# Customer Capital*

*Preliminary and Incomplete*

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**Abstract**

Firms spend significant resources on creating and maintaining long-term customer relationships. We explore the role of this customer capital - a form of intangible capital - for firm valuation and physical investment. We build a general equilibrium model of long-term customer relationships, where existing customers are partially locked-in due to frictional product markets, and firms advertise, competing for customers by offering discounts. Calibrating the model, we find that it reproduces the low correlation of investment with Tobin's Q, and the high correlation of investment with cash flows found in data. Consistent with our model, we find that in the data investment is more correlated with cash flows, and less with Q, in industries where customer capital (as proxied by advertising expenses) is more important.

## 1 Introduction

In the neoclassical adjustment cost model of investment[1] firms accumulate physical capital, such as plants or equipment, to maximize the present discounted value of profits. The value of the firm is driven by its quantity of capital, and there is a one-to-one relationship between investment and the ratio of firm value to the capital stock, i.e. Tobin's Q. The data challenge this theory, however, with a low correlation between investment and Tobin's Q. This observed low correlation is particularly puzzling because even in extensions of the model specifically designed to break the direct relationship between investment and Tobin's Q, the theory still turns out to work well quantitatively (Gomes (2001)). While there are many potential mechanisms to break the Q-theory, such as fixed costs, financing constraints, or decreasing returns to scale in production, to replicate the failure of investment regressions in data studies generally need to appeal to measurement error in Q (e.g. Erickson and Whited (2000), Eberly, Rebelo, and Vincent (2008)).[2]

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[1] E.g. Abel (1982), Hayashi (1982), Summers (1981).

[2] There is a large literature on Q-theory and the cash flow sensitivity (see Caballero (1999) for a survey).

One natural candidate explanation for the discrepancy between theory and data is that the interpretation of capital has been too narrow. Firms spend substantial resources accumulating also "intangible capital," such as patents, organizational processes, human capital, and brand names. These investments are expensed and the accumulated capital is, by and large, not recorded in balance sheet statements.[3] At the same time, there is ample anecdotal evidence that these investments matter both for the valuation and decision-making of firms.[4]

In this paper, we focus on a specific type of intangible capital, the customer base. Due to search frictions in the product market, attracting customers requires incurring sales and marketing costs, but these customers later become partially locked in, creating positive profits for the firm. Firms derive value from these long-term customer relationships, i.e. their "customer capital."A particularly interesting property of this customer capital is its complementarity to physical capital: to make a sale, a firm needs *both* physical production capacity *and* customers. This complementarity will turn out to be central for our results, implying that the composition of assets - physical vs. customer capital - matters.

We build a general equilibrium model of production and investment with a frictional product market,[5] which leads to long-term customer relationships emerging. The frictions arise from differences in products across producers, which buyers can only evaluate by inspection. To attract new customers, and to allow the inspection to take place, firms must spend resources on marketing and sales efforts. We assume firms can fully communicate prices to everyone through advertising, but that the product inspection is subject to congestion, as the sales personnel of the firm can only handle a limited number of customers per period. We assume that once the customer is locked-in, the firm raises the price to a point making the customer indifferent between staying and finding a new supplier, but to attract new customers, firms compete with initial discounts. The model is calibrated and solved numerically.

Examples of products motivating the model are newspapers subscriptions and cell phones services. Newspapers offer discounts to entice new customers. They are then able to charge a price above the marginal cost of production for an extended period of time.[6] Cell phone providers also compete for customers by offering contracts with an initial discount (the phone is offered free of charge).[7] We view the model useful also for thinking about products sold without explicit contracts.

We show that the frictional product market leads to a Tobin's Q above one, as the firm derives value from its customer base. As we let frictions diminish, firm behavior approaches that of the

[3]Similarly, intangible investment and capital are not measured in the national income accounts (NIPA) or the fixed assets tables.

[4]Several authors have argued for the importance of intangible capital recently (e.g. Atkeson and Kehoe (2005), Hall (2001), McGrattan and Prescott (2007)).

