Lecture 4: Campbell-Shiller Decomposition

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Campbell and Shiller decomposition

- Introduce a log-linear approximation to the present-value identity (prices = Present Discounted Value [PDV] of dividends).
- Use this approximation to discuss the **sources of stock price volatility**.
- This formula has proven extremely useful in applied work, because it is easy to use it in conjunction with linear time series models (e.g. VARs). Also known as “Discount Rate - Cash Flow Decompositions”

Present Discounted Values

- Start from the definition of return:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}.$$

- Rewrite this as "price equal PDV of dividends" by rewriting and iterating forward:

$$\begin{aligned} P_t &= \frac{D_{t+1} + P_{t+1}}{R_{t+1}} \\ &= \frac{D_{t+1}}{R_{t+1}} + \frac{D_{t+2}}{R_{t+1} R_{t+2}} + ... \\ &= \sum_{k=1}^{\infty} \frac{D_{t+k}}{R_{t,t+k}}, \end{aligned}$$

where $R_{t,t+k}$ is the **realized** return on the asset (not the risk-free rate!!!) from t to $t+k$:

$$R_{t,t+k} = R_{t+1} \times ... \times R_{t+k}.$$

Log Returns.

- Use lowercase letters for logs: $r_t = \log(1 + R_t)$, $p_t = \log(P_t), d_t = \log(D_t)$:
- Compute log-return as:

$$
\begin{aligned}
r_{t+1} &= \log(P_{t+1} + D_{t+1}) - \log(P_t) \\
&= p_{t+1} + \log\left(1 + \frac{D_{t+1}}{P_{t+1}}\right) - p_t \\
&= p_{t+1} - p_t + \log(1 + \exp(d_{t+1} - p_{t+1}))
\end{aligned}
$$

Approximation

- Do a first-order Taylor approximation to the function $f(x) = \log(1 + \exp(x))$ around $x = \overline{d} - \overline{p}$ where a denotes the sample average value:

$$\log\left(1 + \exp\left(d_{t+1} - p_{t+1}\right)\right) \simeq k + (1 - \rho)\left(d_{t+1} - p_{t+1}\right),$$

with:

$$\begin{aligned} \rho &= \frac{1}{1 + \exp\left(\overline{d - p}\right)} \\ k &= -\log \rho + (1 - \rho)\log(1/\rho - 1). \end{aligned}$$

- This yields:

$$r_{t+1} = \rho p_{t+1} - p_t + k + (1 - \rho) d_{t+1},$$

$$p_t = \rho p_{t+1} + k + (1 - \rho) d_{t+1} - r_{t+1}.$$

PDV

- Iterating forward yields:

$$\begin{aligned} p_t &= \frac{k}{1-\rho} + \sum_{j\geq 1} \rho^{j-1} \left((1-\rho)\, d_{t+j} - r_{t+j} \right), \\ &= \frac{k}{1-\rho} + \sum_{j\geq 0} \rho^{j} \left((1-\rho)\, d_{t+1+j} - r_{t+1+j} \right). \end{aligned}$$

- This identity holds for any returns, prices and dividends ex-post. (And thus ex-ante, if you take conditional expectations of this equation.) It is an accounting identity without any behavorial assumption.

Price-Dividend Ratio:

- From the log-linearized return equation we have:

$$p_t - d_t = \rho(p_{t+1} - d_{t+1}) + k + \Delta d_{t+1} - r_{t+1}.$$

- Iterating forward:

$$p_t - d_t = \frac{k}{1-\rho} + \sum_{j \geq 0} \rho^j \left(\Delta d_{t+1+j} - r_{t+1+j}\right)$$

- Variation in the price-dividend ratio occurs because of variation in dividend growth or discount factors.

Return innovations

- Start with the return equation

$$r_{t+1} = \rho p_{t+1} - p_t + k + (1-\rho) d_{t+1},$$

  Now apply apply $E_{t+1} - E_t$ to both sides:

$$r_{t+1} - E_t r_{t+1} = \rho \left(p_{t+1} - E_t p_{t+1} \right) + (1-\rho) \left(d_{t+1} - E_t d_{t+1} \right)$$

- From the price-dividend expression

$$\begin{aligned} p_{t+1} - E_t p_{t+1} &= \left(E_{t+1} - E_t \right) \sum_{j \geq 0} \rho^j \left(\Delta d_{t+2+j} - r_{t+2+j} \right) \\ &+ d_{t+1} - E_t d_{t+1} \end{aligned}$$

  which implies:

$$\begin{aligned} r_{t+1} - E_t r_{t+1} &= \rho \left(E_{t+1} - E_t \right) \sum_{j \geq 0} \rho^j \left(\Delta d_{t+2+j} - r_{t+2+j} \right) \\ &+ \left(d_{t+1} - E_t d_{t+1} \right) \end{aligned}$$

Return innovations

- We then have the expression

$$\begin{aligned} r_{t+1} - E_t r_{t+1} &= (E_{t+1} - E_t) \sum_{s \geq 0} \rho^s \Delta d_{t+1+s} \\ &- (E_{t+1} - E_t) \sum_{s \geq 1} \rho^s r_{t+1+s}. \end{aligned}$$

- An unexpectedly good stock return must occur because either the current dividend went up, or expectations of future dividends go up, or because expectations of future returns go down.
- The first two terms are a standard "cash flow effect" and the second is an expected return or risk premium effect: the price goes up if the risk premium or risk-free interest rate go down.

