A stochastic model for the formation of channel networks in tidal marshes

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[1] Salt marshes are often dissected by a network of channels formed by tidal oscillations and related water fluxes. In this note we present a stochastic model able to simulate the formation and evolution of tidal networks in salt marshes. The model is based on a simplification of the shallow water equations that enables the determination of the water surface gradients on the marsh platform and related bottom shear stresses. The governing equations are then solved with a random walk algorithm in an enclosed domain representing the marsh platform and its physical boundaries. Model results show how the development of the network depends on the tidal forcing, the critical shear stress for erosion, as well as the local heterogeneities in vegetation and sediment substrate. *INDEX TERMS:* 1815 Hydrology: Erosion and sedimentation; 3020 Marine Geology and Geophysics: Littoral processes; 3210 Mathematical Geophysics: Modeling; 4235 Oceanography: General: Estuarine processes; 4560 Oceanography: Physical: Surface waves and tides (1255). **Citation:** Fagherazzi, S., and T. Sun (2004), A stochastic model for the formation of channel networks in tidal marshes, *Geophys. Res. Lett.*, *31*, LXXXXX, doi:10.1029/2004GL020965.

1. Introduction

[2] Tidal channels are ubiquitous in saltmarshes and are of critical importance for the exchange of water, sediments, and nutrients, between the marsh and the ocean. Tidal channels form because tidal oscillations move large volumes of water on the marsh surface. Since saltmarshes are located in the intertidal zone, tidal fluxes are often constrained in few tens of centimeters of water, producing elevated shear stresses at the marsh bottom that lead to erosion and local incision. Once the channel is initiated by local scour, tidal fluxes concentrate in it thus increasing the channel dimensions. In a short period of time the channel becomes the only avenue for water exchanges between the ocean and the marsh platform [*Fagherazzi and Furbish*, 2001]. The transfer of momentum between marsh surface and channel [see *Fagherazzi et al.*, 2003] augments the channel discharge that cyclically scours the channel bottom thus preventing infilling. As a result, tidal channels systematically dissect marsh platforms, and often create a dendritic network somehow resembling the fluvial network of terrestrial watersheds. Tidal networks have basic geometric properties common to other natural networks [*Fagherazzi et al.*, 1999] but lack of scale invariance characteristics that are peculiar of fluvial patterns [*Rinaldo et al.*, 1999a, 1999b]. The absence of scale invariance can be ascribed to the numerous physical and biological processes that shape the tidal channels and act at the same temporal and spatial scales. These intertwined processes hinder the tendency of the system to develop a self-organized configuration [*Rinaldo et al.*, 1999b]. Tidal channels are also highly sinuous, with meanders that are geometrically similar to the meanders developing in rivers [*Marani et al.*, 2002]. However, the bidirectionality of the discharge in tidal channels often implies a meander evolution that departs from terrestrial meanders [*Fagherazzi et al.*, 2004].

[3] Recent studies have determined the water directions on the marsh platform and the delineation of the drainage area of each tidal channel [*Rinaldo et al.*, 1999a], thus enabling a theoretical characterization of drainage density in salt marshes [*Marani et al.*, 2003]. Based on the watershed delineation for tidal channels reported by *Rinaldo et al.* [1999a], we have built a numerical model able to simulate the development of tidal networks in saltmarshes. The model lays the foundations for a systematic study of the role of physical and biological processes on network formation and characteristics.

2. The Model

[4] The hydrodynamic model is based on the simplification of the shallow water equations introduced by *Rinaldo et al.* [1999a]. Given the low velocities of water on the marsh platform, only the pressure term and the friction term are not negligible in the momentum equations, leading to the two following simplified expressions:

$$u = -\frac{h}{\Lambda}\frac{\partial \eta}{\partial x} \qquad v = -\frac{h}{\Lambda}\frac{\partial \eta}{\partial y} \tag{1}$$

