

Uncertainty, Financial Frictions, and Irreversible Investment

Simon Gilchrist*

Jae W. Sim[†]

Egon Zakrajšek[‡]

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Abstract

Using macro- and micro-level data, we establish three stylized facts: (1) fluctuations in idiosyncratic uncertainty can have a large effect on aggregate investment; (2) the impact of uncertainty on investment occurs largely through changes in credit spreads; and (3) financial shocks—identified vis-à-vis orthogonal innovations to credit spreads—have a strong effect on investment, irrespective of the level of uncertainty. These findings cast doubt on the traditional “wait-and-see” transmission channel of uncertainty shocks and point to financial distortions as the main mechanism through which fluctuation in uncertainty affect macroeconomic outcomes. We explore this hypothesis within a general equilibrium model, featuring heterogeneous firms that face time-varying uncertainty, nonconvex capital adjustment costs, and financial market frictions. We show that our model successfully replicates the stylized facts concerning the macroeconomic implications of uncertainty and financial shocks. By influencing the effective supply of credit, both shocks exert a powerful effect on aggregate investment and generate countercyclical credit spreads and procyclical leverage, dynamics consistent with the data and counter to those implied by the technology-driven real business cycle models.

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*DEPARTMENT OF ECONOMICS BOSTON UNIVERSITY AND NBER. Email: sgilchri@bu.edu

[†]OFFICE OF FINANCIAL STABILITY, FEDERAL RESERVE BOARD. Email: jae.w.sim@frb.gov

[‡]DIVISION OF MONETARY AFFAIRS, FEDERAL RESERVE BOARD. Email: egon.zakrajsek@frb.gov

1 Introduction

The countercyclical behavior of the cross-sectional dispersion of economic variables such as labor income, business cashflows, productivity, and stock returns—commonly used to measure time-varying economic uncertainty—is one of the stylized facts of business cycle fluctuations.¹ In macroeconomics, the framework of irreversible investment provides the traditional channel through which changes in uncertainty—by altering the “option value” of investing—affect economic activity; see, for example, Bernanke [1983]; Bertola and Caballero [1994]; Abel and Eberly [1994, 1996]; Caballero and Pindyck [1996]; and more recently, Bloom [2009] and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry [2011].²

Financial distortions, reflecting departures from the Modigliani-Miller paradigm of frictionless financial markets, create an additional channel through which fluctuations in uncertainty can affect macroeconomic outcomes. In the canonical framework used to price risky debt (Merton [1974]), the return on levered equity resembles—under limited liability—the payoff of a call option, whereas the bondholders face the payoff structure that mimics that of an investor writing a put option. An increase in the riskiness of the firm’s assets thus benefits equity holders at the expense of bondholders, implying a rise in the default-risk premium to compensate bondholders for heightened uncertainty. To the extent that external finance—both through the debt and equity markets—is subject to agency and/or moral hazard problems, an increase in uncertainty will raise the user cost of capital, inducing a decline in investment spending.

In this paper, we develop a quantitative model that allows us to analyze the interaction of uncertainty and irreversible investment in a general equilibrium setting featuring an array of financial market frictions. Specifically, we embed a tractable bond-contracting model into a capital accumulation problem, in which heterogeneous firms employ a decreasing returns-to-scale production technology that is subject to a persistent idiosyncratic shock, the variance of which is allowed to vary over time according to a stochastic law of motion. The capital accumulation problem encompasses two forms of nonconvex adjustment frictions emphasized in the work of Abel and Eberly [1994, 1996], Caballero, Engel, and Haltiwanger [1995], Doms and Dunne [1998], Caballero [1999], Cooper, Haltiwanger, and Powers [1999], and Cooper and Haltiwanger [2006]: partial irreversibility and fixed adjustment costs.

On the financial side, firms make investment decisions subject to a full range of choices regarding their capital structure—internal funds, debt, and equity financing—in an environment where external funds are costly because of agency problems in financial markets.³ The partial invest-

¹See, for example, Campbell and Taksler [2003], Storesletten, Telmer, and Yaron [2004], Eisfeldt and Rampini [2006], and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry [2011].

²Despite its intuitive appeal, the effect of uncertainty on investment in the presence of irreversibilities can be theoretically ambiguous. As shown, for example, by Abel [1983], Veracierto [2002], and Bachmann and Bayer [2009], the effect depends importantly on the assumptions regarding the initial accumulation of capital, market structure, and the equilibrium setting.

³Cooley and Quadrini [2001], Hennessy and Whited [2007], and Philippon [2009] consider similar contracting frameworks, though only in partial equilibrium. Bernanke, Gertler, and Gilchrist [1999] do allow for general equilibrium feedback effects but consider only debt financing.

ment irreversibility plays two roles in our model. First, it creates the so-called option value of waiting. In response to increased uncertainty, the investment inaction region expands, which causes some firms to delay exercising their growth options—that is, they adopt the “wait-and-see” posture. Second, the implied illiquidity of capital assets limits the firm’s debt capacity because in the case of costly default, the liquidation of capital reduces the recovery value of corporate debt claims.

The main contribution of our paper is to examine in a unified framework the implications of the interaction between the capital and debt overhang problems for business cycle dynamics. Our quantitative general equilibrium model allows us to answer the following question: How much of the impact of fluctuations in uncertainty on aggregate investment reflects nonconvex frictions associated with irreversibility—the traditional wait-and-see effect—and how much of it can be attributed to distortions in financial markets?

We motivate this question by a set of new empirical results that highlight the relationship between uncertainty, investment, and credit spreads on corporate bonds, a commonly-used indicator of the degree of financial frictions. First, we construct a novel proxy for idiosyncratic uncertainty using high-frequency firm-level stock market data, a measure that arguably reflects exogenous changes in uncertainty, rather than the endogenous effects of informational and contractual frictions that have been theoretically linked to the countercyclical dispersion in economic returns (cf. Eisfeldt and Rampini [2006] and Jurado, Ludvigson, and Ng [2013]). We use this proxy to examine the interaction between economic activity, uncertainty, and credit spreads within a structural vector autoregression (SVAR) framework.

The results from this macro-level analysis indicate that credit spreads are an important conduit through which fluctuations in uncertainty are propagated to the real economy. Unanticipated increases in uncertainty lead to a significant widening of credit spreads and a decline in output that is driven primarily by the protracted drop in the investment component of aggregate demand. In contrast, financial disturbances—identified vis-à-vis orthogonalized shocks to credit spreads—have a large effect on output and investment, irrespective of the level of uncertainty in the economy.

We complement this analysis by constructing a firm-level panel data set that combines information on prices of individual corporate bonds trading in the secondary market with our estimates of firm-specific uncertainty and the issuers’ income and balance sheet statements. The results from this micro-level analysis are consistent with our earlier findings: Conditional on the firm’s leverage, profitability, and other indicators of creditworthiness, our firm-specific uncertainty proxy is an important determinant—both economically and statistically—of credit spreads on the firm’s outstanding bonds.

Furthermore, we find that conditional on investment fundamentals—that is, proxies for the marginal product of capital—increases in idiosyncratic uncertainty are associated with a substantial decline in the rate of capital formation. However, once the information content of credit spreads is taken into account, the impact of uncertainty on business investment is significantly attenuated. Capital expenditures, by contrast, remains highly responsive to movements in credit spreads. The

combination of our macro and micro-level analysis thus supports the notion that financial distortions are an important part of the mechanism through which uncertainty shocks affect business investment.

Simulations of our general equilibrium model accord well with the empirical results. By altering the effective supply of credit, financial distortions amplify the response of aggregate investment to uncertainty shocks. Although the risk-free rate falls in response to increased uncertainty, this easing of financial conditions is more than offset by the sharp and persistent increase in credit spreads. The resulting increase in the user cost of capital leads firms to slash capital expenditures and delever, creating a quantitatively important channel through which fluctuations in uncertainty shape business cycle dynamics. In an economy without financial distortions, by contrast, this user cost of capital channel is completely absent, and the response of aggregate investment to uncertainty shocks is an amalgam of the wait-and-see decisions of individual firms.

The combination of costly reversible investment and financial market frictions also allows us to identify a potential new source of aggregate disturbances: shocks to the liquidation value of capital. With partial irreversibility, an adverse shock to the resale value of capital will impinge on the debt capacity of firms by reducing the collateral value of capital assets, creating an interaction between the capacity and debt overhang problems. According to our simulations, these “liquidity” shocks have the potential of being an important source of cyclical fluctuations in an economy with financial frictions. A relatively small amount of variability in the resale value of capital generates plausible volatility in the model’s key endogenous aggregates and delivers realistic comovements between main macroeconomic quantities. Importantly, our simulations also show that when the business cycle is driven by either uncertainty or liquidity shocks, the level of credit spreads and their dispersion are strongly countercyclical, comovements consistent with the data.

In an economy with financial distortions, uncertainty and liquidity shocks induce a deterioration in the quality of borrowers’ balance sheets, which causes the effective supply of credit to shift inward, thus making quantities and spreads move in the opposite direction. Although our model does not include a formal financial intermediary sector, disturbances that affect the collateral value of capital assets can be viewed as a tractable way to model disruptions in the credit-intermediation process arising from shocks that impair the “risk-bearing” capacity of financial institutions. Thus our model generates macroeconomic dynamics that are consistent with the empirical evidence of Stock and Watson [2012], who show that the toxic combination of adverse uncertainty and credit supply shocks were the two primary factors behind the collapse of U.S. economic activity during the “Great Recession.”

The roadmap for the remainder of the paper is as follows. Section 2 lays out the empirical evidence. In Section 3, we present the model. The calibration of the model is discussed in Section 4. The model-implied impulse response functions and conditional business cycle moments are presented in Section 5. In Section 6, we offer a brief conclusion.

2 Empirics

The absence of an objective and widely accepted measure of uncertainty complicates any empirical analysis aimed at estimating the effect of fluctuations in uncertainty on economic activity. Among the many proxies used in this burgeoning literature, we find the realized or option-implied volatility of equity returns; the cross-sectional dispersion of firm profits, stock returns, or productivity; the cross-sectional dispersion of survey-based forecasts of macroeconomic indicators; and indexes based on the frequency of “uncertainty-related” words or phrases that occur across a large number of news sources.

Given this paper’s focus on the interaction between uncertainty, financial conditions, and investment, we look to asset prices—which, in principle, should encompass all aspects of the firm’s environment that the investors view as important—to infer fluctuations in uncertainty. Moreover, by using information from financial markets—specifically from the stock market—we can rely on a standard asset pricing framework to purge our measure of uncertainty of forecastable variation. Using this proxy, we show that financial conditions, as summarized by the level of credit spreads, significantly influence the response of investment to fluctuations in uncertainty. In fact, our empirical analysis, which is based on both aggregate time series and firm-level panel data, indicates that the impact of uncertainty on investment occurs primarily through changes in credit spreads, a result consistent with the presence of significant financial frictions.

2.1 Time-Varying Idiosyncratic Uncertainty

We utilize daily firm-level stock returns to construct our benchmark estimate of time-varying uncertainty. Specifically, from the Center for Research in Security Prices (CRSP) data base, we extracted daily stock returns for all U.S. nonfinancial corporations with at least 1,250 trading days (essentially five years) of data. This selection criterion yielded a panel of 11,303 firms over the period from July 1, 1963 (1963:Q3) to September 30, 2012 (2012:Q3). To ensure that our results were not driven by a small number of extreme observations, we eliminated all firm/day observations with a daily absolute return in excess of 50 percent.

Our estimate of uncertainty is based on the following three-step procedure. First, we remove the forecastable variation in daily excess returns using the standard (linear) factor model:

$$(R_{it_d} - r_{t_d}^f) = \alpha_i + \beta_i' \mathbf{f}_{t_d} + u_{it_d}, \quad (1)$$

where i indexes firms and t_d , $d = 1, \dots, D_t$, indexes trading days in quarter t . In equation (1), R_{it_d} denotes the (total) daily return of firm i , $r_{t_d}^f$ is the risk-free rate, and \mathbf{f}_{t_d} is a vector of observable risk factors. In implementing the first step, we employ a 4-factor model—namely, the Fama and French [1992] 3-factor model, augmented with the momentum risk factor proposed by Carhart [1997].⁴

⁴The 4-factor model is used widely in applications that require a model of expected returns; see Fama and French [2004] for a thorough discussion. Although the small-minus-big (SMB), the high-minus-low (HML), and the momentum (MOM) risk factors are not motivated by predictions about state variables of concern to investors, this empirical

In the second step, we calculate the quarterly firm-specific standard deviation of daily idiosyncratic returns, according to

$$\sigma_{it} = \sqrt{\left[\frac{1}{D_t} \sum_{d=1}^{D_t} (\hat{u}_{it_d} - \tilde{u}_{it})^2 \right]}; \quad t = 1, \dots, T, \quad (2)$$

where \hat{u}_{it_t} denotes the OLS residual—the idiosyncratic return—from (1) and $\tilde{u}_{it} = D_t^{-1} \sum_{d=1}^{D_t} \hat{u}_{it_d}$ is the sample mean of daily idiosyncratic returns in quarter t . Thus, σ_{it} implied by equation (2) is an estimate of time-varying equity volatility for firm i , a measure that is purged of the forecastable variation in expected returns and thus is less likely to reflect the countercyclical nature of contractual and informational frictions.

In the final step, we assume that our firm-specific measure of uncertainty follows an autoregressive process of the form:

$$\log \sigma_{it} = \gamma_i + \delta_i t + \rho \log \sigma_{i,t-1} + v_t + \epsilon_{it}, \quad (3)$$

where γ_i denotes a firm fixed effect intended to control for the cross-sectional heterogeneity in σ_{it} , while the firm-specific term $\delta_i t$ captures secular trends in the idiosyncratic risk of publicly-traded U.S. firms documented by Campbell, Lettau, Malkiel, and Xu [2001]. In the aggregate, our uncertainty proxy corresponds to the sequence of estimated time fixed effects \hat{v}_t , $t = 1, \dots, T$, which captures shocks to idiosyncratic uncertainty that are common to all firms. The presence of this common variation—an empirical analogue of the time-varying volatility of a firm-specific productivity shock in our theoretical model—is essential because if fluctuations in σ_{it} were themselves entirely idiosyncratic, the macroeconomic impact of uncertainty shocks should wash out in the aggregate.

We estimate equation (3) by OLS, which yields an estimate of $\rho = 0.443$, an indication that uncertainty tends to be fairly persistent.⁵ The specification also fits the data quite well, explaining more than 75 percent of the variation in the dependent variable. The solid line in Figure 1 shows our estimate of time-varying uncertainty derived from the estimated time fixed effects in equation (3). The dotted line is the spread between the 10-year yield on BBB-rated corporate bonds and the 10-year Treasury yield, a barometer of strains in financial markets.⁶ As evidenced by the correlation coefficient of 0.42, there is a significant degree of comovement between our uncertainty proxy and

framework provides a convenient way to abstract from the well-documented patterns in average returns, which allows to construct a plausibly more exogenous proxy for idiosyncratic uncertainty. The daily risk factors and the risk free rate were obtained from the Kenneth R. French’s website <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

⁵Because the average firm is in the panel for more than 60 quarters, the bias of the OLS estimator, owing to the presence of a lagged dependent variable, firm fixed effects, and incidental trends is likely to be negligible; see Phillips and Sul [2007].

⁶The usefulness of corporate bond credit spreads as indicators of financial market conditions and their considerable predictive power for economic activity—both for the U.S. and other advanced countries—has been demonstrated by Gertler and Lown [1999], Gilchrist, Yankov, and Zakrajšek [2009], Helbling, Huidrom, Kose, and Otrók [2011], Gilchrist and Zakrajšek [2012], Meeks [2012], Bleaney, Mizen, and Veleanu [2012], and Faust, Gilchrist, Wright, and Zakrajšek [2012].

financial conditions. Moreover, both uncertainty and credit spreads are clearly countercyclical, typically increasing noticeably before and during recessions and spiking in periods of widespread financial distress.

2.2 Macro-Level Evidence

In this section, we use a SVAR to analyze the dynamic interaction between uncertainty, financial conditions, and economic activity.⁷ Specifically, we estimate a VAR consisting of the following eight endogenous variables: the logarithm of real business fixed investment (i_t); the logarithm of real personal consumption expenditures (PCE) on durable goods (c_t^D); the logarithm of real PCE on nondurable goods and services (c_t^N); the logarithm of real GDP (y_t); the logarithm of the GDP price deflator (p_t); the nominal effective federal funds rate (m_t)—an indicator of the stance of monetary policy; the 10-year BBB-Treasury credit spread (s_t); and our proxy for uncertainty (\hat{v}_t). The VAR is estimated over the 1963:Q3–2012:Q3 period using four lags of each endogenous variable.⁸

The primary focus of this exercise is quantify the effect of uncertainty shocks on financial conditions and economic activity. To identify these disturbances, we employ a standard recursive ordering technique, in which shocks to uncertainty have an immediate impact on credit spreads and short-term interest rates, but they affect economic activity and prices with a lag (Identification Scheme I); to provide a point of comparison, we rely on the same recursive ordering to examine the impact of shocks to credit spreads—that is, “financial shocks”—that are orthogonal to the contemporaneous level uncertainty and other macroeconomic variables. We also consider a VAR specification that reverses this causal ordering, which allows us to examine the implications of uncertainty shocks conditional on the information contained in the current level of credit spreads (Identification Scheme II).

Panel (a) in Figure 2 shows the impulse response functions of selected variables to an uncertainty shock, while Panel (b) shows the responses of the same variables to a financial shock, where both shocks are orthogonalized using the first identification scheme. Given these identifying assumptions, an unanticipated increase in uncertainty is associated with an immediate widening of credit spreads. Moreover, this uncertainty shock has significant adverse consequences for economic growth, as output declines almost immediately, reaching a trough about a year after the initial spike in uncertainty. Consistent with the standard irreversibility argument, the impact of the uncertainty shock falls primarily on the investment component of aggregate demand: Business expenditures on fixed capital fall steadily, bottoming out 1.5 percent below the trend six quarters after the shock, while consumer spending on durable goods also declines significantly.

⁷Variants of this approach can be found in Popescu and Smets [2010], Bekaert, Hoerova, and Lo Duca [2010], and Bachmann, Elstner, and Sims [2013].

⁸The very end of our sample (i.e., 2009:Q1–2012:Q3) includes a period during which the federal funds rate has been very close to zero. By constraining the actions of the central bank, the zero lower bound on nominal interest rates impinges on the unconstrained variability and overall distribution of the funds rate that would otherwise arise in response to the shocks to the economy. This constraint could potentially affect the impulse response functions reported below. However, omitting the zero-rate policy phase from our sample had no discernible effect on the VAR results reported in the paper.

As shown in Panel (b) of Figure 2, a financial shock, which leads to an increase of almost 30 basis points in the 10-year BBB-Treasury spread, is also associated with a significant contraction in economic activity. In addition to depressing the investment component of aggregate demand, this shock also induces a significant decline in nondurable goods consumption. The innovation in credit spreads, however, has an economically negligible and statistically insignificant effect on our uncertainty proxy.

Figure 3 shows the implications of these two shocks orthogonalized using an alternative scheme, in which credit spreads are ordered *before* our uncertainty measure. In that case, an unanticipated increase in uncertainty has a noticeably less adverse effect on the real economy (Panel (a)). Although both business fixed investment and consumer spending on durable goods fall in response to an increase in uncertainty, the declines are much smaller and less persistent compared with those in Panel (a) of Figure 2. In fact, the response of output to an uncertainty shock is statistically indistinguishable from zero.

Shocks to credit spreads, in contrast, have significant and long-lasting effects on economic activity (Panel (b)). A one-standard-deviation shock to the 10-year BBB-Treasury spread is associated with an immediate jump in uncertainty, a substantial fall in real GDP and a protracted decline in business fixed investment, as well as in major categories of consumer spending. Indeed, the magnitude and shape of the responses of output and its major components to financial shocks are very similar to those shown in the corresponding panel of Figure 2.

The time-series evidence presented above shows that an increase in uncertainty leads to an economically and statistically significant widening of credit spreads and a drop in output, where the latter is driven by a protracted decline in the major investment categories of aggregate demand. The analysis also suggests that financial distortions—as summarized by the level of credit spreads in the economy—may be an important part of the transmission mechanism propagating uncertainty shocks to the real economy. Indeed, our results show that once such disturbances are orthogonalized with respect to the contemporaneous level of credit spreads, the impact of uncertainty shocks on economic activity is significantly attenuated.

2.3 Micro-Level Evidence

To provide additional evidence regarding the role of financial frictions as an important determinant of investment dynamics in response to fluctuations in uncertainty, we now turn to firm-level data. Specifically, we construct a panel covering the 1973–2012 period and consisting of more than 1,100 publicly-traded U.S. nonfinancial firms included in both the CRSP and S&P’s Compustat databases. The distinguishing characteristic of these firms is that a significant portion of their outstanding liabilities is in the form of long-term bonds that are actively traded in the secondary market. We use the secondary market prices of individual securities to construct firm-level credit spreads, which are then matched to the issuer’s income and balance sheet data. (The description of the bond-level data set and the details regarding the construction of credit spreads and other firm-specific variables are contained in Appendix A.)

As in Leahy and Whited [1996] and Bloom, Bond, and Van Reenen [2007], our empirical strategy involves regressing business fixed investment on the firm-specific measure of uncertainty, while controlling for the fundamental determinants of investment spending. In contrast to the earlier research, our panel regressions also include credit spreads at the level of an *individual* firm, which allows us to parse out the effect of uncertainty on investment conditional on this important indicator of the firm’s financial condition.

2.3.1 Uncertainty and Credit Spreads

Before jumping to the main results of this analysis, we document the strong relationship between credit spreads and idiosyncratic uncertainty that is present at the micro level, a relationship that mirrors the time-series evidence presented above. Specifically, we estimate the following credit-spread regression:

$$\log s_{it}[k] = \beta_1 \log \sigma_{it} + \beta_2 R_{it}^E + \beta_3 [\Pi/A]_{it} + \beta_4 \log [D/E]_{i,t-1} + \theta' \mathbf{X}_{it}[k] + \epsilon_{it}[k], \quad (4)$$

where $s_{it}[k]$ denotes the credit spread of a bond issue k in period t , a security that is a liability of firm i . In addition to our estimate of idiosyncratic uncertainty σ_{it} , credit spreads are allowed to depend on the firm’s recent financial performance, as measured by the realized quarterly stock return R_{it}^E and the ratio of operating income to assets $[\Pi/A]_{it}$.⁹ By contrast, the ratio of the book value of total debt to the market value of the firm’s equity—denoted by $[D/E]_{it}$ —captures the strength of the firm’s balance sheet. The vector $\mathbf{X}_{it}[k]$ contains variables capturing (pre-determined) bond-specific characteristics that could influence bond yields through either liquidity or term premiums, including the bond’s duration, the par amount outstanding, the bond’s age, the bond’s (fixed) coupon rate, and an indicator variable that equals one if the bond is callable and zero otherwise.¹⁰

Table 1 summarizes these results. According to column 1, an increase in uncertainty leads to a significant widening of credit spreads—the elasticity estimate of 0.730 implies that an increase in uncertainty of 10 percentage points in quarter t is associated with an immediate jump in credit spreads of about 50 basis points. The coefficients on the remaining key variables are also economically and statistically highly significant and have their expected signs: Strong profitability performance, as evidenced by a high realized return on equity or an increase in the ratio of operating income to assets, is associated with a narrowing of credit spreads, whereas an increase in leverage leads to a rise in credit spreads.

