Basic flow field in a tidal basin

Sergio Fagherazzi
Department of Environmental Sciences, University of Virginia, Virginia, USA

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[1] A simplified model for tidal flow in a basin is presented. The model is based on the assumption of a flat water surface oscillating synchronously in the tidal basin. Under this hypothesis the depth-averaged continuity equation becomes a Poisson equation that can be easily resolved at each instant of the tidal cycle. This formulation, which is particularly valid for small, deep basins, provides a simplified solution of the depth-integrated shallow water equations and suggests a possible approach to model long-term morphodynamic evolution of tidal basins. The model is tested in San Diego Bay, California, and the results are briefly discussed. *INDEX TERMS:* 4560 Oceanography: Physical: Surface waves and tides; 4235 Oceanography: General: Estuarine processes; 4255 Oceanography: General: Numerical modeling

1. Introduction

[2] Tidal motion in a basin produces a complex flow field that depends on the basin shape and bathymetry. Calculation of water fluxes caused by the tidal oscillation is not an easy task, and it is commonly performed solving the two-dimensional shallow water equations [*Velyan*, 1992]. On the other hand, the advantage of a simplified solution of the problem is twofold. The solution can be directly utilized in models where the complexity of the problem requires a high simplification of the processes involved, as in models of long-term morphodynamic evolution, where the study of the hydrodynamics is coupled with that of sediment transport. At the same time the simplified solution can be utilized as a starting point for studies that make use of perturbation techniques to shed light on the structure of the full set of equations. A simplified solution needs to have three characteristics in order to be effective: a) a strong physical basis, b) an origin from general principle (conservation of mass, momentum, or energy), c) if a full set of equations already describes the problem, the solution has to satisfy these equations. A typical example of a complex problem where a simplified solution is crucial for modeling purposes is water flow in a river. Here a steady uniform solution can be derived from a balance between the gravity force and bottom friction. Drainage basin evolution models directly use this solution [*Howard*, 1994], whereas models of river meandering take the uniform flow as a starting point (zeroth order solution) for analyses based on perturbation techniques [*Blondeaux and Seminara*, 1985; *Ikeda et al.*, 1981].

[3] Contrary to rivers, the tidal motion in a basin is driven by the oscillation of water elevation, which is intrinsically unsteady. An intuitive approach is to consider the water surface to be flat and to oscillate synchronously with the tide at the inlet. This hypothesis was already proposed for salt-marshes [*Boon*, 1975], utilized to study the equilibrium bottom configuration in a tidal basin [*Schuttelaars and de Swart*, 1999], the channel network in salt-marshes [*Rinaldo et al.*, 1999a, 1999b], and the cross sectional shape of tidal creeks [*Fagherazzi and Furbish*, 2001]. In the present analysis a method is presented for calculating the flow field in a bidimensional embayment under this hypothesis.

2. Method

[4] The integration of the Reynolds equations over the vertical direction leads to the system of shallow water equations, consisting of two equations for the conservation of momentum plus the continuity equation [e.g. *Dronkers*, 1964]. Herein the hypothesis of flat surface makes it possible to neglect the two momentum equations [*Schuttelaars and de Swart*, 1999], so that the flow field can be resolved utilizing only the continuity equation, which reads:

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \tag{1}$$

where q_x and q_y are the discharges per unit width in the x- and y-direction, h is the water depth and t is time.

[5] It is important to notice that in (1) the temporal variation in water depth $\partial h/\partial t$ is the same in each point of the basin and is equal to the water depth variation at the inlet. If I introduce the potential function Φ defined as:

$$q_x = \frac{\partial \Phi}{\partial x}, \quad q_y = \frac{\partial \Phi}{\partial y} \tag{2}$$

the continuity equation then becomes a Poisson equation:

$$\nabla^2 \Phi = -\frac{dh_0}{dt} \tag{3}$$

which can be solved in the tidal basin at each time step knowing the variation of water elevation at the mouth dh_0/dt. A similar equation, without potential function, has been utilized in *Boon* [1975]. However, *Boon* [1975] only focused on a salt marsh channel, and he was obliged to assign the contributing area for each channel cross section in order to calculate the discharge. Here the introduction of a potential function makes it possible to calculate the water discharge at each point of a tidal basin, independently of the basin shape.