Where u and v are the water velocities on the marsh surface in the x- and y- directions respectively, h is the average water depth, η is the local water elevation above the average water level in the marsh, and Λ is a friction parameter derived by a linearization of the quadratic friction term. Substitution of equation (1) in the continuity equation leads to the following Poisson equation that allows the determination of the water surface elevation in the marsh area between channels [*Rinaldo et al.*, 1999a]:

$$\nabla^2 \eta = k \qquad \text{with}: \ k = \frac{\Lambda}{h^2}\frac{dh}{dt} \tag{2}$$

where dh/dt is the variation of average water depth on the marsh as a function of time, and can be set equal to the tidal forcing. We basically assume that, to a first order of approximation, the water surface on the marsh is flat with elevation h above m.s.l. equal to the tidal elevation at the

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marsh boundaries; η is then the difference between the real water elevation at each point of the marsh and the average elevation h.

[5] Equation (2) is valid for a flat marsh surface of limited extension, and needs to be coupled to the water elevation in the tidal channels to fully describe the distribution of water in time in a salt marsh. To close the problem *Rinaldo et al.* [1999a] noticed that the propagation of the tide in the channels is much faster than the propagation on the marsh surface, so that the water level in the channels can be considered flat ($\eta = 0$) to a first approximation.

[6] Equation (2) is particularly suitable to model the formation of tidal channels in a salt marsh. In fact, once the water elevation is known at each point of the marsh surface, it is possible to determine the bottom shear stresses through the equations:

$$\tau_x = -gh\frac{\partial\eta}{\partial x}; \quad \tau_y = -gh\frac{\partial\eta}{\partial y} \tag{3}$$

and then scour (i.e., transform in channel) each point of the marsh in which the shear stress is higher than the critical shear stress for erosion. In reality the water elevation changes during the tide (the term dh/dt in equation (2)), and the shear stress varies accordingly. To simplify the problem we suppose that the channel incision takes place during a short period of time after the tide starts receding and dh/dt is maximum. Under these conditions, it is reasonable to assume that both h and dh/dt are constant, and equation (2) becomes a Poisson equation with a constant source term, which can be resolved once suitable boundary conditions are specified.

[7] In this model it is assumed that the growth of tidal channel networks in marshes is dominated by the headward extension of the channels [*Pestrong*, 1965]. *Knighton et al.* [1992] observed that channel development takes place in marshes when a diffuse flow over the surface of the marsh becomes concentrated through localized scour at the head of the channel. Extension by headward erosion may proceed rapidly, and it has been observed that, under some conditions, first order channels can extend their lengths by more than 200 m in 130 years [*Collins et al.*, 1986].

[8] Herein the rate of channel headward extension ζ is set proportional to the excess shear stress at the channel head:

$$\zeta = \beta(\tau - \tau_{cr}) \tag{4}$$

where τ_{cr} is the critical shear stress.

[9] In order to solve equation (2) and calculate the bottom shear stress with (3) we use a random walk algorithm, similar to that used in the well studied Diffusion-Limited Aggregation (DLA) model [*Witten and Sander*, 1981]. This model has already been utilized to study a wide variety of phenomena in which a randomly branched pattern grows under the control of a scalar field that can be described by a Laplace equation. In the DLA model particles are added one at a time to a growing cluster of particles. In each stage of the simulation, a particle is released from a randomly chosen site on a distant boundary that encloses the growing cluster (the boundary of the computational domain) and the particle then performs a random walk in the domain. If the particle contacts the growing aggregate, it is stopped in the position of contact and incorporated into the aggregate. A new stage of the simulation is then initiated by the release of a new particle from the outer boundary. The wide range of applications of the DLA model arises because the probability of finding the random walker in a small region in the computational domain is identical to the integral of the Laplacian field, provided that the rules used to launch and terminate the random walks correspond to the boundary conditions that determine the Laplacian field [*Kadanoff*, 2000].