⁹Operating income before depreciation and amortization (OIBDA) is a commonly used measure of financial performance. Because it reflects only the income earned from regular operations, OIBDA is considered to be a good indicator of profitability in continuing business operations.

¹⁰Specification (4) is similar to that used by Bharath and Shumway [2008] to predict credit default swap and corporate bond yield spreads. As in their paper, our main explanatory variables—volatility, profitability, and leverage—correspond to the “naïve” constituents of the distance-to-default, which, according to the Merton [1974] model, should be a sufficient statistic for default. Sections A.1 and A.2 of the data appendix contains details regarding the construction of variables used in this analysis.

The inclusion of bond-specific fixed credit-rating effects (column 2) improves the fit of the regression substantially—evidently they contain important additional information about the borrowers’ credit quality—though it lowers the coefficient on uncertainty noticeably; nevertheless, the effect remains economically and statistically highly significant, with a 10-percentage-point increase in uncertainty leading to a widening of spreads of about 30 basis points. Note that his results is robust to the inclusion of fixed industry effects (column 3), which control for any systematic differences in recovery rates across industries.

The specification in column 4 also controls for macroeconomic developments by adding a full set of time dummies to the regression. Although the magnitude of the coefficient on uncertainty diminishes further, the impact of uncertainty on credit spreads remains important: A 10-percentage-point increase in our measure of idiosyncratic uncertainty is associated with a rise in credit spreads of 15 basis points, statistically a highly significant response that is remarkably consistent with the VAR-based response shown in Figure 2. These micro-level results provide strong corroborating evidence that fluctuations in uncertainty influence financial conditions by significantly altering the level of credit spreads in the economy.

2.3.2 Uncertainty, Credit Spreads, and Investment

We now turn to the main question of the paper—the link between investment, uncertainty, and financial frictions. Our empirical investment equation is given by the following specification:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta \log Z_{it} + \eta_i + \lambda_t + \epsilon_{it}, \quad (5)$$

where $[I/K]_{it}$ denotes the investment rate of firm i in period t ; σ_{it} is our measure of idiosyncratic uncertainty; s_{it} is the credit spread on the portfolio of bonds issued by firm i ; and Z_{it} is a proxy for the marginal product of capital, a variable that measures firm i ’s future investment opportunities.¹¹ In addition to uncertainty, credit spreads, and investment fundamentals, equation (5) includes a fixed firm effect η_i and a fixed time effect λ_t —the former controls for systematic differences in the average investment rate across firms, while the latter captures a common investment component reflecting macroeconomic factors, which can influence firm-level investment through movements in either output or interest rates.

We measure the investment fundamentals Z_{it} using either the the current sales-to-capital ratio $[Y/K]_{it}$ or the current operating-income-to-capital ratio $[\Pi/K]_{it}$, two widely used proxies for the marginal product of capital; see Gilchrist and Himmelberg [1998] for detailed discussion.¹² As

¹¹The log-log nature of specification (5) reflects the fact that the firm-level investment rates, uncertainty, and credit spreads are highly positively skewed, a feature of the data that is significantly ameliorated through the use of a logarithmic transformation. Section A.2 of the data appendix contains details regarding the construction of variables used in this exercise.

¹²Taking logs of $[Y/K]_{it}$ is straightforward, but because operating income may be negative, we use $\log(c + [\Pi/K]_{it})$ —where c is chosen so that $(c + [\Pi/K]_{it}) > 0$ for all i and t —when relying on the operating income to measure the firm’s investment opportunities. In principle, the estimated elasticities may depend on the constant c . In practice, however, reasonable variation in c has no effect on the estimated elasticities.

an alternative forward-looking measure of investment fundamentals, we also consider Tobin’s Q , denoted by Q_{it} .

The result in columns 1–3 of Table 2 indicate a significant role for uncertainty in the capital accumulation process—the coefficient on uncertainty is statistically highly significant, regardless of the measure of investment fundamentals. The estimated elasticities of investment demand with respect to uncertainty lie in the range between -0.17 and -0.08 , indicating that an increase in uncertainty of 10 percentage points is associated with the decline in the investment rate between 0.5 and 1.1 percentage points. However, once the credit spreads are added to the regression (columns 4–6), the marginal effect of uncertainty on investment is virtually eliminated. Credit spreads, in contrast, are statistically and economically highly important determinants of investment spending, with a 100-basis-point rise in credit spreads implying a drop in the investment rate between 1.4 and 1.9 percentage points.

A well-documented result from the empirical investment literature is the fact that lagged investment rate is economically an important determinant of current investment spending; see, for example, Gilchrist and Himmelberg [1995] and Eberly, Rebelo, and Vincent [2012] for evidence and detailed discussion. Accordingly, we also consider a dynamic specification of the form:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta_1 \log Z_{it} + \theta_2 \log[I/K]_{i,t-1} + \eta_i + \lambda_t + \epsilon_{it}. \quad (6)$$

In this case, we eliminate fixed firm effects using the forward orthogonal deviations transformation of Arellano and Bover [1995] and estimate the resulting specification using GMM. Within this dynamic framework, both uncertainty and credit spreads are treated as endogenous and are instrumented with their own lagged values.

The results of this exercise are presented in Table 3. According to columns 1–3, fluctuations in uncertainty have economically large and statistically significant effects on capital spending. Taking into account lagged investment dynamics, the estimated elasticities imply that an increase in uncertainty of 10 percentage points depresses the investment rate between 2.7 and 4.0 percentage points in the long run. However, the adverse effect of increased uncertainty on investment spending is more than halved once the information content of credit spreads is taken into account—estimates of long-run elasticities in columns 4–6 imply that the same-sized increase in uncertainty lowers the investment rate between 1.1 and 1.8 percentage points. In contrast, a jump of 100 basis points in credit spreads is estimated to shave off between 1.4 and 2.1 percentage points from the rate of capital formation in the long run.

3 Model

In this section, we develop a quantitative general equilibrium model, the key properties of which are consistent our empirical findings. In the model, heterogeneous firms face time-varying idiosyncratic uncertainty, nonconvex capital adjustment costs—including both partial investment irreversibility and fixed investment costs—and financial distortions in the debt and equity markets. Within this

framework, we focus on two nontraditional sources of business cycle fluctuations: Shocks from the first source affect the volatility of the idiosyncratic technology process, whereas shocks from the second source directly reduce the collateral value of the firms' capital assets and thus impinge on their borrowing capacity.

The presence of nonconvex adjustment frictions implies that uncertainty and capital liquidity shocks have real consequences for macroeconomic outcomes regardless of the structure of financial markets. In our model, however, the presence of financial distortions induces a change in the effective supply of credit in response to both types of disturbances, an effect that significantly amplifies the initial impact of each shock on investment spending. In fact, our model generates macroeconomic dynamics that are consistent with the following empirical results:

- Uncertainty shocks have a significant effect on business investment;
- Swings in uncertainty lead to large movements in credit spreads;
- Fluctuations in credit spreads exert a significant influence on the aggregate business cycle, irrespective of the level of uncertainty.

To the extent that credit spreads provide an useful gauge of the degree of frictions in financial markets, the combination of our empirical analysis and model simulations provides compelling evidence that financial distortions are a crucial part of the transmission mechanism through which uncertainty shocks are propagated through the real economy.

3.1 Production

The key set of agents in our model economy consists of a continuum of heterogeneous firms that produce goods used for consumption or can also be used in the production of the capital input. Firms producing the final good—denoted by y —combine labor h and capital k using a decreasing returns-to-scale (DRS) production function. The production is subject to an aggregate and idiosyncratic technology shocks—denoted by a and z , respectively—and requires a payment of fixed operating costs $F_o > 0$ that are proportional to firm size as measured by its existing capital stock.

Formally, the technological assumption are summarized by a production function

$$y = (az)^{(1-\alpha)\chi} (k^\alpha h^{1-\alpha})^\chi - F_o k; \quad 0 < \alpha < 1 \text{ and } \chi < 1, \quad (7)$$

where α is the value-added share of capital, and the parameter χ governs the degree of decreasing returns in production. The normalization parameter $(1 - \alpha)\chi$ associated with the exogenous technology shocks ensures that the firm's profit function is linear in technology shocks a and z :

$$\begin{aligned} \pi(a, z, w, k) &\equiv \max_{k \geq 0} \{ (az)^{(1-\alpha)\chi} (k^\alpha h^{1-\alpha})^\chi - F_o k - wh \}; \\ &= az\psi(w)k^\gamma - F_o k, \end{aligned} \quad (8)$$

where w is the (real) wage and

$$\gamma = \frac{\alpha\chi}{1 - (1 - \alpha)\chi} \quad \text{and} \quad \psi(w) = [1 - (1 - \alpha)\chi] \left[\frac{(1 - \alpha)\chi}{w} \right]^{\frac{(1 - \alpha)\chi}{1 - (1 - \alpha)\chi}}.$$

The combination of decreasing returns-to-scale and fixed operating costs implies that the firm can earn strictly positive (or negative) profits in equilibrium. In principle, this can generate non-trivial firm dynamics through entry and exit. To keep the model tractable, however, we do not explicitly model the firm's endogenous entry/exit decision. As in Cooley and Quadrini [2001] and Veracierto [2002], we assume that a constant fraction $1 - \eta$ of firms exogenously exits the industry in each period and that the exiting firms are replaced by new firms within the same period. This stochastic overlapping generation structure provides a convenient way to motivate the use of leverage by firms in the steady state without introducing a corporate income tax shield.

The process governing the evolution of the aggregate technology shock a is standard, as we assume that it follows a continuous Markov process:

$$\log a' = \rho_a \log a + \log \epsilon'_a; \quad |\rho_a| < 1 \text{ and } \log \epsilon'_a \sim N(-0.5\omega_a^2, \omega_a^2). \quad (9)$$

In contrast, the idiosyncratic technology shock z is assumed to follow an N -state Markov chain process with *time-varying* volatility. We let $i, j = 1, \dots, N$ index the states of this process and let $p_{i,j}$ denote the transition probability of moving from state i in the current period to state j in the subsequent period.

Importantly, as we show in Section B.1 of the model appendix, the Markov chain of the idiosyncratic technology shock is constructed in such a way that its conditional mean is not affected by fluctuations in volatility. Its conditional variance, however, is a linear function of the realization of the time-varying volatility process, which is given by a continuous Markov process of the form

$$\log \sigma'_z = (1 - \rho_\sigma) \log \tilde{\sigma}_z + \rho_\sigma \sigma_z + \log \epsilon'_\sigma; \quad |\rho_\sigma| < 1 \text{ and } \log \epsilon'_\sigma \sim N(-0.5\omega_\sigma^2, \omega_\sigma^2). \quad (10)$$

Within our framework, an increase in σ_z represents a mean-preserving spread (MPS) of z' , in that the support of the distribution of the idiosyncratic technology shock is evolving stochastically over time: An increase in uncertainty today increases the dispersion of z tomorrow and vice versa.

3.2 Capital Accumulation

Firms accumulate capital subject to two types of nonconvex adjustment frictions: fixed costs and partial irreversibility. Formally, the total cost of capital adjustment is given by

$$p(k', k) = F_k k \times \mathbf{1}[k' \neq (1 - \delta)k] + (p^+ \times \mathbf{1}[k' \geq (1 - \delta)k] + p^- \times \mathbf{1}[k' \leq (1 - \delta)k]) (k' - (1 - \delta)k), \quad (11)$$

where $\mathbf{1}[\cdot]$ is an indicator function and $0 < \delta < 1$ denotes the depreciation rate. The term $F_k k$ represents the fixed costs associated with investment expenditures, which are assumed to be proportional to the size of a firm. As emphasized by Cooper and Haltiwanger [2006], these fixed adjustment costs capture the inherent indivisibility of physical capital and potential increasing returns to both the installation of new capital and restructuring of productive capacity during period of intensive investment. The second term corresponds to the costly reversible investment framework of Abel and Eberly [1996], whereby the liquidation value of installed capital p^- is a fraction of initial purchase price p^+ . The assumption that $p^-/p^+ < 1$ captures the notion of capital specificity and implies that installed capital is less liquid than uninstalled capital.

To explore the implications of time-varying liquidity of capital assets, we let p^- follow a continuous Markov process given by

$$\log p^{-'} = (1 - \rho_{p^-}) \log \tilde{p}^- + \rho_{p^-} \log p^- + \epsilon'_{p^-}; \quad |\rho_{p^-}| < 1 \text{ and } \log \epsilon'_{p^-} \sim N(-0.5\omega_\kappa^2, \omega_\kappa^2). \quad (12)$$

In this framework, the costly reversible investment plays a dual role. First, in an environment of time-varying uncertainty, the partial irreversibility creates an option value of waiting—that is, in periods of heightened uncertainty, the investment inaction region expands, and firms delay exercising their investment/disinvestment options. Second, the illiquidity of capital asset limits the debt capacity of firms because the implied liquidation cost reduces the collateral value of capital assets. A novel feature of our approach is in the potential interaction between the capacity and debt overhang problems in a unified quantitative business cycle framework; see Shleifer and Vishny [1992], Eisfeldt [2004], and Manso [2008] for more theoretical approaches.

3.3 The Firm’s Problem

We assume that at the beginning of each period, all economic agents in the model observe the realization of the idiosyncratic and aggregate productivity shocks, z and a , respectively; at the same time, they also observe the level of uncertainty σ_z and the resale value of capital p^- . The timing assumptions are such that this period’s uncertainty level σ_z determines the distribution of ϵ'_z in the subsequent period. From a perspective of agents in the model, an increase in uncertainty today represents “news” regarding the distribution of profits tomorrow. To streamline the notation, we define the aggregate state of the economy as a vector $\mathbf{s} = [a, \sigma_z, p^-, \mu]$, where μ is a joint distribution of the idiosyncratic technology, capital, and net liquid asset positions (to be defined below) across heterogeneous firms.

To finance investment projects, firms use a combination of internal and external funds, where the sources of external funds are debt and equity. Relative to internal funds, external funds command a premium, either because of the direct cost of issuing equity, or in the case of debt, because of the agency costs associated with default.

3.3.1 Debt Finance

Debt finance available to the firm consists of a sequence of one-period, zero-coupon bonds. The debt contract specifies the par value of the issue b' and the price q , yielding the total amount of debt financing equal to qb' in each period. By combining the proceeds from debt issuance with other sources of funds, the firm purchases capital to be used in production. In the subsequent period—after observing the realization of shocks—the firm decides whether or not to fulfill its debt obligation. If the firm decides not to default, it pays the face value of the debt b' to the lender and makes its production and financial decisions for the next period. If the firm chooses to default, it enters a debt-renegotiation process with bond investors.

The renegotiation process is conducted under limited liability by assuming that there exists a lower bound to the net worth of the firm—denoted by \bar{n} —below which the firm cannot promise to pay back any outstanding liability.¹³ The realized net worth next period is defined as the sum of net profits and the market value of undepreciated capital less the face value of debt:

$$n' = a' z' \psi(w(\mathbf{s}')) k'^{\gamma} - F_o k' + p^-(1 - \delta) k' - b'. \quad (13)$$

Note that the value of capital in place is evaluated at the resale value p^- , rather than its book value p^+ . By combining the expression for net worth with the default condition $n' \leq \bar{n}$, we can define a level of idiosyncratic technology that triggers default—denoted by z^D —conditional on the tomorrow's aggregate state \mathbf{s}' and individual state (k', b') , as

$$z^D(k', b'; \mathbf{s}') \equiv \frac{\bar{n} + b' + F_o k' - p^-(1 - \delta) k'}{a' \psi(w') (k')^{\gamma}}. \quad (14)$$

Under limited liability, the new level of debt renegotiated by the firm and bond investors—denoted by b^R —cannot exceed the amount of debt $\bar{b}(k', z'; \mathbf{s}')$ that is consistent with the assumed lower bound on net worth:

$$b^R \leq \bar{b}(k', z'(\sigma_z); \mathbf{s}') \equiv a' z'(\sigma_z) \psi(w') k'^{\gamma} - F_o k' + p^-(1 - \delta) k'. \quad (15)$$

We assume that the firm does not have any bargaining power during the renegotiation process. This assumption implies that the renegotiated debt is determined by the upper bound on the amount of debt that bond investors can recover in the case of default—that is, $b^R = \bar{b}(k', z'(\sigma_z); \mathbf{s}')$.

The default entails a dead-weight loss, captured by bankruptcy costs that are assumed to be

¹³This type of bond contract is similar to that of Merton [1974], Cooley and Quadrini [2001], and Hennessy and Whited [2007]. However, in our setup, a default occurs when the net worth of the firm n hits the lower bound \bar{n} , whereas in the aforementioned papers, a default occurs when the value of the equity v hits the lower bound \bar{v} . If the technology shock follows an i.i.d. process and the analysis is conducted in partial equilibrium, the two assumptions are equivalent. However, if the technology shock is persistent or the firm's value function has other arguments (e.g., aggregate state variables), the two assumptions are no longer equivalent. The decision to use a lower bound for the net worth to determine the default threshold is a simplifying assumption that allows us to avoid the computationally intensive task of inverting the value function to compute the default boundary $n(z)$ in each iteration of the dynamic programming routine.

proportional to the amount of liquidated capital. Thus the actual recovery in the case of default is given by $b^R - \xi(1 - \delta)k'$, where the parameter $0 < \xi < 1$ governs the magnitude of the bankruptcy costs and hence the degree of frictions in the corporate bond market. Therefore, the recovery rate \mathcal{R} in the case of default is given by

$$\mathcal{R}(k', b', z'(\sigma_z); \mathbf{s}') = \frac{\bar{b}(k', z'(\sigma_z), \mathbf{s}')}{b'} - \xi(1 - \delta)\frac{k'}{b'}. \quad (16)$$

Standard no-arbitrage arguments then imply the following bond pricing formula:

$$q_i(k', b'; \mathbf{s}') = \mathbb{E} \left\{ m(\mathbf{s}, \mathbf{s}') \left[1 + \sum_{j \in \mathcal{D}} p_{i,j} [\mathcal{R}(k', b', z'_j(\sigma_z); \mathbf{s}') - 1] \right] \middle| \mathbf{s} \right\}, \quad (17)$$

where $m(\mathbf{s}, \mathbf{s}')$ is the stochastic discount factor of the representative household and

$$\mathcal{D} = \{j \mid j \in \{1, \dots, N\} \text{ and } z'_j(\sigma_z) \leq z^D(k', b'; \mathbf{s}')\} \quad (18)$$

is the set of states of the idiosyncratic technology shock z , in which, if realized, the firm will default on its debt obligations.

Note that the exogenous exit rate $1 - \eta$ does not appear in the bond pricing formula. This is because we assumed that the exit shock is realized *after* the firms make their repayment/default decisions. Consequently, the exit shock does not directly affect the returns of bond investors. The credit spreads that arise in our framework are thus not a direct result of the exogenous exit. However, the exit process does affect credit spreads indirectly by influencing the firms' choice of leverage. Because firms survive with probability η , their effective discounting factor in the steady state is $\eta\beta$. When the firm has no debt, its marginal borrowing costs are close to $1/\beta$ —the risk-free rate—as the likelihood of default is essentially zero. This marginal borrowing rate is lower than the firm's internal discounting rate $1/(\eta\beta)$, which represents the cost of issuing new shares in the absence of any additional issuance costs. As a result, the firms are induced to hold a positive amount of debt in equilibrium, with the amount of leverage depending on η .

3.3.2 Equity Financing

We now turn to equity financing, starting with the definition of dividends (d):

$$d \equiv az_i\psi(w)k^\gamma - F_o k - p(k', k) - b + q_i b' + e, \quad (19)$$

where e denotes the value of newly issued shares when positive and the value of share repurchases when negative. We posit that the firms face a minimum dividend constraint,

$$d \geq \bar{d} \geq 0. \quad (20)$$

Thus, when the firm’s internal funds and the proceeds raised with the bond issue fall short of its financing needs, the firm must raise outside equity—that is, $e > 0$ —to satisfy the dividend constraint.¹⁴

In the absence of financial distortions, the notional amount of equity issuance e reduces the value of existing shares by the same amount. To introduce frictions in the equity market, we assume that seasoned equity issuance is costly, in the sense that the value of existing shares is reduced by more than the amount of newly issued shares. As in Gomes [2001], Cooley and Quadrini [2001], Hennessy and Whited [2007], and Bolton, Chen, and Wang [2011], we capture this distortion by assuming a constant marginal cost of equity issuance. Formally, the loss in the value of existing shares associated with the amount e of newly issued equity is given by

$$\bar{\varphi}(e) \equiv e + \varphi \max\{e, 0\}, \quad (21)$$

where $\varphi > 0$ measures the degree of frictions in the market for seasoned equity. Share repurchases (i.e., $e < 0$), in contrast, are assumed equivalent to dividend payments and thus do not involve any additional costs.

3.3.3 Recursive Problem Formulation

To formulate the firm’s optimization problem recursively, we define a composite state variable—the net liquid asset position (x)—according to

$$x \equiv az\psi(w)k^\gamma - F_o k - b = n - p^-(1 - \delta)k. \quad (22)$$

The firm’s dividend can then be rewritten as $d = x - p(k', k) + q_i b' + e$, which allows us to express the value of equity as $v_i(k, x; \mathbf{s})$, where the subscript i denotes the firm’s relative position in the discrete distribution of the idiosyncratic technology level z in the current period. In combination with the realized level of uncertainty σ_z , this is the only information needed to predict the subsequent values of the idiosyncratic technology shock.¹⁵

¹⁴As documented by Fama and French [2005], in an average year between 1973 and 2002, almost 60 percent of dividend-paying firms also issued new shares, on net, evidence that seems difficult to reconcile with the assumption of frictionless financial markets. However, another possibility—one that does not require departures from the Modiglian–Miller paradigm—is that the lower bound dividends \underline{d} in equation (20) may be strictly positive for a significant fraction of firms. Indeed, this is the interpretation we adopt in this paper.

¹⁵An equivalent way of formulating the problem would be involve specifying the value of equity as $v_{i,j}(k, b; \mathbf{s})$, where i is the firm’s relative position in the distribution of z in the previous period and j is the relative position in the current period. In that case, we would need to keep track of both i and j because at any point in time, the realization of the idiosyncratic technology shock depends on i , the relative position in the distribution of z previously; j , the current relative position; and σ_z , the current realization of volatility. However, only the last two elements are needed to make predictions regarding the relative position of the idiosyncratic technology shock in the subsequent period. The first element is needed to recover the current realization of z and the liquid asset position x . By defining x as the composite state variable allows us to avoid this complication, which can be quite time consuming in large-scale computational problems.

The firm's problem can be then formulated recursively as

$$\begin{aligned}
v_i(k, x; \mathbf{s}) = & \min_{\phi} \max_{d, e, k', b'} \left\{ d + \phi(d - \underline{d}) - \bar{\varphi}(e) \right. \\
& \left. + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \max \{ v_j(k', x'(\sigma_z); \mathbf{s}'), v_j(k', x^R(\sigma_z); \mathbf{s}') \} \mid \mathbf{s} \right] \right\} \quad (23)
\end{aligned}$$

subject to (17), (19), and $\mathbf{s}' = \Gamma(\mathbf{s})$; $i, j = 1, \dots, N$,

where ϕ is the Lagrange multiplier associated with the dividend constraint (20) and $\mathbf{s}' = \Gamma(\mathbf{s})$ is the law of motion governing the evolution of the aggregate state, which we describe below. Note that the continuation value of the firm is bounded below by the default/renegotiation value $v_j(k', x^R(\sigma_z); \mathbf{s}')$, where

$$x^R(\sigma_z) = a' z'(\sigma_z) \psi(w') k'^{\gamma} - F k' - b^R.$$

By directly differentiating the value function defined by (23) with respect to e , we obtain the first-order condition for equity issuance

$$1 + \phi = 1 + \varphi \times \mathbf{1}(e > 0), \quad (24)$$

an expression with a straightforward interpretation: The firm will issue new shares if and only if the dividend constraint binds.

