutions of the first-order condition (7). Existence and uniqueness of such a solution depend on further properties of the objective function; in principle a second order analysis may be employed to obtain conditions under which (6) has a unique solution. That direction is not pursued in the present paper beyond assuming existence of a solution.

V. BLOCKING PROBABILITIES

Let S denote the set of feasible cell loads. That is,

$$S = \{\mathbf{n} \in \mathbb{Z}_+^{N|} : \mathbf{n} \text{ satisfies condition (1)}\}.$$

For any set of arrival rates $\lambda = (\lambda_i : i \in N)$ the vector of cells loads evolves according to a Markov process whose states belong to S . This process is obtained by truncating the state space of a reversible process that corresponds to cells loads when interference limitations are ignored; in particular its equilibrium distribution π_λ is given by

$$\pi_\lambda(\mathbf{n}) = G \prod_{i \in N} \frac{\lambda_i^{n_i}}{n_i!}, \quad \mathbf{n} \in S,$$

where G is a constant which ensures that π_λ is a probability vector.

Let $\mathbf{e}(i) = (e_j(i) : j \in N)$ be such that $e_j(i) = 1$ if $j = i$ and $e_j(i) = 0$ otherwise. The blocking probabilities can then be expressed as

$$B_i(\lambda) = \sum_{\mathbf{n}: \mathbf{n} + \mathbf{e}(i) \notin S} \pi_\lambda(\mathbf{n}).$$

Each $B_i(\cdot)$ is differentiable, however despite its appealing form, further manipulation of the above expression is hindered by difficulties in computing the normalization constant G . To gain more insight on the blocking probabilities we proceed with a "reduced load approximation," which has proved useful in analysis of blocking in circuit-switched telephony [6]:

Reduced load approximation: We shall approximate $B_i(\lambda)$ by the quantity $\hat{B}_i(\lambda)$ defined by

$$\hat{B}_i(\lambda) = 1 - \prod_{j \in N} (1 - b_j(\lambda))^{w_{ij}} \quad (8)$$

where the numbers $b_j(\lambda), j \in N$, satisfy the equalities

$$b_j(\lambda) = E \left((1 - b_j(\lambda))^{-1} \sum_{i \in N} w_{ij} \lambda_i \prod_{k \in N} (1 - b_k(\lambda))^{w_{ki}}, \kappa_j \right) \quad (9)$$

and $E(\cdot, \cdot)$ denotes the Erlang blocking formula

$$E(\rho, \kappa) = \left(\sum_{m=0}^{\kappa} \frac{\rho^m}{m!} \right)^{-1} \frac{\rho^\kappa}{\kappa!}$$

for all $\rho > 0$ and positive integer κ . The set of equations (9) has a unique solution [6]; hence the approximation is well-defined, and furthermore the solution is differentiable in λ [5, Lemma 2.2]. The reduced load approximation above can be better motivated by first replacing the feasibility condition (1) by

$$\sum_{i \in N} n_i w_{ij} \leq \kappa_j. \quad (10)$$

Note that this condition is more stringent than (1) in that it limits the interference on idle cells as well. Under the feasibility condition (10), κ_j can be regarded as capacity of cell j and w_{ij} can be regarded as the units of capacity reserved from cell j per call in progress in cell i . The expression (8) then suggests that $\hat{B}_j(\lambda)$ is the blocking probability at cell i in a hypothetical model where each *unit* of capacity is available independently with probability $1 - b_j(\lambda)$ at link j , and furthermore availability of capacity is independent from link to link. Such a model is consistent only if the parameters $b_j(\lambda)$ satisfy the fixed-point relation (9).

The approximate blocking probabilities $\hat{B}_j(\lambda)$ are known to be asymptotically exact for the feasibility condition (10) along a limiting regime where the network arrival rates λ_j and thresholds κ_j increase in proportion [6]. While condition (10) leads to higher blocking than condition (1), the disparity may arguably be expected to vanish in the same limiting regime as increasing the arrival rates reduces the chances of finding cells at idle state. This intuition is confirmed in Section VII for moderate values of model parameters, via numerical verification of the reduced load approximation.

