Utility Maximizing Entrepreneurs and the Financial Accelerator*

Mikhail Dmitriev† and Jonathan Hoddenbagh‡

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In the financial accelerator literature developed by Bernanke, Gertler and Gilchrist (1999) and others, risk neutral entrepreneurs maximize next period returns on equity rather than expected utility. We depart from this literature and embed risk neutral utility maximizing entrepreneurs in an otherwise standard financial accelerator model. Using this framework, we solve for two optimal lending contracts: a variable rate contract and a fixed rate contract. In contrast to the standard model, we find that shocks to financial frictions have no effect under the optimal variable rate contract. We then show that under the optimal fixed rate contract, financial frictions remain a driver of business cycles. Our results emphasize the crucial role of insurance markets in business cycles.

Keywords: Financial accelerator; financial frictions; risk; optimal contract; agency costs.

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†Boston College. Email: dmitriev@bc.edu. Web: http://www.mikhaildmitriev.org
‡Boston College. Email: jon.hoddenbagh@gmail.com. Web: https://www2.bc.edu/jonathan-hoddenbagh/
1 Introduction

There is a long line of literature examining financial frictions in macroeconomic models. In one
of the foundational papers in this literature, Bernanke, Gertler and Gilchrist (1999) — here-
after BGG — derive a variable rate loan contract between risk averse lenders and risk neutral
borrowers in the costly state verification (CSV) framework of Townsend (1979).

Although this

loan contract has become the standard contract for CSV models of financial frictions, it is not
optimal.

In this paper we derive the optimal variable rate loan contract in the CSV model for a set of
one-period contracts with deterministic monitoring. The optimal contract adds two additional
channels to the traditional BGG framework: consumption insurance and utility maximizing
entrepreneurs. First, we introduce insurance markets between lenders and borrowers following
early criticism of BGG by Chari (2003), later formalized by Carlstrom, Fuerst and Paustian
(2012), hereafter CFP. As CFP show, this channel strengthens the financial accelerator. Second,
risk neutral entrepreneurs in our model maximize their utility instead of next period returns
on equity. We show that this second channel dampens the financial accelerator. To the best
of our knowledge, previous work has failed to include utility maximizing entrepreneurs in the
contracting problem between lenders and borrowers.

Under the optimal variable rate contract, we find that shocks to the volatility of entrepreneurs’
idosyncratic productivity — what we call “risk” shocks — have little to no impact on the real
economy, in contrast with the standard BGG contract. In a bid to overcome this, we depart
from the literature’s focus on variable rate contracts and derive the optimal fixed rate contract,
which amplifies and propagates shocks in a quantitatively similar manner to the BGG con-
tract. In particular, under the optimal fixed rate contract risk shocks have a significant impact
on the real economy. The optimal fixed rate contract thus incorporates utility maximizing
entrepreneurs while generating a strong financial accelerator.

Our model is standard and consists of a risk averse representative household, risk neutral
entrepreneurs and aggregate risk. Entrepreneurs borrow money from the representative house-
hold and purchase capital to use in production. Entrepreneurs are identical ex ante but differ
depending on the ex post realization of an idiosyncratic productivity shock. Both agents have
full information about the distribution of idiosyncratic shocks ex ante, so there is no adverse
selection problem. Borrowers observe the realization of their idiosyncratic shock, but lenders
do not: they need to pay monitoring costs to observe it.

In the BGG contract borrowers guarantee a constant safe rate of return to lenders in order
to maximize returns on their equity. As a result, borrowers absorb all risk in the economy.

\footnote{In a variable rate loan contract the lending rate is state contingent. Variable rate contracts are standard in
the literature. In contrast, under a fixed rate loan contract the lending rate is not state contingent.}

\footnote{An excellent list of references for partial equilibrium multiperiod contracts includes Monnett and Quintin
stochastic monitoring.}
It should be noted that this is an assumption and not an equilibrium condition. Because of underlying moral hazard, negative shocks to entrepreneurs net worth lead to a tightening of financial constraints and higher default rates. This results in the so-called financial accelerator: the BGG contract amplifies macroeconomic fluctuations in an otherwise standard representative agent dynamic stochastic general equilibrium (DSGE) model.

Recent work by CFP shows that the BGG contract is suboptimal because it does not provide appropriate consumption insurance for risk averse households. In CFP, risk neutral entrepreneurs find it optimal to offer lenders a variable rate contract with a negative covariance between the rate of return and the lender’s consumption. In other words, risk neutral borrowers sell insurance to the risk averse lender, a dynamic which is absent in BGG. This insurance channel strengthens the financial accelerator. During a recession when consumption is low, the CFP loan contract offers lenders a higher lending rate and higher of return in order provide consumption insurance. The combination of already low net worth and lending rates during a recession increases volatility. Entrepreneurs have to pay a lending rate exactly when their net worth is already low. Because capital depends on the size of net worth, the dynamics of net worth directly affect investment, leading to a sharp decline in investment during recessions. The stabilizing effect of the insurance channel, a rise in household consumption during a recession, is outweighed by a much stronger decline in investment. The overall effect of consumption insurance is to amplify business cycle fluctuations.

We take the consumption insurance channel of CFP and introduce utility maximizing entrepreneurs into the CSV framework. In both BGG and CFP, entrepreneurs maximize their expected next period equity, defined as net worth minus entrepreneurial wages, but expected utility depends on the expected discounted sum of all future equity. In the optimal contract, entrepreneurs maximize current and future capital returns. This dynamic is absent in BGG, CFP and to our knowledge the entire CSV literature.

How does the variable rate loan contract change when entrepreneurs maximize utility rather than next period equity? In CFP, entrepreneurs sell as much insurance to the household as they can because insurance does not effect current returns. Under the optimal contract, utility maximizing entrepreneurs sell less insurance because they are concerned not only about current returns but also future returns, which are impacted by insurance claims. Although entrepreneurs do not know whether shocks will be positive or negative, they do know that the model is stationary: they expect a positive technology shock (or positive monetary shock) to lead to an initial increase in the price of capital followed by a decline back to the steady state value. The change in the price of capital will thus be positive in the first period and negative in all other periods.

The entrepreneur’s desire to insure against negative shocks arises from the autocorrelation structure of capital returns. If a positive shock to capital returns predicts positive future capital returns relative to the steady state, risk neutral entrepreneurs demonstrate risky behaviour by
selling more insurance and leveraging up. If, on the other hand, a positive shock to capital returns predicts negative future capital returns, risk neutral entrepreneurs will behave in a more risk averse manner by selling less insurance.

For standard calibrations under the optimal variable rate contract, we find that the amplifying effect of the “consumption insurance channel” and the dampening effect of the “utility maximizing entrepreneurs channel” cancel each other out for technology shocks so that the quantitative predictions of the model roughly coincide with BGG. On the other hand, the financial accelerator is negligible for risk shocks. This contrasts with BGG, where risk shocks explain a significant portion of business cycle fluctuations and generate the desired comovement of macroeconomic variables. For example, Christiano, Motto and Rostagno (2009) employ the BGG contract and emphasize the importance of risk shocks in generating business cycle fluctuations. Under the optimal variable rate contract, the importance of risk shocks disappears completely.

To restore the strength of the financial accelerator, we depart from the literature and propose a fixed rate lending contract. Under the optimal fixed rate contract, the financial accelerator attains BGG levels for all types of shocks. Counterintuitively, the optimal contract with fixed lending rates is equivalent quantitatively to the BGG contract with variable lending rates. This paradox arises from the relative similarity of the state contingent interest rates offered in the BGG contract: regardless of the state that occurs, the lending rate will be almost uniform. The uniformity of state contingent lending rates is a result of BGG’s assumption that entrepreneurs bear all risk in the lending contract. Under the optimal fixed rate contract, risk neutral entrepreneurs bear the majority of risk as an equilibrium condition, not as an assumption.

1.1 Related Literature

There is a large literature on the role of financial frictions in macroeconomics, particularly on how such frictions amplify and propagate shocks, which is the idea of the financial accelerator. In much of the literature, borrowers and lenders are unable to buy or sell insurance (i.e. there is no hedging).

One early example is Kiyotaki and Moore (1997), who show that feedback between collateral prices and loans leads to amplification. But Suarez and Sussman (1997) predicted and Krishnamurthy (2003) later proved that the feedback loop in Kiyotaki and Moore (1997) disappears when agents are able to hedge, weakening the amplification mechanism of the financial friction. To restore amplification, Krishnamurthy (2003) constrained hedging by the size of aggregate collateral. Consequently, the main source of amplification in the case of Kiyotaki and Moore is aggregate collateral, in the spirit of Holmstrom and Tirole (1998).

The benchmark technique for modeling financial frictions has become the CSV framework of Townsend (1979). Early contributions to this strand of the literature include the work of
Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), the aforementioned BGG model and many others. The CSV framework is quite popular because of its strong financial accelerator, which was thought to be robust to hedging. However, we show that the relative strength of the financial accelerator in CSV models is impacted by the introduction of hedging on both the lender and borrower side. In the optimal contract households hedge against consumption risk, which increases the accelerator, while utility maximizing entrepreneurs hedge against low future returns to capital, which decreases the accelerator. For technology shocks, these two effects more or less cancel each other out so that amplification is similar to BGG. However, for risk shocks the accelerator virtually disappears. One possible remedy for amplification in CSV models, especially for risk shocks, is to explicitly model financial intermediation with a constraint on hedging as in Krishnamurthy (2003).

Similar concerns about the financial accelerator and the impact of hedging arise in the costly state enforcement literature. For example, Kiyotaki and Gertler (2010) build a model where non-financial firms face no hedging constraints, but financial intermediaries must pay a constant safe interest rate to the household, which is similar to the original BGG model. In this family of model borrowers and lenders maximize utility, but the explicit guarantee of safe returns makes the contract different from the optimal. We are not aware of research, investigating what happens for an optimal contract with variable and fixed lending rates.

Suarez and Sussman (1997) extend the Stiglitz and Weiss (1981) framework. They preserve asymmetric information between borrowers and lenders while keeping a full set of financial instruments to hedge risk, but they still obtain amplification. However, their model is very stylized, and whether their results can be extended to the quantitatively testable DSGE model, remains an open question.

There is also a rich literature on pecuniary externalities which constrain hedging and lead to inefficient decentralized allocations where agents underinsure. Topics in this vein include sudden stops for emerging economies, such as Caballero and Krishnamurthy (2001, 2003, 2004), Jeanne and Korinek (2013) and Bianchi (2011), as well as research on macroprudential policies, including Stein (2012), Jeanne and Korinek (2010) and Bianchi and Mendoza (2011). The robustness of pecuniary externalities to contracts with variable lending rates remains an open question. Our emphasis is stronger on the role of insurance and hedging, but in its absence many interesting results can be obtained.

Our paper is also related to the growing body of medium sized DSGE models with financial frictions. To the best of our knowledge, the literature follows the BGG framework and employs entrepreneurs who maximize operational equity rather than expected utility. Examples include Villaverde (2009, 2010) and Christiano, Motto and Rostagno (2013). Again, our results differ from the conclusions of this literature, as risk shocks under the optimal variable rate contract have very little impact on the real economy.
2 The Optimal Lending Contract in Partial Equilibrium

Our main contribution in this paper is to introduce utility maximizing entrepreneurs into an otherwise standard CSV model of financial frictions. In this section we outline the key differences between the dynamically optimal loan contract chosen by utility maximizing entrepreneurs and the alternative loan contracts in BGG and CFP in a partial equilibrium setting. Here we assume that entrepreneurs take the price of capital and the expected return to capital as given. In Section 3 we endogenize these variables in general equilibrium.