Similarly, directly differentiating equation (23) with respect to b' yields

$$(1 + \phi) [q_i(k', b'; \mathbf{s}) + q_{i,b}(k', b'; \mathbf{s}) b'] = -\eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}^c} p_{i,j} v_{j,b}(k', x'(z_{i,j}(\sigma_z), k', b'); \mathbf{s}') \mid \mathbf{s} \right],$$

where \mathcal{D}^c denotes the complement of the default set defined in equation (18). The associated Benveniste-Scheinkman condition is given by $v_{i,x}(k, x; \mathbf{s}) = -(1 + \phi)$, which allows us to express the first-order condition for debt issuance as

$$\begin{aligned}
q_i(k', b'; \mathbf{s}) + q_{i,b}(k', b'; \mathbf{s}) b' &= \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}^c} p_{i,j} \left(\frac{1 + \phi'}{1 + \phi} \right) \mid \mathbf{s} \right] \\
&= \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}^c} p_{i,j} \left(\frac{1 + \varphi \times \mathbf{1}(e' > 0)}{1 + \varphi \times \mathbf{1}(e > 0)} \right) \mid \mathbf{s} \right], \quad (25)
\end{aligned}$$

where the second equality follows directly from equation (24). The left side of equation (25) represents the marginal cost of debt finance, while the expression on the right equals the expected present value of an additional dollar of debt on the firm's balance sheet. By increasing its leverage, the firm improves its current cashflow but increases the chance of a future liquidity shortfall, thereby raising the likelihood that it will have to issue costly new shares in the future. Under

limited liability, the firm cares about future cashflows only to the extent that it avoids default, an aspect of the firm's financial policy captured by the summation over the states in the non-default set \mathcal{D}^c .

3.3.4 Optimal Capital Choice

The presence of nonconvex adjustment costs in the capital accumulation process (see equation (11)) complicates the derivation of the conditions characterizing the firm's optimal choice of capital. To highlight the role of uncertainty and illiquidity—and their interaction—in determining the optimal choice of capital, we focus below on the case where the fixed cost of investment F_k is assumed to be zero. The corresponding first-order conditions for the more general case with $F_k > 0$ are provided in Section B.2 of the model appendix.

Assuming that $F_k = 0$, the dynamic programming problem given by equation (23) can be separated into the following two ancillary problems:

$$v_i(k, x; \mathbf{s}) = \max \{v_i^+(k, x; \mathbf{s}), v_i^-(k, x; \mathbf{s})\}, \quad (26)$$

where

$$v_i^+(k, x; \mathbf{s}) = \min_{\phi, \lambda^+} \max_{d^+, e^+, k^+, b^+} \left\{ d^+ + \phi(d^+ - \underline{d}) - \bar{\varphi}(e^+) + \lambda^+[k^+ - (1 - \delta)k] \right. \\ \left. + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \max \{v_j(k^+, x^+(\sigma_z); \mathbf{s}'), v_j(k^+, x^{R^+}(\sigma_z); \mathbf{s}')\} \mid \mathbf{s} \right] \right\} \quad (27)$$

subject to (17), (19), and $\mathbf{s}' = \Gamma(\mathbf{s})$; $i, j = 1, \dots, N$;

and

$$v_i^-(k, x; \mathbf{s}) = \min_{\phi, \lambda^-} \max_{d^-, e^-, k^-, b^-} \left\{ d^- + \phi(d^- - \underline{d}) - \bar{\varphi}(e^-) - \lambda^-[k^- - (1 - \delta)k] \right. \\ \left. + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \max \{v_j(k^-, x^-(\sigma_z); \mathbf{s}'), v_j(k^-, x^{R^-}(\sigma_z); \mathbf{s}')\} \mid \mathbf{s} \right] \right\} \quad (28)$$

subject to (17), (19), and $\mathbf{s}' = \Gamma(\mathbf{s})$; $i, j = 1, \dots, N$.

In these two ancillary value functions, λ^+ and λ^- denote the Lagrange multipliers associated with constraints $k^+ - (1 - \delta)k \geq 0$ and $k^- - (1 - \delta)k \leq 0$, respectively. The value function (27) expresses the value of the firm that is committed to a non-negative sequence of capital expenditures—regardless of the optimality of doing so—whereas (28) considers the opposite case, namely a firm whose capital spending plan includes only disinvestment (or no new capital expenditures). The optimality of the firm's actual investment policy is ensured by equation (26), and the values of the growth and contraction options are given by $v_i(k, x; \mathbf{s}) - v_i^-(k, x; \mathbf{s})$ and $v_i(k, x; \mathbf{s}) - v_i^+(k, x; \mathbf{s})$, respectively.

In Section B.2 of the model appendix, we show that the investment Euler equations associated with the expansion and contraction problems can be expressed as

$$Q_i^+(k, x; \mathbf{s}) = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^+; s') + (1 - \delta)Q'_j(k^+, x'(\sigma_z); \mathbf{s}')] \mid \mathbf{s} \right] \\ + q_{i,k}(k^+, b^+; \mathbf{s})b^+ - \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^+; s') + (1 - \delta)p^-] \mid \mathbf{s} \right]; \quad (29)$$

and

$$Q_i^-(k, x; \mathbf{s}) = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^-; s') + (1 - \delta)Q'_j(k^-, x'(\sigma_z); \mathbf{s}')] \mid \mathbf{s} \right] \\ + q_{i,k}(k^-, b^-; \mathbf{s})b^- - \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^-; s') + (1 - \delta)p^-] \mid \mathbf{s} \right], \quad (30)$$

where $i, j = 1, \dots, N$, and the shadow value of capital—that is, the marginal Q —associated with the expansion and contraction problems is given by

$$Q_i^+(k, x; \mathbf{s}) = p^+ - \frac{\lambda_i^+(k, x; \mathbf{s})}{1 + \phi_i(k, x; \mathbf{s})} \quad (31)$$

and

$$Q_i^-(k, x; \mathbf{s}) = p^- + \frac{\lambda_i^-(k, x; \mathbf{s})}{1 + \phi_i(k, x; \mathbf{s})}. \quad (32)$$

The marginal Q in equation (31) represents the marginal cost of expanding capacity, whereas that in equation (32) measures the revenue generated by liquidating a unit of capital. The absence of arbitrage implies that the marginal cost of investment cannot exceed the purchase price of new capital goods p^+ , and that the marginal value of reducing production capacity cannot fall below the liquidation value of capital p^- . Hence, the marginal Q associated with this capital accumulation problem is truncated from above by p^+ and below by p^- :

$$Q_i(k, x; \mathbf{s}) = \min \left\{ p^+, \max \left\{ p^-, p^+ - \frac{\lambda_i^+(k, x; \mathbf{s})}{1 + \phi_i(k, x; \mathbf{s})} \right\} \right\} \\ = \min \left\{ p^+, \max \left\{ p^-, p^- + \frac{\lambda_i^-(k, x; \mathbf{s})}{1 + \phi_i(k, x; \mathbf{s})} \right\} \right\}. \quad (33)$$

The firm's optimal choice of capital reflects the target capital stocks $k_i^+(k, x; \mathbf{s})$ and $k_i^-(k, x; \mathbf{s})$ implied by the ancillary problems (27) and (28), respectively, and is given by

$$k'_i(k, x; \mathbf{s}) = \min \left\{ k_i^-(k, x; \mathbf{s}), \max \left\{ k_i^+(k, x; \mathbf{s}), (1 - \delta)k \right\} \right\}. \quad (34)$$

As a result, the firm can find itself in one of three possible situations. First, $v_i^+(k, x; \mathbf{s}) > v_i^-(k, x; \mathbf{s})$,

which implies that an expansion of capital stock is optimal; in that case, the expansion constraint is not binding ($\lambda_i^+(k, x; \mathbf{s}) = 0$), the contraction constraint is binding, ($\lambda_i^-(k, x; \mathbf{s}) > 0$), and investment dynamics are determined by the Euler equation (29). Second, a reduction of production capacity is optimal—that is, $v_i^-(k, x; \mathbf{s}) > v_i^+(k, x; \mathbf{s})$; in that case, the expansion constraint is binding ($\lambda_i^+(k, x; \mathbf{s}) > 0$), the contraction constraint is not binding, ($\lambda_i^-(k, x; \mathbf{s}) = 0$), and disinvestment dynamics are determined by equation (30). And lastly, inaction is optimal because $v_i^+(k, x; \mathbf{s}) = v_i^-(k, x; \mathbf{s})$; in that case, $k^+ = k^- = (1 - \delta)k$, and the dynamics of capital accumulation are governed by either (29) or (30), as $Q_i^+(k, x; \mathbf{s}) = Q_i^-(k, x; \mathbf{s})$.

As shown by Abel and Eberly [1996] in a continuous time setting, these types of investment dynamics are a standard feature of the costly reversible investment problem. When the expansion constraint is not binding because of limited production capacity, the firm invests until the marginal value of capital equals its purchase price. If, on the other hand, the contraction constraint is not binding because of excess production capacity, the firm disinvests to the point where the marginal value of capital equals its liquidation value. When neither of these policies is optimal, the firm simply allows its capital stock to shrink through depreciation. As can be seen above, these investment dynamics in our model are further complicated by the fact that the two target capacity levels $k_i^+(k, x; \mathbf{s})$ and $k_i^-(k, x; \mathbf{s})$ —which bring the firm’s efficiency level to p^+ or p^- , depending on which constraint is binding—depend on the firm’s financial condition, as summarized by its net liquid asset position x .

The effect of financial conditions on capital accumulation is also evident in the investment Euler equations. For example, consider equation (29), where the first line captures the standard neoclassical effect—an increase in profits resulting from an additional unit of capital along with the capital gain. The first term in the second line shows the impact of today’s investment decision on the current borrowing cost: Positive investment today immediately boosts—for any borrowed amount—the bond investors’ recovery value in the event of default, which lowers the marginal cost of debt and thereby increases the firm’s debt capacity. The second term reflects the assumption of limited liability because the firm does not care about the states of nature in which it is forced to declare a costly default.¹⁶ Finally, note that all future benefits of today’s investment are discounted by an additional stochastic discounting factor $(1 + \phi'_j)/(1 + \phi_i)$, which measures the shadow value of internal funds tomorrow relative to today.

These additional terms create channels through which disturbances, such as uncertainty shocks and shocks to the liquidation value of capital, can affect aggregate investment. According to the canonical framework used to price risky debt, the payoff structure of levered equity resembles the payoff of a call option, while the bondholders face a payoff structure that is equivalent to that of an investor writing a put option. As a result, an increase in payoff uncertainty benefits equity holders at the expense of bondholders, prompting an adjustment in borrowing terms. If the increase in borrowing costs required to leave the bond investors no worse off is greater than the potential gain

¹⁶Upon default, the firm liquidates its capital. As a result, the value of capital is evaluated at its market liquidation value p^- , rather than at its internal value Q_i .

to the equity holders, an increase in uncertainty will lead to a decline in aggregate investment. A negative shock to the liquidation value of capital can also exert a powerful effect on aggregate investment because a lower liquidation value directly reduces the debt capacity of firms and thus increases the cost of capital for any given level of borrowing.

3.4 Market Clearing Conditions and Aggregation

To close the model, this section specifies conditions required to clear the labor and goods markets; it also explains how the distribution of the idiosyncratic technology shocks, capital, and net liquid asset positions across heterogeneous firms evolves over time.

We begin with the problem of the representative household, who solves

$$W(\mathbf{s}) = \max_{b', s', c, h} \{u(c, h) + \beta \mathbb{E}[W(\mathbf{s}') | \mathbf{s}]\}, \quad (35)$$

subject to a budget constraint,

$$\begin{aligned} c + \int (qb' + p_s s') \mu(dz, dk, dx) &= wh + \int [\tilde{R}^b + (d + \tilde{p}_s)s] \mu(dz, dk, dx) \\ &+ \int F_o k \mu(dz, dk, dx). \end{aligned} \quad (36)$$

In equation (35), the period-specific utility function $u(c, h)$ is assumed to be strictly increasing and strictly concave in consumption c and strictly decreasing and strictly concave in hours worked h . In the budget constraint (36), $\mu(dz, dk, db)$ represents the joint distribution of the idiosyncratic technology, capital, and net liquid asset positions across heterogeneous firms; $\tilde{R}^b = [1 + \mathbf{1}[z \in \mathcal{D}(k, b, z)]] \times [\mathcal{R}(k, b, z(\sigma_z)) - 1]b$ is the realized gross return on corporate bonds issued in the previous period (i.e., qb); p_s is the ex-dividend value of equity; \tilde{p}_s is the current market value of equity; and $0 \leq s \leq 1$ is the household's share of outstanding equity. The equity valuation terms are linked by the accounting identity:

$$\tilde{p}_s = p_s - \bar{\varphi}(e),$$

where $\bar{\varphi}(e) \geq e$ captures the cost of issuing new shares. Note that the fixed cost of operation is rebated to the household in a lump-sum fashion.

The dynamic efficiency conditions associated with the household's problem are given by

$$p_s(k, x, z; \mathbf{s}) = \mathbb{E} \left[\beta \frac{u'(c', h')}{u(c, h)} [d' - \bar{\varphi}(e') + p'_s(k', x', z'; \mathbf{s}')] \mid z, \mathbf{s} \right]; \quad (37)$$

and

$$q(k', b', z; \mathbf{s}) = \mathbb{E} \left[\beta \frac{u'(c', h')}{u(c, h)} [1 + \mathbf{1}[z' \in \mathcal{D}'(k', b'; \mathbf{s}')] \times [\mathcal{R}'(k', b', z'; \mathbf{s}') - 1]] \mid z, \mathbf{s} \right], \quad (38)$$

while the static optimization condition characterizing the labor supply decision implies that $w =$

$-u_h(c, h)/u_c(c, h)$.

Conditions ensuring that the labor and goods market clear can then be expressed as

$$h^s(\mathbf{s}) = \int h^d(z, k; \mathbf{s})\mu(dz, dk, dx); \quad (39)$$

$$c(\mathbf{s}) = \int y(z, k, h; \mathbf{s})\mu(dz, dk, dx) - \int p(k'(k, x, z; \mathbf{s}), k)\mu(dz, dk, dx). \quad (40)$$

Note that implicit in the goods market clearing condition (40) is the assumption that the dilution costs associated with seasoned equity issuance take the form of *discount* sales, implying that the gains and losses of new and existing shareholders offset each other—as a result, the dilution costs do not affect the aggregate resource constraint. The stock market clearing condition is simply $s' = s = 1$. The bond market then clears by Walras' law.¹⁷

Finally, the law of motion for μ —the joint distribution of $z \in Z$, $k \in K$, and $x \in X$ —is given by

$$\mu'(Z, K, X, \mathbf{s}') = \int \mathbf{1}[z_j \in Z] \times \mathbf{1}[k'_i \in K] \times \mathbf{1}[x'_j(k'_i, \min\{b'_i, b^R\}, \mathbf{s}) \in X] p_{i,j} \mu(dz, dk, dx, \mathbf{s}) G(\mathbf{s}, d\mathbf{s}'), \quad (41)$$

where $i, j = 1, \dots, N$ indexes the states of the Markov chain governing the evolution of the idiosyncratic technology shock z and G is the transition function of the state vector \mathbf{s} . Following the literature on computable general equilibrium with heterogeneous agents (see Krusell and Smith [1998]), we adopt the assumption of bounded rationality—that is, the agents use only a finite number of moments of the joint distribution to forecast equilibrium prices. Specifically, we assume that the agents use only the first moments of log-linearized laws of motions to predict three prices: (1) the marginal utility of the representative household $u_c(\mathbf{s})$; (2) the real wage $w(\mathbf{s})$; and (3) the purchase price of capital $p^+(\mathbf{s})$.

Because our specification for preferences of the representative household specifies an infinitely elastic labor supply, the equilibrium wage can be backed out from the marginal utility of consumption. As a result, it is sufficient for the agents in the model to forecast the aggregate debt after renegotiation (\tilde{b}), the aggregate stock of capital (\tilde{k}), and the marginal utility of consumption ($u_c(\mathbf{s})$).¹⁸ To do so, we assume that the agents uses the following system of log-linear forecasting

¹⁷Implicit in these aggregation conditions is also an assumption that firms that exit the market because of an exogenous shock are replaced by entrants that inherit all the technical and financial characteristics of the exiting firms. This assumption significantly simplifies the law of motion characterizing the evolution for the measure of firms.

¹⁸Because the relationship between the aggregate net liquid asset position \tilde{x} and aggregate debt \tilde{b} is linear, it would be equivalent if the agents projected \tilde{x} as opposed to \tilde{b} . However, in solving the individual firm problems, using the net liquid asset position as a state variable reduced the dimensionality of the state space.

rules:

$$\begin{bmatrix} \log \tilde{b}' \\ \log \tilde{k}' \\ \log \tilde{c} \end{bmatrix} = \underset{(3 \times 1)}{\mathbf{\Gamma}_0} + \underset{(3 \times 2)}{\mathbf{\Gamma}_1} \begin{bmatrix} \log \tilde{b} \\ \log \tilde{k} \end{bmatrix} + \underset{(3 \times 3)}{\mathbf{\Gamma}_2} \begin{bmatrix} \log \sigma_z \\ \log a \\ \log p^- \end{bmatrix}. \quad (42)$$

Consistency with the general equilibrium conditions requires that the perceived aggregate laws of motion are accurate, in the sense that the forecast errors implied by the system (42) are arbitrarily small. To achieve this consistency, we initialize $\mathbf{\Gamma}_0$, $\mathbf{\Gamma}_1$, and $\mathbf{\Gamma}_2$ with arbitrary values and then simulate the model using Monte Carlo methods with randomly drawn aggregate and idiosyncratic shocks; in the simulation, we let the agents learn from their errors and update the forecasting rules until full convergence. An important aspect of this algorithm involves letting all the market clears, even when the agents' perceived laws of motion are “inaccurate”—that is before full convergence.¹⁹

4 Calibration

For the most part, our calibration of the model relies on parameter values that are standard in the literature. First, the time period in the model equals one quarter. We set α , the value-added share of capital in the Cobb-Douglas production function, to 0.3. Together with the estimation procedure described in Section A.4 of the data appendix, this implies an estimate of decreasing returns-to-scale—the parameter χ —of about 0.85, a value that is within the range of values used in the literature. The quarterly depreciation rate δ is set equal to 0.025. The quasi-fixed costs of production $F_o = 0.05$, which implies that fixed costs equal about 10 percent of sales.²⁰ With this choice, the dividend-payout ratio (i.e., the ratio of dividends to net income) in the model is 50 percent, roughly the same as the average dividend-payout ratio of the S&P 500 firms in the postwar period.

The estimation procedure used to determine the degree of decreasing returns-to-scale is also used to calibrate the Markov chain for the idiosyncratic technology shock z and the process governing its stochastic volatility. Specifically, we assume two states for the idiosyncratic technology shock and using the Markov-chain approximation method of Tauchen [1986], we calibrate the persistence of the process to be 0.80, somewhat less than that implied by the data (see Table A-4 in the data appendix).

The solid line in Figure 4 depicts a measure of uncertainty based on shocks to the profit function

¹⁹In the simulation, we assume that at any point in time, the model economy contains 10,000 heterogeneous firms. We simulate this economy for 1,100 periods by feeding in randomly drawn aggregate and idiosyncratic shocks. In updating the agents' perceived aggregate laws of motion, we drop the initial 100 observation and use the rest to estimate the system of equations (42). The updated laws of motion are then used to update the policy rules implied by the agents' respective dynamic programming problems; these optimization problems are solved numerically using a value function iteration with linear interpolations for the off-the-grid points. See Miao [2006] for the conditions under which a unique equilibrium exists for the this type of an economy.

²⁰According to Compustat data, the median ratio of sales, general, and administrative (SG&A) expenses to sales is about 20 percent. A portion of SG&A costs is accounted for by investment in intangible capital, which is counted as investment by the Bureau of Economic Analysis but is recorded as “expenses” in Compustat. We assume that one half of this ratio reflects fixed costs of production.

that is used to calibrate the time-varying volatility of the process for the idiosyncratic technology shock. Note that like its counterpart derived from equity prices, this uncertainty proxy is highly countercyclical and comoves closely with credit spreads (the dotted line). The steady-state level of uncertainty $\tilde{\sigma}_z$ is set to 15 percent (30 percent annualized), which is equal to the sample mean of the uncertainty proxy shown in Figure 4.²¹ Using this proxy to estimate an empirical counterpart to equation (10) yields $\hat{\rho}_z = 0.75$ (with the 95-percent confidence interval of [0.64, 0.86]) and $\hat{\omega}_\sigma = 0.02$. We set the persistence of the time-varying volatility process to 0.90, a value at the high end of the estimated range, but consistent with the value used by Bloom [2009]. To generate fluctuations in uncertainty in the range between 20 and 40 percent (annualized)—a range consistent with the variability of our uncertainty proxy over the 1976–2012 period—we set the quarterly standard deviation of uncertainty shocks to 0.04.²²

The most daunting calibration challenge concerns the process for the liquidation value of capital given in equation (12). Because we are not aware of any data source that tracks the resale value of fixed capital at the macro level, we estimate an AR(1) specification for p^- using as a proxy the ratio of the price index of used car sales relative to that of new car sales, two components of the monthly CPI published by the Bureau of Labor Statistics (BLS). This yields $\hat{\rho}_{p^-} = 0.97$ and $\hat{\omega}_\kappa = 0.015$, which are the values used in our calibration.²³ The liquidation value of capital in steady state \tilde{p}^- is set equal to 0.5, which implies a steady-state level for the book-value of leverage of one-half, the same as the average leverage calculated from the Compustat data.²⁴ With this calibration, an adverse liquidity shock of one standard deviation reduces the resale value of capital about 3.5 percent. The purchase price of capital p^+ is normalized to one.

Following Prescott [1986], we set the persistence of the aggregate technology shock—the parameter ρ_a in equation (9)—to 0.95; as is usual in the literature, the volatility of innovations of the TFP process—the parameter ω_a —is set to 0.0075 at the quarterly rate. The integration of all exogenous AR(1) processes in the model is approximated by Gaussian quadratures.

Given the calibration of the processes for the idiosyncratic uncertainty and the liquidation value of capital assets, we set the degree of frictions in the corporate bond market—the bankruptcy cost parameter ξ —to generate an average credit spread of 160 basis points, which corresponds to the median of the BBB-Treasury spread shown in Figure 1. Accordingly, we let $\xi = 0.10$, a value consistent with that used by Bernanke, Gertler, and Gilchrist [1999] and the micro-level evidence of

²¹Setting the steady-state level of uncertainty to 30 percent (annualized) may appear to be somewhat low compared with other notable studies in this area; see, for example, Cooper and Haltiwanger [2006]. However, these studies typically rely on plant-level data, while we use Compustat income and balance sheet data, which are subject to a considerable degree of smoothing reflecting firm-level aggregation. In fact, our calibration of the steady-state level of uncertainty is consistent with other studies based on Compustat data (e.g., Hennessy and Whited [2007]).