[5]Recently several papers have used the idea of frictions in product markets to explain a variety of empirical puzzles (Drozd and Nosal (2008), Arkolakis (2008), Arseneau and Chugh (2008), Kleshchelski and Vincent (2008)). Our paper is also related to a literature on pricing with markups, which emphasizing the role of customer markets (e.g., Bils (1989), Nakamura and Steinsson (2008)). The idea of switching costs as a source of firm monopoly power and firm value has long been recognized in the IO literature (e.g., Klemperer (1995).

[6]The New York Times (Oct. 1, 2007) reports that "According to the Newspaper Association of America, the average cost of getting a new subscription order, including discounts, was $68 in 2006".

[7]In some industries, it appears to be common practice to evaluate the value by measuring the number of customers, the retention rate, and the margin per customer.

neoclassical adjustment cost model with Q approaching unity. Our main finding is that the model generates a time-varying wedge between marginal Q and Tobin's Q which implies: (i) a low correlation of investment with Tobin's Q, and (ii) that firm cash flow is likely to have predictive power over Tobin's Q in an investment regression. When search frictions are small, Q-theory regressions work perfectly in the model, i.e. the $R^2$ is 100% and the slope coefficient reflects the physical adjustment costs. When frictions are significant, however, firms become constrained by their customer base in the short-run. When a firm is hit by a good productivity shock, it first concentrates on increasing its customer base, rather than increasing its physical production capacity. Investment rises only as the accumulating customer base allows. Unlike marginal Q, which traces the hump-shaped response of investment, Tobin's Q rises immediately as the value of the existing customer base of the firm rises. Firm profits can be a better predictor of investment because they also increase with a delay, both because of the time it takes for the customer base to expand, and because the costs of expanding are deducted from profits per standard accounting rules.

Finally, we provide an empirical test in Compustat data. Identifying industries with high advertising expenditures as ones where firms face above average frictions in the product market, the theory implies that in those industries we should see: (i) Tobin's Q have less predictive power for investment, and (ii) cash flows have more predictive power for investment, than for the average firm. This is confirmed by the data.

Section 1 presents our model and section 2 studies its quantitative implications. Section 3 presents the result of our empirical test. Section 4 concludes.

*Literature review: to be added*

# 2 Model of Customer Capital

This section presents the model, starting with the production firms and then considering the consumers.

## 2.1 Production Firms

Production is carried out by a continuum of firms each period. Each firm decides each period: (i) how much to spend on sales effort, (ii) what kind of offer to advertise to attract new customers, and (iii) how much to invest in production capacity. Together with the customer base inherited from the previous period, the first two choices determine how many customers the firm has this period. As each customer buys one unit, this determines how much output needs to be produced.

Production technologies are Cobb-Douglas: $y = f(k, l, z) = zk^{\alpha_k} l^{1-\alpha_k}$, with $\alpha_k \in (0, 1)$. Producers rent labor $l$ at a competitive market with wage $w$. Capital $k$ is accumulated via investment $i$ according to $k_{t+1} = (1 - \delta_k)k_t + i_t$, where $\delta_k$ is the depreciation rate of capital. Investment goods are purchased at a (frictionless) competitive market for goods at a price normalized to one, but installing this new capital leads to adjustment costs, $\phi(i, k)$. Firm-specific productivity $z$ follows a Markovian stochastic process with a bounded support and a continuous and monotone transition function $Q$.

The products the firms produce are not identical, but differentiated in such a way that potential new customers must inspect the product in person to determine whether it meets their (idiosyncratic) needs or not. To attract these potential customers, and to allow this inspection to take place, firms must spend resources on marketing and sales efforts. For concreteness, we measure these firm specific efforts by $s$, the measure of sales personnel of the firm. We assume $s$ is associated with a convex cost $\kappa(s)$.[8]

Product inspection is affected by congestion. If a firm's pricing attracts $x$ potential customers this period, the measure of new customer relationships born is given by $M(x, s) = \xi x^{\alpha} s^{1-\alpha}$, with $\xi > 0, \alpha \in (0, 1)$. This measure increases in potential customers, but at a diminishing rate with fixed sales personnel. The measure increases in sales personnel, but at a diminishing rate with fixed customers. Increasing both leads to constant returns. Using $\theta = \frac{s}{x}$ to denote sales personnel per potential customer, the rate at which the producer gets new customers (per sales person) is $q(\theta) = \xi\theta^{-\alpha}$, and the rate at which potential customers find suppliers is $\mu(\theta) = \xi\theta^{1-\alpha}$.

