Uncertainty, Financial Frictions, and Irreversible Investment

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Empirical Observations

- Economic uncertainty is time-varying and countercyclical.
  Campbell et al. (2001); Storesletten et al. (2004); Eisfeldt & Rampini (2006);
  Bloom (2009); Bloom et al. (2011)

- Credit spreads on corporate bonds are countercyclical.
  Gertler & Lown (1999); Gilchrist, Yankov & Zakrajšek (2009)

- Credit spreads predict economic activity.
  Philippon (2009); Gilchrist & Zakrajšek (2012); Faust et al. (2012); Bleaney et al. (2012)
Motivation

- Existing literature:
  - Investment-uncertainty nexus motivated by irreversibility.
    Bernanke (1983); Abel & Eberly (1994,1996); Caballero & Bertola (1994); Caballero & Pindyck (1996); Bloom (2009); Bloom et al. (2011)
  - We examine the interaction between uncertainty and investment in the context of imperfect financial markets and irreversibility.
  - We also analyze macroeconomic implications of fluctuations in capital liquidity.
    Shleifer & Vishny (1992); Eisfeldt (2004); Manso (2008)
Uncertainty, Financial Frictions, and Irreversible Investment

Introduction

- Standard debt contract:
  - Payoff from holding a risky bond is a concave function of the (stochastic) project return.

- Mean-preserving spread in the distribution of shocks:
  - Perfect financial markets:
    - expected defaults $\uparrow \Rightarrow$ credit spreads $\uparrow \Rightarrow$ no impact on $I$
  - Imperfect financial markets:
    - expected defaults $\uparrow \Rightarrow$ credit spreads $\uparrow \Rightarrow$ cost of capital $\uparrow \Rightarrow I \downarrow$
Our Paper

- Provides new empirical evidence on the link between uncertainty, business investment, and credit spreads.
- Develops a quantitative GE model that replicates key empirical relationships in the data:
  - Embeds a costly reversible investment framework in a GE model with frictions in both the debt and equity markets.
  - Generalizes previous GE frameworks.
    - Kiyotaki & Moore (1997); BGG (1999); Jermann & Quadrini (2009)
  - Allows for heterogeneity across firms in the economy.
    - Chugh (2010); Arellano, Bai & Kehoe (2010); Kahn & Thomas (2010); Midrigan & Xu (2010); Christiano et al. (2013)
Key Results

- The impact of uncertainty shocks on business investment occurs primarily through changes in credit spreads.
- Model implications in response to uncertainty shock:
  - Financial frictions magnify the impact of uncertainty shocks relative to a model with irreversible investment only.
  - Negative comovement between credit spreads and investment.
  - Positive comovement between net worth and investment.
- Model also implies substantial economic fluctuations in response to capital liquidity shocks.
A New Uncertainty Proxy

- There is no objective measure of uncertainty.
- Informational and/or contractual frictions can generate countercyclical dispersion of economic returns.
  
  Eisfeldt & Rampini (2006); Jurado et al. (2013)
- Use information from the stock market to infer fluctuations in uncertainty:
  - Cross Section: 11,303 U.S. nonfinancial corporations
  - Time Series: July 1, 1963 to September 30, 2012
- Use a standard asset pricing framework to purge our uncertainty proxy of forecastable variation.
THREE-STEP ESTIMATION PROCEDURE

- Standard (linear) factor model of asset returns:

\[(R_{itd} - r_{td}^f) = \alpha_i + \beta'_{i}f_{td} + u_{itd}\]

  - **Risk factors**: market excess return, SMB, HML, MOM

- Idiosyncratic uncertainty:

\[\sigma_{it} = \left(\frac{1}{D_t} \sum_{d=1}^{D_t} (\hat{u}_{itd} - \bar{u}_{it})^2\right)^{1/2}; \quad \bar{u}_{it} = \frac{1}{D_t} \sum_{d=1}^{D_t} \hat{u}_{itd}\]

- Dynamic volatility model:

\[\log \sigma_{it} = \gamma_i + \delta_i t + \rho \log \sigma_{i,t-1} + v_t + \epsilon_{it}\]

  - **Benchmark uncertainty estimate**: \(\hat{v}_t, t = 1, \ldots, T\).
Uncertainty, Financial Frictions, and Irreversible Investment

Empirics

**Uncertainty & Credit Spreads**

![Graph showing Uncertainty and Credit Spreads](image)