At time $t$, entrepreneur $j$ purchases capital $K_t(j)$ at a unit price of $Q_t$. At time $t+1$, the entrepreneur rents this capital to perfectly competitive wholesale goods producers. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase capital:

$$Q_tK_t(j) = N_t(j) + B_t(j).$$

(1)

After buying capital, the entrepreneur is hit with an idiosyncratic shock $\omega_{t+1}(j)$ and an aggregate shock $R_{t+1}^k$, so that entrepreneur $j$ is able to deliver $Q_tK_t(j)R_{t+1}^k\omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega(j)$ is a log-normal random variable with distribution $\log(\omega(j)) \sim N(-\frac{1}{2}\sigma^2_\omega, \sigma^2_\omega)$ and mean of one.

Following BGG, we assume entrepreneurs are risk neutral and die with constant probability $1 - \gamma$. Upon dying, entrepreneurs consume all operational equities, which are equal to net worth minus wages. If entrepreneurs survive they do not consume anything, and they supply labor and earn wages which they later reinvest. Entrepreneur $j$’s value function is

$$V_t^e(j) = (1 - \gamma) \sum_{s=1}^{\infty} \gamma^s C_{t+s}^e$$

(2)

where $C_{t+s}^e$ is the entrepreneur’s consumption,

$$C_t^e(j) = N_t(j) - W_t^e$$

(3)

defined as wealth accumulated from operating firms, equal to net worth without entrepreneurial real wages $W_t^e$. The timeline for entrepreneurs is plotted in Figure 1.
2.1 Borrower and Lender Payoffs

The contract between the lender and borrower follows the familiar CSV framework. We assume that the lender cannot observe the realization of idiosyncratic shocks to entrepreneurs unless he pays monitoring costs which are a fixed percentage of total assets. Given this friction, the risk neutral borrower offers the risk averse lender a contract with an adjustable interest rate subject to macroeconomic conditions.

The entrepreneur repays the loan only when it is profitable to do so. In particular, the entrepreneur will repay the loan only if, after repayment, he has more assets than liabilities. We define the cutoff productivity level \( \bar{\omega}_{t+1}(j) \), also known as the bankruptcy threshold, as the minimum level of productivity necessary for an entrepreneur to repay the loan:

\[
\frac{B_t(j) Z_{t+1}(j)}{\text{Cost of loan repayment}} = \bar{\omega}_{t+1}(j) \frac{R_{t+1}^k Q_t K_t(j)}{\text{Minimum revenue for loan repayment}}.
\]

If \( \omega_{t+1}(j) < \bar{\omega}_{t+1} \) the entrepreneur defaults and enters bankruptcy; if \( \omega_{t+1}(j) \geq \bar{\omega}_{t+1}(j) \) he repays the loan. The cutoff productivity level allows us to express the dynamics of net worth for a particular entrepreneur \( j \):

\[
N_{t+1}(j) = Q_t K_t(j) R_{t+1}^k \max \left\{ \omega_{t+1}(j) - \bar{\omega}_{t+1}(j), 0 \right\} + W_{t+1}^e.
\]

The gross rate of return for the lender, \( R_{t+1} \), also depends on the productivity cutoff. For idiosyncratic realizations above the cutoff, the lender will be repaid the full amount of the loan \( B_t(j) Z_{t+1}(j) \). For idiosyncratic realizations below the cutoff, the entrepreneur will enter bankruptcy and the lender will pay monitoring costs and take over the entrepreneur’s assets, ending up with \( (1 - \mu) K_t(j) R_{t+1}^k \omega_{t+1}(j) \). More formally, the lender’s ex post return is

\[
B_t(j) R_{t+1}(j) = \begin{cases} 
B_t(j) Z_{t+1} & \text{if } \omega_{t+1}(j) \geq \bar{\omega}_{t+1}(j) \\
(1 - \mu) K_t(j) R_{t+1}^k \omega_{t+1}(j) & \text{if } \omega_{t+1}(j) < \bar{\omega}_{t+1}(j)
\end{cases}
\]

Taking into account that loans to entrepreneurs are perfectly diversifiable, the lenders return
on a loan $R_{t+1}$ to entrepreneur $j$ is defined as

$$B_1(j)R_{t+1} \equiv Q_tK_t^j R_{t+1}^k h(\bar{\omega}_{t+1}(j)),$$

(7)

where $h(\bar{\omega}_{t+1})$ is the share of total returns to capital that go to the lender. We define this share as

$$h(\bar{\omega}_t) = \left\{ \begin{array}{c} \bar{\omega}_t + (1 - \mu) \int_0^{\bar{\omega}_t} f(\omega) d\omega \\ \bar{\omega}_t \end{array} \right\} \text{ Share to lender if loan pays}$$

$$\text{Share to lender if loan defaults}$$

(8)

where $f$ is the probability density function and $F$ is the cumulative distribution function of the log-normal distribution of idiosyncratic productivity.

In order to simplify the entrepreneur’s optimization problem, we introduce the concept of leverage, $\kappa_t$, defined as the value of the entrepreneur’s capital divided by net worth:

$$\kappa_t(j) \equiv Q_tK_t^j / N_t(j).$$

(9)

2.2 Variable Rate Loan Contracts: BGG, CFP and The Optimal Variable Rate Loan Contract

The differences between the BGG contract, the CFP contract and the optimal variable rate contract arise from two sources: the lender’s participation constraint and the borrower’s objective function.

First, the lender’s participation constraint in BGG differs from CFP and the variable rate contract. The participation constraint arises from the household Euler equation and stipulates the minimum rate of return that entrepreneurs must offer to lenders to receive a loan. In BGG, the participation constraint has the following form:

$$\beta E_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} \right\} R_{t+1} = 1.$$

(10)

Under this participation constraint, entrepreneurs pay a constant safe rate of return to the lenders, $R_{t+1}$, which ignores the risk averse representative household’s desire for consumption insurance. In contrast, the participation constraint in CFP and the optimal variable rate contract is:

$$\beta E_t \left\{ \frac{U_{C,t+1}R_{t+1}}{U_{C,t}} \right\} = 1.$$

(11)

As CFP show, the above expression implies that households prefer a state contingent rate of return that is negatively correlated with household consumption. Quite simply, households like consumption insurance. In recessions, households desire a higher rate of return because their marginal utility of consumption is high, and vice versa in booms. Consumption insurance in CFP and the variable rate contract serves to amplify the financial accelerator.
Second, the borrower’s objective function in BGG and CFP differs from the optimal variable rate contract. Entrepreneurs in BGG and CFP maximize next period net worth, defined in equation (5). If we substitute the expression for leverage from (9) into (5), we have the entrepreneur’s objective function in BGG and CFP:

\[
\kappa_t(j) N_t(j) E_t \left\{ R_{t+1}^k \max \left[ \omega_{t+1}(j) - \bar{\omega}_{t+1}(j), 0 \right] \right\}. \tag{12}
\]

In contrast, under the dynamically optimal contract entrepreneurs maximize utility, given by (2). As we’ve mentioned before, utility maximizing entrepreneurs behave in a “risk averse” manner because they are concerned not only about current capital returns but also future capital returns. As such, they stabilize macroeconomic volatility and dampen the financial accelerator.

We now have all of the ingredients necessary to set up the entrepreneur’s optimization problem and solve for the three different loan contracts: (1) the BGG contract; (2) the CFP contract; and, (3) the optimal variable rate contract.

**Proposition 1** To solve for the BGG contract, entrepreneurs choose their state contingent cutoff \( \bar{\omega}_{t+1}(j) \) and leverage \( \kappa_t(j) \) to maximize next period net worth (12) subject to (5), (7) and (10). The first order conditions to this problem are given by

\[
0 = N_t E_t \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}(j)) \right\} - \frac{U_{c,t}}{\kappa_t} E_t \left\{ \lambda_{t+1} \right\}, \tag{13}
\]

\[
0 = N_t g'(\bar{\omega}_{t+1}(j)) + \lambda_{t+1} \beta E_t \left\{ (U_{c,t+1})' h'(\bar{\omega}_{t+1}(j)) \right\}, \tag{14}
\]

where \( g = \int_{\bar{\omega}(j)}^{\infty} \omega f(\omega) d\omega - \bar{\omega}(j) \left[ 1 - F(\bar{\omega}(j)) \right]. \)

**Proof** See Appendix B. ■

Note that we cannot eliminate the Lagrange multiplier from these first order conditions, since first order condition with respect to capital has lagrangian multiplier with expected sign, while first order condition with respect to productivity cutoff has Lagrange multiplier without expectation. BGG contract differs from other variable rate contract by having state contingent constraints on productivity cutoff, while in other contracts there will be one non-state contingent participation constraint. As a result, in the BGG contract leverage cannot be expressed as a function of the productivity cutoff even after log-linearization, unlike the CFP and optimal contract with a variable lending rate below. We first look at the CFP contract.

**Proposition 2** To solve for the CFP contract, entrepreneurs choose their state contingent cutoff \( \bar{\omega}_{t+1}(j) \) and leverage \( \kappa_t(j) \) to maximize (12) subject to (5), (7) and (11). The solution
to this problem is given by

\[-\beta \frac{U_{c,t+1}}{U_{c,t}} h'(\tilde{\omega}_{t+1}) = \frac{g'(\bar{\omega}_{t+1})}{\kappa_t \mathbb{E}_t \{ R^k_{t+1} g(\bar{\omega}_{t+1}) \}}. \tag{15}\]

**Proof** See Appendix C. ■

Observe that the state contingent cutoff \( \tilde{\omega}_{t+1}(j) \) in (15) does not depend on any \( t + 1 \) variables apart from the marginal utility of consumption \( U_{c,t+1} \). This illustrates the importance of consumption insurance in the CFP contract. In BGG the productivity cutoff depends not on the realization of consumption but on next period capital returns, in order to satisfy the state contingent budget constraint guaranteeing safe returns to the lender. In CFP entrepreneurs have only one budget constraint to satisfy, and move resources across states to satisfy this constraint. Since their constraint guarantees a certain level of utility to the household, entrepreneurs give up resources in those states that are more valued by the household, and move resources to those states that are less valued by the household. This provides higher utility for households on average. Since entrepreneurs must guarantee only a reserve level of utility in CFP, they are able to offer a smaller amount of resources in all states, thus maximizing their own expected next period equity while giving the household the reserve level of utility.

Having described the BGG and CFP contracts in detail, we turn our attention to the optimal variable rate contract. As we discussed above, the optimal contract takes the consumption insurance channel from CFP and adds utility maximizing entrepreneurs.

**Proposition 3** To solve for the optimal variable rate contract, entrepreneurs choose their state contingent cutoff \( \tilde{\omega}_{t+1}(j) \) and leverage \( \kappa_t(j) \) to maximize (2) subject to (3), (5), (7) and (11). The solution to this problem is given by

\[-\beta \frac{U_{c,t+1}}{U_{c,t}} h'(\tilde{\omega}_{t+1}) = \frac{g'(\bar{\omega}_{t+1}) \Psi_{t+1}}{\kappa_t \mathbb{E}_t \{ R^k_{t+1} g(\bar{\omega}_{t+1}) \}}. \tag{16}\]

where

\[\Psi_t = 1 + \gamma \kappa_t \mathbb{E}_t \{ g(\bar{\omega}_{t+1}) R^k_{t+1} \Psi_{t+1} \}. \tag{17}\]

**Proof** See Appendix D. ■

Relative to the CFP contract in (15), the only additional term in the optimal contract in (16) is \( \Psi_{t+1} \). \( \Psi_{t+1} \) reflects an increase in expected consumption in \( t + 2 \) from investments made in \( t + 1 \). Heuristically, \( \Psi_{t+1} \) behaves qualitatively in the same way as \( \mathbb{E}_{t+1} \{ R^k_{t+2} \} \). If we log-linearize the first order conditions for the optimal rate contract, they will differ from the CFP contract only by the term \( \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1} \) on the right hand side of the equation, which heuristically means that the dynamically optimal contract differs from the statically optimal contract by \( \mathbb{E}_{t+1} \hat{R}^k_{t+2} - \mathbb{E}_t \hat{R}^k_{t+2} \).