[6] In order to solve (3), I have to specify suitable boundary conditions. At the border with the mainland (boundary $\Gamma 1$), I impose the water flux equal to zero. At the basin inlets (boundary $\Gamma 2$), the discharge normal to the inlet cross section is exactly calculated knowing the water surface variation inside the basin, and will be derived solving (3). What instead is not identified is the value of the discharge in the direction tangential to the inlet section, which will depend in general on the inlet shape and on the interaction between basin and open sea.

[7] To complete the boundary conditions I then need to specify the value of the gradient of Φ in the direction tangent to the inlet cross section. Since the discharge per unit width inside the basin is the gradient of the potential function, I can add to the potential function an arbitrary constant without changing the result. I then set this constant in order to have the potential function equal to zero in a specified point of the inlet section, and derive the value

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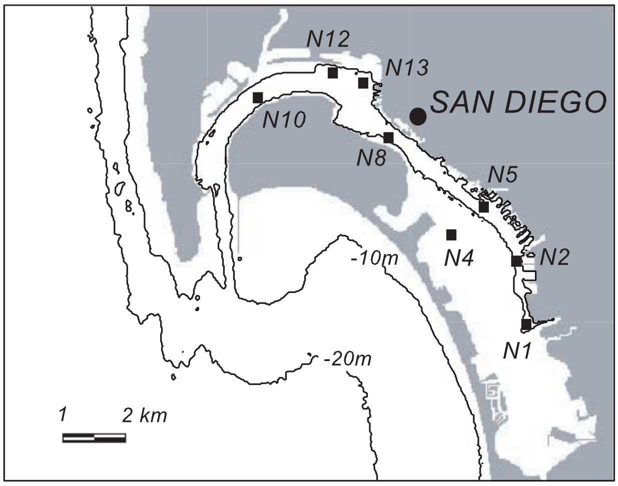

Figure 1. San Diego Bay, California. N1 to N13 are the locations where NOAA collected velocity data in 1983. The bathimetric data show that a deep channel is present in front of the city waterfront.

for the other section points as a function of the discharge in the tangential direction. The b.c. become:

$$\frac{\partial \Phi}{\partial n} = 0 \quad \text{in} \quad \Gamma_1 \tag{4}$$

$$\Phi = \Phi_0 \quad \text{in} \quad \Gamma_2 \quad \text{with} \quad \frac{\partial \Phi_0}{\partial r} = q_r \tag{5}$$

where n is the normal direction to the boundary directed inward, r the tangent direction to the boundary, and q_r the value of the discharge at the inlet in the direction tangent to the cross section. Herein we assume the discharge to be perpendicular to the inlet cross section, so that $q_r = 0$.

3. Results and Discussion

[8] As a test example I apply the method to San Diego Bay, California (Figure 1). San Diego bay is a tidal basin connected to the Pacific Ocean by an inlet with an artificial jetty that controls beach erosion. Since freshwater flow in the bay is low as well as wind magnitude, currents are predominantly produced by tides [*Wang et al.*, 1998]. The astronomical tide in San Diego bay is mixed, with the amplitude of the semidiurnal component M2 equal to 52 cm (period 12.42 hr) and amplitude of the diurnal component K1 equal to 35 cm (period 23.93 hr) [*Wang et al.*, 1998]. Equation (3) can be solved at any instant of the tidal cycle, and, since it is linear, it is possible to calculate separately the flow for each tidal component and add together the corresponding results. Here I show results for the M2 component when the time derivative of the water elevation is maximum, i.e.:

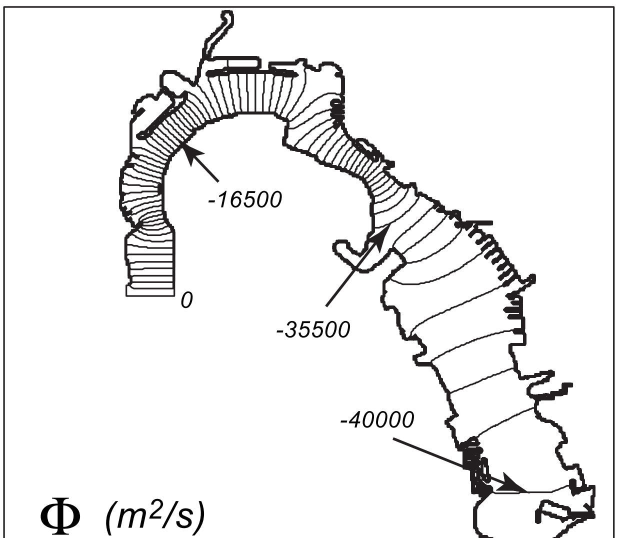

Figure 2. Contour lines for the potential function Φ in San Diego Bay during flood.

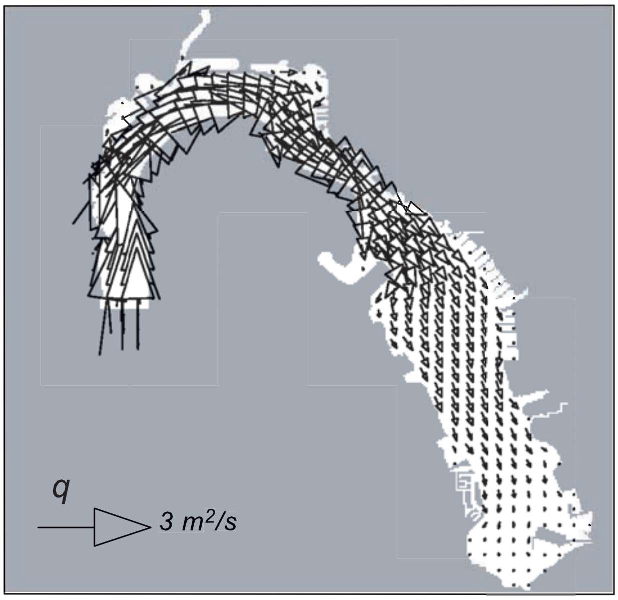

Figure 3. Maximum discharge during flood calculated from the potential function.

$$h = a \sin\left(\frac{2\pi}{T} t\right); \quad \left.\frac{dh_0}{dt}\right|_{\max} = a \frac{2\pi}{T} = 7.255 \ 10^{-5} \ \text{m/s} \tag{6}$$

where a is the tidal amplitude and T the period. As boundary conditions, I impose zero flux at the border with the mainland. I also assume that the flow at the inlet is perpendicular to the inlet cross section, so that the tangential discharge is zero as well as the tangential gradient of the potential. As a result of this hypothesis I impose the potential function equal to zero for the boundary points at the inlet. The solution of the potential function is reported in Figure 2; the discharge per unit width is shown in Figure 3.

[9] It is important to note that the computation of the discharge is independent of bottom elevation. With the hypothesis of flat water level, the water enters and exits from the inlet independently of the value of the bottom elevation in the basin. The water depth instead comes into play in the determination of the averaged vertical velocity, using the equations:

$$u_x = q_x / h; \qquad u_y = q_y / h \tag{7}$$

where u_x and u_y are the vertically averaged velocities in the x- and y-directions, and h the water depth. Furthermore, since the Laplacian operator is symmetric, the flow in ebb is identical to the flow in flood.

