EC337: Economics of Legal Issues, Spring 2009

Problem Set #1 Solutions

February 2, 2009

1. **(a)** For Bert and Ernie, cookies and relaxation are perfect complements. One interpretation is that they eat cookies when and only when they are relaxing. They only enjoy the cookies they have enough time eat while relaxing and only enjoy relaxation when they are eating cookies.

   (b) In order to choose R to maximize $\min\{24-R, R\}$, choose $24-R = R$. Simplifying this expression gives $R = 12$. That the choice of $24 - R = R$ is optimal can be seen by comparing the bundle $(C, R) = (16, 8)$ with the bundle $(C, R) = (12, 12)$. In the former case, Bert (or Ernie) only has enough relaxation time to enjoy 8 trays of cookies and so enjoys $u = 8$. In the latter case, Bert (or Ernie) has enough relaxation time to enjoy all 12 trays of cookies and so enjoys $u = 12$. The same argument can be extended to show that $(C, R) = (12, 12)$ yields more utility than other combination of work and relaxation.

   (c) As determined in part (b), $(u_B, u_E) = (12, 12)$ when each consumes his own cookies.

   (i) For any Pareto optimum, $C_B = R_B$ and $C_E = R_E$. The intuition is fairly straightforward. Suppose that there is some allocation where $C_E \neq R_E$ is Pareto optimal. If $C_E > R_E$, it would be possible to reduce both the amount Ernie works and the number of cookies he consumes a little in order to increase his utility by giving him more relaxation time to enjoy some of his "excess" cookies. The same kind of reasoning (in reverse) works when $C_E < R_E$.

   Of course, not all such points are Pareto optima. Only points on the utility possibility frontier can be be Pareto optima. In this case, it turns out that the utility possibility frontier coincides exactly with the requirement that none of the cookies baked are wasted. In other words, $48 = (C_E + C_B) + (R_E + R_B)$ must also hold. Substituting the first two equalities into this resource constraint gives $24 = C_E + C_B$. So, the Pareto frontier is defined by $24 - C_B = C_E$. Thus, $(C_B, R_B) = (C_E, C_E) = (12, 12)$ is Pareto optimal.

   Note: this might seem a bit odd because we always examined the Pareto and utility possibility frontiers in "utility space" for the examples considered in class. But it's not. We can simply "translate" the line from "consumption space" to "utility space" because $u_B = C_B$ and $u_E = C_E$ for any Pareto optimum. So, the Pareto frontier is $24 - u_B = u_E$. If you drew out the graph, answering the other questions below would not have been difficult. You just needed to follow the approach discussed in class.

   (ii) Yes. Both Bert and Ernie obtain $u_B = u_E = 12$ and the point is Pareto optimal as per the explanation given in (i).

(iii) The most straightforward thing to do is to notice that $u_B + u_E = 24$ for any point on the line $24 = C_E + C_B$ as discussed in the note above. In other words, all Pareto optimal allocations are also Utilitarian! This is the insight Elmo had when he thought about the problem.

If, instead, you choose to maximize $u_B + u_E = \min\{C_B, R_B\} + \min\{C_E, R_E\}$ subject to $24 = C_E + C_B$, you should substitute the restriction into the problem. The problem then becomes a problem of maximizing $\min\{C_B, R_B\} + \min\{24 - C_B, R_E\}$. Since every Pareto optimum must also have $C_B = R_B$ and $C_E = R_E$, you could also subsitute these restrictions into the problem. This makes problem to choose C_B in order to maximize $\min\{C_B, C_B\} + \min\{24 - C_B, 24 - C_B\}$. Note that any choice of C_B between 0 and 24 will do. In other words, any point on the line $24 = C_E + C_B$ is utilitarian.

(d) You should choose C to maximize $\min\{C, 24 - \frac{1}{3}C\}$. Using the same kind of argument as in part (b) above, you should set the amount of relaxation and cookies equal in order to obtain $C = 24 - \frac{1}{3}C$. Simplifying this expression gives $\frac{4}{3}C = 24$ or $C = 18$.

(e) Where each consumes his own cookies, $(C_B, R_B, W_B) = (18, 18, 6)$ and $(C_E, R_E, W_E) = (12, 12, 12)$. In terms of utility, Bert obtains $u_B = 18$ and Ernie obtains $u_E = 12$.