VI. CHARACTERIZATION OF PRICES

We next exploit the tractable nature of reduced load approximations to obtain approximate expressions for the optimal price of spectrum. Conclusions of this section are valid under the following simplifying assumption:

Assumption 3: (Exactness of reduced load approximation) $B_i(\lambda) = \hat{B}_i(\lambda)$ for each cell i and all call arrival rates $\lambda = (\lambda_i : i \in N)$.

Theorem 6.1: Under Assumption 3 an inner solution p^* of the seller's problem (6) satisfies

$$p^* = \sum_{i \in L} \left(1 - \hat{B}_i(\lambda(p^*)) \right) \alpha_i(p^*) \gamma_i(p^*) \quad (11)$$

where

$$\gamma_i(p^*) = \frac{\varepsilon_i(p^*)}{F'(p^*)} \sum_{j \in N} w_{ij} \frac{d}{d\kappa_j} Q(\lambda(p^*)) \quad (12)$$

and $\varepsilon_i(p^*) = p^* \alpha'_i(p^*) / \alpha_i(p^*)$ is the price elasticity of demand in cell i .

Theorem 6.1 can be interpreted for the three pricing philosophies alluded in Section III as follows:

Flat price: The form (11) suggests that optimal flat price per unit time for region L is the same as the revenue generated from the region L per unit time by charging each admitted call in cell $i \in L$ an amount $\gamma_i(p^*)$. In parsing the expression (12) for this quantity it is helpful to interpret $\frac{d}{d\kappa_j} Q(\lambda(p^*))$ as the reduction in the seller's revenue from region $N - L$ due to unit reduction in the interference threshold of cell j , or equivalently due to imposing unit interference on cell j . An accepted call in cell $i \in L$ then leads to a reduction of $w_{ij} \frac{d}{d\kappa_j} Q(\lambda(p^*))$ in seller's revenue. The form (12) in turn indicates that the *per-call* price $\gamma_i(p^*)$ balances the attendant loss of revenue, up to a multiplicative quantity that depends on the price elasticity of demand in cell i and the revenue function F .

Price per demand: If the seller's revenue is given by (2) then

$$F'(p) = \sum_{i \in L} \alpha_i(p) (1 + \varepsilon_i(p)),$$

and rearrangement of equalities (11) and (12) yields

$$\sum_{i \in L} \alpha_i(p^*) \varepsilon_i(p^*) \left(1 - \hat{B}_i(\lambda(p^*)) \times \left(p^* \frac{1 + \varepsilon_i^{-1}(p^*)}{1 - \hat{B}_i(\lambda(p^*))} - \sum_{j \in N} w_{ij} \frac{d}{d\kappa_j} Q(\lambda(p^*)) \right) \right) = 0.$$

The insight offered by this equality can perhaps be clarified by considering pricing of a single cell, in which case $L = \{i\}$ and

$$p^* = \left(1 - \hat{B}_i(\lambda(p^*)) \right)^{-1} \left(1 + \varepsilon_i^{-1}(p^*) \right)^{-1} \sum_{j \in N} w_{ij} \frac{d}{d\kappa_j} Q(\lambda(p^*)).$$

In particular the optimal per-demand price p^* is proportional to the marginal cost of the seller due to an *accepted* call, discounted at rate equal to acceptance probability.

Price per honored demand: In the case when the seller's revenue is given by (3) a relatively more explicit characterization of p^* can be obtained by defining $U(p)$ as the *overall* revenue of the seller after the transaction at price p . That is,

$$U(p) = \sum_{i \in N} \left(1 - \hat{B}_i(\lambda(p)) \right) \lambda_i(p) r_i(p)$$

where

$$r_i(p) = \begin{cases} p & \text{if } i \in L \\ 1 & \text{if } i \in N - L; \end{cases} \quad (13)$$

and in turn

$$F(p) - C(p) = U(p) - R(p).$$

Proposition 6.1: (Optimal price per honored demand) If F is given by (3) then under Assumption 3 an inner solution p^* of the seller's problem (6) satisfies

$$\sum_{i \in L} \alpha'_i(p^*) \left(1 - \hat{B}_i(\lambda(p^*)) \times \left(p^* \left(1 + \varepsilon_i^{-1}(p^*) \right)^{-1} - \sum_{j \in N} w_{ij} \frac{d}{d\kappa_j} U(\lambda(p^*)) \right) \right) = 0.$$

Hence if $L = \{i\}$ is comprised of a single cell then

$$p^* = (1 + \varepsilon_i^{-1}(p^*))^{-1} \sum_{j \in N} w_{ij} \frac{d}{d\kappa_j} U(\lambda(p^*)).$$

Note that here the form of the optimal price does not include a discount at the acceptance probability since the price is already applied to accepted calls.