10
To simplify matters, assume for the moment that lenders are risk neutral. If so, the CFP contract implies that the productivity cutoff will not depend on the realization of the shocks:

$$-\beta h'(\tilde{\omega}_{t+1}) = \frac{g'(\tilde{\omega}_{t+1})}{\kappa_t \mathbb{E}_t \{ R_{t+1}^k g(\tilde{\omega}_{t+1}) \}}.$$  \hspace{1cm} (18)

On the other hand, under the dynamically optimal contract we have

$$-\beta h'(\tilde{\omega}_{t+1}) = \frac{g'(\tilde{\omega}_{t+1}) \Psi_{t+1}}{\kappa_t \mathbb{E}_t \{ R_{t+1}^k g(\tilde{\omega}_{t+1}) \Psi_{t+1} \}},$$  \hspace{1cm} (19)

where default is sensitive to shocks that increase $\Psi_{t+1}$ or $\mathbb{E}_t R_{t+1}^k$. What matters for the default rate and the productivity cutoff is the update of expectations about future capital returns. If the entrepreneur’s forecast of future capital returns increases such that $\mathbb{E}_{t+1} R_{t+2}^k > \mathbb{E}_t R_{t+2}^k$, then the entrepreneur would like to pay a lower interest rate on loans, and vice versa in the opposite case. On the other hand in CFP the productivity cutoff, default rate, and share of asset returns going to entrepreneurs is constant if households are risk neutral.

### 2.3 The Optimal Fixed Rate Loan Contract

The literature assumes that borrowers and lenders agree on a loan contract in period $t$ that specifies a unique repayment rate for each possible state of the world in period $t+1$. This is what we call a contingent or variable rate contract. But what if borrowers and lenders are unable to write a state-contingent lending contract? Instead, what if borrowers and lenders can only write a contract in period $t$ with one fixed lending rate $Z_t(j)$ for all states of the world in $t+1$? We solve for the optimal fixed rate loan contract in this section. The productivity cutoff $\tilde{\omega}$ is defined differently when loans are restricted to a fixed rate:

$$\frac{B_t(j) Z_t(j)}{\text{Cost of loan repayment}} = \frac{\tilde{\omega}_{t+1}(j) R_{t+1}^k Q_t K_t(j)}{\text{Minimum revenue for loan repayment}},$$  \hspace{1cm} (20)

where instead of the period $t+1$ lending rate $Z_{t+1}(j)$ we have the period $t$ lending rate $Z_t(j)$. Now we are ready to define the optimal contract with fixed lending rates.

**Proposition 4** To solve for the optimal fixed rate contract, entrepreneurs choose their loan rate $Z_t(j)$ and leverage $\kappa_t(j)$ to maximize (2) subject to (3), (5), (7) and (11), (20). The solution to this problem is given by

$$-\beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} h'(\tilde{\omega}_{t+1}) \right\} = \frac{\mathbb{E}_t \left\{ g'(\tilde{\omega}_{t+1}) \Psi_{t+1} \right\}}{\kappa_t \mathbb{E}_t \left\{ R_{t+1}^k g(\tilde{\omega}_{t+1}) \Psi_{t+1} \right\}}.$$  \hspace{1cm} (21)
where

\[ \Psi_t = 1 + \mathbb{E}_t \left\{ \gamma \kappa_t R^k_{t+1} g(\bar{\omega}_{t+1}) \Psi_{t+1} \right\} . \]  

(22)

**Proof**  See Appendix E. ■

Table 1 outlines the solutions to the CFP contract, the variable rate optimal contract and the fixed rate optimal contract. One can easily see the similarities and differences between the contracts. For example, the optimality condition for the fixed rate contract is equal to the expectation of the optimality condition for the variable rate contract. Although the fixed rate optimality condition does not define the dynamics of the productivity cutoff, which are defined by (20), it is necessary to compute the optimal leverage ratio. In this respect the fixed rate contract is more similar to the BGG contract, where the dynamics of the productivity cutoff are effectively defined by the participation constraint, given by (7) and (10). On the other hand, in the CFP contract and the variable rate contract, the dynamics of the productivity cutoff and hence bankruptcies depend on the optimality conditions, which differ by the term \( \Psi_t \) as we explained above.

### Table 1: Contract Optimality Conditions

<table>
<thead>
<tr>
<th>Contract Type</th>
<th>Optimality Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFP Contract:</td>
<td>[ \frac{g'\left(\bar{\omega}<em>{t+1}\right)}{\kappa_t \mathbb{E}<em>t \left{ R^k</em>{t+1} g(\bar{\omega}</em>{t+1}) \right}} = -\beta h'\left(\bar{\omega}<em>{t+1}\right) \frac{U</em>{c,t+1}}{U_{c,t}} ]</td>
</tr>
<tr>
<td>Variable Rate Contract:</td>
<td>[ \frac{g'\left(\bar{\omega}<em>{t+1}\right) \Psi</em>{t+1}}{\kappa_t \mathbb{E}<em>t \left{ R^k</em>{t+1} g(\bar{\omega}<em>{t+1}) \Psi</em>{t+1} \right}} = -\beta h'\left(\bar{\omega}<em>{t+1}\right) \frac{U</em>{c,t+1}}{U_{c,t}} ]</td>
</tr>
<tr>
<td>Fixed Rate Contract:</td>
<td>[ \frac{\mathbb{E}<em>t \left{ g'\left(\bar{\omega}</em>{t+1}\right) \Psi_{t+1} \right}}{\kappa_t \mathbb{E}<em>t \left{ R^k</em>{t+1} g(\bar{\omega}<em>{t+1}) \Psi</em>{t+1} \right}} = -\beta \mathbb{E}<em>t \left{ h'\left(\bar{\omega}</em>{t+1}\right) \frac{U_{c,t+1}}{U_{c,t}} \right} ]</td>
</tr>
</tbody>
</table>

3 **The Model in General Equilibrium**

We now embed the four loan contracts in a standard dynamic New Keynesian model. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers. Entrepreneurs buy capital from capital producers and then rent it out to perfectly competitive wholesalers, who sell their goods to monopolistically competitive retailers. Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. Retailers have price-setting power and are subject to Calvo price rigidities. Households bundle the retail goods in CES fashion into a final consumption good. A graphical overview of the model is provided in Figure 2 below. The dotted lines denote financial flows, while the solid lines denote real flows (goods, labor, and capital).
Figure 2: Overview of the Model

- Retailers
  - Wholesale Goods
  - Payments
  - Retail Goods
  - Retail Goods

- Wholesalers
  - Capital, Entrepreneurial Wage
  - Capital Rent

- Entrepreneurs
  - Payment
  - Capital Rent, Entrepreneurial Wage

- Households
  - Dividends
  - Payment
  - Retail Goods
  - Payments

- Financial Intermediaries
  - Deposits
  - Repayment
  - Loans

- Capital Producers
  - Payments for Capital

- Capital
  - Repayment

- Payments for Consumption
3.1 Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money

$$\max_{E_t} \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \zeta \log \left( \frac{M_{t+s}}{P_{t+s}} \right) - \chi H_{t+s}^{1+\eta} \right] \right\}, \quad (23)$$

where $C_t$ is household consumption, $M_t/P_t$ denotes real money balances, and $H_t$ is household labor effort. The budget constraint of the representative household is

$$C_t = W_t H_t - T_t + \Pi_t + R_t \frac{D_t}{P_t} - \frac{D_{t+1}}{P_t} + \frac{M_{t-1} - M_t}{P_t} + \frac{B_{t-1} R^n_t - B_t}{P_t}, \quad (24)$$

where $W_t$ is the real wage, $T_t$ is lump-sum taxes, $\Pi_t$ is profit received from household ownership of final goods firms distributed in lump-sum fashion, and $D_t$ are deposits in financial intermediaries (banks) that pay a contingent nominal gross interest rate $R_t$, and $B_t$ are nominal bonds that pay a gross nominal non-contingent interest rate $R^n_t$.

Households maximize their utility (23) subject to the budget constraint (24) with respect to deposits, labor, nominal bonds and money, yielding three first order conditions: The Fisher equation defines the relationship between the nominal and real interest rates

$$U_{C,t} = \beta E_t \left\{ R_{t+1} U_{C,t+1} \right\}, \quad (25)$$

$$W_t U_{C,t} = \chi H_t^\eta, \quad (26)$$

$$U_{C,t} = \beta R^n_t E_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}, \quad (27)$$

$$U_{C,t} = \zeta \frac{1}{m_t} + \beta E_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}. \quad (28)$$

We define the gross rate of inflation as $\pi_{t+1} = P_{t+1}/P_t$, and real money balances as $m_t = M_t/P_t$.

3.2 Retailers

The final consumption good is made up of a basket of intermediate retail goods which are aggregated together in CES fashion by the representative household:

$$C_t = \left( \int_0^1 c_{it}^\frac{1-\varepsilon}{\varepsilon} \, dt \right)^\frac{\varepsilon}{1-\varepsilon}. \quad (29)$$

Demand for retailer $i$’s unique variety is

$$c_{it} = \left( \frac{p_{it}}{P_t} \right)^{-\varepsilon} C_t, \quad (30)$$
where $p_{it}$ is the price charged by retail firm $i$. The aggregate price index is defined as

$$P_t = \left( \int_0^1 p_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \quad (31)$$

Each retail firm chooses its price according to Calvo (1979) in order to maximize net discounted profit. With probability $1 - \theta$ each retailer is able to change its price in a particular period $t$. Retailer $i$'s objective function is

$$\max \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \frac{p_{it}^* - P_{t+s}^w}{P_{t+s}} \left( \frac{p_{it}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \right\}, \quad (32)$$

where

$$\Lambda_{t,s} \equiv \beta^s \frac{U_{C,t+s}}{U_{C,t}} \quad (33)$$

is the household (i.e. shareholder) intertemporal marginal rate of substitution and $P_{t}^w$ is the wholesale goods price. The first order condition with respect to the retailer’s price $p_{it}$ is

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} (p_{it}^* / P_{t+s})^{-\varepsilon} Y_{t+s} \left[ p_{it} - \frac{\varepsilon}{\varepsilon - 1} P_{t+s}^w \right] \right\} = 0. \quad (34)$$

From this condition it is clear that all retailers which are able to reset their prices in period $t$ will choose the same price $p_{it}^* = P_{t}^* \forall i$. The price level will evolve according to

$$P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1 - \theta) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (35)$$

Dividing the left and right hand side of (35) by the price level gives

$$1 = \left[ \theta \pi_{t-1}^\varepsilon + (1 - \theta) (p_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \quad (36)$$

where $p_t^* = P_t^*/P_t$. Using the same logic, we can normalize (34) and obtain:

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} p_{t+s}^w \right\}, \quad (37)$$

where $p_{t+s}^w = \frac{P_{t+s}^w}{R_t}$ and $p_{t+s} = P_{t+s}/P_t$.

### 3.3 Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage $W_t$ and $W_t^e$ and rent capital from entrepreneurs at rental rate $R_t^e$. Note that capital purchased in period $t$ is used in period $t + 1$. 