**Note:** Credit spread is the (nonfinancial) 10-year BBB-Treasury spread.
SVAR Analysis

- 8-variable VAR(4) system:
  - $i_t = \log$ of real business fixed investment
  - $c_t^D = \log$ of real PCE on durable goods
  - $c_t^N = \log$ of real PCE on nondurable goods & services
  - $y_t = \log$ of real GDP
  - $p_t = \log$ of the GDP price deflator
  - $\hat{v}_t =$ economic uncertainty
  - $s_t = 10$-year BBB-Treasury corporate bond spread
  - $m_t =$ effective (nominal) federal funds rate

- Implications of two types of shocks:
  - **Uncertainty**: orthogonalized innovations in $\hat{v}_t$
  - **Financial**: orthogonalized innovations in $s_t$

- **Identification Scheme I**: $(i_t, c_t^D, c_t^N, y_t, p_t, \hat{v}_t, s_t, m_t)$
- **Identification Scheme II**: $(i_t, c_t^D, c_t^N, y_t, p_t, s_t, \hat{v}_t, m_t)$
IMPLICATIONS OF AN UNCERTAINTY SHOCK

Identification scheme I

**Business fixed investment**

**PCE - durables**

**PCE - nondurables & services**

**GDP**

**Uncertainty**

**Credit spread**

**Quarters after shock**

---

**Percent**

---

**Quarters after shock**

---

**Percent**

---

**Quarters after shock**

---

**Percent**

---

**Quarters after shock**

---

**Uncertainty**

---

**Credit spread**

---

**NOTE**: The shaded bands represent the 95-percent confidence intervals.
IMPLICATIONS OF A FINANCIAL SHOCK

Identification scheme I

- Business fixed investment
- PCE - durables
- PCE - nondurables & services
- GDP
- Uncertainty
- Credit spread

Quarters after shock:
- 0  2  4  6  8 10 12

Percent:
- -2.5  -2.0  -1.5  -1.0  -0.5  0.0  0.5  1.0

Percentage points:
- -0.8  -0.6  -0.4  -0.2  0.0  0.2  0.4  0.6  0.8

NOTE: The shaded bands represent the 95-percent confidence intervals.
Implications of an Uncertainty Shock

Identification scheme II

NOTE: The shaded bands represent the 95-percent confidence intervals.
IMPLICATIONS OF A FINANCIAL SHOCK

Identification scheme II

NOTE: The shaded bands represent the 95-percent confidence intervals.
Firm-Level Panel Analysis

- Examine the link between uncertainty, credit spreads, and business investment using a **firm-level** panel dataset.
- CRSP/Compustat panel of U.S. nonfinancial firms matched with prices of outstanding corporate bonds traded in the secondary market.
- Lehman/Warga & Merrill Lynch issue-level data:
  - **Sample period**: Jan1973–Sep2012 (month-end)
  - 1,164 U.S. nonfinancial issuers
  - 6,725 senior unsecured, fixed-coupon issues
  - 385,062 bond/month observations
  - **Information**: price, issue date, maturity, coupon, issue size, etc.
Credit-spread regression:

\[
\log s_{it}[k] = \beta_1 \log \sigma_{it} + \beta_2 R^E_{it} + \beta_3 [\Pi/A]_{it} + \beta_4 \log[D/E]_{i,t-1} + \theta' X_{it}[k] + \epsilon_{it}[k]
\]

- \( s_{it}[k] \) = credit spread on bond \( k \) (issued by firm \( i \))
- \( [D/E]_{i,t} \) = debt-to-equity ratio
- \( R^E_{it} \) = realized return on equity
- \( [\Pi/A]_{it} \) = OIBDA-to-assets ratio
- \( X_{it}[k] \) = bond-specific control variables
  (par value, coupon, duration, age, callable indicator)
### Uncertainty & Credit Spreads

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log \sigma_{it}$</td>
<td>0.730</td>
<td>0.459</td>
<td>0.484</td>
<td>0.216</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.046)</td>
<td>(0.049)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$R_{it}^E$</td>
<td>-0.095</td>
<td>-0.112</td>
<td>-0.109</td>
<td>-0.053</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.025)</td>
<td>(0.024)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$[\Pi/A]_{it}$</td>
<td>-4.100</td>
<td>-1.835</td>
<td>-1.500</td>
<td>-1.318</td>
</tr>
<tr>
<td></td>
<td>(0.698)</td>
<td>(0.502)</td>
<td>(0.475)</td>
<td>(0.385)</td>
</tr>
<tr>
<td>$\log[D/E]_{i,t-1}$</td>
<td>0.212</td>
<td>0.056</td>
<td>0.049</td>
<td>0.078</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.474</td>
<td>0.641</td>
<td>0.648</td>
<td>0.797</td>
</tr>
<tr>
<td>$p$-value: credit rating effects</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$p$-value: industry effects</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$p$-value: time effects</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**NOTE**: Robust standard errors in parentheses.
Uncertainty, Credit Spreads & Investment