Computation of optimal cell price: The derivatives that appear in the above expressions possess certain properties that can be useful in computing p^* . For notational convenience let us denote quantity $\frac{d}{d\kappa_j} U(\lambda(p))$ by $c_j(p)$. Note that $c_j(p)$ is the reduction in the overall revenue of the seller due to a unit reduction in the interference threshold of cell j . To obtain a more explicit characterization of this quantity, let $\eta_j(p)$ denote the increase in the (unit) blocking probability $b_j(\lambda(p))$ at cell j per unit decrease in the interference threshold of the cell. Let

$$\rho_{ij}(p) = \lambda_i(p) \prod_{k \neq j} (1 - b_k(\lambda(p)))^{w_{ik}}$$

denote the intensity of calls at cell i after being thinned due to blocking at cells other than j . Such a call is accepted with probability $(1 - b_j(\lambda(p)))^{w_{ij}}$ in which case it returns revenue $r_i(p)$. Unit decrease in κ_j results in a reduction

$$(1 - b_j(\lambda(p)))^{-1} \sum_{i \in N} \rho_i(p) w_{ij} \eta_j(p) r_i(p)$$

in revenue obtained from such calls. Here

$$\rho_i(p) = \rho_{ij}(p) (1 - b_j(\lambda(p)))^{w_{ij}}$$

is the rate of accepted calls at cell i . By way of blocking in its neighborhood, a call has further consequences in operation of other cells. Since each blocked call in cell i can be associated with increasing the threshold of cell l by w_{il} units, a call in cell j leads to an increase of

$$(1 - b_j(\lambda(p)))^{-1} \sum_{i \in N} \rho_i(p) w_{ij} \eta_j(p) \sum_{l \in N-j} w_{il} c_l(p)$$

in the revenue obtained from other cells in the network. Therefore $c_j(p)$ can be written in the form

$$c_j(p) = \varphi_j(p) \sum_{i \in N} w_{ij} \rho_i(p) \left(r_i(p) - \sum_{l \in N-j} w_{il} c_l(p) \right)$$

where $\varphi_j(p) = \eta_j(p) (1 - b_j(\lambda(p)))^{-1}$. A similar relation can be written for the derivatives $\frac{d}{d\kappa_j} Q(\lambda(p))$ as well.

It can be shown that for a fixed value of p the above relations identify the values $c_j(p)$ uniquely [5]. Also in cases when an inner solution p^* exists, properly damped versions of the recursion

$$p^{k+1} = (1 + \varepsilon_i^{-1}(p^k))^{-1} \sum_{j \in N} w_{ij} c_j^k \quad (14)$$

$$c_j^{k+1} = \varphi_j(p^k) \sum_{i \in N} w_{ij} \rho_i(p^k) \left(r_i(p^k) - \sum_{l \in N-j} w_{il} c_l^k \right) \quad (15)$$

Cell No.	Blocking Probability	
	Reduced load approximation	Simulated under condition (1)
1	0.358	0.305 ± 0.003
2-7	0.279	0.259 ± 0.003
8-18 (even)	0.107	0.102 ± 0.002
9-19 (odd)	0.159	0.150 ± 0.002

TABLE I

BLOCKING PROBABILITIES OF THE 19-CELL SYSTEM REPRESENTED BY THE GRAPH IN FIGURE 1 AND PARAMETERS: $\kappa = 5.0$, $w_{ij} = 0.5$, AND $w_{ii} = 1.0$ FOR ALL i, j . ARRIVAL RATES ARE EQUAL FOR ALL THE CELLS WITH VALUE 1.0. CONFIDENCE LEVEL FOR THE SIMULATED RESULTS IS 95%.

may be expected to converge, thereby yielding p^* . In fact, in Section VII we give an example where the recursion turns useful in computing the optimal price.