[10] Random walk simulations can be also used to solve equation (2). The probability $p_{i,j}$ that the site (i, j) of the domain will be occupied by a random walker becomes:

$$\begin{cases} p_{i,j} = \frac{1}{4}\left(p_{i-1,j} + p_{i,j-1} + p_{i+1,j} + p_{i,j+1}\right) + k \\ \quad \text{if} \quad \frac{1}{4}\left(p_{i-1,j} + p_{i,j-1} + p_{i+1,j} + p_{i,j+1}\right) + k < 1 \\ p_{i,j} = 1 \quad \text{if} \quad \frac{1}{4}\left(p_{i-1,j} + p_{i,j-1} + p_{i+1,j} + p_{i,j+1}\right) + k \geq 1 \end{cases} \tag{5}$$

where $p_{i-1,j}$, $p_{i,j-1}$, $p_{i+1,j}$, $p_{i,j+1}$ are the probabilities that the four nearest neighbor sites are occupied. This indicates that a scalar field obeying equation (2) can be sampled by random walkers if the walkers not only enter lattice sites from nearest neighbor sites, but are also injected directly into the lattice during the simulation. Thus on a mathematical basis the utilization of random walkers with a probability distribution equal to equation (5) in the domain is equivalent to resolving the partial differential equation (2). We can also intuitively link the number of random walkers that hit the tidal network to the velocity of water during the ebb peak. The locations impacted by many walkers correspond to the channel banks subject to high water velocities during ebb flow.

[11] To simulate (2) sites in the computational domain (the region in which equation (2) is to be solved) are selected at random, with equal probability, and random walks are initiated at all of the selected sites. When a random walk exits the computational domain through the channel boundaries or the seaward boundary of the marsh, the random walk is terminated to satisfy the boundary condition $\eta(\mathbf{x}) = 0$ where $\mathbf{x}$ is the position on the domain perimeter. If the walker enters a site that belongs to the landward boundary of the marsh, it is reflected to represent the absence of flow into or out of the marsh along this boundary. As the number of randomly injected walkers is increased, the scalar field is sampled more completely. In the $N \to \infty$ limit the water elevation η above the mean water elevation can be calculated as:

$$\eta(i,j) = \frac{n(i,j)N_\Omega k}{N} \tag{6}$$

where $n(i, j)$ is the number of times that the lattice site at position (i, j) has been visited by a random walker, and N_Ω is the number of lattice sites in the region representing the unchannelized floodplain, and N is the number of random walkers utilized in the simulation. Since $\eta = 0$ on all the sites that represent the channel network, the gradient at the perimeter of the computational domain in the direction perpendicular to the channel boundary (pointing away from the channel) is equal to $\nabla\eta_{i,j} = \eta_{i,j}$.

[12] After the scalar field has been calculated (or adequately sampled in the simulations) the velocity of the boundary of the computational domain, which corresponds to the boundary of the channels or the seaward edge of the

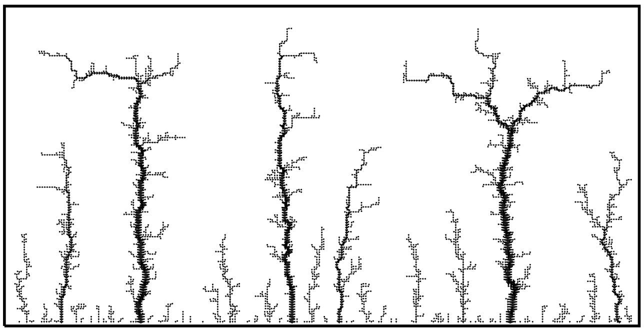

Figure 1. Simulation of tidal networks development in a lattice domain 256 × 128. The upper, left, and right boundaries are considered land whereas the lower boundary separates the salt marsh from the ocean. The simulation parameters are $c = 0.00225$, $S_c = 2.0$, $k = 0.33$.

marsh, is calculated; and then the boundary is moved. Equation (4) indicates that the velocity of the boundary, which represents the rate of channel extension, is proportional to:

$$V(\mathbf{x}) \propto \nabla\eta(\mathbf{x}) - c \quad \text{with} \quad c = \frac{\tau_{cr}}{gh} \tag{7}$$

where $V(\mathbf{x})$ is the velocity of the boundary at position $\mathbf{x}$ in the direction normal to the channel boundary, and c is a constant that depends on the critical shear stress for erosion.