²²Using this calibration, a shock of one standard deviation implies an increase in uncertainty of about 8 percentage points (annualized), a magnitude that is roughly consistent with the VAR evidence presented in Section 2.2.

²³These estimates are expressed at the quarterly rate and have been converted from monthly estimates based on the 1998:M2–2011:M11 sample period.

²⁴The book-value of leverage in the model corresponds to $b/(\pi + p^+k)$. To calculate an empirical analogue using quarterly Compustat data, we let b equal the book-value of total debt; π gross profits; and p^+k the book-value of (gross) property, plant, and equipment. Over the 1976:Q1–2012:Q3 period, the average leverage in the U.S. nonfinancial corporate sector was 0.49, according to this metric.

Levin, Natalucci, and Zakrajšek [2004] and one that implies a relatively modest degree of additional loss for the lender from bankruptcy. Concerning the survival probability, we set $\eta = 0.95$, a value consistent with the survey of the Business Employment Dynamics conducted by the BLS.²⁵

The estimates of the cost of seasoned equity issuance vary substantially in the literature, from a low of 0.08 in Gomes [2001] to a high of 0.30 in Cooley and Quadrini [2001]. We make a conservative choice by letting $\varphi = 0.12$. Given this value, we choose the lower bound of dividends \bar{d} such that 15 percent of firms, on average, issue new shares in each quarter of the simulation, a proportion that is roughly in line with that implied by the quarterly Compustat data. Finally, we set the lower bond on net worth $\bar{n} = 0$.

Concerning the preferences of the representative household, we specify a standard functional form:

$$u(c, h) = \frac{c^{1-\theta} - 1}{1-\theta} - \zeta \frac{h^{1+\vartheta}}{1+\vartheta};$$

and then choose the simplest specification to reduce the computational burden. Accordingly, we set the coefficient of constant relative risk aversion $\theta = 1$ and assume indivisible labor by setting $\vartheta = 0$, which implies an infinite Frisch elasticity of labor supply; we keep ζ , the weight of the disutility of labor hours, as a free parameter and choose its value so that the real wage in the steady state can be normalized to one. Finally, we set the household's rate of time preference $\beta = 0.99$, which implies an annualized risk-free rate of 4 percent. The parameter values used in the calibration are summarized in Table 4.

5 Model Simulations

5.1 Capital Adjustment Dynamics

In this section, we describe the capital adjustment dynamics in our model. The top panel of Figure 5 display the contours of the firm's investment policy function assuming partial irreversibility only, while the policy function in the bottom panel also takes into account the fixed capital adjustment costs.²⁶ As expected, both policy functions feature (S, s) -type adjustments, where the "S"-shape includes two distinct distinct target capital levels: A level associated with the capital expansion problem and a level associated with the capital contraction problem, which are separated by an investment inaction region. The presence of financial frictions implies that the (S, s) rule governing investment spending will also depend on the firm's (net) liquid asset position.

Focusing first on Panel (a), a somewhat simpler case, consider a firm in a strong financial position (i.e., $x \gg 0$). In that case, the firm's choice of capital tomorrow, as a function of its current production capacity, is well described by the standard (S, s) rule with two target stocks separated

²⁵According to this survey, the average yearly survival rate for the establishments that were born between 1994 and 2009 is 0.784, which implies a quarterly survival rate of 0.942.

²⁶For illustrative purposes, we assume that the liquidation value of capital is 80 percent of the initial purchase price in both examples, which is appreciably above the steady-state value of 50 percent used in our calibration. The fixed investment costs $F_k = 0.01$, the value used in the calibration.

by the inaction region. The presence of financial distortion, however, induces two distinct plateaus in both the capital expansion and contraction problems. The upward sloping region between the two plateaus corresponds to the state space where the capital targets respond positively to an improvement in the firms' financial position. Holding the level of idiosyncratic technology fixed, these regions are in fact characterized by a continuum of targets as summarized by the firm's overall position (x, k) .

As shown in Panel (b), the introduction of fixed adjustment costs expands the investment inaction region. To economize on these costs, the firm allows its capital stock to depreciate below the level that would in the absence of fixed costs trigger a positive investment response; similarly, the firm is willing to tolerate a capital overhang before adjusting its productive capacity. As a result, the adjustment of capital occurs in discrete jumps, which generates lumpy investment dynamics.

The uneven topography of the policy function in the region associated with the capital expansion problem provides a sense of how costly external finance complicates investment dynamics in that case. Note also that financial conditions have less of an effect on the capital contraction problem. In those states, the firm allows the capital overhang to persist longer, which implies that the resulting contraction in production capacity leaves the firm with sufficient holdings of liquid assets, so that its capital contraction target—and therefore disinvestment—is less sensitive to current financial conditions. It is important to note that for illustrative purposes, we constructed the policy functions in the above two examples over a wide range of capacity levels. In the actual simulations of the model, however, the firms almost never reduce their productive capacity—other than in default/liquidation events—because even a small degree of irreversibility (20 percent in the above two examples) makes the sale of installed capital almost always an unprofitable activity.

5.2 Macroeconomic Implications

In this section, we examine the behavior of key endogenous variables in response to the three aggregate shocks in our model: an aggregate technology shock (a); a shock to the volatility of the idiosyncratic technology shock (σ_z); and a shock affecting the liquidation value of capital assets (p^-). Our benchmark economy features a full set of financial distortions. To assess the role of financial frictions, we also solve a version of the model without financial distortions. In that version, the firms face the same nonconvex capital adjustment costs as in the benchmark case, except that they finance investment expenditures using only internal funds and equity, where the issuance of the latter is not subject to any dilution costs. As a result, the stock of outstanding corporate debt is no longer an aggregate state variable, and the forecasting rules in equation (42) are modified accordingly (see Section B.3 of the model appendix for details).

When constructing the impulse response functions, we take into account nonlinearities that are inherent in each model. Specifically, letting $i = 1, \dots, N$ index the N heterogeneous firms in the economy and $h = 1, \dots, H$ index the H periods of the impulse response horizon, we let $\mathcal{Z} = \{z_{ih} \mid i = 1, \dots, N \text{ and } h = 1, \dots, H\}$ denote the associated set of idiosyncratic technology

states implied by the Markov chain with time-varying volatility²⁷. Using the set \mathcal{Z} , we construct two model simulations over the H periods: (1) a simulation perturbed by an aggregate shock; and (2) a simulation without an aggregate shock. Let x_{ih} denote a generic model variable (e.g., capital stock k_{ih}) and index these two simulations by x_{ih}^1 and x_{ih}^0 , respectively. Note that the same set of idiosyncratic technology states \mathcal{Z} underlies the construction of x_{ih}^1 and x_{ih}^0 , for all $i = 1, \dots, N$ and $h = 1, \dots, H$. The only difference between these two simulations is that we introduce an aggregate shock at h^* in the first simulation, which is then allowed to die out in an AR(1) fashion over the remainder of the impulse response horizon.

To remove the effect of the sampling variation of the aggregate shock, we then repeat the above procedure a large number of times. The model-implied impulse response function of the aggregate variable x in response to an aggregate shock—denoted by \hat{x}_h —is then calculated according to

$$\hat{x}_h = 100 \times \log \left[\frac{\sum_{m=1}^M \sum_{i=1}^N x_{m,ih}^1}{\sum_{m=1}^M \sum_{i=1}^N x_{m,ih}^0} \right]; \quad h = 1, \dots, H, \quad (43)$$

where M denotes the number of simulation repetitions.²⁸ In constructing the impulse response functions in this manner, we are capturing the nonlinear aspects of the model—at least at the level of an individual firm—and allowing these nonlinearities to affect aggregate outcomes, even though the perceived aggregate laws of motion used to solve for $x_{m,ih}^1$ are assumed to be log-linear.²⁹ In addition, by explicitly aggregating across heterogeneous firms at each horizon h , our procedure allows us to construct impulse responses of moments that are based on firm-level data, such as a measure of aggregate TFP, a proportion of inactive firms, a measure of “lumpy” investment, and cross-sectional dispersion of credit spreads.

Aggregate technology shock: Figure 6 depicts the behavior of the model’s main endogenous variables in response to a positive aggregate technology shock of one standard deviation. The solid lines represent the impulse response functions of the economy with financial frictions, while the dotted lines correspond to the impulse responses of the economy without any financial distortions. The comparison of the two sets of responses reveals that financial distortions have a minimal effect on the magnitude and shape of the responses of economic activity to the aggregate technology shock. In both model economies, the unanticipated increase in aggregate TFP leads to a strong

²⁷See Section B.1 of the model appendix

²⁸Note that all aggregate shocks, with the exception of the uncertainty shock, have no effect on \mathcal{Z} , the set of idiosyncratic technology states. By design, however, the impact of the uncertainty shock today will have an effect on the dispersion of the idiosyncratic technology shocks in the future. Hence, $z_{m,ih}^1 \neq z_{m,ih}^0$ for this type of aggregate shock. However, the *relative* position of each individual firm in the distribution of the idiosyncratic technology shock will be the same as in the case when the economy is not perturbed by an uncertainty shock. For all other aggregate shocks, $z_{m,ih}^1 = z_{m,ih}^0$, for all m, i , and h .

²⁹If the agents’ perceived aggregate laws of motion fitted perfectly, taking into account the model nonlinearities when computing impulse response functions should, in principle, not make any difference. And although the very high R^2 values reported in Table B-1 of the model appendix indicate a highly accurate fit, even such small deviations from a perfect fit can, as we show below, lead to important differences between impulse responses computed according to our method and those implied by the log-linear approximate laws of motion.

and persistent increase in consumption, investment, and hours worked, dynamics that are similar to those of the canonical RBC framework.

In both cases, the responses of key economic aggregates to the aggregate TFP shock are also consistent with the work of Sim [2006], Khan and Thomas [2008], and Bachmann, Caballero, and Engel [2008], who find that the presence of nonconvex adjustment frictions does not imply significant departures from the dynamics of a standard RBC economy. A novel feature of our analysis is the fact that introducing financial distortions in the model—the significance of which can be judged by the difference in the impulse responses between the two models—also does not seem to change this conclusion. While, the technology-induced boom allows the firms to lever up, the resulting increase in the demand for credit also significantly boosts credit spreads and the risk-free rate. In turn, these changes in financial conditions restrain the expansion of corporate balance sheets and damp the financial accelerator mechanism arising from financial frictions.

The top panel of Table 5 compares the business cycle moments of the two economies, conditional on the aggregate technology shocks only. Consistent with our impulse response results, conditional moments of the key endogenous quantities are very similar across the two model specifications. Moreover, in spite of the presence of nonconvex adjustment frictions and financial distortions, the benchmark economy exhibits the hallmark features of an RBC model, with the investment about three times more volatile than output and highly correlated with output, consumption, and hours worked.

Uncertainty shock: The macroeconomic implications of an uncertainty shock are shown in Figure 7.³⁰ The comparison of the investment response across the two models reveals a striking result: More than three-quarters of the impact of the uncertainty shock on investment is due to financial distortions. To understand how the increase in uncertainty interacts with the different frictions of the benchmark model, we decompose the correlation between the firm-level investment adjustments and idiosyncratic uncertainty into the adjustment at intensive and extensive margins. We measure the adjustment at the extensive margin by calculating, for each period, the fraction of firms with positive investment expenditures ($\text{Freq}[I^+]$). As another metric, we also consider “lumpy” investment, defined as a proportion of firms with capital expenditures in excess of 10 percent of the book-value of installed capital ($\text{Freq}[\text{lumpy-}I^+]$). The intensive margin, by contrast, is defined as the average positive capital expenditures ($\text{Avg}[I^+]$) in each period.³¹

According to the standard nonconvex adjustment theory, aggregate investment dynamics, especially in response to fluctuations in uncertainty, will primarily reflect the firms’ adjustment at the extensive margin—a jump in uncertainty raises the option value of waiting, which increases the

³⁰In this scenario, the volatility of the idiosyncratic technology shock (σ_z) jumps 6 percentage points from its steady-state level (30 percent, annualized) upon the impact of the shock—a shock of about one standard deviation—and then reverts back to its average level following the process in equation (10).

³¹In this experiment, the liquidation value of capital is fixed at its steady-state value of 0.5. With this calibration, liquidating capital is almost never optimal, unless the realization of the idiosyncratic technology shock is unusually bad and the firm has a significant capital overhang problem, a combination that generates large losses due to fixed operating costs. As a result, disinvestment at the firm level plays a minor role in the determination of the dynamics of aggregate investment, which allows us to focus on positive investment expenditures only.

proportion of firms in the inactive region.³² According to the top panel of Table 6, this certainly seems to be the case in the model without financial frictions: The frequency of positive investment adjustments is negatively correlated with the movements in uncertainty, implying that an increase in uncertainty tends to push more firms in the inaction region; the frequency of lumpy investment episodes is also negatively correlated with fluctuations in uncertainty. In contrast, the correlation between the average level of capital expenditures and uncertainty is essentially zero, a result consistent with the conventional wisdom that the response of aggregate investment to uncertainty shocks in a model with nonconvex adjustment frictions reflects primarily adjustment at the extensive margin.

These results change substantially when the model includes financial distortions. In that case, most of the adjustment in aggregate investment occurs at the intensive margin. The correlation of almost -0.7 between the average size of capital expenditures and uncertainty implies that uncertainty shocks leads to a large reduction in the target level of capital for active firms, rather than to a reduction in the number of active firms. In fact, both the frequency of positive capital expenditures and lumpy investments are positively correlated with fluctuations in uncertainty, a dynamic that works against generating a large decline in aggregate investment.

As shown in Figure 7, these differences in the adjustment dynamics of aggregate investment are due entirely to financial frictions. While the increase in uncertainty induces an appreciable decline in the risk-free rate, this easing of financial conditions is more than offset by the sharp increase in credit spreads, which jump about 175 basis points upon impact and remain elevated for a considerable period of time. The resulting deterioration in borrowing terms implies a significant increase in the user cost of capital, which leads firms to slash capital expenditures and delever, key reasons why most of the firm-level adjustment occurs at the intensive margin. In the model without financial frictions, by contrast, this user cost of capital channel is completely absent. The response of the risk-free rate to the uncertainty shock is economically trivial, while the proportion of inactive firms increases significantly.

The impulse responses from the benchmark model are thus consistent with our empirical evidence, which showed that fluctuations in uncertainty are associated with large swings in credit spread, which, in turn, are an important determinant of investment spending. We interpret the combination of our empirical evidence and model simulations as indicating that general equilibrium models steeped in the Modigliani–Miller paradigm of frictionless financial markets are likely missing a crucial part of the mechanism through which uncertainty shocks are propagated to the real economy.

The middle panel of Table 5 summarizes the conditional business cycle moments of this experiment. The high relative volatility of investment suggests that fluctuations in uncertainty can be an important driver of this cyclically-sensitive component of aggregate demand. However, as evidenced by the low conditional volatility of output, this source of disturbances, is unable to generate sufficiently volatile aggregate business cycles. In both economies, a substantial portion of the drop

³²See, for example, Bernanke [1983], Bertola and Caballero [1994], and Bloom [2009].

in investment in response to increased uncertainty is offset by an increase in consumption, which dampens output fluctuations. This negative comovement occurs because the uncertainty shock—unlike the aggregate TFP shock—does not directly affect the resource constraint of the economy. The counterfactual conditional negative correlation between investment and consumption is an artifact of the closed-economy setting, which does not allow the household to invest in a foreign asset and thus should not be taken as evidence against the importance of uncertainty shocks as a driving force of business cycle fluctuations.

Capital liquidity shock: The dynamic behavior of the two economies when hit by an adverse liquidity shock is shown in Figure 8.³³ The differences in macroeconomic outcomes between the two models are striking. In the model without financial frictions, a drop in the liquidation value of capital has essentially no effect on economic activity. The same shock, by contrast, induces a severe recession in the economy with financial distortions. Business investment plunges more than 15 percent upon the impact of the shock, and the hours worked decline sharply and remain below steady state for a prolonged period of time. Even consumption—after the initial increase—declines noticeably and stays subdued over the remainder of the forecast horizon. These dynamics translate into a decline in aggregate output of more than 0.5 percent upon impact and ensure that the resulting recovery is slow and protracted.

Financial frictions are again the key propagation mechanism of this shock, as the drop in the liquidation value of capital assets immediately curtails the firms’ debt capacity and induces a significant and persistent widening of credit spreads. The resulting deterioration in business creditworthiness increases the demand for safe assets and leads to a drop in the risk-free rate, a “flight-to-quality” phenomenon that is a hallmark feature of financial crises. The decline in the risk-free rate is of the same magnitude as the increase in credit spreads, which indicates that fluctuations in the default-risk component of corporate bond prices—as opposed to swings in risk-free base rates—are the source of the information content of the “bond market’s q ” for business fixed investment (cf. Gilchrist and Zakrajšek [2007] and Philippon [2009]).

The bottom panel of Table 6 shows the correlations of the different investment adjustment margins with the liquidation value of capital. Not surprisingly, these correlations are essentially zero in the economy with frictionless financial markets. However, when financial distortions are present, the average (positive) capital expenditure is strongly positively correlated with fluctuations in the resale value of capital, reflecting the tight link between liquidity shocks and the firms’ debt capacity. On the extensive margin, the correlation between the liquidation value of capital and the frequency of positive investment expenditures is negative, whereas the correlation with lumpy investment is positive. Evidently, an improvement in the liquidity of the secondary market for capital—an improvement in the sense of higher resale value—induces firms to economize on transaction costs arising from the fixed capital adjustment costs by increasing the average size of

³³In this scenario, a shock reduces the liquidation value of the firms’ capital assets (p^-) 5 percent upon impact, a shock of about 1.5 standard deviations; the liquidation value of capital is then allowed to revert back to its steady-state value of 0.5 following the process in equation (12).

investment expenditures and by more frequently making large capital purchases.

According to the conditional moments shown in the bottom panel of Table 5, such liquidity shocks have the potential of being an important source of cyclical fluctuations in an economy with financial frictions. In our simulation, the resale value of capital fluctuates between 85 percent and 115 percent of its steady-state value, a relatively modest amount of variability in light of much larger swings in the value of commercial real estate assets, for example. Nevertheless, this reasonable amount of volatility in the resale value of capital generates economically realistic amounts of variability in key endogenous aggregates: The conditional standard deviation of aggregate output implied by the benchmark model closely matches the volatility of U.S. real GDP, and the relative conditional volatilities of consumption and hours worked are also close to their respective empirical counterparts; that said, the model does generate business fixed investment that is too volatile relative to what is observed in the data.

The benchmark model also delivers more realistic comovements between main macroeconomic quantities. The degree of comovement between investment and output is very close to that observed in the data, while the correlation between consumption and output, though still a bit on the high side, is more reasonable. Importantly, shocks to the liquidation value of capital generate a positive correlation between investment and consumption, though for the same reasons as discussed above, the degree of comovement is still notably below that implied by the actual data.

Role of nonlinearities: We close this section by briefly discussing the nonlinear aspects of our benchmark model. Recall that our method for computing the impulse response functions took into account the nonlinearities in the firms' investment and financial policies that arise naturally in an economy with irreversible investment, fixed capital adjustment costs, and financial distortions. We chose this computationally intensive approach, in part, because the response of the model to various aggregate shocks may not be reflected fully by the log-linear specification of the agents' perceived aggregate laws of motion.

The economic significance of the model's inherent nonlinearities is illustrated in Figure 9. The solid lines show the response of aggregate investment to the specified shock constructed according to equation (43), while the dotted line depicts the corresponding responses based solely on the agents' perceived aggregate laws of motion. The latter method clearly understates the sensitivity of aggregate investment to all three shocks—the impact of technology and uncertainty shocks on investment is off by as much as 25 percent. These differences in impulse responses suggest that the agents' perceived laws of motion may be misspecified in spite of the very high R^2 values reported in Table B-1 of the model appendix. However, exploring nonlinear forms for the laws of motion—for example, by including higher-order moments—is from a computational perspective prohibitively expensive, and we leave this issue for future research.

5.3 Cyclical Properties of Credit Spreads

One of the defining feature of the U.S. business cycle is the strong negative correlation between output growth and yield spreads on the various forms of external finance. Another notable, though less emphasized, feature is the countercyclical behavior of the *dispersion* in credit spreads (see Figure A-1 in Section A.1 of the data appendix). Table 7 compares the cyclical behavior of credit spreads—both the level and dispersion—implied by our benchmark model with the U.S. data.

Assuming that aggregate technology shocks are the sole source of economic fluctuations (column 1) implies that both the average level of credit spreads and their cross-sectional dispersion are strongly procyclical, results that run counter to the data. The procyclical behavior of credit spreads in the model arises because a positive aggregate TFP shock induces firms to lever up to take advantage of new profitable investment opportunities. This increase in leverage, however, lowers the firms' collateralizable net worth, implying a deterioration in their creditworthiness that induces a widening of credit spreads. The counterfactual procyclicality of credit spreads in a technology-driven business cycle is a general feature of general equilibrium models with costly external finance, a fact pointed out by Gomes, Yaron, and Zhang [2003] in their critique of the financial accelerator mechanism.

When the business cycle is driven by either uncertainty or liquidity shocks (columns 2–3), the level of credit spreads and their dispersion are strongly countercyclical, comovements strongly supported by the data. These two types of economic disturbances share a common feature in that they both impair the borrowers' creditworthiness. In response, the supply curve of loanable funds shifts inward, causing the quantity of credit and its price to move in the opposite direction, which generates the countercyclical behavior in credit spreads.³⁴ An adverse technology shock, by contrast, induces a downward shift in the demand for credit, which makes quantities and prices move in the same direction, thereby generating procyclical credit spreads.

These results have a number of important implications for research into the causes and sources of business cycle fluctuations. From a theoretical perspective, the procyclical behavior of credit spreads in financial accelerator models need not be a fatal flaw, as researchers might be looking at wrong shocks, rather than using a wrong model. On the empirical side, our analysis argues that a successful estimation and identification of standard monetary DSGE models with financial markets should include measures of credit spreads—along with other potential financial variables—in the list of observables, a point underscored by the recent work of Christiano, Motto, and Rostagno [2013].

³⁴The counterfactual positive correlation between credit spreads and net worth in the case when uncertainty shocks are the sole source of business cycle fluctuations reflects the rapid deleveraging of the business sector when faced with more stringent borrowing terms implied by increased uncertainty.

6 Conclusion

In this paper, we argued that financial distortions due agency problems between financial market participants are an important part of the transmission mechanism by which fluctuations in uncertainty affect aggregate investment. We explored the quantitative significance of this mechanism in a context of a general equilibrium model with heterogeneous firms facing time-varying idiosyncratic uncertainty, nonconvex capital adjustment costs—including both partial investment irreversibility and fixed investment costs—and financial distortions in the debt and equity markets. As in the standard framework, the partial investment irreversibility in our model induces firms to adopt a wait-and-see attitude in response increased uncertainty. At the same time, the implied illiquidity of capital assets reduces the firms' debt capacity because in the case of costly default, the liquidation of capital lowers the recovery value of corporate debt claims.

Our simulations indicate that financial distortions are a powerful conduit through which uncertainty shocks affect aggregate investment. A jump in uncertainty leads to a sharp and persistent widening of credit spreads—implying an increase in the user cost of capital—which induces firms to simultaneously slash capital expenditures and delever. This quantitatively important channel is absent in an economy without financial distortions, where the significantly more-attenuated response of investment to uncertainty shocks reflects solely the aggregation of the standard wait-and-see decisions of individual firms. These model-implied dynamics are consistent with both our macro and micro-level empirical evidence, which shows that movements in corporate bond credit spreads are economically and statistically an important part of the uncertainty-investment nexus.