A firm that seeks to attract new customers places an advertisement in the newspaper, announcing its prices to everyone in the economy. The firm influences the measure of potential customers it attracts through pricing, and may have an incentive to offer low prices to attract more customers. Once a customer has adopted the product, this incentive is gone, and the firm maximizes profits by raising the price for that customer to a level that makes the customer indifferent between staying with the firm and finding a new supplier.[9] This value is determined by the equilibrium value of allocating an additional unit of time to searching for a new supplier. As this value is the same for all customers with all firms, the price $p$ charged is also the same. However, firms can compete for new customers by offering an initial discount $\Delta$ in their advertisement. They anticipate that a larger discount will attract more potential customers per sales person – an offer of $\Delta$ leads to market tightness $\theta = \Theta(\Delta)$, where the function $\Theta : \mathbb{R} \to \mathbb{R}_+$ is strictly decreasing and convex. The function $\Theta$ will be determined as part of the equilibrium, based on the optimal search decisions of customers (see Section 2.2.1).[10]

A firm with an existing customer base $n$ that chooses discount $\Delta$ and sales personnel $s$, attracts $sq(\Theta(\Delta))$ new customers this period. Such a firm generally produces for $y = n+sq(\Theta(\Delta))$ customers this period, hiring exactly the amount of labor needed to produce this output, given the existing

[8]Perhaps because it becomes difficult to find more effective sales personnel.

[9]The firm cannot commit to lower prices after the customer has adopted the product. However, ex ante the customer understands this, and because he is indifferent with respect to the timing of prices, it does not affect the allocation. It does affect firm value however, raising Tobin's Q. It is assumed that producers have a large number of customers, leaving individual customers with no bargaining power to negotiate a lower price with the firm.

[10]Customers prefer larger discounts, but because producers offering them attract many potential customers per sales person, it becomes time-consuming for the consumer to inspect the product and establish a customer relationship with a producer offering a large discount. Customers take into account this time cost in choosing among discounts and in the limit customer flows adjust such that customers are indifferent between higher and lower discounts. The higher the discount, the more potential customers per sales person, i.e. lower $\theta$ (see Section 2.2.1 for more).

stock of capital. The producer's Bellman equation reads

$$V(k,n,z;w,\Theta) = \max_{s\geq 0,\Delta,i,l,y} \{py - lw - \kappa(s) - sq(\Theta(\Delta))\Delta - \phi(i,k) + \beta \int V(k',n',z';w,\Theta)Q(z,dz')\} \quad (1)$$

where

$$y \leq n + sq(\Theta(\Delta)), \quad (2)$$
$$y = f(k,l,z), \quad (3)$$
$$n' = (1-\delta_n)y, \quad (4)$$
$$k' = (1-\delta_k)k + i. \quad (5)$$

The firm sells the output $y$ to its customers at price $p$. It pays the wages of production labor, the costs of sales personnel, and gives the discounts promised to new customers. It also pays the costs of investing into production capacity, in place next period. As the producer's sales are constrained by the size of its customer base, equation (2) states that the firm's production output must be less or equal to that customer base. Equation (3) determines the labor needed to produce the desired output $y$. Equation (4) is the law of motion for the customer base, where $\delta_n$ is the depreciation rate of customers, representing attrition due to idiosyncratic reasons. Finally, equation (5) is the law of motion for capital.

While in general producers choose positive levels of sales personnel, those facing a sufficiently bad productivity shock may choose not to seek new customers, allowing their customer base to shrink with attrition. If the shock is particularly bad, they may even choose to produce less than their existing stock of customers would allow selling.[11] How prevalent this situation is depends on the customer retention rate $1-\delta_n$ and how large and persistent firm level shocks are. It is less likely to happen if the retention rate is low, or if negative shocks are small and/or temporary.