VII. NUMERICAL STUDY

A. Accuracy of the Reduced Load Approximation

We start our numerical study by showing an example of the accuracy of the *reduced load approximation* in computing the cell blocking probabilities. Recall that the approximation is based on the more stringent feasibility condition (10) compared to (1). For this purpose we use a 19-cell hexagonal lattice model with the corresponding graph shown in Figure 1. The cells are assumed to have equal interference thresholds, that is, $\kappa_i = 5.0$ for $i \in N$. We shall assume that a call generates half of the interference in neighboring cells relative to its own cell. More specifically, $w_{ij} = 0.5$ for each edge such that $i \neq j$ and $w_{ii} = 1.0$ for all nodes i .

We first consider the network under the traffic demand of $\nu_i = 1.0$ arrivals per unit time per cell, and solve the fixed point equations (9) using repeated substitutions. Approximate blocking probabilities $\bar{B}_i(\lambda)$ are then computed via (8) and are given in Table I. To verify their accuracy, we simulate the network process under the feasibility condition (1), where idle cells can have unlimited interference. The duration of the simulation process is taken to be long enough so that each cell receives around 5000 call requests over the period of the process. Table I presents the resulting proportion of blocked calls along with 95% confidence intervals.

The disparity between approximate and simulated values for blocking probabilities appear acceptable in view of the practically prohibitive computational complexity of exact analysis. Moreover, the approximate results are asymptotically exact under certain limiting regimes as argued in Section V.

We next turn to the sensitivity of optimal price to errors in the blocking probabilities due to reduced load approximation. Our investigation here involves computing optimal price of a single cell using the reduced load approximation and also using the exact equilibrium distribution of the network process. We adopt the 7-cell topology whose graph representation is

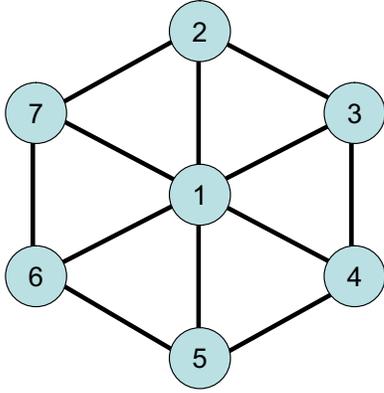


Fig. 2. Graph representation for the 7-cell topology studied in Section VII.

shown in Figure 2, where cell 1 is for sale. A smaller topology would be useful to avoid otherwise lengthy simulation processes to compute the exact profit for every given price. In fact, for this small topology the equilibrium distribution can be exactly computed in a relatively short time.

For the network in Figure 2, we continue adopting the parameters $\kappa_i = 5.0$, $w_{ij} = 0.5$ for each edge such that $i \neq j$ and $w_{ii} = 1.0$ for all nodes i . The traffic demand of the seller prior to the transaction is taken as

$$\nu_i = \begin{cases} 0 & \text{if } i = 1 \\ 1 & \text{if } i = 2, \dots, 7, \end{cases} \quad (16)$$

and the demand function α_1 in cell 1 is taken as

$$\alpha_1(p) = p^{-2}. \quad (17)$$

Figure 3 shows exact and approximated profit of the seller for different prices for cell 1. The figure shows that the profit maximization problem admits a unique solution for this particular setup. The disparity in the profit $F(p) - C(p)$ appears small and exact and approximate optimal prices are remarkably close, both values approximately equal to 1.4.

B. Computation of Optimal Prices

We continue our numerical study by computing the optimal price for the 7-cell network shown in Figure 2 using the recursion (14), (15). Indeed, for a fixed value of p , it can be shown that sufficiently damped form of (15) always converges [5]. The same argument does not seem to carry out to the case when price recursive formula (14) is incorporated. However, we show by an example that for the polynomial demand functions given in (17) convergence indeed occurs.

Figure 4 shows the convergence path to the optimal price p^* for the network shown in Figure 2, under the demand function (17) and per honored demand pricing. The convergence occurs after relatively small number of iterations, namely 20 for this example. The resulting optimal price is 1.37 and it matches the value observed in Figure 4.