- Investment regression:

\[ \log\left(\frac{I}{K}\right)_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta \log Z_{it} + \eta_i + \lambda_t + \epsilon_{it} \]

- Investment fundamentals \((Z)\):
  - \([Y/K]_{it} = \text{sales-to-capital ratio}\)
  - \([\Pi/K]_{it} = \text{operating-income-to-capital ratio}\)
  - \(Q_{it} = \text{Tobin’s Q}\)
  - \([I/K]_{i,t-1} = \text{lagged investment rate}\)
### Uncertainty, Credit Spreads & Investment

#### Static specification

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
<tbody>
<tr>
<td>log $\sigma_{it}$</td>
<td>-0.169</td>
<td>-0.081</td>
<td>-0.157</td>
<td>-0.036</td>
<td>0.022</td>
<td>-0.062</td>
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<tr>
<td></td>
<td>(0.036)</td>
<td>(0.034)</td>
<td>(0.034)</td>
<td>(0.035)</td>
<td>(0.033)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>log $s_{it}$</td>
<td></td>
<td></td>
<td>-0.206</td>
<td>-0.172</td>
<td>-0.152</td>
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<td></td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td></td>
</tr>
<tr>
<td>log $[Y/K]_{it}$</td>
<td>0.558</td>
<td></td>
<td></td>
<td>0.535</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td></td>
<td></td>
<td>(0.045)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log $[\Pi/K]_{it}$</td>
<td></td>
<td>1.166</td>
<td></td>
<td></td>
<td>1.075</td>
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<td></td>
<td></td>
<td>(0.086)</td>
<td></td>
<td></td>
<td>(0.088)</td>
<td></td>
</tr>
<tr>
<td>log $Q_{i,t-1}$</td>
<td></td>
<td></td>
<td>0.715</td>
<td></td>
<td></td>
<td>0.645</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.040)</td>
<td></td>
<td></td>
<td>(0.041)</td>
</tr>
<tr>
<td>$R^2$ (within)</td>
<td>0.325</td>
<td>0.307</td>
<td>0.297</td>
<td>0.349</td>
<td>0.323</td>
<td>0.310</td>
</tr>
</tbody>
</table>

**NOTE:** Robust standard errors in parentheses.
## Uncertainty, Credit Spreads & Investment

### Dynamic specification

<table>
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<th>Explanatory Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log $\sigma_{it}$</td>
<td>-0.272</td>
<td>-0.179</td>
<td>-0.199</td>
<td>-0.123</td>
<td>-0.078</td>
<td>-0.106</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.059)</td>
<td>(0.060)</td>
<td>(0.057)</td>
<td>(0.054)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>log $s_{it}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.101</td>
<td>-0.068</td>
<td>-0.080</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>log $[I/K]_{i,t-1}$</td>
<td>0.568</td>
<td>0.576</td>
<td>0.538</td>
<td>0.565</td>
<td>0.567</td>
<td>0.535</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.023)</td>
<td>(0.029)</td>
<td>(0.027)</td>
<td>(0.023)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>log $[Y/K]_{it}$</td>
<td>0.446</td>
<td>-</td>
<td>-</td>
<td>0.452</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td></td>
<td></td>
<td>(0.053)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log $[\Pi/K]_{it}$</td>
<td>-</td>
<td>0.918</td>
<td>-</td>
<td>-</td>
<td>0.908</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.144)</td>
<td></td>
<td></td>
<td>(0.135)</td>
<td></td>
</tr>
<tr>
<td>log $Q_{i,t-1}$</td>
<td>-</td>
<td>-</td>
<td>0.548</td>
<td>-</td>
<td>-</td>
<td>0.507</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.045)</td>
<td></td>
<td></td>
<td>(0.042)</td>
</tr>
</tbody>
</table>

**NOTE**: Robust standard errors in parentheses.
SUMMARY OF EMPirical Evidence

- Three stylized facts:
  - Fluctuations in uncertainty can have a large effect on aggregate investment.
  - The impact of uncertainty on business investment occurs largely through changes in credit spreads.
  - Financial shocks have a strong effect on aggregate investment, irrespective of the level of uncertainty.