15
Following BGG, the production function of the representative wholesaler is given by

\[ Y_t = A_t K_{t-1}^\alpha (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)}, \]  

(38)

where \( A_t \) denotes aggregate technology, \( K_t \) is capital, \( H_t \) is household labor, \( H_t^e \) is entrepreneurial labor, and \( \Omega \) defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to

\[ Y_t = A_t K_{t-1}^\alpha H_t^{(1-\alpha)\Omega}. \]  

(39)

One can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be

\[ \frac{P_w}{P_t} = \frac{1}{x_t}, \]  

(40)

where \( x_t \) is the variable markup charged by final goods producers. The objective function for wholesalers is then given by

\[ \max_{H_t, H_t^e, K_{t-1}} \frac{1}{x_t} A_t K_{t-1}^\alpha (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} - W_t H_t - W_t^e H_t^e - R_t^e K_{t-1}. \]  

(41)

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

\[ \frac{1}{x_t} \alpha Y_t K_{t-1} = R_t^e, \]  

(42)

\[ \Omega (1 - \alpha) \frac{Y_t}{H_t} = W_t, \]  

(43)

\[ \Omega (1 - \alpha) \frac{Y_t}{H_t^e} = W_t^e. \]  

(44)

### 3.4 Capital Producers

The perfectly competitive capital producer transforms final consumption goods into capital. Capital production is subject to adjustment costs, according to

\[ K_t = I_t + (1 - \delta) K_{t-1} - \frac{\phi_k}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}, \]  

(45)

where \( I_t \) is investment in period \( t \), \( \delta \) is the rate of depreciation and \( \phi_k \) is a parameter that governs the magnitude of the adjustment cost. The capital producer’s objective function is

\[ \max_{I_t} K_t Q_t - I_t, \]  

(46)
where \( Q_t \) denotes the price of capital. The first order condition of the capital producer’s optimization problem is
\[
\frac{1}{Q_t} = 1 - \phi_K \left( \frac{I_t}{K_{t-1}} - \delta \right).
\]

(47)

3.5 Lenders

One can think of the representative lender in the model as a perfectly competitive bank which costlessly intermediates between households and borrowers. The role of the lender is to diversify the household’s funds among various entrepreneurs. The bank takes nominal household deposits \( D_t \) and loans out nominal amount \( B_t \) to entrepreneurs. In equilibrium, deposits will equal loanable funds \( (D_t = B_t) \). Households, as owners of the bank, receive a state contingent real rate of return \( R_{t+1} \) on their “deposits” — which equals the rate of return on loans to entrepreneurs.\(^3\)

Households choose the optimal lending rate according to their first order condition with respect to deposits:
\[
\beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} R_{t+1} \right\} = \mathbb{E}_t \left\{ \Lambda_{t+1} R_{t+1} \right\} = 1.
\]

As we discussed above, the lender prefers a return that co-varies negatively with household consumption, which amplifies the financial accelerator.

3.6 Entrepreneurs

We’ve already described the entrepreneur’s problem in detail in Section 2. Entrepreneurs choose their cutoff productivity level and leverage according to: (13) and (14) in BGG; (15) in CFP; and (16) and (17) in the dynamically optimal contract.

Wholesale firms rent capital at rate \( R^k_{t+1} = \frac{aY_t}{X_tK_{t-1}} \) from entrepreneurs. After production takes place entrepreneurs sell undepreciated capital back to capital goods producers for the unit price \( Q_{t+1} \). Aggregate returns to capital are then given by
\[
R^k_{t+1} = \frac{1}{X_t} \frac{aY_{t+1}}{K_t} + Q_{t+1}(1-\delta) \frac{Q_t}{Q^*}\frac{1}{K_{t-1}}.
\]

(48)

Consistent with the partial equilibrium specification, entrepreneurs die with probability \( 1-\gamma \), which implies the following dynamic for aggregate net worth:
\[
N_{t+1} = \gamma N_t K_t R^k_{t+1} g(\bar{\omega}_{t+1}) + W^{e}_{t+1}.
\]

(49)

3.7 Goods market clearing

We have goods market clearing
\[
Y_t = C_t + I_t + G_t + C^e_t + \mu G(\bar{\omega}_t) R^k_{t} Q_{t-1} K_{t-1},
\]

(50)

\(^3\)Note that lenders are not necessary in the model, but we follow BGG and CFP in positing a perfectly competitive financial intermediary between households and borrowers.
where $\mu G(\bar{\omega}) = \int_{0}^{\bar{\omega}} \mu f(\omega) \omega d\omega$ is the fraction of capital returns that go to monitoring costs, paid by lenders.

### 3.8 Monetary Policy

We assume that there is a central bank which conducts monetary policy by choosing the nominal interest rate $R^n_t$ according to a Taylor Rule. The Taylor Rule for monetary policy is given by

$$R^n_t = (R^n_{t-1})^\rho \pi^\xi_{t-1} e^{\epsilon^n_t}$$  \hspace{1cm} (51)

where $\rho$ and $\xi$ determine the relative importance of the past interest rate and past inflation in the central bank’s interest rate rule. Shocks to the nominal interest rate are given by $\epsilon^{R^n}$.

### 3.9 Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon^A_t,$$  \hspace{1cm} (52)

$$\log(G_t/Y_t) = (1 - \rho^G) \log(G_{ss}/Y_{ss}) + \rho^G \log(G_{t-1}/Y_{t-1}) + \epsilon^G_t,$$  \hspace{1cm} (53)

$$\log(\sigma_{\omega,t}) = (1 - \rho^{\omega}) \log(\sigma_{\omega,ss}) + \rho^{\omega} \log(\sigma_{\omega,t-1}) + \epsilon^\omega_t$$  \hspace{1cm} (54)

where $\epsilon^A$, $\epsilon^G$ and $\epsilon^{id}$ denote exogenous shocks to technology, government spending and idiosyncratic volatility, and $G_{ss}$ and $\sigma_{\omega,ss}$ denote the steady state values for government spending and idiosyncratic volatility respectively. Recall that $\sigma^2_{\omega}$ is the variance of idiosyncratic productivity, so that $\sigma_{\omega}$ is the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the Taylor Rule in (51).

### 3.10 Equilibrium

The model has 20 endogenous variables and 20 equations. The endogenous variables are: $Y$, $H$, $C$, $\Lambda$, $C^e$, $W$, $W^e$, $I$, $Q$, $K$, $R^n$, $R^k$, $R$, $p^*$, $X$, $\pi$, $N$, $\bar{\omega}$, $k$ and $Z$. The equations defining these endogenous variables are: (9), (25), (26), (28), (33), (36), (37), (40), (39), (43), (44), (45), (47), (48), (49), (50), (51), (27), (A.7) and (D.4). The exogenous processes for technology, government spending and idiosyncratic volatility follow (52), (53) and (54) respectively. Nominal interest rate shocks are defined by the Taylor Rule in (51).

### 4 Quantitative Analysis

#### 4.1 Calibration

Our baseline calibration largely follows BGG and CFP. We set the discount factor $\beta = 0.99$, the risk aversion parameter $\sigma = 1$ so that utility is logarithmic in consumption, and the elasticity of labor is 3 ($\eta = 1/3$). The share of capital in the Cobb-Douglas production function is $\alpha = 0.35$. 
Investment adjustment costs are $\phi_k = 10$ to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly depreciation is $\delta = 0.025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = 0.0272$, yielding an annualized business failure rate of three percent, which matches the data. The idiosyncratic productivity term, $\log(\omega(j))$, is assumed to be log-normally distributed with variance of 0.28. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price-setting, we assume the Calvo parameter $\theta = 0.75$, so that only 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. In the monetary policy rule, we set the autoregressive parameter on the nominal interest rate to $\rho = 0.9$ and the parameter on inflation to $\xi = 1.1$. The central bank thus responds to inflation in a robust manner. The persistence of the shocks to technology, government spending and idiosyncratic volatility are $\rho_A = 0.98$, $\rho^G = 0.6$ and $\rho^{id} = 0.9$ respectively.

Following BGG, we consider a one percent technology shock and a 25 basis point shock (in annualized terms) to the nominal interest rate. For the risk shock, we allow the standard deviation of idiosyncratic productivity to increase by one percentage point, from 0.28 by to 0.29.

### 4.2 Quantitative Comparison: BGG and CFP vs the Privately Optimal Contracts

In our quantitative analysis we first compare three allocations: the competitive equilibrium under the BGG contract; the competitive equilibrium under the CFP contract; and the competitive equilibrium under the optimal variable rate contract. Impulse responses for shocks to technology, the nominal interest rate and idiosyncratic volatility are found in this section.

Figure 3 shows impulse responses for a one percent technology shock when prices are sticky.
As we’ve explained before, the key variable of interest is the price of capital. Entrepreneurs find that holding more capital in period one is not profitable after a positive technology shock because its price will be declining over time back to the steady state. As such, forward looking entrepreneurs under the optimal variable rate contract are reluctant to invest in new capital when the shock hits because they anticipate lower capital returns in future periods. In contrast, entrepreneurs in the BGG and CFP models do not look beyond the next period, and thus are more willing to lever up in the wake of a positive technology shock. Consumption insurance is the other driving factor. Lenders in the CFP and optimal variable rate contract will settle for a lower rate of return in a boom in order to ensure a higher rate of return in a recession, which amplifies the financial accelerator. This is especially noticeable in CFP, as the bankruptcy threshold plummets and net worth spikes, amplifying the financial accelerator. In the optimal variable rate model, the negative autocorrelation of $R^k$ dominates the consumption insurance channel, so that the financial accelerator is actually lower than the CFP and BGG models.

The difference between the three allocations is most noticeable in Figure 4, which plots impulse responses for a one percent shock to the nominal interest rate when prices are sticky.
Because the monetary shock is less persistent than the technology shock, due in part to the aggressive response of the central bank to inflation, the price of capital depreciates back to its steady state value very quickly after an initial rise. As a result, capital returns are positive in the first period, but negative thereafter. This leads to an even sharper difference between the response of entrepreneurs in the three models. In particular, the bankruptcy rate in the optimal variable rate model is essentially constant, such that net worth and entrepreneurs consumption respond very little to the nominal interest rate shock. Since consumption is so stable, there is little role for the consumption insurance channel. Risk neutral entrepreneurs thus behave in a “risk-averse” manner under the variable rate contract because of the strong negative autocorrelation on returns to capital. Forward looking entrepreneurs stabilize consumption and output, leading to a very small financial accelerator. In contrast, the CFP contract with consumption insurance leads to a decline in the lending rate following the rise in consumption, which amplifies the response of output, consumption and other macroeconomic aggregates to the interest rate shock.

In Figure 5 we plot impulse responses for a one percentage point increase in idiosyncratic volatility $\sigma_\omega$ with persistence $\rho_\omega = 0.9$. 
This is what we defined earlier as a risk shock. In all three models, increased volatility leads to an initial decline in investment and output. The consumption path is close to zero for all three contracts, but increases slightly under CFP and the optimal contract. Because of the consumption insurance channel in CFP and the optimal contract, the lending rate declines following a risk shock, which boosts entrepreneurs net worth and investment. As a result output declines much less than in BGG. An additional factor is at work under the optimal variable rate contract: the initial decrease in the price of capital and its appreciation in the future provide attractive opportunities for forward looking entrepreneurs to invest in new capital, so that they agree with households to have lower lending rate, which leads to a smaller decline in output than for BGG and CFP, and leads to quick recovery of the economy. Overall, we see that under the optimal variable rate contract risk shocks have smaller impact on the real economy and may even boost output over a longer time horizon.