When deciding how to set the price $p$, the producer understands that there is a cutoff price level above which the customer will be driven to search for another supplier. As long as the price remains weakly below this cutoff, however, raising the price has no effect on demand. To maximize profit, the firm sets the price equal to this cutoff, which is determined in Section 2.2.1.

Problem (1) determines producer decision rules $s(k,n,z;w,\Theta)$, $\Delta(k,n,z;w,\Theta)$, $l^d(k,n,z;w,\Theta)$, $i(k,n,z;w,\Theta)$, $y(k,n,z;w,\Theta)$ for each firm, together with the value function $V(k,n,z;w,\Theta)$.[12]

### 2.1.1 Choosing Sales Effort

The optimal level of sales personnel is characterized by the first order condition

$$\frac{\kappa'(s)}{q(\theta)} + \Delta = p - wl_2(k,y,z) + \beta(1-\delta_n) \int V_2(k',n',z';w,\Theta)Q(z,dz'). \quad (6)$$

[11] Customers are indifferent between staying with the current producer and finding a new one, so they will simply find another supplier. We assume that customers retain no memory of past suppliers and their products.

[12] We assume the value function is differentiable. There may be an issue with differentiability of $V$ w.r.t. $n$ when $s=0$ is optimal. We conjecture that the argument in Campbell and Fisher (2001) can be adapted, i.e. the conditional expectation of the value is differentiable in $n$, which is enough for our purposes.

The left-hand side represents the cost of acquiring an additional customer: given the discount $\Delta$ used, the firm must pay the costs of $\frac{1}{q(\Theta(\Delta))}$ additional units of sales personnel per new customer, as well as giving them the discount. The right-hand side represents the increase in firm value from an additional customer: today's revenue increases by $p$, today's costs increase according to the marginal cost of production $wl_2(k, y, z)$, and tomorrow's customer base is larger by $1 - \delta_n$ customers.

The optimal discount is characterized by the first-order condition:

$$1 = \frac{q'(\theta)\Theta'(\Delta)}{q(\theta)}[p - wl_2(k, y, z) - \Delta + \beta(1 - \delta_n) \int V_2(k', n', z'; w, \Theta)Q(z, dz')]. \tag{7}$$

The left hand side represents the per-new-customer cost of raising the discount, while the right hand side represents the corresponding increase in value: the rate at which the firm establishes new customer relationships increases according to $\frac{q'(\theta)\Theta'(\Delta)}{q(\theta)}$,[13] with each new customer delivering the increase in value in the brackets. While a larger discount reduces the profitability of new customers, it also reduces costs of getting new customers by attracting more customers per sales person. The outcome of the tradeoff depends on how severely congestion affects the formation of new customer relationships, captured by the elasticity of the matching function. If the sales personnel cannot accommodate many customers per period, then offering large discounts to attract those customers is not profitable.

Combining equations (6) and (7) yields

$$\frac{\kappa'(s)}{q(\theta)} = \frac{q(\theta)}{q'(\theta)\Theta'(\Delta)}. \tag{8}$$

Once we specify the function $\Theta$, we will see that firms choosing a higher level of sales effort also use bigger discounts.

### 2.1.2 Relation to Investment Regressions

The first-order condition for investment $i$ reads

$$\phi_1(i, k) = \beta \int V_1(k', n', z'; w, \Theta)Q(z, dz'), \tag{9}$$

relating investment today to expected marginal Q next period, $\int V_1(k', n', z; w, \Theta)Q(z, dz')$. If the adjustment cost is quadratic, as generally specified, optimal $\frac{i}{k}$ becomes a linear function of marginal Q. Marginal Q is not observed in data, and is generally proxied by average Q, $V(k', n', z'; w)/k'$. A large empirical literature shows that this equation is not supported by the data – average Q fails to explain investment behavior well, and instead the firm's cash-flow appears to matter. We will show that these empirical observations can be rationalized by frictional product markets, which introduce a systematic wedge between marginal and average Q.