The unified treatment of nonconvex capital adjustment costs and financial market frictions also highlights the potential role that shocks to the liquidation value of capital play in business cycle fluctuations. With partial investment irreversibility, an adverse shock to the resale value of capital curtails the debt capacity of firms by reducing the collateral value of their capital assets, creating an interaction between the capital and debt overhang problems. According to the model simulations such capital liquidity shocks can be a powerful driver of economic fluctuations. And lastly, in addition to matching a number of important business cycle moments, our simulations also show that when the the business cycle is driven by either uncertainty or capital liquidity shocks, the level of credit spreads and their dispersion are strongly countercyclical, comovements consistent with the data and contrary to those implied by technology-driven real business cycle models.

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Table 1: Uncertainty and Credit Spreads

Explanatory Variable	(1)	(2)	(3)	(4)
$\log \sigma_{it}$	0.730 (0.041)	0.459 (0.046)	0.484 (0.049)	0.216 (0.021)
R_{it}^E	-0.095 (0.026)	-0.112 (0.025)	-0.109 (0.024)	-0.053 (0.009)
$[\Pi/A]_{it}$	-4.100 (0.698)	-1.835 (0.502)	-1.500 (0.475)	-1.318 (0.385)
$\log[D/E]_{i,t-1}$	0.212 (0.024)	0.056 (0.013)	0.049 (0.013)	0.078 (0.011)
Adjusted R^2	0.474	0.641	0.648	0.797
Credit Rating Effects ^a	-	0.000	0.000	0.000
Industry Effects ^b	-	-	0.000	0.000
Time Effects ^c	-	-	-	0.000

NOTE: Sample period: 1973:M1–2012:M9 at a quarterly frequency. No. of firms = 1,124; No. of bonds = 6,373; and Obs. = 107,482. The dependent variable in all specifications is $\log(s_{it}[k])$, the logarithm of the credit spread of bond k (issued by firm i) in month t . All specifications include a constant, a vector of bond-specific control variables $\mathbf{X}_{it}[k]$ (not reported) and are estimated by OLS. Robust asymptotic standard errors reported in parentheses are double-clustered in the firm (i) and time (t) dimension, according to Cameron, Gelbach, and Miller [2011].

^a p -value for the exclusion test of credit-rating fixed effects.

^b p -value for the exclusion test of industry fixed effects.

^c p -value for the exclusion test of time fixed effects.

Table 2: Uncertainty, Credit Spreads, and Investment
(Static Panel Data Specification)

Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)
$\log \sigma_{it}$	-0.169 (0.036)	-0.081 (0.034)	-0.157 (0.034)	-0.036 (0.035)	0.022 (0.033)	-0.062 (0.034)
$\log s_{it}$	-	-	-	-0.206 (0.021)	-0.172 (0.021)	-0.152 (0.021)
$\log[Y/K]_{it}$	0.558 (0.046)	-	-	0.535 (0.045)	-	-
$\log[\Pi/K]_{it}$	-	1.166 (0.086)	-	-	1.075 (0.088)	-
$\log Q_{i,t-1}$	-	-	0.715 (0.040)	-	-	0.645 (0.041)
R^2 (within)	0.325	0.307	0.297	0.349	0.323	0.310

NOTE: Sample period: 1973:M1–2012:M9 at an annual frequency; No. of firms = 772; Obs. = 8,557. The dependent variable in all specifications is $\log[I/K]_{it}$, the logarithm of investment rate of firm i in year t . All specifications include time fixed effects (not reported) and firm fixed effects, which are eliminated using the within transformation, with the resulting specification estimated by OLS. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. Parameter estimates for $\log[\Pi/K]_{it}$ and the associated standard errors are adjusted for the fact that $\log[\Pi/K]_{it}$ is computed as $\log(0.5 + [\Pi/K]_{it})$.

Table 3: Uncertainty, Credit Spreads, and Investment
(Dynamic Panel Data Specification)

Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)
$\log \sigma_{it}$	-0.272 (0.062)	-0.179 (0.059)	-0.199 (0.060)	-0.123 (0.057)	-0.078 (0.054)	-0.106 (0.057)
$\log s_{it}$	-	-	-	-0.101 (0.031)	-0.068 (0.031)	-0.080 (0.032)
$\log[I/K]_{i,t-1}$	0.568 (0.028)	0.576 (0.023)	0.538 (0.029)	0.565 (0.027)	0.567 (0.023)	0.535 (0.023)
$\log[Y/K]_{it}$	0.446 (0.056)	-	-	0.452 (0.053)	-	-
$\log[\Pi/K]_{it}$	-	0.918 (0.144)	-	-	0.908 (0.135)	-
$\log Q_{i,t-1}$	-	-	0.548 (0.045)	-	-	0.507 (0.042)
L-R effect: uncertainty	-0.630 (0.142)	-0.421 (0.137)	-0.430 (0.125)	-0.282 (0.130)	-0.180 (0.124)	-0.228 (0.124)
L-R effect: spreads	-	-	-	-0.233 (0.070)	-0.156 (0.069)	-0.171 (0.066)

NOTE: Sample period: 1973:M1–2012:M9 at an annual frequency; No. of firms = 758; Obs. = 6,615. The dependent variable in all specifications is $\log[I/K]_{it}$, the logarithm of investment rate of firm i in year t . All specifications include time fixed effects (not reported) and firm fixed effects, which are eliminated using the forward orthogonal deviations transformation. The resulting specification is estimated by GMM using a one-step weighting matrix; see Arellano and Bover [1995] for details. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. Parameter estimates for $\log[\Pi/K]_{it}$ and the associated standard errors are adjusted for the fact that $\log[\Pi/K]_{it}$ is computed as $\log(0.5 + [\Pi/K]_{it})$.

Table 4: Model Calibration Summary

Model Parameter	Value
<i>Production and capital accumulation</i>	
Value-added share of capital (α)	0.30
Decreasing returns-to-scale (χ)	0.85
Fixed costs of production (F_o)	0.05
Fixed costs of investment (F_k)	0.01
Depreciation rate (δ)	0.025
Purchase price of capital (p^+)	1.00
<i>Financial markets</i>	
Lower bound on net worth (\bar{n})	0.00
Survival probability (η)	0.95
Bankruptcy costs (ξ)	0.10
Equity issuance costs (φ)	0.12
<i>Representative household</i>	
Relative risk aversion (θ)	1.00
Relative disutility of hours worked (ζ)	1.00
Frisch elasticity of labor supply ($1/\vartheta$)	∞
<i>Exogenous shocks</i>	
Persistence of the idiosyncratic technology shock process ^a	0.80
Steady-state level of idiosyncratic uncertainty ($\tilde{\sigma}_z$)	0.15
Persistence of the idiosyncratic uncertainty process (ρ_σ)	0.90
Volatility of innovations of the idiosyncratic uncertainty process (ω_σ)	0.04
Persistence of the aggregate technology shock process (ρ_a)	0.98
Volatility of innovations of the aggregate technology shock process (ω_a)	0.0075
Steady-state liquidation value of capital (\tilde{p}^-)	0.50
Persistence of the liquidation value of capital process (ρ_{p^-})	0.98
Volatility of innovations of the liquidation value of capital process (ω_κ)	0.015

NOTE: Period in the model equals one quarter. Values of parameters with a time dimension are expressed at quarterly rates. See text for details.

^a The idiosyncratic technology shock follows a 2-state Markov chain process with time-varying volatility; see Section B.1 of the model appendix for details.

Table 5: Model-Implied Conditional Business Cycle Moments

<i>Conditional on Technology Shocks Only</i>								
	RSTD(I)	RSTD(C)	RSTD(H)	STD(Y)	Corr(I, Y)	Corr(C, Y)	Corr(H, Y)	Corr(C, I)
Model w/ FF	2.60	0.95	0.12	2.47	0.63	0.99	0.46	0.54
Model w/o FF	2.90	0.98	0.12	2.32	0.53	0.99	0.22	0.53
<i>Conditional on Uncertainty Shocks Only</i>								
	RSTD(I)	RSTD(C)	RSTD(H)	STD(Y)	Corr(I, Y)	Corr(C, Y)	Corr(H, Y)	Corr(C, I)
Model w/ FF	13.5	0.63	0.88	0.23	0.65	0.49	0.78	-0.33
Model w/o FF	14.6	0.63	0.71	0.10	0.76	0.71	0.78	0.10
<i>Conditional on Liquidity Shocks Only</i>								
	RSTD(I)	RSTD(C)	RSTD(H)	STD(Y)	Corr(I, Y)	Corr(C, Y)	Corr(H, Y)	Corr(C, I)
Model w/ FF	6.71	0.60	0.55	1.11	0.61	0.88	0.86	0.16
Model w/o FF	14.3	0.63	0.69	0.09	0.78	0.73	0.78	0.14
<i>Memo: Data</i> ^a	2.79	0.42	0.60	1.12	0.63	0.56	0.65	0.43

NOTE: The model-implied conditional moments are based on 1,000 quarters of simulated data for 10,000 firms: Y = output; I = investment; C = consumption; and H = hours worked. RSTD(x) denotes the relative standard deviation of x —that is, the standard deviation of variable $x = I, C, H$, relative to the standard deviation of output (i.e., $\text{STD}(x)/\text{STD}(Y)$). All model-implied aggregate series are expressed in percent deviations from their respective steady-state values; see text for the definition of shocks and other details.

^a Sample period: 1962:Q1–2012:Q3. Variable definitions: Y = nonfarm business sector output (c-w \$2005); I = business investment in equipment & software (c-w \$2005); C = PCE on nondurable goods & services (c-w \$2005); and H = index of hours worked for all persons (nonfarm business sector). Actual data are transformed into stationary series by log-differencing.

Table 6: Extensive and Intensive Investment Adjustment Margins

<i>Conditional on Uncertainty Shocks Only</i>		
Selected Correlations	Model w/o FF	Model w/ FF
Corr(Freq[I^+], σ_z)	-0.135	0.210
Corr(Freq[lumpy- I^+], σ_z)	-0.215	0.101
Corr(Avg[I^+], σ_z)	-0.076	-0.673
<i>Conditional on Liquidity Shocks Only</i>		
Selected Correlations	Model w/o FF	Model w/ FF
Corr(Freq[I^+], p^-)	-0.051	-0.837
Corr(Freq[lumpy- I^+], p^-)	-0.058	0.378
Corr(Avg[I^+], p^-)	-0.038	0.837

NOTE: The model-implied moments are based on 1,000 quarters of simulated data for 10,000 firms. Freq[I^+] = frequency of positive investment expenditures; Freq[lumpy- I^+] = frequency of positive “lumpy” investment expenditures, where lumpy investment is defined as capital expenditures greater than 10 percent of the book value of currently installed capital; and Avg[I^+] = average positive investment expenditure.

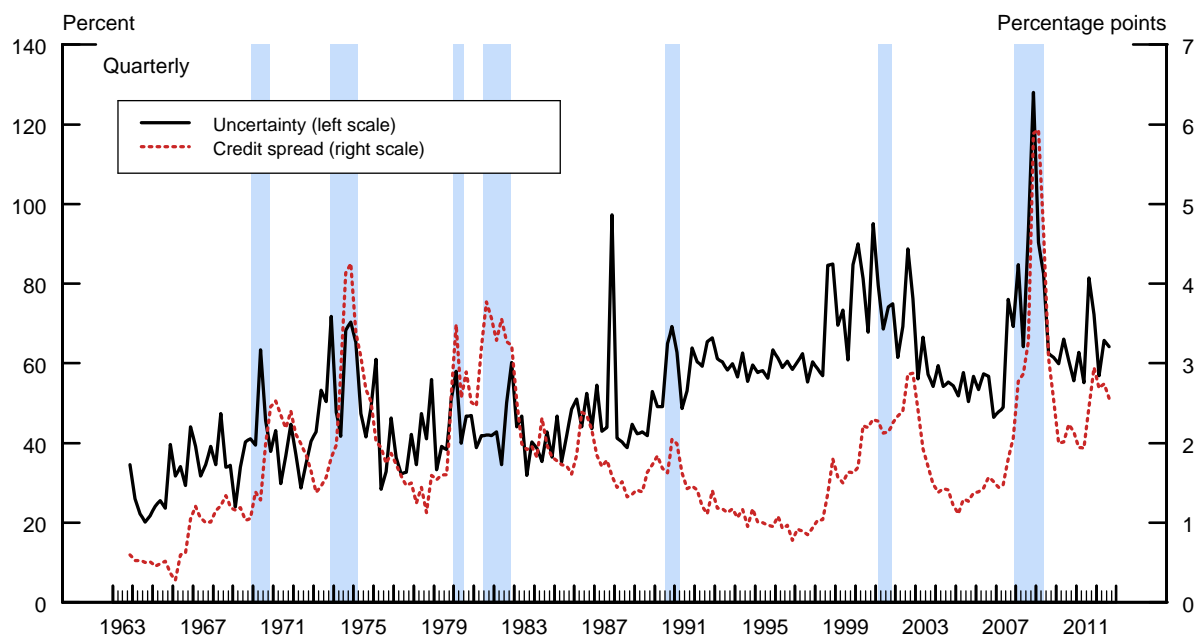
Table 7: Cyclical Properties of Credit Spreads
(*Model vs. Data*)

Selected Correlations	Conditional on the Type of Shock			
	Technology	Uncertainty	Liquidity	Data
Corr(S, Y)	0.927	-0.811	-0.938	-0.457
Corr(S, I)	0.626	-0.515	-0.577	-0.531
Corr(S, C)	0.916	-0.368	-0.816	-0.498
Corr(S, NW)	-0.813	0.802	-0.703	-0.297
Corr(STD(S), Y)	0.933	-0.832	-0.950	-0.245

NOTE: The model-implied moments are based on 1,000 quarters of simulated data for 10,000 firms: S = credit spread; Y = output; I = investment; C = consumption; and NW = net worth. With the exception of the credit spread, all model-implied aggregate series are expressed in percent deviations from their respective steady-state values; see text for the definition of shocks and other details.

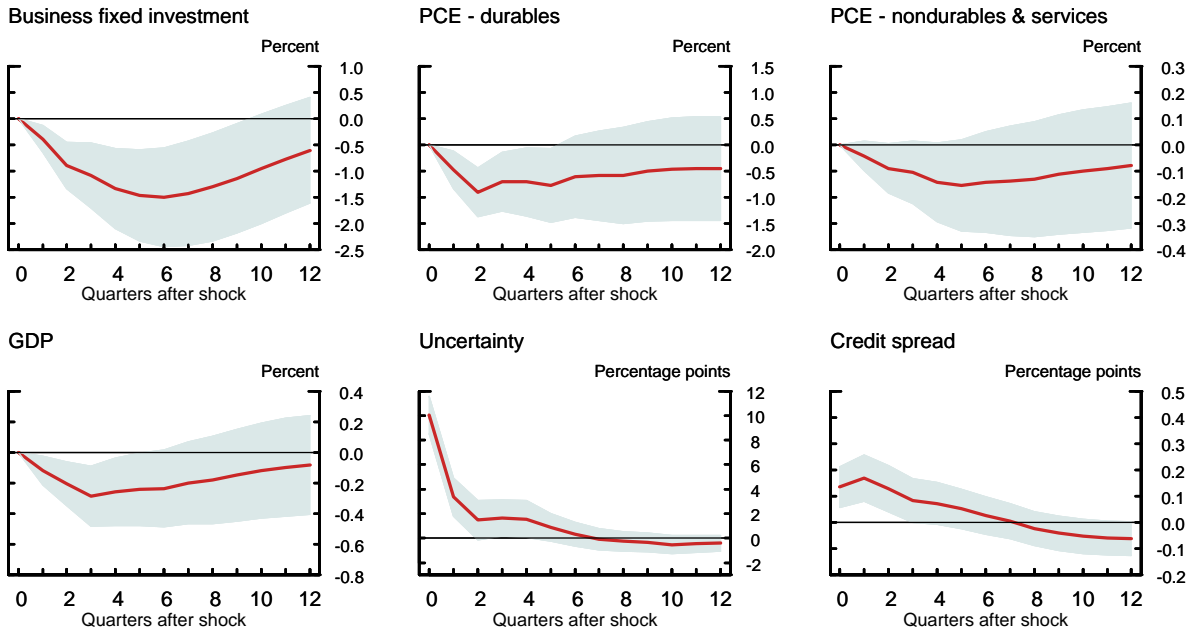
^a Sample period: 1962:Q1–2012:Q3 (unless noted otherwise). Variable definitions: S = 10-year BBB-Treasury corporate bond spread; Y = nonfarm business sector output (c-w \$2005); I = business investment in equipment & software (c-w \$2005); NW = market-value of net worth in the nonfinancial corporate sector (deflated by the GDP price deflator, 2005 = 100); and STD(S) = cross-sectional standard deviation of credit spreads (Sample period: 1973:Q1–2012:Q3). With the exception of the credit spread (S) and the cross-sectional dispersion of credit spreads (STD(S)), actual data are transformed into stationary series by log-differencing.

Figure 1: Uncertainty and Credit Spreads

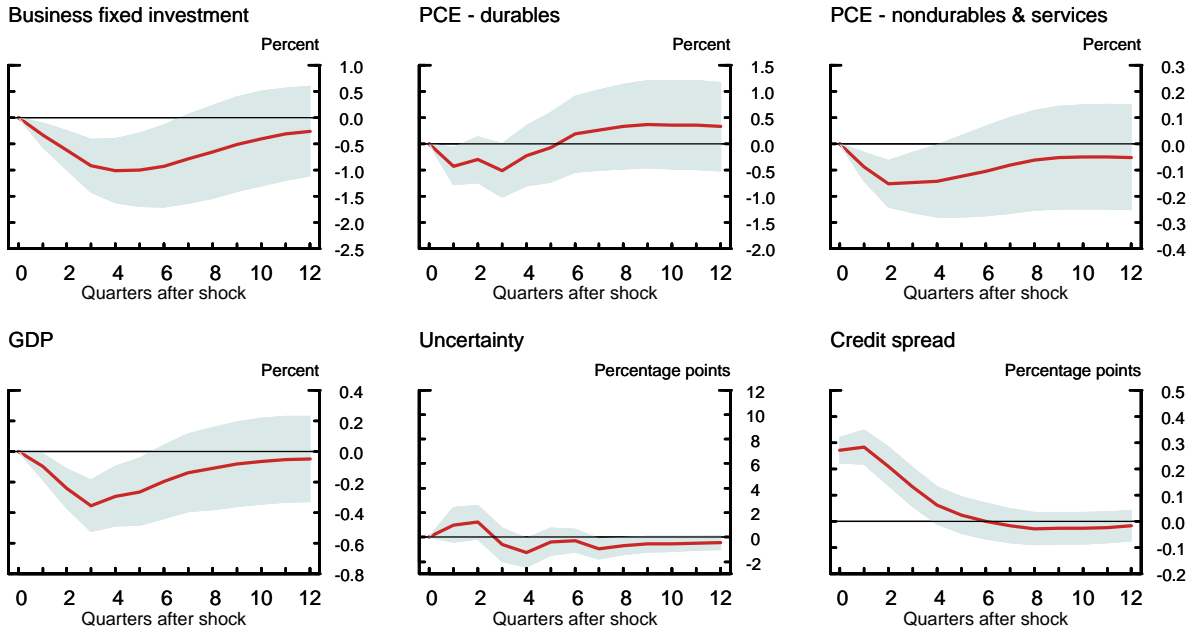


NOTE: Sample period: 1963:Q4–2012:Q3. The solid line depicts the estimate of idiosyncratic uncertainty (in annualized percent) based on firm-level equity returns (see text for details). The dotted line depicts the spread between the 10-year yield on BBB-rated nonfinancial corporate bonds and the 10-year Treasury yield. The shaded vertical bars denote the NBER-dated recessions.

Figure 2: Macroeconomic Implications of Uncertainty and Financial Shocks
(Identification Scheme I)



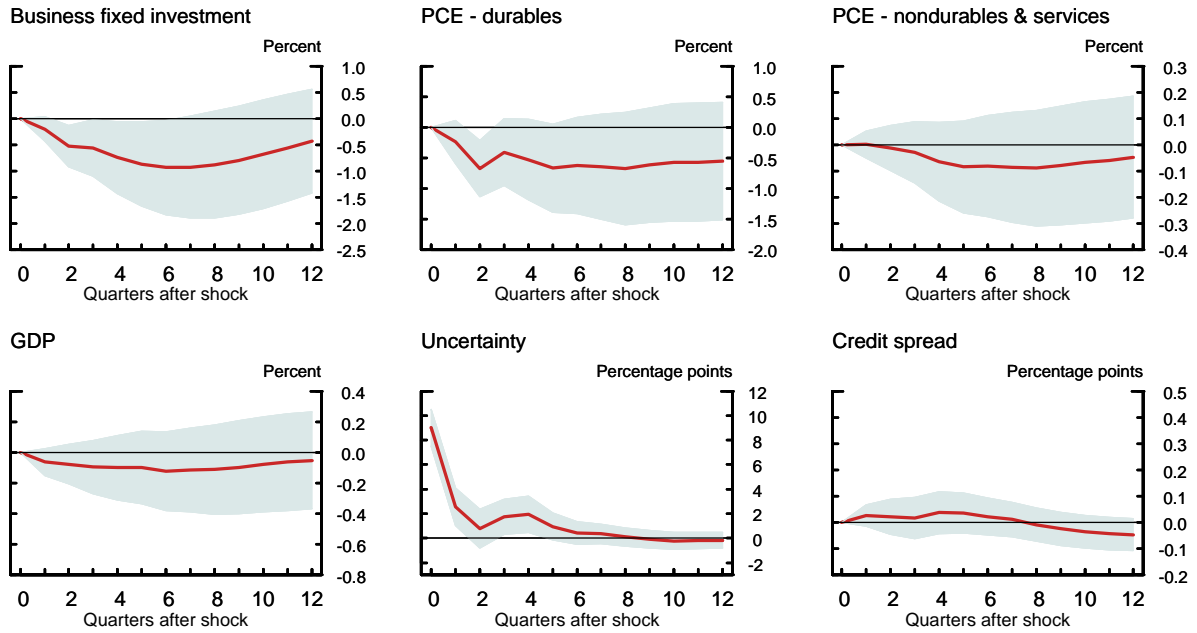
(a) Response of selected macroeconomic variables to an uncertainty shock



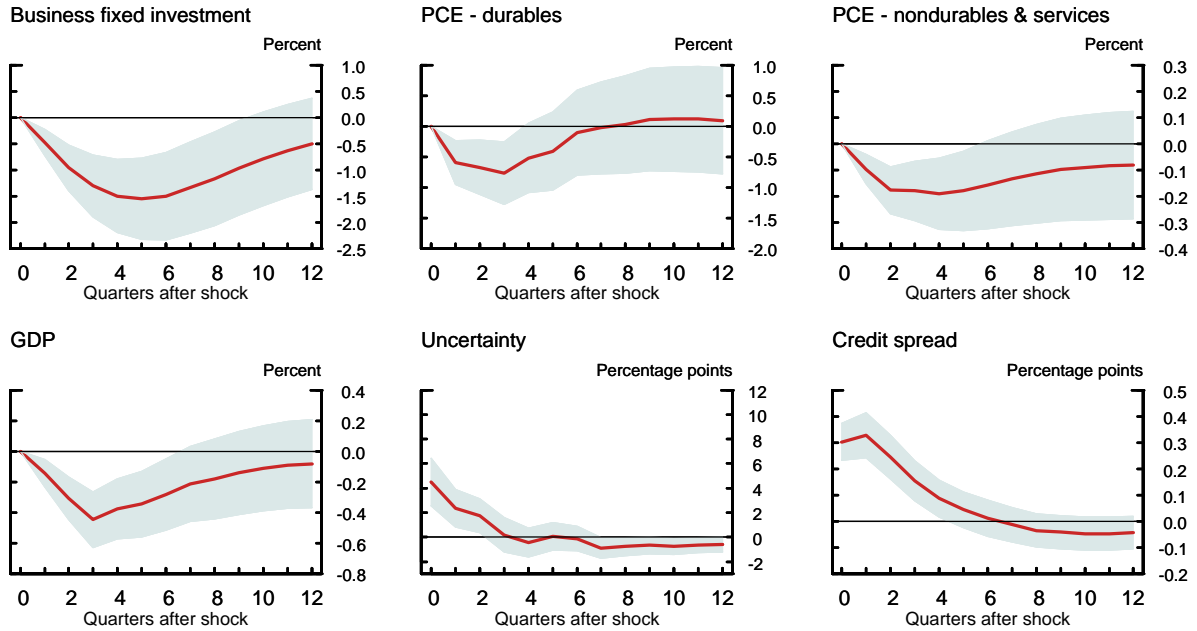
(b) Response of selected macroeconomic variables to a financial shock

NOTE: The panels in Figure 3(a) depict impulse responses of selected macroeconomic and financial indicators to an orthogonalized 1-standard-deviation shock to our estimate of idiosyncratic uncertainty; the panels in Figure 3(b) depict the corresponding impulse response to an orthogonalized 1-standard-deviation shock to the 10-year BBB-Treasury credit spread. Identification scheme I corresponds to the following recursive ordering of the VAR system: $(i_t, c_t^D, c_t^N, y_t, p_t, s_t, \sigma_t, m_t)$; see text for details. The shaded bands represent the 95-percent confidence intervals based on 1,000 bootstrap replications.