4.3 Quantitative Comparison: BGG vs the Optimal Fixed Rate Contract

We now turn our attention to comparing the optimal fixed rate contract with the BGG contract. Figure 3 plots impulse responses for a one percentage point increase in productivity.
Figure 6: Technology Shock

The impulse responses demonstrate the close relationship between the two contracts. The financial accelerator is almost identical in both cases. This similarity is also shown in Figure 7, which plots impulse responses for a one percentage point increase in idiosyncratic volatility $\sigma_\omega$ with persistence $\rho_\omega = 0.9$. Again, the difference between the BGG contract and the optimal fixed rate contract is negligible.
The figures make it clear that there is little to distinguish the two contracts, in spite of the fact that one lending rate is state contingent (BGG) and one is not (the fixed rate contract). But why is this the case? The similarity between the two contracts arises from the low variance of the state contingent lending rates in BGG. Even though lending rates are state contingent, they do not differ very much from state to state. Table 2 demonstrates this explicitly.

Table 2: Volatility of the Lending Rate

<table>
<thead>
<tr>
<th>Model/Shock</th>
<th>$A_t$</th>
<th>$R_{n,t}$</th>
<th>$\sigma_{id,t}$</th>
<th>Std.Dev.$(Z_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFP</td>
<td>-2.290</td>
<td>-0.254</td>
<td>-0.513</td>
<td>2.360</td>
</tr>
<tr>
<td>BGG</td>
<td>0.019</td>
<td>-0.006</td>
<td>0.016</td>
<td>0.025</td>
</tr>
<tr>
<td>Optimal Variable Rate</td>
<td>0.197</td>
<td>0.168</td>
<td>-0.646</td>
<td>0.696</td>
</tr>
<tr>
<td>Optimal Fixed Rate Contract</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$^1$ The first three columns show the response of the lending rate in percent deviation from the steady state for a one standard deviation shock to technology ($A_t$), the nominal interest rate ($R_{n,t}$), and the risk shock ($\sigma_{id,t}$). The fourth column shows the conditional standard deviation of the lending rate.
The lending rate in the BGG contract is much less volatile than the lending rate in the CFP contract or the variable rate optimal contract. The lending rate in BGG has a standard deviation of 0.025 percent of the steady state level, which is quantitatively similar to the fixed rate standard deviation of zero.

The intuition for this result follows from the low percentage of defaults in the standard calibration. When the default rate is low, bankruptcies have a small impact on the dynamics of the lending rate. To illustrate the intuition more clearly, we plot the contracting problem in Figure 8, which shows the share of gross capital returns that go to borrowers, lenders and monitoring costs with respect to the bankruptcy threshold $\bar{\omega}$.

![Figure 8: Share of Gross Capital Returns](image)

When the bankruptcy threshold is small, its increase leads to a one for one increase in the lenders share of gross capital returns, as can be seen from the forty five degree line of Figure 8. If the lending rate is fixed, then the bankruptcy threshold is defined as $\bar{\omega}_{t+1}(j) = \frac{\kappa_{t-1}}{\kappa_{t-1}R_{t+1}} Z_t$, so that the threshold moves one for one with the realization of aggregate returns to capital. But because $\bar{\omega}_t(j)$ also moves one for one with the lenders share for small values, an increase in capital returns by one percent leads to a decrease in the share of lenders by one percent. Lenders thus receive a fixed return. As a result, the fixed lending rate yields a fixed rate of return when aggregate shocks do not increase the default probability to a high level. When the default probability is high, a fixed lending rate no longer yields constant returns because the lenders share does not move one for one with respect to a shock to capital returns.

To clearly illustrate the difference between the optimal fixed rate contract and BGG, we consider a calibration which significantly increases the percentage of defaults. We achieve this
by raising the death rate of entrepreneurs. Figure 9 plots impulse responses when the quarterly death rate of entrepreneurs is fifteen percent.

Figure 9: Technology Shock With $\gamma = 0.85$

![Graphs showing impulse responses for various economic variables like Output, Consumption, Labor, Bankruptcy Threshold, Net Worth, and Price of Capital for BGG and Optimal Fixed Rate contracts.

Note: All impulse responses are plotted as percent deviations from steady state.

In this calibration returns to capital reach six percent, and the default rate rises to twenty percent per quarter. Although this calibration is not plausible, one can see noticeable differences between BGG and the optimal fixed rate contract. Here the fixed rate contract generates less volatility, because lenders take a significant share of the losses. For realistic calibrations, the optimal fixed rate contract and the BGG contract generate similar business cycle dynamics. However, it is important to remember that this similarity results from equilibrium conditions and the chosen calibration, rather than by assumption.

5 Conclusion

This paper contributes to the literature on financial frictions in macroeconomics by adding utility maximizing entrepreneurs to the celebrated financial accelerator models of BGG and CFP. We derive two optimal loan contracts: a fixed rate contract and a variable rate contract.

Under the optimal variable rate loan contract, we find that risk neutral entrepreneurs behave in a “risk averse” way when they maximize expected utility rather than next period returns...
on capital. While risk averse households want to smooth their consumption, risk neutral entrepreneurs desire a loan contract that facilitates high net worth when expected future returns on capital are high. Entrepreneurs thus prefer a variable rate loan contract with a lower repayment rate in states that promise attractive investment returns in the future. Counterintuitively, in a model with standard capital adjustment costs, states with negative technology shocks promise higher future returns to investments. This happens because entrepreneurs acquire cheap capital that will become more expensive in future periods. The opposite is true in booms. As a result, entrepreneurs want a lending rate that co-varies positively with the business cycle, so that the interest on loans is low in recessions and high in booms. This “risk averse” behavior dampens macroeconomic volatility relative to the traditional financial accelerator model and wipes out the impact of risk shocks. Our results are similar to Krishnamurthy (2003), who demonstrated that hedging eliminates the financial accelerator in a three period version of Kiyotaki and Moore (1997).

Preserving the strength of the financial accelerator, especially for risk shocks, requires the introduction of some constraints on hedging. The fixed rate contract does exactly that: it constrains hedging by preventing households from purchasing consumption insurance (via a lending rate that co-varies negatively with the business cycle) and preventing entrepreneurs from hedging against credit risk (via a lending rate that co-varies positively with the business cycle). Consequently, the optimal fixed rate contract restores the accelerator and the impact of risk shocks on the real economy. Without these constraints, hedging by both borrowers and lenders renders the impact of financial shocks negligible.

Our findings have important implications. When there are no constraints on hedging and insurance with respect to aggregate shocks, financial frictions provide little amplification even in the presence of asymmetric information. On the other hand, when there are constraints on hedging and insurance, financial frictions generate significant amplification.
References


A Normalization of the Contract

We can also formulate the optimal contract using normalized variables. For example, one can substitute leverage into the right hand side of equation (5) to obtain

\[ N_{t+s}(j) = N_{t+s-1}(j) \kappa_{t+s-1}(j) R_{t+s}^e \max \{ \omega_{t+s}(j) - \bar{\omega}_{t+s}(j), 0 \} + W_{t+s}^e = N_{t+s-1}(j) R_{t+s}^e(j) + W_{t+s}^e, \]

where \( R_t^e(j) = \kappa_{t-1}(j) R_t^e \max \{ \omega_t(j) - \bar{\omega}_t(j), 0 \} \) is the entrepreneur’s ex post realized return. Iterating this equation backward generates

\[
N_{t+s}(j) = N_{t+s-1}(j) R_{t+s}^e(j) + W_{t+s}^e = N_t(j) \hat{R}_{t,t+s}^e + W_{t+1}^e \hat{R}_{t+1,t+s}^e + \ldots + W_{t+s}^e
\]

\[
= N_t(j) \hat{R}_{t,t+s}^e + \sum_{i=1}^{t} W_t^{e,i} \hat{R}_{t+i,t+s}^e;
\]

where \( \hat{R}_{t,t+s}^e = R_{t+1}^e R_{t+2}^e \ldots R_{t+s}^e \) and \( \hat{R}_{t+s,t+s}^e = 1 \). Intuitively, \( \hat{R}_{t,t+s}^e \) is the entrepreneur’s ex post accumulated rate of return on projects from period \( t \) through period \( t+s \). For example, suppose the entrepreneur invests one dollar in period \( t \) and continues to reinvest his profits in new projects in each subsequent period. In period \( t+s \), the entrepreneur will have accumulated \( \hat{R}_{t,t+s}^e \) from his initial one dollar investment. We can substitute (A.2) into the value function (2) and obtain

\[
V_t^e(j) = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) R_{t+1}^e + \sum_{s=2}^{\infty} \gamma^{s-1} \left( N_t(j) \hat{R}_{t,t+s}^e + \sum_{i=1}^{s-1} W_{t+i}^e \hat{R}_{t+i,t+s}^e \right) \right\}.
\]

(A.3)

Because the entrepreneur will optimize with respect to leverage and the productivity cutoff we want to express (A.3) as a function of the leverage and the productivity cutoff. In the first step, we separate terms to get

\[
V_t^e(j) = (1 - \gamma) N_t(j) \mathbb{E}_t \left\{ \hat{R}_{t,t+1}^e \sum_{s=1}^{\infty} \gamma^{s-1} \hat{R}_{t+1,t+s}^e \right\} + (1 - \gamma) \mathbb{E}_t \left\{ \sum_{s=1}^{\infty} \gamma^{s-1} \left( \sum_{i=1}^{s-1} W_{t+i}^e \hat{R}_{t+i,t+s}^e \right) \right\}.
\]

(A.4)

where we used \( \hat{R}_{t,t+s}^e = \hat{R}_{t+1,t+s}^e \hat{R}_{t+1,t+s}^e \). Net worth enters the value function as a constant multiplicative term and has no effect on the entrepreneur’s choice of leverage \( \kappa_t(j) \) or cutoff \( \bar{\omega}_t(j) \); both enter only through \( \hat{R}_{t,t+1}^e \). Using the law of iterated expectations \( \mathbb{E}_t(x_{t+1}) = \mathbb{E}_t[\mathbb{E}(x_{t+1}|\Omega_{agg,t+1})] \) and the independence of idiosyncratic productivity from aggregate productivity, we can replace the realizations of idiosyncratic productivity with their expectation and
get

\[ V_t^e(j) = (1 - \gamma) N_t(j) \mathbb{E}_t \left[ \tilde{R}_{t,t+1}^{e,agg} \left( \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^{e,agg} \right) + (1 - \gamma) \mathbb{E}_t \left[ \sum_{s=2}^{\infty} \gamma^{s-1} \left( \sum_{t=1}^{s-1} W_t^{e} \tilde{R}_{t+1,t+s}^{e,agg} \right) \right] \right], \]

(A.5)

where \( \tilde{R}_{t,t+s}^{e,agg} = R_{t+1}^{e,agg} R_{t+2}^{e,agg} \ldots R_{t+s}^{e,agg} \) with \( \tilde{R}_{t+s,t+s}^{e,agg} = 1 \), and \( R_{t+1}^{e,agg} = \kappa_t R_{t+1}^k g(\tilde{\omega}_{t+1}) \) is the entrepreneur’s ex post realized rate of return expressed as a function of aggregate productivity and leverage, with \( g(\tilde{\omega}_{t+1}) = \int_{\tilde{\omega}_{t+1}}^{\infty} [\omega - \tilde{\omega}_{t+1}] f(\omega) d\omega. \)

If we divide the lender’s ex post returns in equation (7) by \( N_t(j) \) we get

\[ \left[ \kappa_t(j) - 1 \right] R_{t+1} = \kappa_t(j) R_{t+1}^k h(\tilde{\omega}_{t+1}(j)). \]

(A.6)

Now, if we substitute (A.6) into the Euler equation for the representative household (11), we have

\[ \beta \mathbb{E}_t \left\{ U_{C,t+1} \kappa_t(j) R_{t+1}^k h(\tilde{\omega}_{t+1}(j)) \right\} = \left[ \kappa_t(j) - 1 \right] U_{C,t}. \]

(A.7)

Before looking at the first order conditions to the optimization problem, it is important to notice that all entrepreneurs will choose the same leverage and state-contingent interest rate regardless of their net worth, due to the homotheticity of the problem. Thus, the entrepreneur index \( j \) is omitted below. We use the following notation: \( BGG \) refers to the suboptimal contract of Bernanke, Gertler and Gilchrist (1999), \( CFP \) refers to the suboptimal contract of Carlstrom, Fuerst and Paustian (2012), \( VariableRate \) refers to the optimal variable rate contract, and \( FixedRate \) refers to the optimal fixed rate contract.