[13] The firm gets new customers at rate $q(\theta)$ per sales person.

### 2.2 Representative Household

The representative household's preferences over consumption $C_t$ and leisure $L_t$ are

$$\sum_{t=0}^{\infty} \beta^t u(C_t, L_t). \tag{10}$$

The household's periodic budget constraint reads

$$C_t + B_t \leq w_t(1 - L_t) + D_t + (1 + r_t)B_{t-1} \text{ for all } t \geq 0, \tag{11}$$

in terms of period $t$ consumption. The household is endowed with one unit of time each period and purchases consumption at price normalized to one and leisure at the market wage $w_t$. The household owns all firms in the economy, receiving the aggregated dividends $D_t$ each period. The household has access to a risk-free bond $B_t$, with return $r_t$. In a stationary equilibrium, $1 + r_t = \frac{1}{\beta}$ and $B_t = 0$ for all $t$. This justifies discounting firm profits above at rate $\beta$. The non-leisure time is divided between market work $L^m$ and shopping for consumption goods $L^s$.

#### 2.2.1 Product Search

Product search is necessary for establishing and maintaining long-term customer relationships with producers, in order to procure consumption goods for the household.[14] In equilibrium the return to searching for new suppliers is zero, so for simplicity the search activity could be suppressed in the budget constraint above.

When the household decides to devote an additional unit of time to finding suppliers, it must decide which firm's product to try. In making this choice the household anticipates that firms offering larger discounts attract more potential customers per sales person, increasing the time cost of finding a suitable product. Given discount $\Delta$ and a ratio of sales personnel to potential customer $\theta$, the present value of product search reads

$$-w + \mu(\theta)[1 - p + \Delta - K_s - \eta w + \beta(1 - \hat{\delta}_n)\frac{1 - p - \eta w}{1 - \beta(1 - \hat{\delta}_n)}].$$

The cost of search is the lost wage income from not working, $w$, and new customer relationships are born at rate $\mu(\theta)$. Once this happens, the firm delivers one unit of the good to the buyer each period for as long as the relationship lasts. In return, the buyer pays the producer $p$ units of consumption each period, as well as incurring the labor cost of maintaining the purchasing relationship over time, $\eta w$. In the first period, the buyer receives the discount[15] and incurs a switching cost $K_s$ in adapting to the new supplier. Purchasing relationships end at rate $\hat{\delta}_n$, either for idiosyncratic reasons or because the firm restricts output due to a severe negative shock.

After the customer relationship has been established and the promised initial discount paid, the producer maximizes profits by raising the price to make the buyer indifferent between staying and

[14] Part of these goods will also be sold as investment goods to firms.

[15] It may be greater than $p$.

finding a new supplier. This implies that $p = 1 - \eta w$, and the present value of search reads

$$-w + \mu(\theta)[\Delta - K_s].$$

As the household optimally chooses how much labor to allocate to search with each of the firms advertising, this present value must be equal across all firms advertising. The flow of potential customers determines $\theta$ such that the values are equal *for all available discounts*. Moreover, in equilibrium there must be zero return to allocating more time to product search, and hence

$$-w + \mu(\theta)[\Delta - K_s] = 0 \tag{12}$$

for any discount offered. This implicitly defines a negative relationship between the size of discount and ratio of sales personnel to potential customer for this discount: $\Theta(\Delta) = \mu^{-1}(\frac{w}{\Delta - K_s})$.[16]

Given sales personnel $s$, equations (8) and (12) determine $(\Delta, \theta)$ as function of $s$. For a strictly convex cost of effective sales personnel, they imply that firms choosing larger sales force also offer larger discounts.[17]