VAR model

- Hence, stock price volatility can come from either volatility of future dividends or volatility of expected future returns. Which of these terms contribute more to volatility empirically?
- Fit a VAR to dividend growth and returns and use the VAR to compute the implied decomposition:

$$\begin{pmatrix} \Delta d_{t+1} \\ r_{t+1} \\ x_{t+1} \end{pmatrix} = A(L) \begin{pmatrix} \Delta d_t \\ r_t \\ x_t \end{pmatrix} + \begin{pmatrix} \varepsilon^d_{t+1} \\ \varepsilon^r_{t+1} \\ \varepsilon^x_{t+1} \end{pmatrix},$$

- Iterating on this VAR one can compute the forecast $E_t \Delta d_{t+1+j}$ and then the revision of the forecast.

Companion form:

- Let
$$y_t = (\Delta d_{t+1}, r_{t+1}, x_{t+1})'$$
and define the 3px1 vector:
$$z_t = [y_t..y_{t-p}]'$$
- Then z_t follows an AR1 process:
$$z_{t+1} = Az_t + \nu_{t+1}$$
with
$$\nu_t = \left[\varepsilon_t' \;\; 0..0\right], \quad E\varepsilon_t\varepsilon_t' = \Sigma.$$

Present values with companion form:

- The expected PDV can be computed as:

$$E_t \sum_{j \geq 0} \beta^j z_{t+j} = \sum_{j \geq 0} \beta^j A^j z_t = (I - \beta A)^{-1} z_t$$

- Premultiplying by the correct row vector to obtain the forecast of the PV of a component of z.

Innovations to PDV

- Using this PDV we have

$$\begin{aligned}(E_{t+1} - E_t)\sum_{j\geq 0}\beta^j z_{t+j} &= z_t + \beta(I-\beta A)^{-1} z_{t+1} - (I-\beta A)^{-1} z_t \\ &= z_t + \beta(I-\beta A)^{-1}(Az_t + \varepsilon_{t+1}) \\ &- (I-\beta A)^{-1} z_t\end{aligned}$$

- The terms involving z_t cancel so that

$$(E_{t+1} - E_t)\sum_{j\geq 0}\beta^j z_{t+j} = \beta(I-\beta A)^{-1}\varepsilon_{t+1}.$$

Results

- Dividends are difficult to predict but returns are predictable as we saw before.
- This implies that changes in discount rates account for all of the volatility of revisions to returns.
- Since the risk-free interest rate does not move much in the data, it means the changes in expected returns are mainly changes in risk premia.
- Implication – we need to study models with time-varying risk premia
    - Campbell-Cochrane, which has time-varying risk aversion.
    - Stochastic volatility in consumption growth which leads to higher risk aversion when consumption volatility is high vs low.

A Simple Model of Predictability:

- Stochastic processes:

$$
\begin{aligned}
x_t &= b x_{t-1} + \delta_t \\
r_{t+1} &= x_t + \varepsilon^r_{t+1} \\
\Delta d_{t+1} &= \varepsilon^d_{t+1}
\end{aligned}
$$

Model Solution:

- Price-dividend ratio:

$$d_t - p_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} \left(r_{t+j} - d_{t+j} \right) = \frac{x_t}{1 - \rho b}$$

- Returns:

$$R_{t+1} = \left(1 + \frac{P_{t+1}}{D_{t+1}} \right) \frac{D_{t+1}/D_t}{P_t/D_t}$$

Log-linearization:

$$r_{t+1} = \rho \left(p_{t+1} - d_{t+1} \right) + \Delta d_{t+1} - \left(p_t - d_t \right)$$

- Prices:

$$\Delta p_{t+1} = - \left(d_{t+1} - p_{t+1} \right) + \left(d_t - p_t \right) + \Delta d_{t+1}$$

VAR representation:

$$
\begin{aligned}
d_{t+1} - p_{t+1} &= b\left(d_t - p_t\right) + \frac{1}{1-\rho b}\delta_{t+1} \\
r_{t+1} &= (1-\rho b)\left(d_t - p_t\right) + \left(\varepsilon^d_{t+1} - \frac{\rho}{1-\rho b}\delta_{t+1}\right) \\
\Delta p_{t+1} &= (1-b)\left(d_t - p_t\right) + \left(\varepsilon^d_{t+1} - \frac{\rho}{1-\rho b}\delta_{t+1}\right)
\end{aligned}
$$

Implications for volatility:

- Assume $\rho = 0.96$ then estimated coefficient of returns on dividend price ratio is $1 - \rho b$ where b is autocorrelation of returns. This implies $b = 0.9$.
- Assume $D/P = 0.04$. Returns as a function of levels:

$$r_{t+1} = \frac{1 - \rho b}{D/P} \frac{D_t}{P_t} + \varepsilon_{rt+1}$$

  - If $b = 0$, a 1% rise in the D/P ratio implies that returns must rise 25%.
  - If $b = 0.9$ then $1 - b = 0.1$ and $1 - \rho b = 0.14$ and a 1% rise in D/P implies prices and returns rise by 2.5 and 3.4% respectively.
  - If $b = 0.96$ then a 1% rise in D/P implies prices and returns rise by 1% and 2% respectively. (This seems like upper-bound on persistence).
- Result: iid dividend growth and persistent price-dividend ratio implies returns must respond more than one for one to the dividend-price change.

Volatility

- Also

$$\sigma(d-p) = \frac{1}{1-\rho b}\sigma(x)$$
$$= 7.4\sigma(x)$$

  Dividend-price ratio should be much more volatile than returns.
- Persistence in returns implies that small variation in returns can have a large effect on the dividend-price ratio – similar to Gordon growth formula

$$P = \frac{D}{r-g}$$