[10] The hypothesis of flat water surface is particularly valid for a small embayment with deep bottom. If the surface of the basin is limited, then the time spent by the tidal wave to propagate from the

Table 1. Comparison Between Model Results and Measurements[a]

Station	M2				K1			
	Model		Measured		Model		Measured	
	V_{max} (cm/s)	Direction deg	V_{max} (cm/s)	Direction deg	V_{max} (cm/s)	Direction deg	V_{max} (cm/s)	Direction deg
N1	4.1	150.3	11.2	178.6	1.4	150.2	3.1	176.6
N2	4.2	163.9	10.8	177.7	1.4	163.9	0.7	187.9
N4	19.9	176.2	11.7	175.9	6.7	176.3	1.2	219.2
N5	6.4	135.4	18.4	134.1	2.2	135.3	5.5	138.1
N8	32.9	121.1	38.2	133.8	11.1	121.1	8.4	139.8
N10	27.5	62.6	29.2	63.3	9.3	62.6	6.8	70.1
N12	16.6	101.5	18.6	105.7	5.6	101.1	6.7	112.2
N13	9.8	145.9	21.2	130.4	3.3	145.9	8.1	134.1

[a] Data reported in *Wang et al.*, 1998.

inlet to the extreme boundaries is negligible with respect to the tidal period, and the water surface is almost in phase everywhere. On the contrary, bottom friction in shallow areas reduces the tidal wave speed and attenuates the tidal peak as a consequence of energy dissipation. In San Diego bay the phase shift between the inlet and the extreme boundaries is only few minutes [*Wang et al.*, 1998], thus justifying the application of the present model. The maximum velocity produced by the model is compared with data collected by NOAA in 11 locations during a tidal current survey conduced in 1983 [*Wang et al.*, 1998]. A harmonic analysis makes it possible to extract the M2 and K1 components of the velocity from the data, and to calculate their maximum value and direction. These values are then compared to the model simulations (Table 1). Where the bottom of the bay is uniform (locations N8, N10, and N12), model results are close to measured values, both in magnitude and direction. In the southern part of the bay, instead, an uneven bottom strongly influences the tidal hydrodynamics. Close to the city waterfront the bay is deeper (with depths around 10 m) whereas in the remaining areas the depth decreases to 4 m (Figure 1). The present model does not utilize the bottom topography when the discharge per unit width is calculated, so that the flow entering or leaving the bay is uniformly distributed in the bay cross section. In reality, as pointed out in *Fagherazzi and Furbish* [2001], differences in depth produce a momentum redistribution with a velocity increase in incised channels and a velocity decrease in shallow areas. As a consequence, the model overestimates the velocity where the water depth is limited (location N4), and underestimates the velocity in the channel (location N1, N2, and N5). The flow direction can be also influenced by the topography, but in the San Diego Bay I do not notice strong disagreement between model results and field data (Table 1). Similar results were also found for the diurnal component K1 (Table 1), with dh_0/dt = 2.447 10^{-5} m/s.

[11] Finally, it is possible to show from (3) that the total volume of water flowing inside the bay in half tidal cycle (i.e. the tidal prism) is exactly equal to the volume of water contained between the two horizontal planes corresponding to the maximum and minimum tidal level. In small basins like San Diego Bay where the tide has everywhere about the same amplitude and phase, the tidal prism calculated by the model is then very close to the real tidal prism.

4. Conclusions

[12] In this analysis I present a simplified model for tidal flow in a basin. The model is based on the assumption of flat water level oscillating synchronously in the whole tidal basin. The solution of the continuity equation under this hypothesis is a Poisson equation with suitable boundary conditions. The formulation is physically based, satisfies the depth averaged shallow water equations, and it is valid for small basins with deep water. At each instant of the tidal cycle the flow field can be derived solving a simple Poisson equation, instead of the more complex 2-dimensional shallow water equations (I only have one unknown Φ against the three unknowns h, q_x, q_y for the shallow water equations). Since the equation solved is linear, it is possible to superimpose the effects of each tidal component and to calculate the flow field at any instant of the tidal cycle independently of previous solutions. On the contrary, for the shallow water equations, the flow field can be determined only carrying the simulation for several tidal cycles. The strong simplification adopted herein makes the present model ideal for studies in long term morphodynamic evolution.

[13] The method, however, is unable to capture the momentum redistribution between shallow and deep areas, which increases the flow velocity in incised channels. Nonetheless the basic solution presented herein can be considered as a simple approximation.

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S. Fagherazzi, Department of Environmental Sciences, University of Virginia, Charlottesville, VA 22903, USA. (sf9t@virginia.edu)