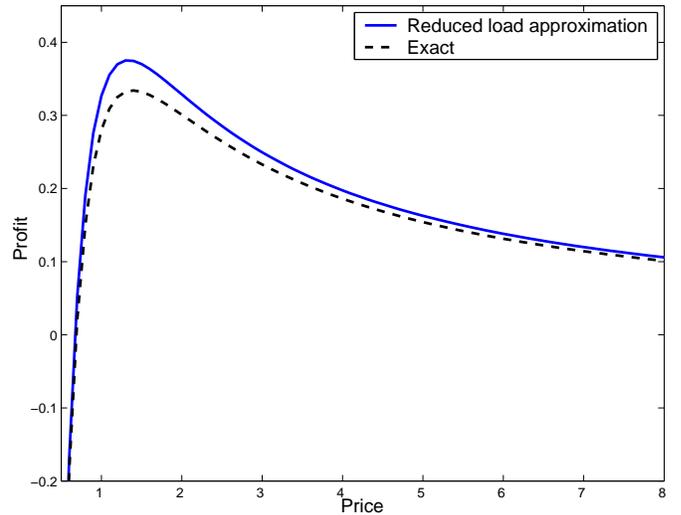


Fig. 3. Approximated and exact profit from selling cell 1 in the system shown in Figure 2 under the demand function (17) and per honored demand price. $\kappa_i = 5.0$, $w_{ij} = 0.5$, $w_{ii} = 1.0$, and ν as given in (16).

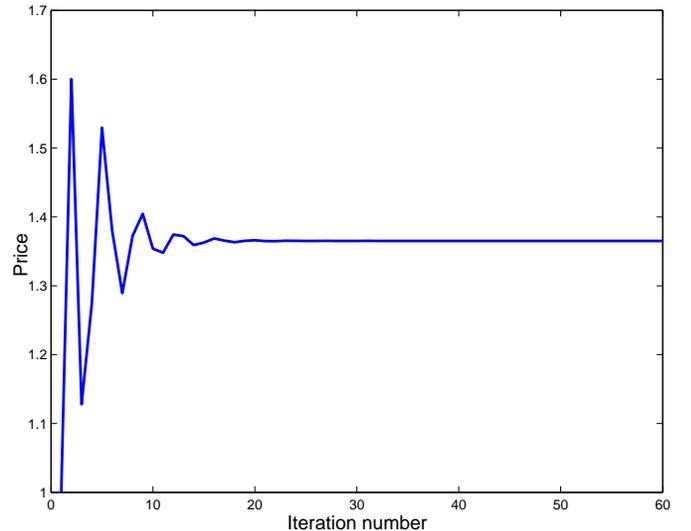


Fig. 4. Convergence path to the optimal price p^* for cell 1 in the network in Figure 2 under the demand function (17) and per honored demand pricing. $\kappa_i = 5$, $w_{ij} = 0.5$, $w_{ii} = 1.0$, and ν_i as given in (16) for all i, j . The limit agrees with the optimal price observed from Figure (3).

C. Comparison with Simple Pricing Techniques

In this section we numerically argue that our proposed pricing technique yields better profit than some other simplistic techniques. Consider first the simple technique where the seller does not count for the cost resulting from the interference caused by the traffic in the sold region. Therefore the profit maximization problem (6) can be modified to be

$$\max_p F(p). \quad (18)$$

In other words, the seller would afford the cost of the interference caused by the buyer's traffic. To understand the conse-

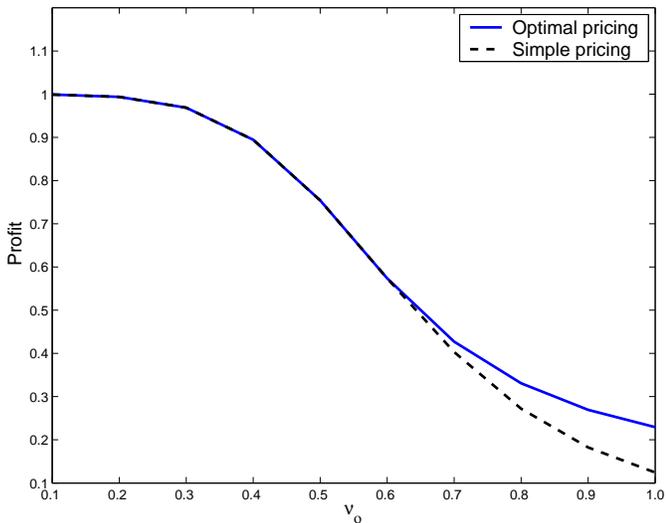


Fig. 5. Profit from the optimal pricing scheme and the simple pricing scheme when cell 1 is sold under the demand function (17) and per honored demand pricing. The seller's traffic is $\nu_i = \nu_o$ for nodes $i \neq 1$. The values are computed using reduced load approximation for $\kappa_i = 5.0$, $w_{ij} = 0.5$ and $w_{ii} = 1.0$.

quences of this technique, we consider the 19-cell network in Figure 1, where we are interested in pricing cell 1 for honored demand. The capacities of the cells are taken to be equal with value $\kappa_i = 5.0$ and all non-zero edge weights to be equal with value $w_{ij} = 0.5$ and $w_{ii} = 1.0$ for all i, j . We assume also price per honored demand with polynomial demand function given by (17).