[13] In the $c = 0$ case, the channel incision can be done simply by filling the perimeter site at position (i,j) as soon as it is contacted by a random walker. This would ensure that the growth probability at site (i, j) is proportional to $\nabla\eta(i, j)$, and the total area of channel incision would be equal to the number of random walkers used in the simulation. However, this algorithm is subject to uncontrolled growth noise given the relatively low number of random walkers. For $c > 0$ the growth of the tidal marsh channels can be instead simulated using a model in which a "score" is kept for each of the perimeter sites. The score can be interpreted as the increasing stress exerted by the flow on the marsh surface. Each time a site is reached by a random walker, its score is increased by 1, and the scores of all the other perimeter sites are decreased by c. A site is filled when its score reaches the value S_c. It is possible to show that the value of S_c is proportional to the inverse of the value β in equation (4), and that the total number of random walkers used in the model is proportional to the elapsed time.

[14] Based on the mapping described above, the model algorithm for tidal channels can be summarized as follows:

[15] 1. Within the computation domain, a lattice site at position (i, j) is chosen randomly with a probability proportional to k defined in equation (2) and a random walker is launched from that site.

[16] 2. If the random walker steps into a site representing a channel or the sea the random walk is terminated, and the score at the site that was last visited by the random walker is increased by 1. The random walker is returned to its last position if it steps over a landward boundary.

[17] 3. The scores of all the channel and seaward boundary sites are decreased by c.

[18] 4. If the accumulated score of a site is greater than S_c, the site becomes part of the channel network.

[19] 5. Steps 1 to 4 are then repeated several times during the simulation.

[20] In the model we also assign a finite width to the developing channels. The channel discharge is proportional to the number of random walkers moving through the channel in a unit amount of time ($q \propto n\Delta t$). Finally, the channel width is assumed to be proportional to the channel discharge raised to a specific power ($w \propto q^{0.77}$), accordingly to the hydraulic geometry studies of *Myrick and Leopold* [1963]. In the current implementation, the width of the channels does not affect the dynamics of the model, but it is only used for visualization purposes.

3. Results and Discussion

[21] Although it is possible to relate the model parameters to the specific characteristics of a saltmarsh, the actual calibration of the model is beyond the scope of this paper.

[22] In Figure 1 we show a simulated tidal channel network in a lattice of size 256 × 128. The lower side of the lattice represents the boundary between the salt marsh and the ocean, whereas at all the other three sides we impose a no flux boundary condition. In the simulation $c = 0.00225$, $S_c = 2.0$, $k = 0.33$.

[23] As it can be seen from Figure 1, the channel network is composed of few major channels and several small channels. This is in agreement with real tidal networks, which have a narrow distribution of channel sizes when compared to fluvial networks [*Fagherazzi et al.*, 1999; *Rinaldo et al.*, 1999a].

[24] Another important characteristic of the simulated tidal network is the presence of large unchannelized areas between the channels. In other words, the network does not fill the entire salt marsh area. In the simulations, after a period of rapid development, the network reaches an equilibrium state and then the channel incision ceases. This behavior is similar to the evolution of fluvial drainage basins where diffusive hillslope processes eventually balance the development of fluvial incisions [*Sun et al.*, 1994]. In a developed tidal network the balance is instead between the friction dominated sheet flow in densely vegetated areas and the open channel flow in the creeks.

[25] An important parameter in the model is the critical shear stress associated with the erosion of saltmarsh sediments and with the headward incision of the channels. Figure 2 shows the development of tidal networks obtained

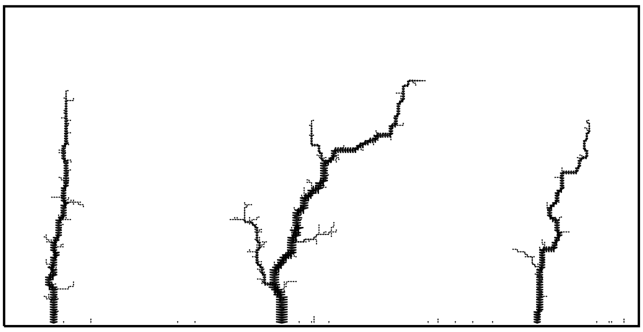

Figure 2. Tidal networks obtained with a higher value of the critical shear stress for erosion of marsh sediments ($c = 0.00675$). A higher critical stress produces few branches, with a decrease in drainage density.