**B BGG Contract**

In the BGG contract, the lender is guaranteed a fixed rate of return. In this case, the entrepreneur’s Lagrangian will be

\[ \mathcal{L}^{BGG} = (1 - \gamma) \mathbb{E}_t \left\{ N_t \kappa_t R_{t+1}^k g(\tilde{\omega}_{t+1}) + \lambda_{t+1} \left[ \beta \mathbb{E}_t \left\{ U_{C,t+1} \right\} k_t R_{t+1}^k h(\tilde{\omega}_{t+1}) - (k_t - 1) U_{C,t} \right] \right\}. \]

The entrepreneur’s first order conditions with respect to \( \kappa_t \) and \( \tilde{\omega}_{t+1} \) are:

\[ \frac{\partial \mathcal{L}^{BGG}}{\partial \kappa_t} = N_t \mathbb{E}_t \left[ R_{t+1}^k g(\tilde{\omega}_{t+1}) \right] - \mathbb{E}_t \lambda_{t+1} \frac{U_{C,t}}{\kappa_t} = 0 \]

(B.1)

\[ \frac{\partial \mathcal{L}^{BGG}}{\partial \tilde{\omega}_{t+1}} = N_t \kappa_t R_{t+1}^k g'(\tilde{\omega}_{t+1}) + \lambda_{t+1} \left[ \beta \mathbb{E}_t (U_{C,t+1}) k_t R_{t+1}^k h'(\tilde{\omega}_{t+1}) \right] = 0 \]

(B.2)
Note that we cannot eliminate the Lagrange multiplier from these first order conditions unless we log-linearize. As a result, the BGG contract cannot be expressed in non-linear terms, unlike the CFP and optimal contracts.

C CFP Contract

For the CFP contract, the entrepreneur’s Lagrangian is

\[ \mathcal{L}^{CFP} = (1 - \gamma) \left\{ N_t \kappa_t \mathbb{E}_t \left[ R_{t+1}^k g(\tilde{\omega}_{t+1}) \right] + \lambda_t \left[ \mathbb{E}_t \left( \beta U_{C,t+1} \kappa_t R_{t+1}^k h(\tilde{\omega}_{t+1}) \right) - (k_t - 1) U_{C,t} \right] \right\}. \]

The first order conditions for \( \kappa_t \) and \( \tilde{\omega}_{t+1} \) are

\[ \frac{\partial \mathcal{L}^{CFP}}{\partial \kappa_t} = (1 - \gamma) \left[ N_t \mathbb{E}_t \left\{ R_{t+1}^k g(\tilde{\omega}_{t+1}) \right\} + \lambda_t \left( \mathbb{E}_t \left\{ \beta U_{C,t+1} R_{t+1}^k h(\tilde{\omega}_{t+1}) - U_{C,t} \right\} \right) \right] = 0, \]

\[ \frac{\partial \mathcal{L}^{CFP}}{\partial \tilde{\omega}_{t+1}} = (1 - \gamma) \left[ N_t \kappa_t R_{t+1}^k g'(\tilde{\omega}_{t+1}) + \lambda_t \beta U_{C,t+1} \kappa_t R_{t+1}^k h'(\tilde{\omega}_{t+1}) \right] = 0. \]

Rearranging these first order conditions, solving in terms of \( \lambda_t \) and setting them equal to each other yields

\[ -\beta \kappa_t \frac{U_{C,t+1}}{U_{C,t}} h'(\tilde{\omega}_{t+1}) = \frac{g'(\tilde{\omega}_{t+1})}{\mathbb{E}_t \left\{ R_{t+1}^k g(\tilde{\omega}_{t+1}) \right\}}. \quad \text{(C.1)} \]

D Optimal Variable Rate Contract

The entrepreneur’s Lagrangian has the following form:

\[ \mathcal{L}^{VariableRate} = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \tilde{R}_{t,t+1}^e,agg \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^e,agg + \sum_{s=2}^{\infty} \gamma^{s-1} \left( \sum_{i=1}^{s-1} W_{t+i}^e \tilde{R}_{t+i,t+s}^e,agg \right) + \sum_{i=0}^{\infty} \lambda_{t+i} \left[ \beta U_{c,t+i} \kappa_{t+i} R_{k,t+i} h(\tilde{\omega}_{t+i}) - (\kappa_{t+i} - 1) U_{c,t+i} \right] \right\}. \]

After substituting our expression for \( R_{t,t+1}^e,agg \) into the entrepreneur’s Lagrangian, the first order condition for \( \mathcal{L}_t \) with respect to \( \kappa_t \) is

\[ \frac{\partial \mathcal{L}^{VariableRate}}{\partial \kappa_t} = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \frac{\partial \tilde{R}_{t,t+1}^e,agg}{\partial \kappa_t} \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^e,agg + \lambda_t \left( \beta U_{C,t+1} R_{t+1}^k h(\tilde{\omega}_{t+1}) - U_{C,t} \right) \right\} = 0. \]

Finding the first order condition with respect to \( \tilde{\omega}_{t+1} \) is less straightforward. To simplify the problem, we remove additive terms without \( \tilde{\omega}_{t+1} \) from the entrepreneur’s Lagrangian:

\[ \mathcal{L}^{VariableRate} = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \tilde{R}_{t,t+1}^e,agg \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^e,agg + \lambda_t \left[ \beta U_{C,t+1} \kappa_t R_{k,t+1} h(\tilde{\omega}_{t+1}) \right] \right\}. \quad \text{(D.1)} \]
Then, using the Law of Iterated Expectations we can write (D.1) as:

\[
\bar{L}_{\text{VariableRate}} = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \bar{R}_{t+1}^{e,agg} \left[ \sum_{s=1}^{\infty} \gamma^{s-1} \bar{R}_{t+1+s}^{e,agg} \right] + \lambda_t \left[ \beta U_{C,t+1} k_t R_{t+1}^k h(\bar{\omega}_{t+1}) \right] \right\} \\
= (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \bar{R}_{t+1}^{e,agg} \mathbb{E}_{t+1} \left[ \sum_{s=1}^{\infty} \gamma^{s-1} \bar{R}_{t+1+s}^{e,agg} \right] + \lambda_t \left[ \beta U_{C,t+1} k_t R_{t+1}^k h(\bar{\omega}_{t+1}) \right] \right\}.
\]

We can express (D.2) in its explicit form, weighted by the probability that specific state \( s_{t+1} \) occurs:

\[
\bar{L}_{\text{VariableRate}} = (1 - \gamma) \sum_{s_{t+1}} \left\{ N_t(j) \bar{R}_{t+1}^{e,agg} (s_{t+1}) \mathbb{E}_{t+1} \left[ \sum_{s=1}^{\infty} \gamma^{s-1} \bar{R}_{t+1+s}^{e,agg} \right] \mathbb{P}(s_{t+1}) \right\}.
\]

Finally, to solve for the first order condition, we take the derivative of (D.3) with respect to the state-contingent cutoff:

\[
\frac{\partial \bar{L}_t}{\partial \bar{\omega}_{t+1}} = (1 - \gamma) N_t(j) \frac{\partial \bar{R}_{t+1}^{e,agg}}{\partial \bar{\omega}_{t+1}} \mathbb{E}_{t+1} \left[ \sum_{s=1}^{\infty} \gamma^{s-1} \bar{R}_{t+1+s}^{e,agg} \right] + \lambda_t \left[ \beta U_{C,t+1} k_t R_{t+1}^k h'(\bar{\omega}_{t+1}) \right] = 0.
\]

One can move \( \lambda \) to the right hand side of both equations and then set the two equations equal to each other:

\[
(1 - \gamma) g'(\bar{\omega}_{t+1}) \frac{N_t k_t R_{t+1}^k \mathbb{E}_{t+1} \left\{ \sum_{s=1}^{\infty} \gamma^{s-1} \bar{R}_{t+1+s}^{e,agg} \right\}}{(1 - \gamma) \frac{1}{k_t} N_t k_t R_{t+1}^k g(\bar{\omega}_{t+1}) (1 + \gamma k_{t+1} R_{t+2}^k g(\bar{\omega}_{t+2}) ...)} = \frac{\beta U_{C,t+1} k_t R_{t+1}^k h'(\bar{\omega}_{t+1})}{\beta \mathbb{E}_t \left\{ U_{C,t+1} R_{t+1}^k h(\bar{\omega}_{t+1}) \right\} - U_{C,t}}.
\]

After cancelling out like terms, and using the fact that \( \Psi = 1 + \gamma \mathbb{E}_t \left\{ k_t R_{t+1}^k g(\bar{\omega}_{t+1}) \Psi_{t+1} \right\} \) we multiply and divide the RHS by \( k_t \):

\[
\frac{g'(\bar{\omega}_{t+1}) \mathbb{E}_{t+1} \Psi_{t+1}}{\mathbb{E}_t \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}) \Psi_{t+1} \right\}} = \frac{\beta k_t U_{C,t+1} h'(\bar{\omega}_{t+1})}{\beta \mathbb{E}_t \left\{ U_{C,t+1} k_t R_{t+1}^k h(\bar{\omega}_{t+1}) \right\} - k_t U_{C,t}} = \frac{\beta k_t U_{C,t+1} h'(\bar{\omega}_{t+1})}{(k_t - 1) U_{C,t} - k_t U_{C,t}}.
\]

where we utilized the participation constraint for lenders in the final step. After rearranging and simplifying, we get

\[
- \beta \frac{U_{C,t+1}}{U_{C,t}} h'(\bar{\omega}_{t+1}) = \frac{g'(\bar{\omega}_{t+1}) \mathbb{E}_{t+1} \Psi_{t+1}}{k_t \mathbb{E}_t \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}) \Psi_{t+1} \right\}}.
\]

(D.4)
E Optimal Fixed Rate Contract

Now, suppose that the interest rate charged on the entrepreneur’s loan is no longer state contingent. In other words, the lender and borrower agree on a fixed interest rate in period \( t \) that must be paid in period \( t + 1 \) regardless of the state that occurs. The entrepreneur’s Lagrangian in this case has the following form:

\[
\mathcal{L}^{FixedRate} = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \nabla^{\text{agg}}_{\tilde{R}_{t+1}^{\text{agg}}} \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^{\text{agg}} + \sum_{i=1}^{\infty} \gamma^{s-1} \left( \sum_{i=1}^{\infty} W_{t+i} \tilde{R}_{t+i,t+s}^{\text{agg}} \right) \right\}
\]

where

\[
\tilde{\omega}_{t+1} = \left( \frac{\kappa_t - 1}{\kappa_t} \right) \left( \frac{Z_t}{R_{k,t+1}} \right). \quad (E.2)
\]