In Figure 5, we show the optimal profit for different traffic rates ν_i to the kept region. We also show the actual revenue when solving (18) for cell 1. For the range of given ν_i , as ν_i increases, the profit gap between the optimal and the simple techniques widens.

Another simplistic approach is to use space guard bands to isolate interference caused by the traffic of the buyer from the rest of the network. For example in the network in Figure 1, traffic in cell 1 can be isolated from the rest of the network by prohibiting traffic to cells (2–7). This implies losing some potential revenue from those cells. However, if the seller prices the traffic in the sold region without counting for the losses, then the seller's profit is clearly suboptimal.

To get an exposure on that, consider the 19-cell network in Figure 1, where cells (2–7) are taken as guard bands for cell 1. We are interested in the price per honored demand for cell 1 facing a demand function given by (17). In Figure 6 we show the profit gained by solving (18) for cell 1 for different traffic intensities on the cells 8–19. As can be noticed, the seller may commit significant losses out of this pricing technique. The percentage loss in profit can be expected to decrease if the sold region L is a large connected component of N that is significantly larger than its boundary, although the loss would still remain positive.

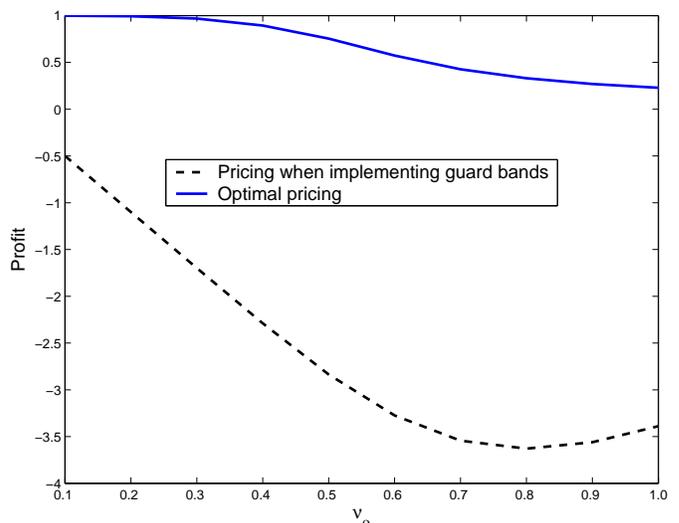


Fig. 6. Profits from the optimal pricing scheme and the pricing scheme with the use of space guard bands, when cell 1 is sold under the demand function (17). The seller's traffic is $\nu_i = \nu_o$ for nodes $i \neq 1$. The values are computed using reduced load approximation for $\kappa_i = 5.0$, $w_{ij} = 0.5$, and $w_{ii} = 1.0$.

VIII. CONCLUSION

We considered the problem of optimal pricing of spectrum in CDMA-based cellular wireless networks. We adopt an economic viewpoint and characterize the optimal price of spectrum in terms of price elasticity of demand: The cellular network is modeled with a weighted graph in which nodes indicate cells and edge weights indicate interference strengths. We study a stochastic model of circuit-switched network traffic with Poisson call arrivals, and express optimal prices by adapting reduced load approximations to the situation at hand. The form of optimal prices suggests that the seller should charge the buyer per accepted call in the sold region, thereby providing a dynamic framework for the transaction. We show that the prices are determined to a large extent by the seller's traffic in the retained region. We finally compare the optimal pricing scheme to more simplistic schemes, and provide numerical results in support of the analytical conclusions.

ACKNOWLEDGMENT

This work was funded in part by NSF through grants ANI-0238397, CCF-0430983, CNS-0435353, ANI-0132802, ANI-0240333, CNS-0435312.

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