## 2.3 Aggregation

We denote the cross-sectional distribution of firms by $\nu(k, n, z)$. Over time this distribution evolves according to a law of motion determined by the productivity process and firm decision-making: For any measurable set $(\mathcal{K}, \mathcal{N}, \mathcal{Z})$, we have

$$\nu'(\mathcal{K}, \mathcal{N}, \mathcal{Z}) = T((\mathcal{K}, \mathcal{N}, \mathcal{Z}), (k, n, z))\nu(k, n, z) \tag{13}$$

where the conditional transition probability $T$ is given by

$$T((\mathcal{K}, \mathcal{N}, \mathcal{Z}), (k, n, z)) = \int \mathbb{I}_{\{-\delta_k k + i(k,n,z;w,\Theta) \in \mathcal{K}, (1-\delta_n) y(k,n,z;w,\Theta) \in \mathcal{N}\}} Q(dz'|z).$$

Focusing on a stationary distribution of firms, we must have $\nu'(\mathcal{K}, \mathcal{N}, \mathcal{Z}) = \nu(\mathcal{K}, \mathcal{N}, \mathcal{Z})$ for all $(\mathcal{K}, \mathcal{N}, \mathcal{Z})$.

Production output in any period is $Y(\nu; w, \Theta) = \int y(k, n, z; w, \Theta) d\nu(k, n, z)$ with the labor demand of production firms given by $L^d(\nu; w, \Theta) = \int l^p(k, n, z; w, \Theta) d\nu(k, n, z)$. Shopping labor is

$$L^s(\nu; w, \Theta) = \eta Y(\nu; w, \Theta) + \int \frac{s(k, n, z; w, \Theta)}{\Theta(\Delta(k, n, z; w, \Theta))} d\nu(k, n, z),$$

where the first term includes the labor needed to maintain existing customer relationships and the second the labor needed to establish new ones.[18]

[16] In the quantitative exercises we use a Cobb-Douglas matching technology, where $q(\theta) = \xi\theta^{-\alpha}$ and $\mu(\theta) = \xi\theta^{1-\alpha}$. We then have $\Theta(\Delta) = (\frac{w}{\xi(\Delta - K_s)})^{\frac{1}{1-\alpha}}$.

[17] They imply a negative relationship between $\frac{\kappa'(s)}{w}$, and $\theta$. In particular, for the Cobb-Douglas case, $\Delta = K_s + [\kappa'(s)\frac{\alpha}{1-\alpha}]^{1-\alpha} w^{\alpha} \frac{1}{\xi}$, and $\theta = \frac{w}{\kappa'(s)} \frac{(1-\alpha)}{\alpha}$.

[18] Notice that because $\theta$ is the ratio of sales personnel to potential customer, the ratio $\frac{s}{\theta}$ is the measure of potential customers.

Investment is given by $I(\nu;w,\Theta) = \int i(k,n,z;w,\Theta)d\nu(k,n,z)$ with costs of investing $\Xi_I(\nu;w,\Theta) = \int \phi(i(k,n,z;w,\Theta),k)d\nu(k,n,z)$. Sales effort is $S(\nu;w,\Theta) = \int s(k,n,z;w,\Theta)d\nu(k,n,z)$ with costs $\Xi_{SA}(\nu;w,\Theta) = \int \kappa(s(k,n,z;w,\Theta))d\nu(k,n,z)$.

The dividend income of the household can be written as

$$\begin{aligned} D(\nu;w,\Theta) =& pY(\nu;w,\Theta) - wL^d(\nu;w,\Theta) - \Xi_{SA}(\nu;w,\Theta) \\ & - \int s(k,n,z;w,\Theta)q(\Theta(\Delta(k,n,z;w,\Theta)))\Delta(k,n,z;w,\Theta)d\nu(k,n,z) - \Xi_I(\nu;w,\Theta), \end{aligned}$$

where $p = 1 - \eta w$.

Focusing on a stationary distribution, the aggregate customer base is constant and the number of customers incurring switching costs equals $\hat{\delta}_n Y(\nu;w,\Theta) = \int s(k,n,z;w,\Theta)q(\theta(\Delta(k,n,z;w,\Theta)))d\nu(k,n,z)$. The total switching costs are $\Xi_{SW}(\nu;w,\Theta) = \hat{\delta}_n Y(\nu;w,\Theta)K_s$.