- Implications: Financial frictions are an important part of the transmission mechanism through which fluctuations in uncertainty are propagated to the real economy.
**Agents & Technological Environment**

- **Representative household**: consumes, works, and saves by investing in stocks and corporate bonds.
- **Heterogeneous firms**: use DRS technology to produce final-good output and accumulate capital.
  - Production subject to persistent aggregate and idiosyncratic technology shocks.
  - Volatility of idiosyncratic technology shocks is time varying.
  - Nonconvex capital adjustment costs:
    - fixed costs
    - costly reversibility $\Rightarrow$ purchase price of capital $> \text{sale price of capital}$
  - Liquidation value of capital follows a stochastic process $\Rightarrow$ capital liquidity shocks.
The household earns a competitive real market wage $w$ by working $h$ hours and saves by purchasing bonds and equity shares of firms in the economy.

Household preferences:

$$u(c, h) = \frac{c^{1-\theta} - 1}{1 - \theta} - \zeta \frac{h^{1+\vartheta}}{1 + \vartheta};$$
**Production Technology**

- Decreasing returns-to-scale and fixed costs of production:
  \[
  y = (az)^{(1-\alpha)\chi} (k^\alpha h^{1-\alpha})^\chi - F_0 k
  \]
  - \(a\) = aggregate technology shock
  - \(z\) = idiosyncratic technology shock
  - \(\chi\) = degree of DRS in production
  - \(F_0\) = fixed operation costs

- Profits are linear in \(a\) and \(z\):
  \[
  \pi(a, z, w, k) = \max_h \left\{ (az)^{(1-\alpha)\chi} (k^\alpha h^{1-\alpha})^\chi - F_0 k - wh \right\}
  \]
  \[
  = az \psi(w) k^\gamma - F_0 k
  \]
**Technology Shocks**

- Aggregate technology shock:
  \[ \log a' = \rho_a \log a + \log \epsilon'_a; \quad \epsilon'_a \sim N(-0.5\omega_a^2, \omega_a^2) \]

- Idiosyncratic technology shock: \(N\)-state Markov chain process with time-varying volatility
  \[ \log \sigma'_z = (1 - \rho_\sigma) \log \tilde{\sigma}_z + \rho_\sigma \log \sigma_z + \epsilon'_\sigma; \quad \epsilon'_\sigma \sim N(-0.5\omega_\sigma^2, \omega_\sigma^2) \]
  - Fluctuations in \(\sigma_z\) do not affect the conditional expectation of \(z'\).
  - An increase in \(\sigma_z\) represent a mean-preserving spread of \(z'\).
**Capital Accumulation**

- Nonconvex capital adjustment:

$$p(k', k) = F_k k \times 1[k' \neq (1 - \delta)k]$$
  $$+ (p^+ \times 1[k' \geq (1 - \delta)k]$$
  $$+ p^- \times 1[k' \leq (1 - \delta)k]) (k' - (1 - \delta)k)$$

  - $F_k = \text{fixed investment adjustment costs}$
  - $p^+ = \text{purchase price of capital}$
  - $p^- = \text{liquidation price of capital}$
  - $p^- / p^+ < 1 \Rightarrow \text{capital specificity}$

- Liquidation price of capital $p^-:$

$$\log p^- = (1 - \rho_{p^-}) \log \tilde{p}^- + \rho_{p^-} \log p^- + \epsilon'_{p^-}$$

  - $\log \epsilon'_{p^-} \sim N(-0.5\omega_\kappa^2, \omega_\kappa^2) = \text{capital liquidity shock}$
Financial Distortions

- Moral hazard and limited liability in credit markets.
- Issuance costs in equity markets.
- Implications:
  - Full set of capital structure choices (i.e., debt vs. equity vs. internal funds)
  - Strategic default and debt renegotiation (i.e., Chapter 11 bankruptcy)
**Net Worth**

- Realized net worth next period equals the sum of net profits and the market value of undepreciated capital less the face value of debt:

\[ n' = a'z'\psi(w(s'))k'^{\gamma} - F_0k' + p'^{-}(1 - \delta)k' - b'. \]

  - The value of capital follows a stochastic process and entails a discount in the amount of \(1 - p'^{-}/p^+\).