After substituting our expression for \( R^{\text{agg}}_{t,t+1} \) into the entrepreneur’s Lagrangian, the first order condition with respect to leverage \( \kappa_t \) is

\[
0 = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \frac{\nabla^{\text{agg}}_{\tilde{R}_{t+1}^{\text{agg}}}}{\nabla_{\kappa_t}} \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^{\text{agg}} + \lambda_t \left[ \beta U_{C,t+1} \left( R_{t+1}^{k} h(\tilde{\omega}_{t+1}) - \frac{Z_t h'(\tilde{\omega}_{t+1})}{\kappa_t} \right) - U_{C,t} \right] \right\} \quad (E.3)
\]

where

\[
\frac{\partial \tilde{R}_{t+1}^{\text{agg}}}{\partial_{\kappa_t}} = \frac{\partial (R_{t+1}^{k} \kappa_t g(\tilde{\omega}_{t+1}))}{\partial_{\kappa_t}} = R_{t+1}^{k} g(\tilde{\omega}_{t+1}) + \frac{Z_t g'(\tilde{\omega}_{t+1})}{\kappa_t}. \quad (E.4)
\]

The entrepreneur’s first order condition with respect to the lending rate \( Z_t \) is

\[
\frac{\partial \mathcal{L}^{FixedRate}}{\partial Z_t} = (1 - \gamma) N_t(j) \mathbb{E}_t \left\{ \frac{\nabla^{\text{agg}}_{\tilde{R}_{t+1}^{\text{agg}}}}{\nabla_{Z_t}} \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^{\text{agg}} + \lambda_t \left[ \beta U_{C,t+1} h'(\tilde{\omega}_{t+1})(\kappa_t - 1) \right] \right\} = 0 \quad (E.5)
\]

where

\[
\frac{\partial \tilde{R}_{t+1}^{\text{agg}}}{\partial_{Z_t}} = \kappa_t R_{t+1}^{k} g'(\tilde{\omega}_{t+1}) \left( \frac{\kappa_t - 1}{\kappa_t R_{k,t+1}} \right) = g'(\tilde{\omega}_{t+1})(\kappa_t - 1). \quad (E.6)
\]

One can move \( \lambda_t \) to the right hand side of (E.3) and (E.5) and then set the two equations equal to each other:

\[
(1 - \gamma) N_t(j) \mathbb{E}_t \left\{ \frac{\partial \tilde{R}_{t+1}^{\text{agg}}}{\partial Z_t} \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^{\text{agg}} \right\} = \beta \mathbb{E}_t \left\{ U_{C,t+1} h'(\tilde{\omega}_{t+1})(\kappa_t - 1) \right\}
\]

\[
(1 - \gamma) \mathbb{E}_t \left\{ N_t(j) \frac{\partial \tilde{R}_{t+1}^{\text{agg}}}{\partial_{\kappa_t}} \sum_{s=1}^{\infty} \gamma^{s-1} \tilde{R}_{t+1,t+s}^{\text{agg}} \right\} = \beta \mathbb{E}_t \left\{ U_{C,t+1} R_{t+1}^{k} h(\tilde{\omega}_{t+1}) - \frac{Z_t h'(\tilde{\omega}_{t+1})}{\kappa_t} \right\} - U_{C,t}
\]

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Now substitute the derivatives from (E.4) and (E.6) into the above expression, and using utilizing the fact that \( \Psi_t = 1 + \gamma \mathbb{E}_t[k_t g(\bar{\omega}_{t+1}) R_{k,t+1} \Psi_{t+1}] \):

\[
\frac{\mathbb{E}_t\left\{ g'(\bar{\omega}_{t+1}) \Psi_{t+1} \right\}}{\mathbb{E}_t\left\{ \left( R_{t+1}^k g(\bar{\omega}_{t+1}) + \frac{Z_t g'(\bar{\omega}_{t+1})}{\kappa_t} \right) \Psi_{t+1} \right\}} = \frac{\mathbb{E}_t\left\{ \beta U_{C,t+1} h'(\bar{\omega}_{t+1}) \right\}}{\mathbb{E}_t\left\{ \beta U_{C,t+1} R_{t+1}^k h(\bar{\omega}_{t+1}) + \frac{\beta U_{C,t+1} Z_t h'(\bar{\omega}_{t+1})}{\kappa_t} - U_{C,t} \right\}}
\]

we can plug in the participation constraint for the denominator of the right hand side to find out

\[
\frac{\mathbb{E}_t\left\{ g'(\bar{\omega}_{t+1}) \Psi_{t+1} \right\}}{\mathbb{E}_t\left\{ \left( R_{t+1}^k g(\bar{\omega}_{t+1}) + \frac{Z_t g'(\bar{\omega}_{t+1})}{\kappa_t} \right) \Psi_{t+1} \right\}} = \frac{\mathbb{E}_t\left\{ \beta U_{C,t+1} h'(\bar{\omega}_{t+1}) \right\}}{\mathbb{E}_t\left\{ \beta U_{C,t+1} Z_t h'(\bar{\omega}_{t+1}) - U_{C,t} \right\}}
\]

(E.7)

Now we divide denominator by the numerator to find out

\[
\frac{1}{\mathbb{E}_t\left\{ g(\bar{\omega}_{t+1}) R_{t+1}^k \Psi_{t+1} \right\} + \frac{Z_t}{k_t}} = \frac{1}{\mathbb{E}_t\left\{ g'(\bar{\omega}_{t+1}) \Psi_{t+1} \right\} - \beta k_t U_{C,t} h'(\bar{\omega}_{t+1})}
\]

(E.9)

then it must hold

\[
-\mathbb{E}_t\left( \beta \frac{U_{c,t+1}}{U_{c,t}} h'(\bar{\omega}_{t+1}) \right) = \frac{\mathbb{E}_t\left\{ g'(\bar{\omega}_{t+1}) \Psi_{t+1} \right\}}{\kappa_t \mathbb{E}_t\left\{ R_{t+1}^k g(\bar{\omega}_{t+1}) \Psi_{t+1} \right\}}
\]

(E.10)
F Complete Log-Linearized Model

F.1 General Equilibrium Equations, Identical For All Contracts

The log-linearized household first order conditions for deposits (25), labor supply (26) and real money balances (28) are given by:

\[-\sigma \left( \mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t \right) + \mathbb{E}_t \hat{R}_{t+1} = 0, \]  
\[\hat{W}_t - \sigma \hat{C}_t = \eta \hat{H}_t, \]  
\[-\sigma \left( \beta \mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t \right) = (1 - \beta) \hat{m}_t + \beta \mathbb{E}_t \hat{\Pi}_{t+1}. \]  

(F.1)

(F.2)

(F.3)

Using (36), (37) and (40), we can write the log-linearized New Keynesian Phillips Curve:

\[\hat{\Pi}_t = -\frac{(1 - \theta)(1 - \theta \beta)}{\theta} \hat{X}_t + \beta \mathbb{E}_t \hat{\Pi}_{t+1}. \]  

(F.4)

The real wage for households (43) and entrepreneurs (44) are, respectively

\[\hat{W}_t = \hat{Y}_t - \hat{H}_t - \hat{X}_t, \]  
\[\hat{W}_e = \hat{Y}_t - \hat{X}_t. \]  

(F.5)

(F.6)

Log-linearization of the motion of capital (45), the price of capital (47), and capital returns (48) gives:

\[\hat{K}_t = \delta \hat{I}_t + (1 - \delta) \hat{K}_{t-1}, \]  
\[\hat{Q}_t = \delta \phi_K (\hat{I}_t - \hat{K}_{t-1}), \]  
\[\hat{R}_{t+1}^k = (1 - \epsilon)(\hat{Y}_{t+1} - \hat{K}_t - \hat{X}_{t+1}) + \epsilon \hat{Q}_{t+1} - \hat{Q}_t. \]  

(F.7)

(F.8)

(F.9)

where \(\epsilon = 1 - \frac{\gamma \kappa}{\kappa + (1 - \delta)}\). Then, using (9), we can write the log-linearized equation for leverage \(\kappa\):

\[\hat{\kappa}_t = \hat{Q}_t + \hat{K}_t - \hat{N}_t \]  

(F.10)

The dynamics of aggregate net worth, defined in (49), can be expressed in log-linear form as:

\[\hat{N}_{t+1} = \epsilon_N (\hat{N}_t + \hat{\kappa}_t + \hat{R}_{t+1}^k + \frac{g'(\bar{\omega})}{g(\bar{\omega})} \hat{\omega}_{t+1}) + (1 - \epsilon_N) \hat{W}^e_{t+1}, \]  

(F.11)

where \(\epsilon_N = \gamma \kappa R^k g(\bar{\omega}).\)

From (3), we have the consumption dynamics for entrepreneurs:

\[\hat{C}^e_t = \hat{N}_{t-1} + \hat{\kappa}_{t-1} + \hat{R}_t^k + \frac{g'(\bar{\omega})}{g(\bar{\omega})} \hat{\omega}_t. \]  

(F.12)
Goods market clearing (50), the Taylor rule for monetary policy (51) and the Fisher equation for the nominal interest rate (27), are respectively:

\[
Y_t = \bar{C}_t + I_t + \bar{G}_t + C^e \bar{C}^e_t + \mu G R^k K \left( \frac{G'(\bar{\omega})}{G(\bar{\omega})} \bar{\omega}_{t+1} + \hat{R}^k_t + \hat{Q}_{t-1} + \hat{K}_{t-1} \right), \tag{F.13}
\]

\[
\hat{R}^n_t = \rho \hat{R}^n_{t-1} + \xi \hat{\pi}_{t-1} + \epsilon R^n_t, \tag{F.14}
\]

\[
\hat{R}^n_t = \hat{R}_t + \hat{\Pi}_t. \tag{F.15}
\]

F.2 BGG

The lender’s participation constraint in BGG follows from log-linearization of (6) and (7):

\[
- \sigma \left( E_t \hat{\dot{C}}_{t+1} - \hat{C}_t \right) + \hat{R}^k_{t+1} + \frac{h'(\bar{\omega})}{h(\bar{\omega})} \bar{\omega}_{t+1} = \frac{1}{\kappa - 1} \hat{\kappa}_t. \tag{F.16}
\]

The log-linearized first order conditions with respect to capital (13) and the bankruptcy threshold \( \bar{\omega} (14) \) are, respectively:

\[
\hat{N}_t + \hat{E}_t \hat{R}^k_{t+1} + \frac{g'(\bar{\omega})}{g(\bar{\omega})} \bar{\omega} E_t \omega_{t+1} = E_t \hat{\dot{\lambda}}_{t+1} - \sigma \hat{C}_t - \hat{\kappa}_t, \tag{F.17}
\]

\[
\hat{N}_t + \frac{g''}{g'} \bar{\omega}_{t+1} = \hat{\lambda}_{t+1} - \sigma \hat{E}_t \hat{C}_{t+1} + \frac{h''}{h'} \bar{\omega}_{t+1}. \tag{F.18}
\]

F.3 CFP

The lender’s participation constraint in CFP follows from log-linearization of (6) and (11):

\[
- \sigma \left( \hat{E}_t \hat{\dot{C}}_{t+1} - \sigma \hat{C}_t \right) + \hat{E}_t \hat{R}^k_{t+1} + \frac{h'(\bar{\omega})}{h(\bar{\omega})} \bar{\omega} E_t \omega_{t+1} = \frac{1}{\kappa - 1} \hat{\kappa}_t. \tag{F.19}
\]

The first order conditions with respect to capital (13) and the bankruptcy threshold \( \bar{\omega} \) are combined into one equation (15). The log-linearized version of this equation is:

\[
- \sigma (\hat{\dot{C}}_{t+1} - \hat{C}_t) + \frac{h''}{h'} \hat{\omega}_{t+1} = \frac{g''}{g'} \bar{\omega} E_t \omega_{t+1} - \hat{\kappa}_t - E_t \hat{R}^k_{t+1} - \frac{g'(\bar{\omega})}{g(\bar{\omega})} \bar{\omega} E_t \omega_{t+1} + \hat{\Psi}_{t+1} - E_t \hat{\Psi}_{t+1}. \tag{F.20}
\]