Figure 3: Macroeconomic Implications of Uncertainty and Financial Shocks
(Identification Scheme II)



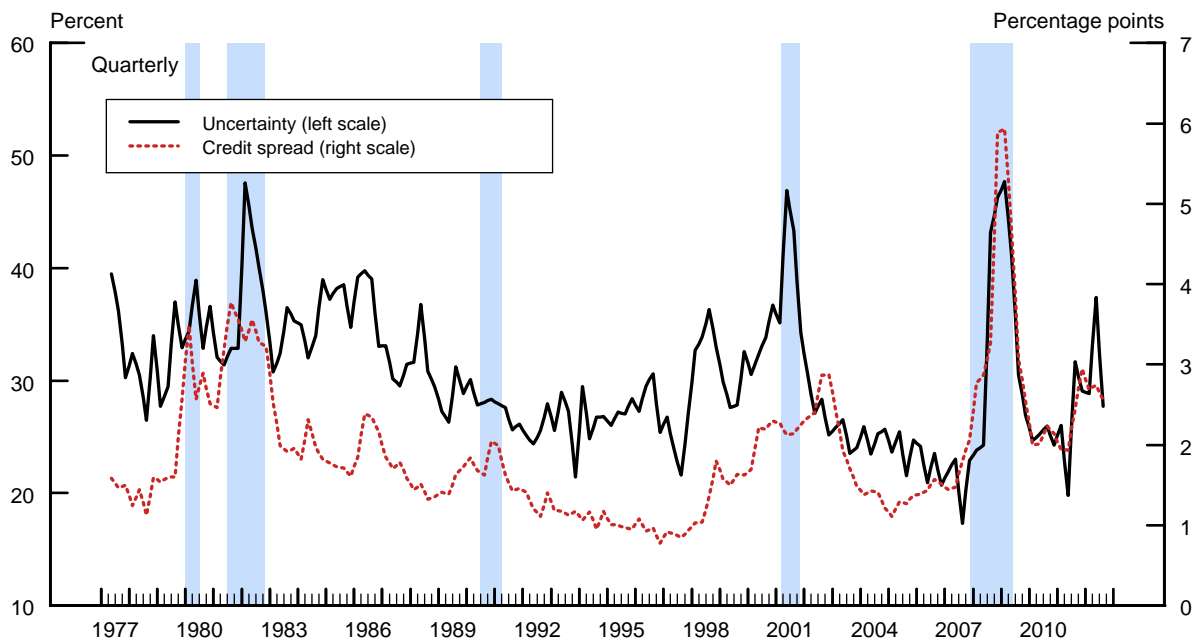
(a) Response of selected macroeconomic variables to an uncertainty shock



(b) Response of selected macroeconomic variables to a financial shock

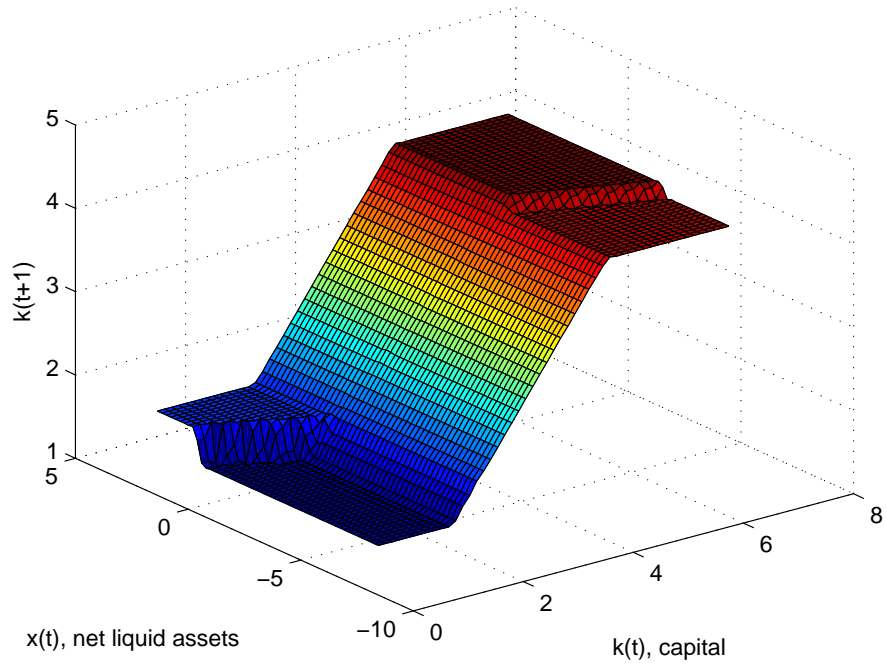
NOTE: The panels in Figure 4(a) depict impulse responses of selected macroeconomic and financial indicators to an orthogonalized 1-standard-deviation shock to our estimate of idiosyncratic uncertainty; the panels in Figure 4(b) depict the corresponding impulse response to an orthogonalized 1-standard-deviation shock to the 10-year BBB-Treasury credit spread. Identification scheme II corresponds to the following recursive ordering of the VAR system: $(i_t, c_t^D, c_t^N, y_t, p_t, \sigma_t, s_t, m_t)$; see text for details. The shaded bands represent the 95-percent confidence intervals based on 1,000 bootstrap replications.

Figure 4: Uncertainty Based on Shocks to the Profit Function

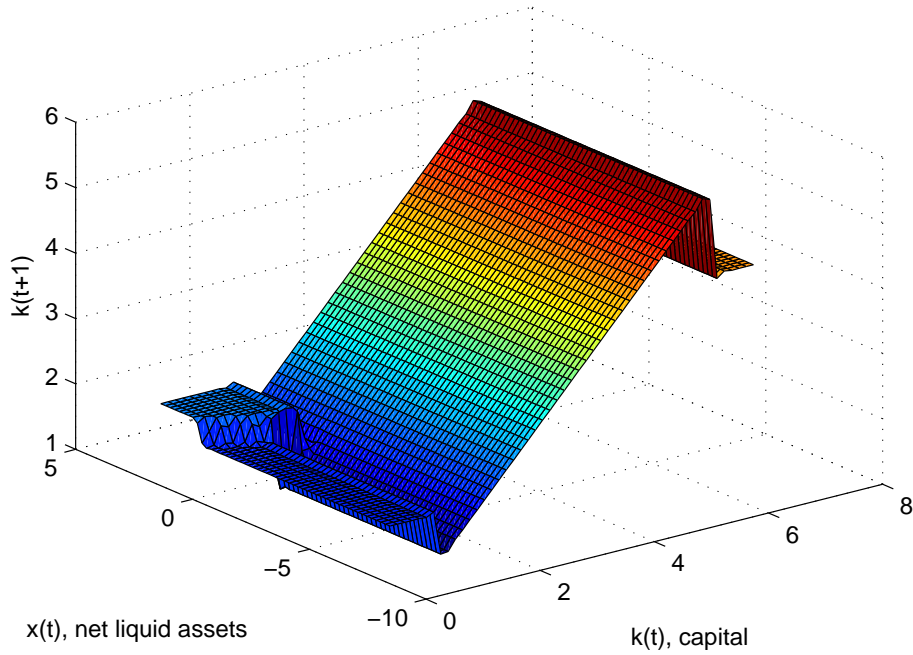


NOTE: Sample period: 1977:Q2–2012:Q3. The solid line depicts the estimate of idiosyncratic uncertainty (in annualized percent) based on shocks to the profit function (see Section A.4 of the data appendix for details). The dotted line depicts the spread between the 10-year yield on BBB-rated nonfinancial corporate bonds and the 10-year Treasury yield. The shaded vertical bars denote the NBER-dated recessions.

Figure 5: (S, s) Investment Policy Functions



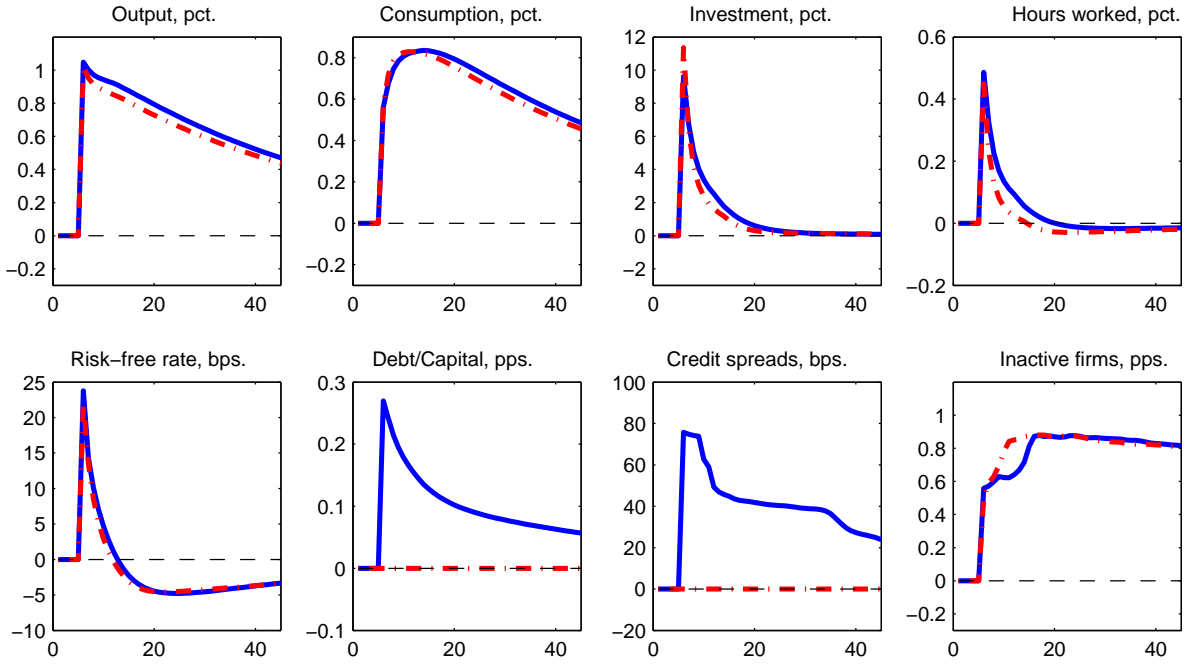
(a) Costly investment reversibility only



(b) Costly investment reversibility and lumpy adjustment

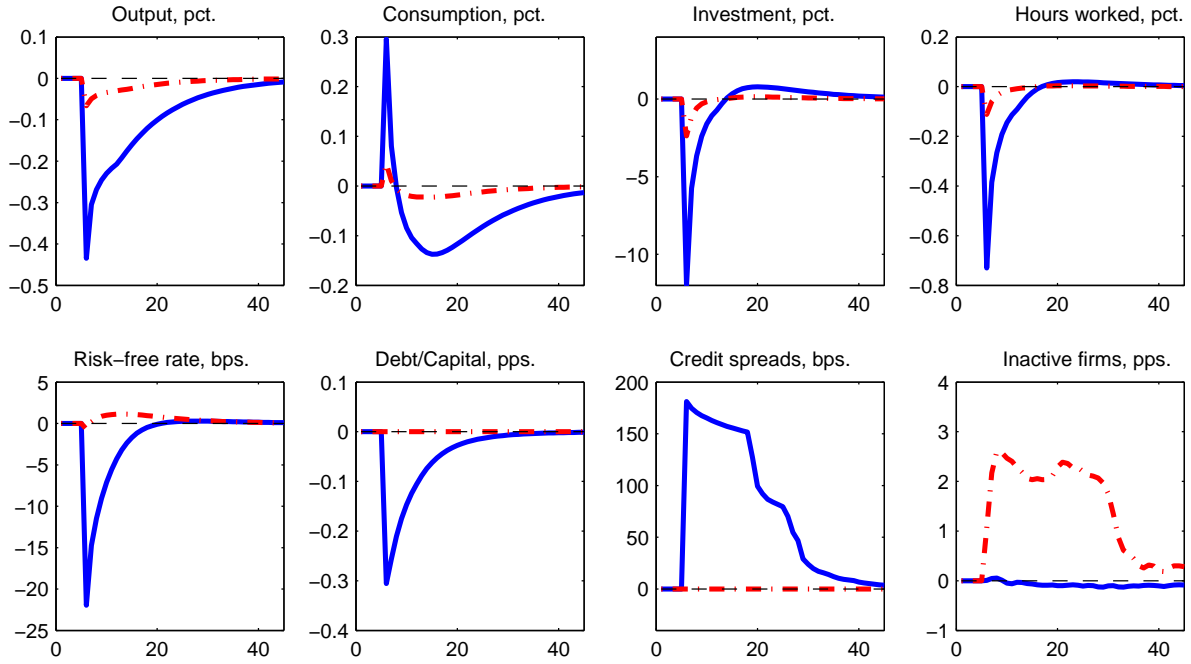
NOTE: Panel (a) depicts the optimal investment policy in the case of no fixed investment adjustment costs (i.e., $F_k = 0$). Panel (b) depicts the optimal investment policy with fixed investment adjustment costs (i.e., $F_k = 0.01$). In both examples, the liquidation value of capital $p^- = 0.8$.

Figure 6: Impact of a Technology Shock



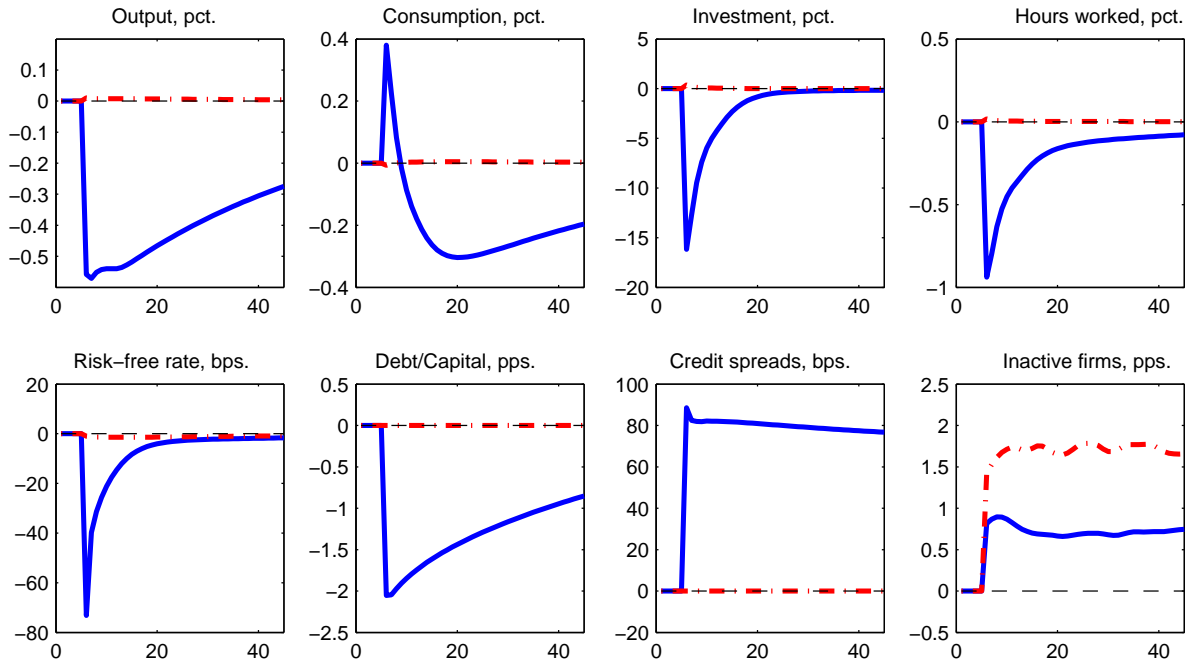
NOTE: Blue solid lines depict the impulse response functions of the model with financial frictions, while the red dashed lines are those of the model without financial frictions. The size of the aggregate technology shock is one standard deviation upon impact, and the shock is then allowed to revert back to steady state, according to the process in equation (9). The impulse response functions are averages of 10,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms (see text for details). Inaction = fraction of firms in the inaction region.

Figure 7: Impact of an Uncertainty Shock



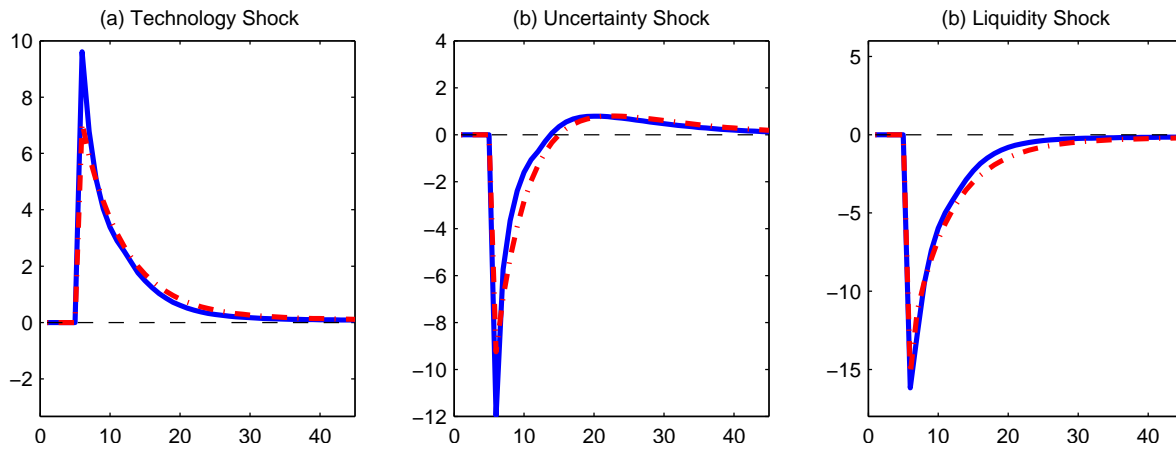
NOTE: Blue solid lines depict the impulse response functions of the model with financial frictions, while the red dashed lines are those of the model without financial frictions. The size of the uncertainty shock is 6 percentage points (annualized) upon impact—about one standard deviation—and the shock is then allowed to revert back to steady state, according to the process in equation (10). The impulse response functions are averages of 10,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms (see text for details). Inaction = fraction of firms in the inaction region.

Figure 8: Impact of a Liquidity Shock



NOTE: Blue solid lines depict the impulse response functions of the model with financial frictions, while the red dashed lines are those of the model without financial frictions. Upon impact, the shock reduces the liquidation value of capital assets 5 percent—about 1.5 standard deviations—and the shock is then allowed to revert back to steady state, according to the process in equation (12). The impulse response functions are averages of 10,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms (see text for details). Inaction = fraction of firms in the inaction region.

Figure 9: Aggregate Investment Dynamics
(Linear vs. Nonlinear Impulse Response Functions)



NOTE: Blue solid lines depict the impulse response functions of aggregate investment to the specified shock implied by the benchmark model with financial frictions, which are computed taking into account the nonlinearities of the firms' investment and financial policies; the red dash-dotted lines are the corresponding impulse response functions based solely on the agents' perceived (log-linear) aggregate laws of motion (see text for details).

Appendices

A Data Appendix

In this appendix, we describe the construction of firm-level credit spreads and other firm-specific variables used in the empirical analysis. Subsection A.1 describes the bond-level data set used to construct firm-specific credit spreads. Subsection A.2 provides the details surrounding the construction of the variables used in the estimation of credit spread regressions, while subsection A.3 describes the construction of the panel data set used to estimate the relationship between investment, uncertainty, and credit spreads. And lastly, subsection A.4 describes the estimation procedure—and the associated results—used to estimate the degree of decreasing return-to-scale in U.S. nonfinancial corporate sector, as well as to construct our proxy for idiosyncratic uncertainty based on firm-level profitability shocks.

A.1 Bond-Level Data

For a sample of U.S. nonfinancial firms covered by the S&P’s Compustat and the Center for Research in Security Prices (CRSP), we obtained month-end secondary market prices of their outstanding securities from the Lehman/Warga and Merrill Lynch databases. As discussed by Gilchrist, Yankov, and Zakrajšek [2009], these two data sources include secondary market prices for a vast majority of dollar-denominated bonds publicly issued in the U.S. corporate cash market. To ensure that we are measuring borrowing costs of different firms at the same point in their capital structure, we limited our sample to only senior unsecured issues with a fixed coupon schedule.

As emphasized by Gilchrist and Zakrajšek [2012], using security-level data allows the construction of credit spreads that are not biased by the maturity/duration mismatch, a problem that plagues credit spread indexes constructed with aggregated data. Specifically, we construct—for each individual bond issue in our sample—a theoretical risk-free security that replicates exactly the promised cash-flows of the corresponding corporate debt instrument. For example, consider a corporate bond k issued by firm i that at time t is promising a sequence of cashflows $\{C(s) : s = 1, 2, \dots, S\}$, which consists of the regular coupon payments and the repayment of the principle at maturity. The price of this bond in period t is given by

$$P_{it}[k] = \sum_{s=1}^S C(s)D(t_s),$$

where $D(t) = \exp(-r_t t)$ is the discount function in period t .