- Net liquid asset position:

\[ x'(\sigma_z) \equiv a'z'(\sigma_z)\psi(w')k'^{\gamma} - F_0k' - b' = n'(\sigma_z) - p'^{-}(1 - \delta)k' \]

- Value of the firm: \( v = v_i(k, x; s) \)
**Endogenous Default**

- Limited liability: lower bound on net worth $\bar{n}$
- Default level of idiosyncratic technology, conditional on the next period’s aggregate state $s'$ and individual state $(k', b')$:

$$z^D(k', b'; s') \equiv \bar{n} + b' + F_k k' - p^{-'}(1 - \delta)k' a' \psi(w')k'\gamma$$

- Set of default states:

$$\mathcal{D} = \{ j | j \in \{1, \ldots, N\} \text{ and } z^j_{j}(\sigma_z) \leq z^D(k', b'; s') \}$$

- Firm defaults if and only if $z^j_{j}(\sigma_z) \in \mathcal{D}$
**Debt Renegotiation**

- With limited liability, renegotiated debt equals the amount consistent with the lower bound of the net worth $\bar{n}$:

$$b^R \leq \bar{b}(k', z'(\sigma_z); s') = a'z'^D(\sigma_z)\psi(w')k'^\gamma - F_0k' + p^{-}(1 - \delta)k'$$

- No bargaining power for firm implies the maximum recovery in equilibrium:

$$b^R = \bar{b}(k', z'(\sigma_z); s')$$

- Subject to bankruptcy costs: $\xi(1 - \delta)k'$; $0 < \xi < 1$
Bond Financing

- Recovery rate in the case of default:
  \[ R(k', b', z'(\sigma_z); s') = \frac{\overline{b}(k', z'(\sigma_z), s')}{b'} - \xi(1 - \delta) \frac{k'}{b'} \]

- Bond price:
  \[ q_i(k', b'; s') = \mathbb{E} \left\{ m(s, s') \left[ 1 + \sum_{j \in D} p_{i,j} [R(k', b', z'_j(\sigma_z); s') - 1] \right] \bigg| s \right\} \]
Letting $e$ represent equity issuance, then dividends satisfy:

$$d \equiv az_i\psi(w)k^\gamma - F_0k - p(k', k) - b + q_i(k', b'; s')b' + e$$

- Frictions in equity market:
  - Dividend constraint:
    $$d \geq \bar{d} \geq 0$$
  - Equity dilution costs:
    $$\bar{\varphi}(e) \equiv e + \varphi \max\{e, 0\}; \quad 0 < \varphi < 1$$
Firm Value Maximization Problem

Discrete choice problem:

\[ v_i(k, x; s) = \max \left\{ v_i^+(k, x; s), v_i^-(k, x; s) \right\}, \]

where

\[ v_i^+(k, x; s) = \min_{\phi, \lambda^+} \max_{d^+, e^+, k^+, b^+} \left\{ d^+ + \phi(d^+ - d) - \varphi(e^+) + \lambda^+[k^+ - (1 - \delta)k] \right\} \]

\[ + \eta \mathbb{E} \left[ m(s, s') \sum_{j=1}^{N} p_{i,j} \max \left\{ v_j(k^+, x^+(\sigma_z); s'), v_j(k^+, x^{R+}(\sigma_z); s') \right\} \right] \]

\[ v_i^-(k, x; s) = \min_{\phi, \lambda^-} \max_{d^-, e^-, k^-, b^-} \left\{ d^- + \phi(d^- - d) - \bar{\varphi}(e^-) - \lambda^-[k^- - (1 - \delta)k] \right\} \]

\[ + \eta \mathbb{E} \left[ m(s, s') \sum_{j=1}^{N} p_{i,j} \max \left\{ v_j(k^-, x^-(\sigma_z); s'), v_j(k^-, x^{R-}(\sigma_z); s') \right\} \right] \]
OPTIMAL CAPITAL STRUCTURE

FOC: equity issuance:

\[ 1 + \phi = 1 + \varphi \times 1[e > 0] \]

FOC: debt issuance:

\[ q_i(k', b'; s) + q_{i,b}(k', b'; s)b' = \eta \mathbb{E} \left[ m(s, s') \sum_{j \in D^c} p_{i,j} \left( \frac{1 + \phi'}{1 + \phi} \right) | s \right] \]

\[ = \eta \mathbb{E} \left[ m(s, s') \sum_{j \in D^c} p_{i,j} \left( \frac{1 + \varphi \times 1(e' > 0)}{1 + \varphi \times 1(e > 0)} \right) | s \right] \]
Optimal Capacity Choice

Capital expansion problem

\[ Q_i^+(k, x; s) = \eta \mathbb{E} \left[ m(s, s') \sum_{j=1}^{N} p_{i,j} \left( \frac{1 + \phi_j'}{1 + \phi_i} \right) \times \left[ \pi_{j,k}(k^+; s') + (1 - \delta) Q'_j(k^+, x'(\sigma_z); s') \right] \mid s \right] \]

\[ + q_{i,k}(k^+, b^+; s) b^+ \]

\[ - \eta \mathbb{E} \left[ m(s, s') \sum_{j \in \mathcal{D}} p_{i,j} \left( \frac{1 + \phi_j'}{1 + \phi_i} \right) \left[ \pi_{j,k}(k^+; s') + (1 - \delta) p^{-j} \right] \mid s \right] \]
Optimal Capacity Choice