F.4 Optimal Variable Rate Contract

The lender’s participation constraint in the optimal variable rate contract is identical to CFP, given by (F.19). Log-linearization of the first order condition (16) and its counterpart (17), yield

\[
- \sigma (\hat{\dot{C}}_{t+1} - \hat{C}_t) + \frac{h''}{h'} \hat{\omega}_{t+1} = \frac{g''}{g'} \bar{\omega} E_t \omega_{t+1} - \hat{\kappa}_t - E_t \hat{R}^k_{t+1} - \frac{g'(\bar{\omega})}{g(\bar{\omega})} \bar{\omega} E_t \omega_{t+1} + \hat{\Psi}_{t+1} - E_t \hat{\Psi}_{t+1} \tag{F.21}
\]
where
\[
\hat{\Psi}_t = \epsilon_N \left( \hat{\kappa}_t + \frac{g'(\bar{\omega})}{g(\bar{\omega})} \bar{\omega} \hat{\omega}_{t+1} + \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{\Psi}_{t+1} \right).
\] (F.22)

**F.5 Optimal Fixed Rate Contract**

The lender’s participation constraint in the optimal fixed rate contract is identical to CFP and the optimal variable rate contract, given by (F.19). The log-linearized version of the first order condition (21) is
\[
\mathbb{E}_t \left[ \hat{R}^k_{t+1} + \left( \frac{g'(\bar{\omega})}{g(\bar{\omega})} - \frac{g''(\bar{\omega})}{g'(\bar{\omega})} + \frac{h''(\bar{\omega})}{h'(\bar{\omega})} \right) \bar{\omega} \hat{\omega}_{t+1} \right] = \mathbb{E}_t \left[ (1 - \kappa) \left( \hat{R}^k_{t+1} + \frac{h'(\bar{\omega})}{h(\bar{\omega})} \bar{\omega} \hat{\omega}_{t+1} \right) + \kappa \sigma (\hat{C}_{t+1} - \hat{C}_t) \right].
\] (F.23)

We also have an additional condition for the fixed rate contract, which restricts the dynamics of the bankruptcy threshold:
\[
\mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{\omega}_{t+1} = \hat{Z}_t + \frac{1}{\kappa - 1} \hat{\kappa}_t.
\] (F.24)
G The Social Planner Equilibrium

The social planner’s objective is to maximize a weighted sum of household and entrepreneur utility. The welfare weights are \( \Omega \) for households and \( 1 - \Omega \) for entrepreneurs. We normalize these weights such that the welfare weight for households is one and the welfare weight for entrepreneurs is \( \epsilon = (1 - \Omega)/\Omega \).

The social planner’s problem is illustrated graphically in Figure 10 for the baseline calibration described in Section 4.1. Given the welfare weights, the planner maximizes the share of gross capital returns that accrue to borrowers relative to lenders by choosing the bankruptcy threshold \( \bar{\omega} \), taking into account the share that is lost to monitoring costs. If the social planner ignores the plight of entrepreneurs and only seeks to maximize household utility, what we call the “Main Street” social planner, the optimal value of \( \bar{\omega} = 0.56 \). In this case, we can see in Figure 10 that the share of gross capital returns going to entrepreneurs consumption (\( C^e \)) will be quite small. On the other hand, if the social planner cares only about entrepreneurs utility, what we call the “Wall Street” social planner, the optimal bankruptcy threshold will be \( \bar{\omega} = 0 \) and entrepreneurs will receive the highest possible level of consumption as a share of capital returns.

More formally, the social planner’s optimization problem is characterized by the following
value function

\[ V^S_t(j) = \epsilon(1-\gamma)\sum_{s=0}^{\infty} \gamma^s \left[ Q_{t+s-1}K_{t+s-1}R_{t+s}^k g(\bar{\omega}_{t+s}) \right] + \sum_{s=0}^{\infty} \beta^s U(C_{t+s}, H_{t+s}), \quad \text{(G.1)} \]

and the following five constraints: the household labor supply condition (26), gross returns to capital (48), capital accumulation (45), the price of capital (47) and goods market clearing (50). The first order conditions are found in Appendix.

If main street social planner would try to choose cutoff minimizing monitoring costs and entrepreneurial consumption, which makes this cutoff and defaults constant over time, then social planner with weighted utilities of entrepreneurs and households would slightly adjust cutoff, depending on the marginal utility of the household, given that marginal utility of entrepreneurs is constant. However, this fluctuations are not significant.

Does decentralized dynamic optimal contract coincide with a social planner equilibrium? Intuitively, we can think about returns on equity for entrepreneurs as profits from having markups. But entrepreneurs maximize their discounted financial flows from markups, having different utility function from the social planner, which generate serious differences with a social planner.

Zero capital adjustment costs provide a simple example demonstrating the difference between dynamic optimal contract and social planner allocation should contain high volatility of defaults and productivity cutoff with respect to technology shocks.

The social planner’s Lagrangian, with a positive weight for entrepreneurs utility ($\epsilon > 0$), is:

\[ \mathcal{L} = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \gamma^s \epsilon(1-\gamma)Q_{t+s-1}K_{t+s-1}R_{t+s}^k g(\bar{\omega}_{t+s}) + \sum_{s=0}^{\infty} \beta^s U(C_{t+s}, H_{t+s}) \\
+ \Lambda_{1,t+s} \left\{ K_{t+s} - I_{t+s} - (1-\delta)K_{t-1} + \frac{\phi K}{2} \left[ \frac{I_{t+s}}{K_{t+s-1}} - \delta \right]^2 K_{t+s-1} \right\} \\
+ \Lambda_{2,t+s} \left\{ \frac{1}{Q_{t+s}} - \left[ 1 - \phi K \left( \frac{I_{t+s}}{K_{t+s-1}} - \delta \right) \right] \right\} \\
+ \Lambda_{3,t+s} \left\{ A_{t+s}K_{t+s-1}^{\alpha}H_{t+s}^{\Omega(1-\alpha)} - C_{t+s} - I_{t+s} - M(\bar{\omega}_{t+s})[\alpha A_{t+s}K_{t+s-1}^{\alpha}H_{t+s}^{\Omega(1-\alpha)} + (1-\delta)K_{t+s-1}Q_t] \right\} \right\} \right\} \quad \text{(G.2)} \]
where \( M(\bar{\omega}_{t+1}) = (1 - \gamma)g(\bar{\omega}_{t+1}) + \mu G(\bar{\omega}_{t+1}) \). The FOC with respect to capital \( K_t \) is:

\[
0 = \Lambda_{1,t} + E_t \left\{ \gamma \epsilon (1 - \gamma) Q_t R_{t+1}^k g(\bar{\omega}_{t+1}) \right. \\
+ \beta \Lambda_{1,t+1} \left[-(1 - \delta) - \phi_K \left( \frac{I_{t+1}}{K_t} - \delta \right) \right. \\
- \left. \beta \Lambda_{2,t+1} \phi_K I_{t+1} \right] \\
+ \beta \Lambda_{3,t+1} \left\{ \alpha A_{t+1} K_t^{\alpha - 1} H_{t+1}^{\Omega(1-\alpha)} - M(\bar{\omega}_t) \right\} \\
\left( G.3 \right)
\]

The FOC with respect to the capital price \( Q_t \) is:

\[
0 = \epsilon (1 - \gamma) K_{t-1} R_{t-1}^k g(\bar{\omega}_t) - \frac{\Lambda_{2,t}}{Q_t^3} + \Lambda_{3,t} (1 - \delta) K_{t-1} M(\bar{\omega}_t) \\
\left( G.4 \right)
\]

Rolling one period ahead, this can be expressed as

\[
\Lambda_{2,t+1} = \Lambda_{3,t+1} (1 - \delta) Q_{t+1}^2 K_t M(\bar{\omega}_{t+1}) + \epsilon (1 - \gamma) Q_{t+1}^2 K_t R_{t+1}^k g(\bar{\omega}_{t+1}). \\
\left( G.5 \right)
\]

The FOC with respect to investment \( I_t \) is:

\[
0 = \Lambda_{1,t} \left[-1 + \phi_K \left( \frac{I_t}{K_{t-1}} - \delta \right) \right] + \Lambda_{2,t} \left[ \phi_K \frac{1}{K_{t-1}} \right] - \Lambda_{3,t} \\
\left( G.7 \right)
\]

and if we roll one period forward we have

\[
0 = \Lambda_{1,t+1} \left[-1 + \phi_K \left( \frac{I_{t+1}}{K_t} - \delta \right) \right] + \Lambda_{2,t+1} \left[ \phi_K K_t \right] - \Lambda_{3,t+1} \\
\left( G.8 \right)
\]

We can rewrite (G.8), substituting in (G.6), as:

\[
\frac{\Lambda_{1,t+1}}{Q_{t+1}} = \Lambda_{2,t+1} \left( \frac{\phi_K}{K_t} \right) - \Lambda_{3,t+1} \\
\Lambda_{1,t+1} = \Lambda_{3,t+1} \left[ \phi_K (1 - \delta) M(\bar{\omega}_{t+1}) Q_{t+1}^3 - 1 \right] + \phi_K \epsilon (1 - \gamma) Q_{t+1}^3 R_{t+1}^k g(\bar{\omega}_{t+1}) \\
\left( G.9 \right)
\]

The FOC with respect to household consumption \( C_t \) is:

\[
0 = U_{C,t} - \Lambda_{3,t} \\
\left( G.10 \right)
\]
The FOC with respect to household labor $H_t$ is:

$$0 = U_{H,t} + \Lambda_{3,t} \left[ \Omega (1 - \alpha) A_t K_{t-1}^\alpha H_{t-1}^{\Omega (1-\alpha)} - M(\bar{\omega}_t) \alpha \Omega (1 - \alpha) A_t K_{t-1}^\alpha H_t^{\Omega (1-\alpha)} \right]$$  \hspace{1cm} (G.11)

The FOC for $\bar{\omega}_t$ is:

$$0 = \epsilon (1 - \gamma) Q_{t-1} K_{t-1} R_t^k g'(\bar{\omega}_t) - \Lambda_{3,t} \left[ (1 - \gamma) g'(\bar{\omega}_t) + \mu G'(\bar{\omega}_t) \right] \left[ \alpha A_t K_{t-1}^\alpha H_t^{\Omega (1-\alpha)} + (1 - \delta) K_{t-1} Q_t \right] = Q_{t-1} K_{t-1} R_t^k$$

which can be rewritten

$$\epsilon (1 - \gamma) g'(\bar{\omega}_t) = U_{C,t} \left[ (1 - \gamma) g'(\bar{\omega}_t) + \mu G'(\bar{\omega}_t) \right]$$  \hspace{1cm} (G.12)

We can also rewrite the FOC for capital:

$$0 = \Lambda_{1,t} + \mathbb{E}_t \left\{ \gamma \epsilon (1 - \gamma) Q_t R_{t+1}^k g'(\bar{\omega}_{t+1}) + \beta \Lambda_{1,t+1} \left[ -(1 - \delta) - \phi_K \left( \frac{I_{t+1}}{K_t} - \delta \right) \frac{I_{t+1}}{K_t} + \phi_K \left( \frac{I_{t+1}}{K_t} - \delta \right)^2 \right] \\
- \beta U_{C,t+1} (1 - \delta) M(\bar{\omega}_t) Q_t^2 \phi_K \frac{I_{t+1}}{K_t} \\
+ \beta U_{C,t+1} \left\{ \alpha A_{t+1} K_{t+1}^\alpha - M(\bar{\omega}_{t+1}) \left[ \alpha^2 A_{t+1} K_{t+1}^{\alpha-1} H_{t+1}^{\Omega (1-\alpha)} + (1 - \delta) Q_{t+1} \right] \right\} \right\}$$  \hspace{1cm} (G.13)