(i) For any Pareto optimum, $C_B = R_B$ and $C_E = R_E$. Again, not all such points are Pareto optima. Only points on the utility possibility frontier can be be Pareto optima. Again, the utility possibility frontier coincides exactly with the requirement that none of the cookies baked are wasted. In other words, the resource constraint $48 = (W_E + W_B) + (R_E + R_B)$ and the **new** production possibility constraint $C_E + C_B = W_E + 3W_B$ must also hold. Substituting the first two equalities into this resource constraint gives $48 = (W_E + W_B) + (C_E + C_B)$. Substituting the production possibility constraint into this equality gives $48 = (W_E + W_B) + (W_E + 3W_E)$. In other words, $48 = 2W_E + 4W_B$ for any Pareto optimal allocation. In the allocation where each consumes his own cookies, $W_E = 12$ and $W_B = 6$ so $2(12) + 4(6) = 48$. Since it is also the case that $C_B = R_B$ and $C_E = R_E$, it follows that the allocation where each consumes his own cookies is Pareto optimal.

Note: The exercise of "translating" the line from "work space" to "utility space" is more complicated. To make it easier, translate "work space" into "relaxation space" by subsituting $24 - W_E = R_E$ and $24 - W_B = R_B$ to obtain $48 = R_E + 2R_B$. You can then translate this into utility space since $u_B = R_B$ and $u_E = R_E$ for any Pareto optimum. This means that the utility possibility frontier is $48 = u_E + 2u_B$ but only up to the line $24 = u_E$! What all this says is pretty intuitive: if Ernie "does all the work by himself" so that $W_E = 24$, Bert gets all the cookies since only he has time to eat them all. Thus, $(u_E, u_B) = (0, 24)$ is one extreme of the utility possibility frontier. If Bert "does all the work by himself" so that $W_B = 12$, Ernie gets 24 of the 36 cookies Bert bakes which means that $(u_B, u_E) = (24, 12)$ is the other extreme. In between, the line has a slope of -2, which reflects the fact that giving Bert an extra hour of relaxation means sacrificing 2 trays of cookies.

Again, if you drew out the graph, answering the other questions should not have been difficult.

(ii) The allocation is not egalitarian since $u_B = 18 \neq 12 = u_E$. An efficient egalitarian allocation must (i) satisfy $u_B = u_E$ and (ii) have $C_B = R_B = 24 - W_B$ and $C_E = R_E = 24 - W_E$. In other words, it must be that $24 - W_B = 24 - W_E$ or $W_B = W_E$. Since an efficient allocation must also have $48 = 2W_E + 4W_B$, it must be that $48 = 6W_B$ or $W_B = 8$. Under this allocation, Bert and Ernie work equally hard $W_B = W_E = 8$ but Bert gives one third of the cookies he makes to Ernie so that

$(C_B, R_B, W_B) = (C_E, R_E, W_E) = (16, 16, 8)$ and $u_B = u_E = 16$.

(iii) Since $48 = 2W_E + 4W_B$, $C_B = R_B$ and $C_E = R_E$ for any Pareto optimal allocation, the problem of maximizing $u_B + u_E$ is to maximize $24 - W_B + 24 - W_E$ subject to the restriction $48 = 2W_E + 4W_B$. Note that any choice where $W_E > 0$ cannot maximize this expression. The intuition is simple. There are enough hours in the day for Bert to do all the required work by himself. That is, you can choose $W_B = 12$ to solve $48 = 4W_B$. Since he is not as efficient as Bert, it would be a waste for Ernie to do any of the baking! In other words, the Utilitarian allocation requires $(C_B, R_B, W_B) = (12, 12, 12)$ and $(C_E, R_E, W_E) = (24, 24, 0)$. In terms of utility, Bert obtains $u_B = 12$ and Ernie obtains $u_E = 24$.

Now standback and think about this result. What is utilitarianism really advocating here?

2. **(a)** It would appear that Armory's behavior *ex ante* should be unaffected by increasing the measure of damages. This might not be the case if the court system is prone to making mistakes however. When courts make mistakes, a policy of "excessive damages" might encourage Armory (or treasure hunters in general) to seek out jewels that are not worth finding. This is related to the problem of "overinvestment in finding" that we are now discussing in the property unit of the course.

   (b) A good reason why it might be sensible to award "excessive" damages is that the situation in *Armory v. Delamirie* may be quite unusual. If you think it that it is unlikely for poor chimney sweep boys to bring civil suits against jewellers, it might be sensible to increase the fine F in order to offset the low likelihood p that "wrongful" jeweler behavior is detected (i.e. contested in court). Since a jeweler's decision to take a jewel worth V is determined by the relationship between V and $\frac{pF}{(1-p)}$, it might be suitable to raise F when p is small. This is the issue of deterrence that will be discussed in the criminal law unit of the course.

   If you look at the answers to parts (a) and (b) together, they suggest that "decoupling" of damages may be appropriate. That is, it may be appropriate to levy "excessive damages" against defendants since this promotes deterrence while only paying "market value" damages to the plaintiff. The difference between the two amounts might even be used to fund the public justice system. While this analysis is admittedly a bit informal, I wanted you to appreciate that *Armory v. Delamirie* actually contains many of the themes we will discuss throughout the semester.