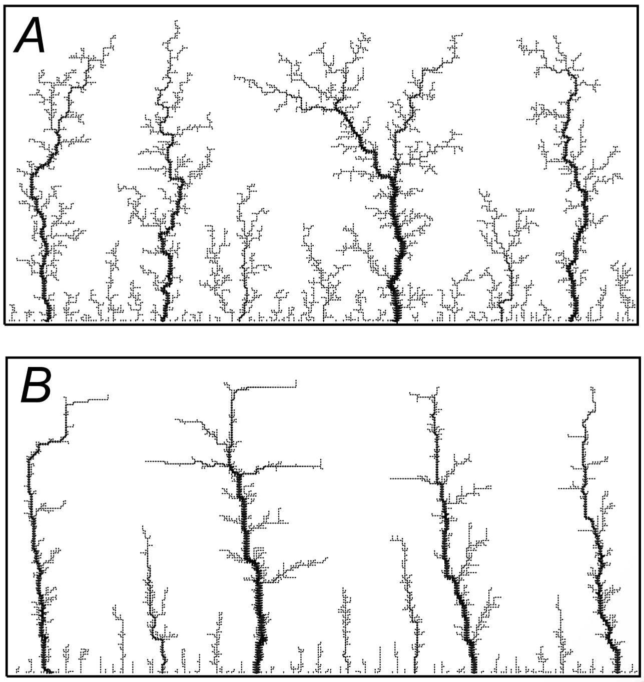

Figure 3. Tidal networks obtained with two different values of channel extension S_c ($S_c = 0$ in Figure 3a, $S_c = 8$ in Figure 3b). With a fast channel incision the channel network is more branched and irregular (Figure 3a) whereas a slow channel incision produces few straight channels (Figure 3b).

with a higher value of the critical shear stress ($c = 0.00675$). A higher critical shear stress leads to fewer branches, and the characteristic spacing between the channels increases, thus indicating that the drainage density is lower in areas with more resistant substrate.

[26] In Figure 3 the tidal channel networks obtained from simulations using different values of Sc are reported ($S_c = 0$ in Figure 3a, $S_c = 8$ in Figure 3b). Different values of S_c affect the time scale of channel incision ($S_c = 1/\beta$ in equation (4)). With a fast channel incision the channel network is more branched and irregular (Figure 3a) whereas a slow channel incision produces few straight channels (Figure 3b). This is because a fast channel incision favors the erosion of the marsh platform in different locations, with the formation of several small channels. Instead, a slow channel incision concentrates the flow in few large channels that capture most of the tidal prism and prevent the formation of the small network structure.

[27] Channel incision in salt marshes does not occur continuously, but is linked to the failure of large blocks at the channel banks [*Gabet*, 1998]. The same behavior has also been documented and studied for the banks of terrestrial rivers [*Darby and Thorne*, 1996] This implies that, although the tidal channel appears to grow smoothly and continuously over long time periods and large spatial areas, locally the process is highly variable and depends on vegetation and substrate heterogeneities. Large values of S_c, (slow rates of channel incision), can be directly linked to a smaller size of slumping blocks, which in turn reduces the randomness of the process and averages away the local substrate heterogeneities. The simulations presented herein suggest that local heterogeneities are important for the development of the small tidal channels, thus influencing the small scale of the network. On the contrary, the development of large channels is controlled by the critical shear stress of the platform sediments.

[28] In conclusion, the presented model represents a first attempt to study the formation and evolution of tidal networks. This formulation can be extended in the future to account for channel meandering, for the role of channel width in the growth of the network, for the influence of vegetation on channel formation, and for the possible refilling and abandonment of the tidal channels. Furthermore our method enables to exploit the numerous published results on LDA for the evolution of tidal networks.

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