Capital contraction problem

\[
Q_i^{-}(k, x; s) = \eta \mathbb{E} \left[ m(s, s') \sum_{j=1}^{N} p_{i,j} \left( \frac{1 + \phi'_j}{1 + \phi_i} \right) \right.
\]
\[
\times \left[ \pi_{j,k}(k^-; s') + (1 - \delta)Q'_j(k^-, x'(\sigma_z); s') \right] \bigg| s \bigg] \bigg] 
\]
\[
+ q_{i,k}(k^-, b^-; s) b^- 
\]
\[
- \eta \mathbb{E} \left[ m(s, s') \sum_{j \in D} p_{i,j} \left( \frac{1 + \phi'_j}{1 + \phi_i} \right) \left[ \pi_{j,k}(k^-; s') + (1 - \delta)p'^- \right] \bigg| s \bigg] \right]
\]
Tobin’s Q

- Tobin’s Q:

\[ Q_i(k, x; s) = \min \left\{ p^+, \max \left\{ p^-, p^+ - \frac{\lambda_i^+(k, x; s)}{1 + \phi_i(k, x; s)} \right\} \right\} \]

\[ = \min \left\{ p^+, \max \left\{ p^-, p^- + \frac{\lambda_i^-(k, x; s)}{1 + \phi_i(k, x; s)} \right\} \right\} \]

- With partial irreversibility, \( Q \) is still monotonic and nonincreasing in \( k \), but truncated from above by \( p^+ \) and below by \( p^- \).
- With fixed investment adjustment costs, \( Q \) is no longer monotonic.
Optimal Investment Policy

- Generalized \((S, s)\) policy:

\[
k'_i(k, x; s) = \min \{k_i^-(k, x; s), \max\{k_i^+(k, x; s), (1 - \delta)k\}\}
\]

- \((S, s)\) capital targets: \(k_i^-(k, x; s), k_i^+(k, x; s)\) are functions of the firm’s overall financial condition \((k, x)\).
HOUSEHOLDS

Household maximization problem:

\[
W(s) = \max_{b', s', c, h} \left\{ u(c, h) + \beta \mathbb{E}[W(s') \mid s] \right\}
\]

subject to a budget constraint,

\[
c + \int (qb' + p_s s') \mu(dz, dk, dx) = wh + \int \left[ \tilde{R}^b + (d + \tilde{p}_s) s \right] \mu(dz, dk, dx) + \int F_{o,k} \mu(dz, dk, dx).
\]

where

\[
\tilde{R}^b \equiv \left[ 1 + \mathbf{1}(z \in D(k, b, z)) \times [R(k, b, z(\sigma_z)) - 1] \right] b
\]
Dynamic Efficiency Conditions

- Consistency between firm and household problems.
- Ex-dividend value of equity:
  \[
  p_s(k, x, z; s) = \mathbb{E} \left[ \beta \frac{u'(c', h')}{u(c, h)} \left[ d' - \bar{\phi}(e') + p_s'(k', x', z'; s') \right] \right] \left| z, s \right|
  \]
- Price of bond:
  \[
  q(k', b', z; s) = \mathbb{E} \left[ \beta \frac{u'(c', h')}{u(c, h)} \left[ 1 + 1[z' \in D'(k', b'; s')] \times [R'(k', b', z'; s') - 1] \right] \right] \left| z, s \right|
  \]
Market Clearing

- Market-clearing conditions:

  Stock market: \[ s' = s = 1 \]

  Labor market: \[ h^s(s) = \int h^d(z, k; s) \mu(dz, dk, dx) \]

  Goods market: \[ c(s) = \int y(z, k, h; s) \mu(dz, dk, dx) \]

  \[ \quad - \int p(k'(k, x, z; s), k) \mu(dz, dk, dx) \]

- Bond market clears by Warlas’ law.
LAW OF MOTION FOR $\mu$

- $\mu$ is the join distribution of $z \in Z$, $k \in K$, and $x \in X$:

$$
\mu'(Z, K, X, s') =
\int 1[z_j \in Z] \times 1[k'_i \in K] \times 1[x'_j(k'_i, \min\{b'_i, b^R\}, s) \in X]
\times p_{i,j} \mu(dz, dk, dx, s) G(s, ds')
$$