To calculate the price of a corresponding risk-free security—denoted by $P_t^f[k]$ —we discount the promised cashflows $\{C(s) : s = 1, 2, \dots, S\}$ using continuously-compounded zero-coupon Treasury yields in period t , derived from the daily estimates of the U.S. Treasury yield curve estimated by Gürkaynak, Sack, and Wright [2007]. The resulting price $P_t^f[k]$ can then be used to calculate the yield—denoted by $y_t^f[k]$ —of a hypothetical Treasury security with identical cashflows as the underlying corporate bond. Consequently, the credit spread $S_{it}[k] = y_{it}[k] - y_t^f[k]$, where $y_{it}[k]$ denotes the yield of the corporate bond k , is thus free of the “duration mismatch” that occurs when the spreads are computed simply by matching the corporate yield to the estimated yield of a zero-coupon Treasury security of the same maturity.

To ensure that our results are not driven by a small number of extreme observations, we eliminated from our data set all observations with credit spreads below 5 basis points and greater than 2,000 basis points. We also eliminated very small corporate issues (par value of less than

\$1 million) and all observations with a remaining term-to-maturity of less than one year or more than 30 years.³⁵ These selection criteria yielded a sample of 6,725 individual securities over the 1973:M1–2012:M9 period. These corporate securities were then matched with their issuer’s quarterly income and balance sheet data from Compustat and daily data on equity valuations from CRSP, a procedure that yielded a sample of 1,164 U.S. nonfinancial firms.

Table A-1: Selected Corporate Bond Characteristics

Variable	Mean	StdDev	Min	P50	Max
No. of bonds per firm/month	2.99	3.69	1.00	2.00	76.0
Mkt. value of issue ^a	339.9	338.9	1.22	249.2	5,628
Maturity at issue (years)	12.8	9.2	1.0	10.0	50.0
Term-to-maturity (years)	11.1	8.5	1.0	8.0	30.0
Duration (years)	6.49	3.28	0.91	6.03	18.0
Credit rating (S&P)	-	-	D	BBB1	AAA
Coupon rate (pct.)	7.06	2.14	0.60	6.88	17.5
Nominal effective yield (pct.)	7.23	2.98	0.22	6.92	30.0
Credit spread (pps.)	2.02	2.23	0.05	1.28	20.0

NOTE: Sample period: 1973:M1–2012:M9; Obs. = 385,062; No. of bonds = 6,725; No. of firms = 1,164. Sample statistics are based on trimmed data.

^a Market value of the outstanding issue deflated by the CPI (2005 = 100).

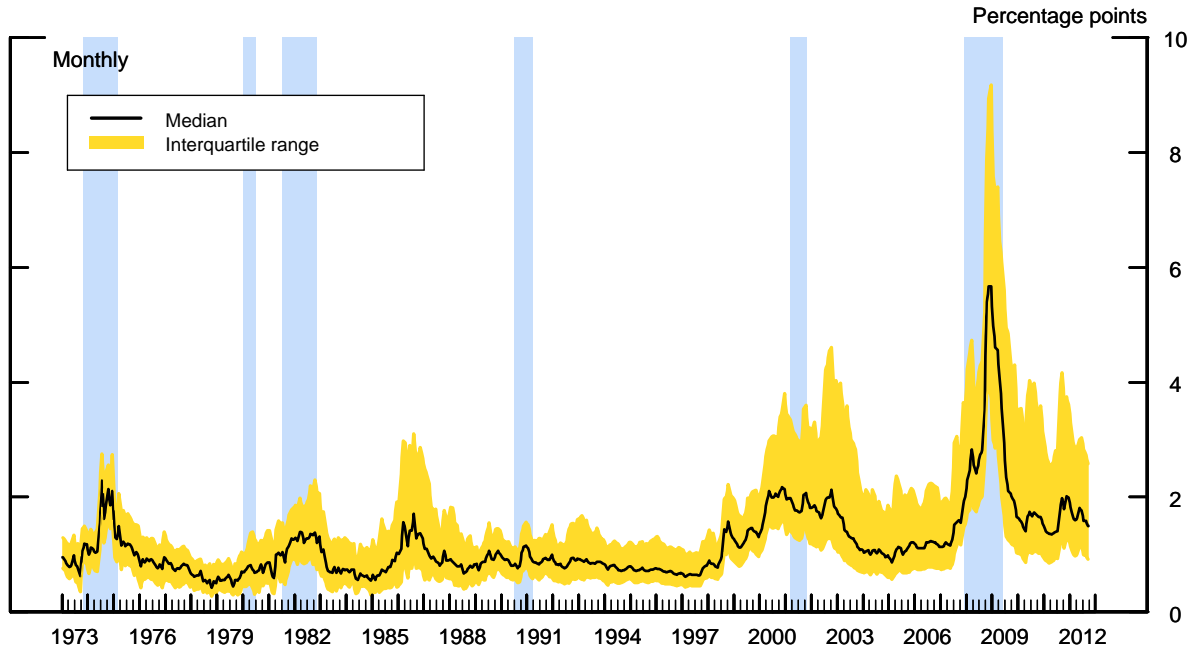
Table A-1 contains summary statistics for the key characteristics of bonds in our sample. Note that a typical firm in our sample has a few senior unsecured issues outstanding at any point in time—the median firm, for example, has three such issues trading in the secondary market in any given month. This distribution, however, is skewed to the right and has a heavy right tail, as some firms can have many more issues trading in the secondary market at a point in time. The size distribution of these issues—as measured by their (real) market value—runs from \$1.2 million to more than \$5.6 billion and has a very similar shape.

Given our focus on the corporate bond market, the maturity of these debt instruments is fairly long—the average maturity at issue of almost 13 years. The average remaining term-to-maturity, by contrast, is about 11 years. In terms of default risk—at least as measured by the S&P credit ratings—our sample spans the entire spectrum of credit quality, from “single D” to “triple A.” At “BBB1,” however, the median observation is still in the investment-grade category. Note that the central tendencies of our sample—in both the maturity and credit-risk dimensions—closely match those of the 10-year BBB-Treasury spread used in the VAR analysis. Lastly, an average bond has an expected return of 202 basis points above the comparable risk-free rate, while the standard deviation of 223 basis points is again indicative of the wide range of credit quality in the sample.

Figure A-1 depicts the time-series evolution of the selected moments of the cross-sectional distribution of credit spreads in our sample. As shown by the solid line, the median credit spread is countercyclical and typically starts to increase before the official onset of recessions. Importantly, the dispersion of credit spreads—as measured by the interquartile range—also tends to lead the business cycle and increases markedly during periods of financial market distress, a pattern consistent with the well-documented countercyclical heterogeneity in firm stock returns, profits, and productivity.

³⁵Calculating credit spreads for maturities of less than one year and more than 30 years would involve extrapolating the Treasury yield curve beyond its support.

Figure A-1: Credit Spreads in U.S. Nonfinancial Corporate Sector



NOTE: Sample period: 1973:M1–2012:M9. The solid line depicts the median spread on senior unsecured bonds in our sample, while the shaded band depicts the corresponding interquartile range. The shaded vertical bars denote the NBER-dated recessions.

A.2 Credit Spread Regressions

While our micro-level data on credit spreads reflect month-end values, the requisite firm-level income and balance sheet items from Compustat are available only quarterly; in addition, our measure of firm-level uncertainty is estimated at a quarterly frequency. The time-series frequency of the panel data set used in the estimation of our credit spread regression is, therefore, quarterly. In constructing the data set used to estimate the credit spread regressions reported in Table 1 in subsection 2.3, we also took into account the fact that the firms’ fiscal years end at different months of the year. As a result, observations in the data set occur at different months of the year but are spaced at regular quarterly (i.e., three-month) intervals.

Letting $v\#$ denote the quarterly Compustat data item number, the key explanatory variables used in the credit spread regressions are defined as follows:

- σ_{it} : *quarterly uncertainty proxy*, constructed using daily idiosyncratic returns over the three months of the firm’s fiscal quarter;
- R_{it}^E : *quarterly stock return*, constructed from daily (total) log returns from the CRSP database over the three months of the firm’s fiscal quarter;
- $[\Pi/A]_{it}$: *ratio of operating income to assets*, defined as operating income before depreciation and amortization ($v21$) in quarter t and scaled by total assets ($v44$) in quarter $t - 1$;
- $[D/E]_{it}$: *ratio of debt to equity*, defined as the book value of debt in current liabilities ($v45$) plus the book value of long-term debt ($v51$)—both in quarter t —and scaled by the market

value of common equity in quarter t as computed from the CRSP database.

Table A-2: Summary Statistic of Selected Firm Characteristics

Variable	Mean	StdDev	Min	P50	Max
σ_{it} (pct.)	29.9	16.7	3.96	25.7	211.4
R_{it}^E (pct.)	7.03	75.2	-635.3	11.5	598.2
$100 \times [\Pi/A]_{it}$	3.67	1.88	-2.00	3.51	9.99
$100 \times [D/E]_{it}$	77.8	102.5	<.01	46.2	1,182.3

NOTE: Sample period: 1973:M1–2012:M9 at a quarterly frequency; Obs. = 39,723; No. of firms = 1,124. Sample statistics are based on trimmed data. Uncertainty (σ_{it}) and quarterly stock returns (R_{it}^E) are annualized.

After deleting observations with missing data items and applying standard filters to remove a small number of outliers, we were left with 1,124 firms for a total of 39,723 firm/quarter observations. Summary statistics of these key firm characteristics are summarized in Table A-2.

A.3 Investment Regressions

In constructing the data set used in the estimation of investment regressions reported in Tables 2–3 in subsection 2.3 of the paper, the Compustat data on capital expenditures are available only at an annual frequency. In constructing this data set, we also take into account the fact that the firms’ fiscal years end at different months of the year. As a result, observations in the data set occur at different months of the year but are spaced at regular yearly (i.e., twelve-month) intervals.

Letting $v\#$ denote the annual Compustat data item number, the key variables used in the investment regressions are defined as follows:

- $[I/K]_{it}$: *ratio of investment to capital*, defined as capital expenditures ($v128$) in year t and scaled by (net) property, plant, and equipment ($v8$) in year $t - 1$;
- σ_{it} : *yearly uncertainty proxy*, constructed using daily idiosyncratic returns over the twelve months of the firm’s fiscal year;
- s_{it} : *yearly average portfolio credit spread*, constructed using month-end credit spreads over the twelve months of the firm’s fiscal year;³⁶
- $[Y/K]_{it}$: *ratio of sales to capital*, defined as (net) sales ($v12$) in year t and scaled by (net) property, plant, and equipment ($v8$) in year $t - 1$;
- $[\Pi/K]_{it}$: *ratio of operating income to capital*, defined as operating income before depreciation and amortization ($v178$) in year t and scaled by (net) property, plant, and equipment ($v8$) in year $t - 1$;
- Q_{it} : *Tobin’s Q*, constructed as the book value of total liabilities ($v181$) plus the market value of common equity from the CRSP database—both in year t —and scaled by the book value of total assets ($v6$) in year t .

³⁶For the firms that have more than one bond issue trading in the secondary market in a given period, we calculate the portfolio spread in that period by computing an average of credit spreads on all of the firm’s outstanding bonds.

Table A-3: Summary Statistic of Selected Firm Characteristics

Variable	Mean	StdDev	Min	P50	Max
σ_{it} (pct.)	30.1	14.6	8.86	26.5	143.8
s_{it} (pps.)	2.11	2.07	0.13	1.38	17.6
$[I/K]_{it}$	0.19	0.12	0.01	0.17	1.00
$[Y/K]_{it}$	3.52	2.83	0.07	2.78	15.0
$[\Pi/K]_{it}$	0.49	0.39	-0.48	0.37	2.50
Q_{it}	1.49	0.76	0.45	1.28	10.7

NOTE: Sample period: 1973:M1–2012:M9 at an annual frequency; Obs. = 8,557; No. of firms = 772. Sample statistics are based on trimmed data.

After deleting observations with missing data items, applying standard filters to remove outliers, and imposing a restriction that a firm needed to be in the panel for a minimum of three years, we were left with 772 firms for a total of 8,557 firm/year observations. Summary statistics of these key firm characteristics are summarized in Table A-3.

A.4 Uncertainty Proxy Based on Profit Shocks

In this section, we describe the procedure used to calibrate the curvature of the profit function and the parameters governing the stochastic volatility process of the idiosyncratic technology shock. Under the assumption of a Cobb-Douglas production function, gross profits (i.e., profits before fixed operation costs) differ from sales only up to a constant. Hence, one can estimate the returns-to-scale using data on either sales or gross profits. We chose sales in order to avoid the occasionally negative gross profit observations.

Letting $v\#$ denote the quarterly data item number, we selected from the Compustat database all U.S. nonfinancial firms with at least 20 quarters of non-missing data on net sales ($v2$) and net property, plant, and equipment ($v42$) over the 1976:Q1–2012:Q3 period, a procedure yielding an unbalanced panel of 9,411 firms for a total of 549,946 observations.³⁷ To ensure that our results were not driven by a small number of extreme observations, we dropped from the sample all observations with the sales-to-capital ratio below 0.01 and above 20.0 and observations with quarterly growth rates of sales and capital above and below 100 percent.

Our empirical counterpart of the profit function in equation (8) in Section 3.1 of the paper is given by

$$\log Y_{it} = c_{it} + \gamma_s \log K_{i,t-1} + \lambda_{jt} + u_{it}, \quad (\text{A-1})$$

where Y_{it} denotes the sales of firm i in quarter t and $K_{i,t-1}$ is the capital stock at the end of quarter $t - 1$. The regression disturbance term u_{it} corresponds to the idiosyncratic technology shock $\log z_{it}$ in our model, while the coefficient γ_s determines—in conjunction with the relative share of capital—the degree of decreasing returns-to-scale in production, according to

$$\chi_s = \frac{\gamma_s}{\alpha + (1 - \alpha)\gamma_s}.$$

As indicated by the subscript s , we allow the curvature of the profit function to differ across production sectors as defined by the 2/3-digit NAICS codes.

Because quarterly firm-level sales are characterized by a strong seasonal pattern, we specify

³⁷Prior to 1976, most firms in Compustat did not report their capital stock data on the quarterly basis.

that the firm-specific term c_{it} in regression (A-1) satisfies:

$$c_{it} = \sum_{n=1}^4 \eta_n \times \mathbf{1}[\text{QTR}_{it} = n],$$

where $\mathbf{1}[\text{QTR}_{it} = n]$ is an indicator function that equals one if firm i 's observation in period t falls in quarter n and zero otherwise—that is, the regression includes a full set of *firm-specific* quarterly dummies. And lastly, industry-specific (3-digit NAICS) time fixed effects—denoted by λ_{jt} —are included in the regression to control for the persistent nature of cyclical profitability shocks within an industry (e.g., McGahan and Porter [1999]).

We use the residuals from the estimation of (A-1) to calibrate the process for the idiosyncratic technology shock. First, the persistence of the process is obtained by estimating the following pooled OLS regression:

$$\hat{u}_{it} = \rho_z \hat{u}_{i,t-1} + \epsilon_{it}, \quad (\text{A-2})$$

Second, if the error term ϵ_{it} in regression (A-2) is distributed normally, then an unbiased estimator of the true standard deviation of ϵ_{it} is given by

$$\hat{\sigma}_{\epsilon,it} = \sqrt{\frac{\pi}{2}} |\hat{\epsilon}_{it}|,$$

which yields an estimate of the volatility of the idiosyncratic technology shock for each firm i in every quarter t .³⁸ To obtain a proxy for the time-varying uncertainty of productivity shocks, the last step involves estimating a panel regression of the form:

$$\log \hat{\sigma}_{\epsilon,it} = \sum_{k=1}^4 \beta_k \log \hat{\sigma}_{\epsilon,i,t-1} + \eta_i + v_t + \zeta_{it},$$

where η_i denotes a fixed firm effect. In keeping with our earlier approach, a measure of uncertainty based on profit shocks shown in Figure 4 corresponds to the sequence of estimated time fixed effects \hat{v}_t , $t = 1, \dots, T$, which captures common fluctuations in the idiosyncratic uncertainty regarding the profitability prospects in the nonfinancial corporate sector.³⁹

The results of the first two steps of our estimation procedure are summarized in Table A-4. According to the entries in the table, the sector-specific estimates of the curvature of the profit function yield economically sensible degree of decreasing returns-to-scale in each broad production sector of the U.S. nonfinancial corporate sector. The estimates of χ lie in a relatively narrow interval, ranging from a low of about 0.81 in agriculture and arts, entertainment, and related services to a high of 0.90 in the mining sector. The average estimate of the decreasing return-to-scale is about 0.85, which is the value used in our calibration. As indicated by the second memo item in the table, our approach implies the persistence of the idiosyncratic technology shock, the coefficient ρ_z in equation (A-2), to be about 0.90.

³⁸This approach is similar to that of Kim and Nelson [1999] and McConnell and Perez Quiros [2000], who estimate the volatility of aggregate output growth in each quarter from a single observation.

³⁹To ease the interpretation, the sequence \hat{v}_t , $t = 1, \dots, T$, has been re-scaled and expressed in annualized percent. In addition, the residual seasonality has been removed by using the X11 filter.

Table A-4: Returns-to-Scale in U.S. Nonfinancial Corporate Sector

Sector (2/3-digit NAICS)	Estimate	[95% Conf. Interval]	Firms	Obs.
Agriculture	0.806	[0.697, 0.882]	42	1,551
Mining	0.906	[0.893, 0.916]	641	31,902
Utilities	0.831	[0.768, 0.873]	250	23,902
Construction	0.813	[0.781, 0.842]	176	7,784
Manufacturing – Durables	0.851	[0.845, 0.857]	3,257	187,213
Manufacturing – Nondurables	0.867	[0.858, 0.875]	1,455	92,547
Wholesale Trade	0.856	[0.839, 0.873]	371	20,848
Retail Trade	0.882	[0.865, 0.898]	577	35,174
Transportation	0.859	[0.837, 0.881]	295	18,014
Information Services	0.838	[0.825, 0.849]	1,095	52,272
Real Estate Services	0.823	[0.796, 0.848]	188	11,481
Professional Services	0.844	[0.828, 0.858]	400	25,433
Administrative Services	0.844	[0.809, 0.866]	205	12,285
Health Care	0.816	[0.782, 0.849]	199	11,776
Arts & Entertainment	0.803	[0.766, 0.856]	66	3,798
Accommodation Services	0.862	[0.840, 0.885]	224	13,965
<i>Memo: Average</i>	0.844	[0.833, 0.852]	-	-
<i>Memo: AR(1)^a</i>	0.898	[0.895, 0.899]	-	-

NOTE: Sample period: 1976:Q1–2012:Q3; No. of firms = 9,441; Obs. = 549,946. Entries in the table denote the estimates of the sector-specific parameter χ_s , measuring the degree of decreasing returns-to-scale, as implied by the OLS estimates of the parameter γ_s from regression (A-1) and the share of capital $\alpha = 0.3$ in a Cobb-Dougllass production function. The 95-percent confidence intervals are based on a cluster bootstrap with 500 replications.

^a Estimate of ρ_z , the parameter governing the persistence of the idiosyncratic technology shock; see equation (A-2) and text for details.

B Model Appendix

In this appendix, we provide details regarding the key elements of our model and its solution method. Section B.1 describes the construction of the Markov chain with time-varying volatility, which governs the evolution of the idiosyncratic technology shock. In Section B.2, we derive the investment Euler equations under costly reversibility. Section B.3 shows that the aggregation method used to compute the solution of the model provides a good approximation for the rational expectations equilibrium. And lastly,

B.1 Markov Chain with Time-Varying Volatility

Consider a discrete-time $N(= 2n)$ -states Markov chain with a transition matrix,

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,N} \\ \vdots & \ddots & \vdots \\ p_{N,1} & \cdots & p_{N,N} \end{bmatrix}, \quad (\text{B-1})$$

where $\sum_{j=1}^N p_{i,j} = 1$ and $p_{i,j} = p_{N-(i-1), N-(j-1)}$ for all $i, j = 1, \dots, N$. Without loss of generality assume that n is an even number. Then the N states of the chain are given by

$$\bar{z} - \frac{\bar{\sigma}}{2}, \dots, \bar{z} - \frac{3\bar{\sigma}}{2(N-1)}, \bar{z} - \frac{\bar{\sigma}}{2(N-1)}, \bar{z} + \frac{\bar{\sigma}}{2(N-1)}, \bar{z} + \frac{3\bar{\sigma}}{2(N-1)}, \dots, \bar{z} + \frac{\bar{\sigma}}{2};$$

where \bar{z} and $\bar{\sigma}^2$ denote the unconditional mean and unconditional variance of the process, respectively. These N equispaced states can, for any period t , be generically written as

$$z_{i,t} = \bar{z} + \left[2 \left(\frac{i-1}{N-1} \right) - 1 \right] \frac{\bar{\sigma}}{2}; \quad i = 1, \dots, N. \quad (\text{B-2})$$

The first two conditional moments of this process are then given by

$$\mathbb{E}[z_{t+1} \mid z_t = z_{i,t}] = \bar{z} + \frac{\mu_i}{2} \bar{\sigma}_z; \quad (\text{B-3})$$

$$\text{Var}[z_{t+1} \mid z_t = z_{i,t}] = \Xi_i \bar{\sigma}_z^2, \quad (\text{B-4})$$

where

$$\mu_i = 2 \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - 1 \quad \text{and} \quad \Xi_i = \sum_{j=1}^N p_{i,j} \left[\frac{j-1}{N-1} - \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) \right]^2.$$

Now consider the same discrete-time Markov chain, except suppose that its volatility follows a stationary process denoted by $\{\sigma_t\}$. In this case, we assume that conditional on observing σ_t in period t , the N equispaced states in period $t+1$ are given by

$$z_{j,t+1} = \bar{z} - \frac{\mu_j}{2} (\sigma_t - \bar{\sigma}) + \left[2 \left(\frac{j-1}{N-1} \right) - 1 \right] \frac{\sigma_t}{2}; \quad j = 1, \dots, N. \quad (\text{B-5})$$

The conditional mean of this modified Markov chain is then given by

$$\begin{aligned} \mathbb{E}(z_{t+1} \mid z_t = z_{i,t}) &= \bar{z} + \frac{\bar{\sigma}}{2} \mu_i + \frac{\sigma_t}{2} \sum_{j=1}^N p_{i,j} \left[2 \left(\frac{j-1}{N-1} \right) - 1 - \mu_i \right] \\ &= \bar{z} + \frac{\bar{\sigma}}{2} \mu_i + \sigma_t \sum_{j=1}^N p_{i,j} \left[\frac{j-1}{N-1} - \sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right] \end{aligned}$$

Note that

$$\begin{aligned}
\sum_{j=1}^N p_{i,j} \left[\frac{j-1}{N-1} - \sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right] &= \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - \sum_{j=1}^N p_{i,j} \left[\sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right] \\
&= \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - \left[\sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right) \right] \sum_{j=1}^N p_{i,j} \\
&= \sum_{j=1}^N p_{i,j} \left(\frac{j-1}{N-1} \right) - \sum_{k=1}^N p_{i,k} \left(\frac{k-1}{N-1} \right), \\
&= 0
\end{aligned}$$

which implies that

$$\mathbb{E}(z_{t+1} \mid z_t = z_{i,t}) = \bar{z} + \frac{\mu_i}{2} \bar{\sigma}. \quad (\text{B-6})$$

Thus, the conditional mean of the modified Markov chain with stochastic volatility is identical to that of the conventional Markov chain with time-invariant volatility (see equation (B-3)). Hence, an increase in volatility represents a mean-preserving-spread (MPS) of z , a property reflecting the presence of the mean-correction term $-0.5\mu_i(\sigma_t - \bar{\sigma})$ in equation (B-5).