## 2.4 Equilibrium

The definition of equilibrium follows Gomes (2001), incorporating the notion of a competitive search equilibrium (Moen (1997) and others) to characterize the frictional product market.

**Definition 1** *A stationary competitive search equilibrium specifies: i) for the household: decision rules* $L^m(w,D(w,\Theta))$, $L^s(w,D(w,\Theta))$, $C(w,D(w,\Theta))$, *ii) for the production firms: decision rules* $s(k,n,z;w,\Theta)$, $\Delta(k,n,z;w,\Theta)$, $i(k,n,z;w,\Theta)$, $l^p(k,n,z;w,\Theta)$, $y(k,n,z;w,\Theta)$ *and value function* $V(k,n,z;w,\Theta)$, *iii) aggregate quantities* $S(w,\Theta)$, $I(w,\Theta)$, $L^p(w,\Theta)$, $Y(w,\Theta)$, $D(w,\Theta)$, *iv) wage* $w$, *v) tradeoff-function* $\Theta(\Delta)$, *vi) price* $p = 1 - \eta w$, *and vii) distribution of firms* $\nu$, *such that*

1. ***Consumers optimize:***
   (a) *The decision rules* $L^m(w,D(w,\Theta))$, $L^s(w,D(w,\Theta))$, $C(w,D(w,\Theta))$, *solve the household problem to maximize* (10) *subject to* (11).
   (b) *For all* $\Delta$,
      i. *the tradeoff* $\Theta(\Delta)$ *satisfies* $-w + \mu(\Theta(\Delta))[\Delta - K_s] = 0$ *whenever* $\Theta(\Delta) > 0$,
      ii. *and* $-w + \mu(\Theta(\Delta))[\Delta - K_s] \leq 0$ *whenever* $\Theta(\Delta) = 0$.
2. ***Firms optimize:*** *The decision rules and value function solve the production firms' Bellman equation.*
3. ***Labor market clears:*** $L^m(w,\Theta) = L^p(w,\Theta)$.
4. ***Consistency:*** *The distribution of firms* $\nu$ *follows the law of motion* (13), *with* $\nu' = \nu$, *and the aggregate variables are consistent with the definitions in Section 2.3. (Add:*$\hat{\delta}_n =$*?)*

By Walras' law, also the goods market clears: $Y = C + \Xi_{SA} + \Xi_I + \Xi_{SW}$.[19]Production output is used up by consumption, sales costs, investment costs, and switching costs.

# 3 Quantitative Analysis

This section demonstrates how the model behaves quantitatively. We examine the contribution of customer capital to the steady-state level of Tobin's Q, as well as the ability of Q to explain firm investment behavior in the face of productivity shocks. Frictional product markets raise the steady-state Q above one, and introduce a systematic wedge between marginal and average Q that implies information on firm cash-flow may be useful for predicting investment.

## 3.1 Calibration

The calibration involves some standard parameters common in the literature, as well as some related to the product/customer search process which are not. For the first set of parameters we adopt commonly used values, and then pick the new parameters to match steady-state implications. (Note that this calibration is preliminary.)

**Standard parameters** We set the period to be one month. The discount factor is set to $\beta = .95$ annually, corresponding to .9957 monthly. The productivity shock is assumed to have a persistence of .95 annually, with a standard deviation of 10%.[20] This persistence is a bit higher than that in Gomes (2001) or Henessy and Whited (2005), but it is consistent with the estimates in Gourio (2008) and Caballero and Engel (1999). Household preferences are assumed to be consistent with balanced growth, i.e. $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma}v(l)$. Focusing on cross-sectional implications, the precise preferences do not matter, however. Finally, in order to make the departure from the Hayashi adjustment cost model minimal, we assume that the production technology has constant returns to scale, with capital share equal to $\alpha_K = .3$. This means that in the limiting case with no frictions, Q-regressions based on model generated data work perfectly, at least in the absence of measurement error.[21] The annual depreciation rate of physical capital is $\delta_k = 10\%$ and the adjustment cost to physical capital is such that the elasticity of investment to Q is unity.