- Solution method for GE models with heterogeneous agents:
  Krusell & Smith (1998)
  - Agents use only a finite number of moments of $\mu$ to forecast equilibrium prices.
**Model Calibration**

- Idiosyncratic productivity process:
  \[
  \log Y_{it} = \theta \log K_{i,t-1} + \eta_i + \lambda_{st} + u_{it} \\
  u_{it} = \rho z u_{i,t-1} + \epsilon_{it}
  \]
  \[\alpha = 0.3 \text{ and } \hat{\theta} = 0.65 \Rightarrow \text{DRS parameter } \chi = 0.85\]

- Idiosyncratic uncertainty process:
  \[
  \log \left[ \sqrt{\frac{\pi}{2}} |\hat{\epsilon}_{it}| \right] = \gamma_i + \sigma_t + \zeta_{it}
  \]
  \[\text{Estimate: } \hat{\sigma}_t = \mu + \rho \sigma \hat{\sigma}_{t-1} + e_t \]
  \[\text{Calibration: } \tilde{\sigma} = 0.15; \rho_{\sigma} = 0.90; \omega_{\sigma} = 0.04\]
Uncertainty Based on Profitability Shocks

- Uncertainty (left scale)
- Credit spread (right scale)

Quarterly data from 1977 to 2010.
Calibration (cont.)

- Resale price of capital: $\tilde{p}^- = 0.5$; $\rho_{p^-} = 0.98$; $\omega_\kappa = 0.015$
- Purchase price of capital: $p^+ = 1$
- Bankruptcy costs: $\xi = 0.10$
- Equity flotation cost: $\varphi = 0.12$
- Survival probability: $\eta = 0.946$
- Quasi-fixed operating cost: $F_o = 0.05$
- Fixed costs of capital adjustment: $F_k = 0.01$
- Depreciation rate: $\delta = 0.045$
- Time discount factor: $\beta = 0.989$
Impact of an Aggregate Technology Shock

**Output, pct.**

**Consumption, pct.**

**Investment, pct.**

**Hours worked, pct.**

**Risk-free rate, bps.**

**Debt/Capital, pps.**

**Credit spreads, bps.**

**Inactive firms, pps.**

**NOTE:** Model w/ financial frictions. Model w/o financial frictions.
### Conditional Business Cycle Moments

Aggregate technology shocks only

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>STD($I$)</th>
<th>STD($C$)</th>
<th>STD($H$)</th>
<th>STD($Y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>With financial frictions</td>
<td>2.60</td>
<td>0.95</td>
<td>0.12</td>
<td>2.47</td>
</tr>
<tr>
<td>Without financial frictions</td>
<td>2.90</td>
<td>0.98</td>
<td>0.12</td>
<td>2.32</td>
</tr>
<tr>
<td><strong>Memo:</strong> Data</td>
<td>2.79</td>
<td>0.43</td>
<td>0.60</td>
<td>1.12</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>With financial frictions</td>
<td>0.63</td>
<td>0.99</td>
<td>0.46</td>
<td>0.54</td>
</tr>
<tr>
<td>Without financial frictions</td>
<td>0.53</td>
<td>0.99</td>
<td>0.22</td>
<td>0.53</td>
</tr>
<tr>
<td><strong>Memo:</strong> Data</td>
<td>0.63</td>
<td>0.56</td>
<td>0.65</td>
<td>0.43</td>
</tr>
</tbody>
</table>

**Note:** All variables are expressed in deviations from their respective steady-state values.
IMPACT OF AN UNCERTAINTY SHOCK

NOTE: Model w/ financial frictions. Model w/o financial frictions.
### Conditional Business Cycle Moments

#### Uncertainty shocks only

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>STD((I))</th>
<th>STD((C))</th>
<th>STD((H))</th>
<th>STD((Y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>With financial frictions</td>
<td>13.5</td>
<td>0.63</td>
<td>0.88</td>
<td>0.23</td>
</tr>
<tr>
<td>Without financial frictions</td>
<td>14.6</td>
<td>0.63</td>
<td>0.71</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Memo:</strong> Data</td>
<td>2.79</td>
<td>0.43</td>
<td>0.60</td>
<td>1.12</td>
</tr>
</tbody>
</table>

#### Comovements

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>With financial frictions</td>
<td>0.65</td>
<td>0.49</td>
<td>0.78</td>
<td>-0.33</td>
</tr>
<tr>
<td>Without financial frictions</td>
<td>0.76</td>
<td>0.71</td>
<td>0.78</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Memo:</strong> Data</td>
<td>0.63</td>
<td>0.56</td>
<td>0.65</td>
<td>0.43</td>
</tr>
</tbody>
</table>

**NOTE:** All variables are expressed in deviations from their respective steady-state values.
Impact of a Capital Liquidity Shock

Output, pct.