Given our assumption that $p_{i,j} = p_{N-(i-1), N-(j-1)}$, the unconditional mean of the process equals

$$\sum_{i=1}^N p_i \mathbb{E}(z_{t+1} \mid z_t = z_{i,t}) = \bar{z} + \frac{\bar{\sigma}}{2} \sum_{i=1}^N p_i \mu_i = \bar{z},$$

which shows that both the conditional and unconditional first moments of the Markov chain with time-varying volatility are not affected by fluctuations in volatility. In contrast, the conditional variance of the process is given by

$$\begin{aligned}
\text{Var}(z_{t+1} \mid z_t = z_{i,t}) &= \sigma_t^2 \sum_{j=1}^N p_{i,j} \left[\left(\frac{j-1}{N-1} \right) - \sum_{j=1}^N p_{i,j} \frac{j-1}{N-1} \right]^2 \\
&= \Xi_i \sigma_t^2.
\end{aligned} \quad (\text{B-7})$$

Thus the conditional volatility of this process depends linearly on the realization of the stochastic process $\{\sigma_t\}$. In this formulation, the support of the distribution of the idiosyncratic technology shock z is evolving stochastically over time, with an increase in σ today inducing a greater dispersion in z tomorrow and vice versa.

B.2 Derivation of the Investment Euler Equations

Recall that the recursive formulation of the firm's problem without the fixed investment costs (i.e., $F_k = 0$) can be written as

$$v_i(k, x; \mathbf{s}) = \max \{ v_i^+(k, x; \mathbf{s}), v_i^-(k, x; \mathbf{s}) \}, \quad (\text{B-8})$$

where the *capital expansion* problem is given by

$$\begin{aligned}
v_i^+(k, x; \mathbf{s}) &= \min_{\phi, \lambda^+} \max_{d^+, e^+, k^+, b^+} \left\{ d^+ + \phi(d^+ - \underline{d}) - \bar{\varphi}(e^+) + \lambda^+ [k^+ - (1 - \delta)k] \right. \\
&\quad \left. + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \max \{ v_j(k^+, x^+(\sigma_z); \mathbf{s}'), v_j(k^+, x^{R^+}(\sigma_z); \mathbf{s}') \} \mid \mathbf{s} \right] \right\};
\end{aligned} \quad (\text{B-9})$$

and the *capital contraction* problem by

$$\begin{aligned}
v_i^-(k, x; \mathbf{s}) = & \min_{\phi, \lambda^-} \max_{d^-, e^-, k^-, b^-} \left\{ d^- + \phi(d^- - \underline{d}) - \bar{\varphi}(e^-) - \lambda^- [k^- - (1 - \delta)k] \right. \\
& \left. + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \max \{ v_j(k^-, x^-(\sigma_z); \mathbf{s}'), v_j(k^-, x^{R^-}(\sigma_z); \mathbf{s}') \} \middle| \mathbf{s} \right] \right\}; \tag{B-10}
\end{aligned}$$

and where

$$\begin{aligned}
d &= az_i \psi(w) k^\gamma - F_o k - p(k', k) - b + q_i b' + e; \\
p(k', k) &= (p^+ \times \mathbf{1}[k' \geq (1 - \delta)k] + p^- \times \mathbf{1}[k' \leq (1 - \delta)k]) (k' - (1 - \delta)k); \\
x(\sigma_z) &= az_j(\sigma_z) \psi(w') (k')^\gamma - F_o k' - b'; \\
x^R(\sigma_z) &= az_j(\sigma_z) \psi(w') (k')^\gamma - F_o k' - b^R; \\
b^R &= a' z'(\sigma_z) \psi(w') (k')^\gamma - F_o k' + p^-(1 - \delta)k'; \\
q_i(k', b'; \mathbf{s}') &= \mathbb{E} \left\{ m(\mathbf{s}, \mathbf{s}') \left[1 + \sum_{j \in \mathcal{D}} p_{i,j} [\mathcal{R}(k', b', z'_j(\sigma_z); \mathbf{s}') - 1] \right] \middle| \mathbf{s} \right\}.
\end{aligned}$$

We first derive the investment Euler equations associated with these two problems assuming $F_k = 0$ and then show how the presence of fixed investment costs modifies the optimality conditions governing the evolution of the firm's capital stock.

B.2.1 Capital Expansion Problem (with $F_k = 0$)

The first-order condition for the capital expansion problem defined in equation (B-9) is given by

$$\begin{aligned}
0 = & \lambda^+ - (1 + \phi)p^+ + (1 + \phi)q_{i,k}(k^+, b^+; \mathbf{s})b^+ \\
& + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}^e} p_{i,j} [v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}') + x_k^+(\sigma_z) v_{j,x}(k^+, x^+(\sigma_z); \mathbf{s}')] \middle| \mathbf{s} \right] \\
& + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} [v_{j,k}(k^+, x^{R^+}(\sigma_z); \mathbf{s}') + x_k^{R^+}(\sigma_z) v_{j,x}(k^+, x^{R^+}(\sigma_z); \mathbf{s}')] \middle| \mathbf{s} \right]. \tag{B-11}
\end{aligned}$$

To derive an investment Euler equation corresponding to this case, it is useful to define two trigger levels for the firm's capital stock—a trigger associated with the expansion problem and a trigger associated with the contraction problem. Formally, the *expansion* trigger is defined as

$$k_i^L(k, x; \mathbf{s}) \equiv \frac{k_i^+(k, x; \mathbf{s})}{1 - \delta}.$$

Thus, when $k \in (0, k_i^L(k, x; \mathbf{s})]$, the firm's stock of capital is small relative to its idiosyncratic technology level and macroeconomic conditions. In that case, the firm's optimal investment policy calls for an expansion of the production capacity to reach the expansion target $k^+(k, x; \mathbf{s})$ immediately. The *contraction* trigger is defined analogously as

$$k_i^U(k, x; \mathbf{s}) \equiv \frac{k_i^-(k, x; \mathbf{s})}{1 - \delta}.$$

If $k \in [k_i^U(k, x; \mathbf{s}), +\infty)$, then the firm is experiencing a capacity overhang and disinvesting of its capital stock to reach the contraction target $k_i^-(k, x; \mathbf{s})$ immediately is optimal. If the firm's production capacity lies in between the expansion and contraction triggers, inaction is optimal.

The possibility of strategic default, however, introduces another element into the firm's optimal investment policy. Specifically, we need to consider a situation in which it is optimal for the firm to default and a situation in which the firm continues as an ongoing concern. We discuss both cases in turn.

Continue as an ongoing concern: First, note that the Benveniste-Scheinkman condition associated with the first-order condition (B-11) is given by

$$v_{j,x}(k^+, x^+(\sigma_z); \mathbf{s}') = 1 + \phi'. \quad (\text{B-12})$$

A complete derivation of the envelope condition characterizing the optimal choice of capital stock in the next period requires the information on whether the capital stock at the beginning of the next period will be in the expansion, contraction, or inaction region. These three different possibilities can be summarized as

$$v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}') = \begin{cases} v_{j,k}^+(k^+, x^+(\sigma_z); \mathbf{s}') = (1 - \delta)(1 + \phi')p^+, & \text{if } k^+ \in (0, k_j^{L'}]; \\ v_{j,k}^+(k^+, x^+(\sigma_z); \mathbf{s}') = (1 - \delta)[(1 + \phi')p^+ - \lambda^{+'}], & \text{if } k^+ \in (k_j^{L'}, k_j^{U'}); \\ v_{j,k}^-(k^+, x^+(\sigma_z); \mathbf{s}') = (1 - \delta)(1 + \phi')p^{-'}, & \text{if } k^+ \in [k_j^{U'}, +\infty). \end{cases}$$

When $k^+ \in (k_j^L(k^+, x^+(\sigma_z); \mathbf{s}'), k_j^U(k^+, x^+(\sigma_z); \mathbf{s}'))$, the expansion value is equal to the contraction value because inaction is optimal. Hence, $v_{j,k}^+(k^+, x^+(\sigma_z); \mathbf{s}')$ in the second line of the above expression can be replaced with $v_{j,k}^-(k^+, x^+(\sigma_z); \mathbf{s}')$.

Directly differentiating the ancillary value functions (B-9) and (B-10) with respect to k , we need to distinguish between the following three cases:

(1) If $k^+ \in (0, k_j^{L'}]$, then

$$v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}') + x_k^+(\sigma_z)v_{j,x}(k^+, x^+(\sigma_z); \mathbf{s}') = (1 + \phi'_j)[\pi_{jk}(k^+; s') + (1 - \delta)p^+].$$

(2) If $k^+ \in (k_j^{L'}, k_j^{U'})$, then

$$v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}') + x_k^+(\sigma_z)v_{j,x}(k^+, x^+(\sigma_z); \mathbf{s}') = (1 + \phi'_j)[\pi_{jk}(k^+; s') + (1 - \delta)] \left[p^+ - \frac{\lambda^{+'}}{1 + \phi'_j} \right].$$

(3) If $k^+ \in [k_j^{U'}, +\infty)$, then

$$v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}') + x_k^+(\sigma_z)v_{j,x}(k^+, x^+(\sigma_z); \mathbf{s}') = (1 + \phi'_j)[\pi_{jk}(k^+; s') + (1 - \delta)p^{-'}].$$

By combining these three cases, we obtain the following envelope condition:

$$v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}') + x_k^+(\sigma_z)v_{j,x}(k^+, x^+(\sigma_z); \mathbf{s}') = (1 + \phi'_j) \left[\pi_{jk}(k^+; s') + (1 - \delta) \times \min \left\{ 1, \max \left\{ p^{-'}, p^+ \left(1 - \frac{\lambda^{+'}}{1 + \phi'_j} \right) \right\} \right\} \right]. \quad (\text{B-13})$$

Choosing to default: First, note that $v_{j,k}(k^+, x^{R^+}(\sigma_z); \mathbf{s}') = v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}')$, as $x^{R^+}(\sigma_z)$ is just a different level of $x^+(\sigma_z)$. Hence, we can apply the same Benveniste-Scheinkman condition (B-12), which yields

$$v_{j,x}(k^+, x^{R^+}(\sigma_z); \mathbf{s}') = 1 + \phi'_j.$$

The strategic default, however, modifies the effect of current investment expenditures on the firm's net liquid asset position in the future. Specifically,

$$\begin{aligned} x_k^+(\sigma_z) &= \pi_{j,k}(k^+; \mathbf{s}') - b_k^{R^+} \\ &= \pi_{j,k}(k^+; \mathbf{s}') - [\pi_{j,k}(k^+; \mathbf{s}') + p^{-'}(1 - \delta)] \\ &= -p^{-'}(1 - \delta) \end{aligned}$$

Hence, in the case of default, we obtain the following envelope condition:

$$\begin{aligned} v_{j,k}(k^+, x^{R^+}(\sigma_z); \mathbf{s}') + x_k^+(\sigma_z)v_{j,x}(k^+, x^{R^+}(\sigma_z); \mathbf{s}') = \\ (1 + \phi'_j) \left[(1 - \delta) \times \min \left\{ 1, \max \left\{ p^{-'}, p^+ \left(1 - \frac{\lambda^{+'}}{1 + \phi'_j} \right) \right\} \right\} - (1 - \delta)p^{-'} \right]. \end{aligned} \quad (\text{B-14})$$

Equating the envelope conditions (B-13) and (B-14) and dividing the resulting expression through by $1 + \phi_i$ yields the investment Euler equation for the capital expansion problem:

$$\begin{aligned} Q_i^+(k, x; \mathbf{s}) = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^+; \mathbf{s}') + (1 - \delta)Q'_j(k^+, x'(\sigma_z); \mathbf{s}')] \middle| \mathbf{s} \right] \\ + q_{i,k}(k^+, b^+; \mathbf{s})b^+ - \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^+; \mathbf{s}') + (1 - \delta)p^{-'}] \middle| \mathbf{s} \right], \end{aligned} \quad (\text{B-15})$$

where $Q_i^+(k, x; \mathbf{s})$ and $Q'_j(k^+, x'(\sigma_z); \mathbf{s}')$ are defined in equations (31) and (33) in the main text, respectively.

B.2.2 Capital Contraction Problem

The first-order condition for the capital contraction problem in equation (B-10) is given by

$$\begin{aligned} 0 = -\lambda^- - (1 + \phi)p^- + (1 + \phi)q_{i,k}(k^-, b^-; \mathbf{s})b^- \\ + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}^c} p_{i,j} [v_{j,k}(k^-, x^-(\sigma_z); \mathbf{s}') + x_k^-(\sigma_z)v_{j,x}(k^-, x^-(\sigma_z); \mathbf{s}')] \middle| \mathbf{s} \right] \\ + \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} [v_{j,k}(k^-, x^{R^-}(\sigma_z); \mathbf{s}') + x_k^{R^-}(\sigma_z)v_{j,x}(k^-, x^{R^-}(\sigma_z); \mathbf{s}')] \middle| \mathbf{s} \right]. \end{aligned} \quad (\text{B-16})$$

Using exactly the same reasoning as above, we can show that the investment Euler equation for the capital contraction problem can be expressed as

$$\begin{aligned} Q_i^-(k, x; \mathbf{s}) = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^-; \mathbf{s}') + (1 - \delta)Q'_j(k^-, x'(\sigma_z); \mathbf{s}')] \middle| \mathbf{s} \right] \\ + q_{i,k}(k^-, b^-; \mathbf{s})b^- - \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^-; \mathbf{s}') + (1 - \delta)p^{-'}] \middle| \mathbf{s} \right], \end{aligned} \quad (\text{B-17})$$

where $Q_i^-(k, x; \mathbf{s})$ is defined in equation (32) in the main text.

B.2.3 Introducing Investment Fixed Costs ($F_k > 0$)

When $F_k > 0$, the presence of fixed investment costs leads to a modification of the Euler equations derived above. Consider the capital expansion problem. The capital expansion and contraction triggers, $k_i^L(k, x; \mathbf{s})$ and $k_i^U(k, x; \mathbf{s})$, are not affected by the presence of investment fixed costs. However, the partial derivative of the value function with respect to k now becomes

$$v_{j,k}(k^+, x^+(\sigma_z); \mathbf{s}') = \begin{cases} v_{j,k}^+(k^+, x^+(\sigma_z); \mathbf{s}') = (1 - \delta)(1 + \phi')(p^+ - F_k), & \text{if } k^+ \in (0, k_j^{L'}]; \\ v_{j,k}^+(k^+, x^+(\sigma_z); \mathbf{s}') = (1 - \delta)[(1 + \phi')p^+ - \lambda^{+'}], & \text{if } k^+ \in (k_j^{L'}, k_j^{U'}); \\ v_{j,k}^-(k^+, x^+(\sigma_z); \mathbf{s}') = (1 - \delta)(1 + \phi')(p^{-'} - F_k), & \text{if } k^+ \in [k_j^{U'}, +\infty). \end{cases}$$

This modification reflects the fact that now the firm has to consider the events in which it will have to pay fixed investment costs tomorrow if the capital stock chosen today—that is, k^+ —falls in either $(0, k_j^{L'}]$ or $[k_j^{U'}, +\infty)$.

Note that because of the fixed investment costs, the value function is not differentiable at $k_j^{L'}$ and $k_j^{U'}$. Those instances, however, correspond to events of measure zero, and because the derivatives are evaluated only inside the expectation operators, the investment Euler equation associated with the capital expansion problem can be written as

$$Q_i^+(k, x; \mathbf{s}) = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^+; \mathbf{s}') + (1 - \delta)\tilde{Q}'_j(k^+, x'(\sigma_z); \mathbf{s}')] \middle| \mathbf{s} \right] \\ + q_{i,k}(k^+, b^+; \mathbf{s})b^+ - \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^+; \mathbf{s}') + (1 - \delta)p^{-'}] \middle| \mathbf{s} \right], \quad (\text{B-18})$$

where the next period's Tobin's Q—in this case denoted by \tilde{Q}' —is no longer given by the min-max truncation in equation (33), but it is defined as

$$\tilde{Q}'_j(k^+, x'(\sigma_z); \mathbf{s}') \equiv \begin{cases} p^+ - F_k, & \text{if } k^+ \in (0, k_j^{L'}]; \\ p^+ - \frac{\lambda_j^+(k^+, x'(\sigma_z); \mathbf{s}')}{1 + \phi_j(k^+, x'(\sigma_z); \mathbf{s}')}, & \text{if } k^+ \in (k_j^{L'}, k_j^{U'}); \\ p^{-'} - F_k, & \text{if } k^+ \in [k_j^{U'}, +\infty). \end{cases} \quad (\text{B-19})$$

Importantly, the Tobin's Q in equation (B-19) is not a monotonic function of the firm's capital stock. For example, consider a firm that is trying to expand its productive capacity starting from a low level of capital. Initially, the expansion constraint does not bind, and the Tobin's Q is given by $p^+ - F_k$. Once the installed level of capital reaches the expansion trigger $k_j^{L'}$, the Tobin's Q discretely jumps to p^+ , as the firm enters the inaction region, stops incurring fixed investment costs F_k , and $\lambda_j^+(k^+, x'(\sigma_z); \mathbf{s}') = 0$ at $k_j^{L'}$. As the firm's capital stock expands beyond the expansion trigger, the expansion constraint becomes binding, and the Tobin's Q starts to decline because $\lambda_j^+(k^+, x'(\sigma_z); \mathbf{s}') > 0$. Note that $p^+ - F_k$ is not necessarily greater than $p^+ - \lambda_j^+/(1 + \phi_j)$ when $k^+ \in (k_j^{L'}, k_j^{U'})$ and is strictly less than p^+ at the expansion trigger $k_j^{L'}$.

Using the same reasoning as above yields the following investment Euler equation for the capital

contraction problem:

$$Q_i^-(k, x; \mathbf{s}) = \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j=1}^N p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^-; \mathbf{s}') + (1 - \delta) \tilde{Q}'_j(k^-, x'(\sigma_z); \mathbf{s}') \mid \mathbf{s} \right] \right. \\ \left. + q_{i,k}(k^-, b^-; \mathbf{s}) b^- - \eta \mathbb{E} \left[m(\mathbf{s}, \mathbf{s}') \sum_{j \in \mathcal{D}} p_{i,j} \left(\frac{1 + \phi'_j}{1 + \phi_i} \right) [\pi_{j,k}(k^-; \mathbf{s}') + (1 - \delta) p^-] \mid \mathbf{s} \right] \right], \quad (\text{B-20})$$

where

$$\tilde{Q}'_j(k^-, x'(\sigma_z); \mathbf{s}') \equiv \begin{cases} p^+ - F_k, & \text{if } k^- \in (0, k_j^{L'}]; \\ p^- + \frac{\lambda_j^-(k^-, x'(\sigma_z); \mathbf{s}')}{1 + \phi_j(k^-, x'(\sigma_z); \mathbf{s}')}, & \text{if } k^- \in (k_j^{L'}, k_j^{U'}); \\ p^- - F_k, & \text{if } k^- \in [k_j^{U'}, +\infty). \end{cases} \quad (\text{B-21})$$

Again, if the Tobin's Q in equation (B-19) is equal to the one in equation (B-21), the firm's optimal investment policy calls for inaction.

B.3 Aggregate Laws of Motion

The dimensionality of the aggregate laws of motion in our model—two endogenous variables and three exogenous states—is quite large. Accordingly, it is important to check how well does the aggregation methodology used to compute the solution of the model approximate the model's true rational expectations equilibrium.

The top panel of Table B-1 shows the aggregate laws of motion implied by the system of equations (42) that are used by the agents to predict future prices in the model with financial frictions. As evidenced by the high R^2 values, the agents' perceived aggregate laws of motion are highly accurate. According to this commonly-used metric, our solution of the model is thus likely a good approximation of the model's true rational expectations equilibrium.

The coefficients of the (log-linear) forecasting rules used by the agents to forecast equilibrium prices also have a number of intuitive properties. For example, the negative coefficient on the stock of debt (\tilde{b}) in the law of motion for the aggregate capital stock reflects the effect of “debt overhang” on macroeconomic outcomes: All else equal, a high level of corporate debt implies less capital accumulation going forward. In contrast, the aggregate debt stock has a positive effect on consumption, a result reflecting the fact that outstanding corporate debt is a part of the representative household's wealth. However, the coefficient on the stock of debt (\tilde{b}) in the law of motion for consumption is very small compared with that of the aggregate capital stock (\tilde{k}), which suggests that at the general equilibrium level, the drag from the debt overhang in the corporate sector reduces the marginal propensity to consume out of claims on corporate debt. As evidenced by the relatively large positive coefficients on the liquidation value of capital (p^-) in the law of motion for both the aggregate capital stock and the aggregate debt stock, liquidity shocks importantly influence the dynamics of capital and debt accumulation in the model.

As discussed in the main text, we also solved a version of the model without financial distortions. In that case, the firms face the same nonconvex capital adjustment costs as before, except that they finance their investment expenditures using only internal funds and equity, where the issuance of the latter is not subject to any dilution costs.⁴⁰ The bottom panel of Table B-1 shows the aggregate

⁴⁰As a result, the stock of outstanding corporate debt is no longer an aggregate state variable.

Table B-1: Agents' Perceived Aggregate Laws of Motion

<i>Model With Financial Frictions</i> ^a							
Dep. Variable	Explanatory Variables						R^2
	Const.	$\log \tilde{k}$	$\log \tilde{b}$	$\log a$	$\log \sigma_z$	$\log p^-$	
$\log \tilde{k}'$	0.0097	0.8545	-0.0093	0.0980	-0.0114	0.0708	0.9986
$\log \tilde{b}'$	0.0804	0.6361	0.0350	0.3324	-0.0348	0.7989	0.9989
$\log c$	-0.7615	0.3475	0.0125	0.3318	0.0113	-0.0746	0.9955
<i>Model Without Financial Frictions</i> ^b							
Dep. Variable	Explanatory Variables						R^2
	Const.	$\log \tilde{k}$	$\log \tilde{b}$	$\log a$	$\log \sigma_z$	$\log p^-$	
$\log \tilde{k}'$	0.0136	0.8044	-	0.0954	-0.0005	0.0012	0.9958
$\log c$	-0.7950	0.3803	-	0.3445	-0.0000	-0.0009	0.9975

NOTE: In both model simulations, we assume that there are 10,000 heterogeneous firms at any point in time. We then simulate the economy for 1,100 quarters by feeding into the specified model randomly drawn aggregate and idiosyncratic shocks. In updating the agents' perceived aggregate laws of motion, we drop the initial 100 quarters and use the remaining observations to estimate the aggregate laws of motion. The updated laws of motion are then used to update the individual policy rules in a numerical dynamic programming problem. The algorithm stops when the changes in the aggregate laws of motion in the subsequent iteration are smaller than the pre-specified tolerance criterion (see text for details).

^a In this version of the model, the firm finances investment expenditure with internal funds, equity, and bonds. Financial frictions imply that equity issuance is subject to dilution costs, while bankruptcy costs imply an additional loss for bond investors in the case of default.

^b In this version of the model, the firm finances investment expenditures with internal funds and equity only, with the issuance of the latter not being subject to any dilution costs. As a result, the aggregate debt \tilde{b} is not a state variable and does not appear in the agents' perceived laws of motion.

laws of motion for the model without financial frictions. According to the goodness-of-fit criteria, the agents' perceived aggregate laws of motion are again highly accurate. Note that in this case, the coefficient on the liquidation value of capital (p^-) in the law of motion for the aggregate capital stock is tiny, which implies that liquidity shocks have a negligible effect on the evolution of the aggregate capital stock in the absence of financial distortions.