Consumption, pct.

Investment, pct.

Hours worked, pct.

Risk-free rate, bps.

Debt/Capital, pps.

Credit spreads, bps.

Inactive firms, pps.

Note: Model w/ financial frictions. Model w/o financial frictions.
## Conditional Business Cycle Moments

Capital Liquidity shocks only

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Relative Volatility</th>
<th>Comovements</th>
</tr>
</thead>
<tbody>
<tr>
<td>With financial frictions</td>
<td>6.71</td>
<td>0.60</td>
</tr>
<tr>
<td>Without financial frictions</td>
<td>14.3</td>
<td>0.63</td>
</tr>
<tr>
<td>Memo: Data</td>
<td>2.79</td>
<td>0.42</td>
</tr>
</tbody>
</table>

NOTE: All variables are expressed in deviations from their respective steady-state values.
### Cyclical Properties of Credit Spreads

Conditional on the Type of Shock

<table>
<thead>
<tr>
<th>Selected Correlations</th>
<th>Technology</th>
<th>Uncertainty</th>
<th>Liquidity</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr($S, Y$)</td>
<td>0.927</td>
<td>-0.811</td>
<td>-0.938</td>
<td>-0.457</td>
</tr>
<tr>
<td>Corr($S, I$)</td>
<td>0.626</td>
<td>-0.515</td>
<td>-0.577</td>
<td>-0.531</td>
</tr>
<tr>
<td>Corr($S, C$)</td>
<td>0.916</td>
<td>-0.368</td>
<td>-0.816</td>
<td>-0.498</td>
</tr>
<tr>
<td>Corr($S, NW$)</td>
<td>-0.813</td>
<td>0.802</td>
<td>-0.703</td>
<td>-0.297</td>
</tr>
<tr>
<td>Corr(STD($S$), $Y$)</td>
<td>0.933</td>
<td>-0.832</td>
<td>-0.950</td>
<td>-0.245</td>
</tr>
</tbody>
</table>

**NOTE:** All variables are expressed in deviations from their respective steady-state values.
Concluding Remarks

- Financial market frictions have an important effect on the investment-uncertainty nexus.
- The presence of financial market frictions significantly amplifies the response of investment and output to both uncertainty and liquidity shocks.
- An important extension of our framework would involve endogenizing the price of capital.
## SUMMARY STATISTICS

Selected bond characteristics

<table>
<thead>
<tr>
<th>Bond Characteristic</th>
<th>Mean</th>
<th>StdDev</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td># of bonds per firm/month</td>
<td>2.99</td>
<td>3.69</td>
<td>1.00</td>
<td>2.00</td>
<td>76.0</td>
</tr>
<tr>
<td>Mkt. value of issue (millions, $2005)</td>
<td>339.9</td>
<td>338.9</td>
<td>1.22</td>
<td>249.2</td>
<td>5,628</td>
</tr>
<tr>
<td>Maturity at issue (years)</td>
<td>12.8</td>
<td>9.2</td>
<td>1.0</td>
<td>10.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Term to maturity (years)</td>
<td>11.1</td>
<td>8.5</td>
<td>1.0</td>
<td>8.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Duration (years)</td>
<td>6.49</td>
<td>3.28</td>
<td>0.91</td>
<td>6.03</td>
<td>18.0</td>
</tr>
<tr>
<td>Credit rating (S&amp;P)</td>
<td>-</td>
<td>-</td>
<td>D</td>
<td>BBB1</td>
<td>AAA</td>
</tr>
<tr>
<td>Coupon rate (pct.)</td>
<td>7.06</td>
<td>2.14</td>
<td>0.60</td>
<td>6.88</td>
<td>17.5</td>
</tr>
<tr>
<td>Nominal effective yield (pct.)</td>
<td>7.23</td>
<td>2.98</td>
<td>0.22</td>
<td>6.92</td>
<td>30.0</td>
</tr>
<tr>
<td>Credit spread (bps.)</td>
<td>202</td>
<td>223</td>
<td>5</td>
<td>128</td>
<td>2,000</td>
</tr>
</tbody>
</table>
(S, s) INVESTMENT POLICY FUNCTION
Partial irreversibility only
(S, s) INVESTMENT POLICY FUNCTION

Partial irreversibility and fixed adjustment costs
CORPORATE BOND CREDIT SPREADS

Median
Interquartile range

Percentage points

Monthly


0 2 4 6 8 10