[19]Note that $\int sq(\theta)\Delta d\nu = \int \frac{s}{\theta}\mu(\theta)\Delta d\nu = \int \frac{s}{\theta}[w + \mu(\theta)K_s]d\nu = w\int \frac{s}{\theta}d\nu + \Xi_{SW}$, by equation (12). Hence, substituting dividends into the household budget gives

$$C = w(1-L) + D = w(L^m + L^s) + pY - wL^p - \Xi_{SA} - w\int \frac{s}{\theta}d\nu - \Xi_{SW} - \Xi_I = w(L^m + L^s) + Y - wL^s - wL^p - \Xi_{SA} - \Xi_{SW} - \Xi_I$$
$$= Y - \Xi_{SA} - \Xi_{SW} - \Xi_I.$$

[20]Because we use a log-linear solution and there is only one shock, the value of $\sigma_z$ has actually no effect on our Q-regression results.

[21]We abstract from decreasing returns to scale in production, a departure from the Hayashi-model which would make Q-regressions misspecified (Gomes (2001), Cooper and Ejarque (2003), Abel and Eberly (2008)). It may be that decreasing returns in physical capital are a proxy for intangible capital.

Table 1: Benchmark calibration.

| Symbol | Meaning | Value |
|---|---|---|
| $\beta$ | discount rate | .95 |
| $\rho_z$ | persistence of productivity | .95 |
| $\sigma_z$ | st.dev. of productivity | .1 |
| $\alpha_K$ | share of capital | .3 |
| $\delta_K$ | depreciation of capital | .1 |
| $\eta_I$ | elasticity of adj. cost | 1 |
| $\delta_N$ | depreciation of customers | .2 |
| $\kappa$ | sales cost parameter | $1.211\times10^5$ |
| $\zeta$ | sales cost parameter | 2 |
| $\alpha$ | matching function parameter | .113 |
| $\xi$ | matching function parameter | |

Notes: The table presents the annual values.

**Customer parameters** We now turn to the parameters which govern customer acquisition. Following Hall (2008) and the data cited therein, the depreciation rate of the customer base is assumed to be $\delta_n = 20\%$ per year. The convex cost of sales effort is assumed to have a power form: $\kappa(s) = \kappa\frac{s^{1+\zeta}}{1+\zeta}$. We adopt a Cobb-Douglas matching function: $M(x,s) = \xi x^{\alpha} s^{1-\alpha}$. This leaves four parameters: the coefficient $\xi$ and elasticity $\alpha$ of the matching function, and the coefficient $\kappa$ and elasticity $\zeta$ of the adjustment cost function. Our current calibration sets $\zeta = 2$, and we consider the sensitivity to this parameter. We set three targets for the calibration: first, the share of advertising in GDP is on average 5% (Hall 2008). Second, the average time spent searching is set to 1% of total time available, which amounts to 15 minutes per day. Finally, the average time to find a product is set to be 10 hours. (To be changed to markup-target.) These three targets pin down $\kappa, \xi$ and $\alpha$. In addition, we set $\eta$ and $K_s$ to zero for now, exploring their contribution later.

Table 1 summarizes the parametrization. We solve the model by log-linearizing around the non-stochastic steady-state.

### 3.2 Firm Value in a Frictional Product Market

The degree of friction in the product market can be viewed as captured by the value of $\xi$, the coefficient on the matching function. If we let $\xi$ grow, frictions diminish as the rate of matching grows, and the model approaches the neoclassical model: sales effort and time spent searching diminish, as do discounts. The customer base loses its value, as the cost of getting new customers falls, and Tobin's Q approaches one.[22]

Figure 1 shows the impact of the friction on steady-state statistics of the model, plotting out the steady-state as a function of $1/\xi$. Increasing the friction implies firms spend more on sales effort and

[22] Of course, away from the steady-state, Q may not be one, even in a model without search frictions. Our adjustment cost formulation implies that the adjustment cost is zero when the firm is just replacing its capital stock.

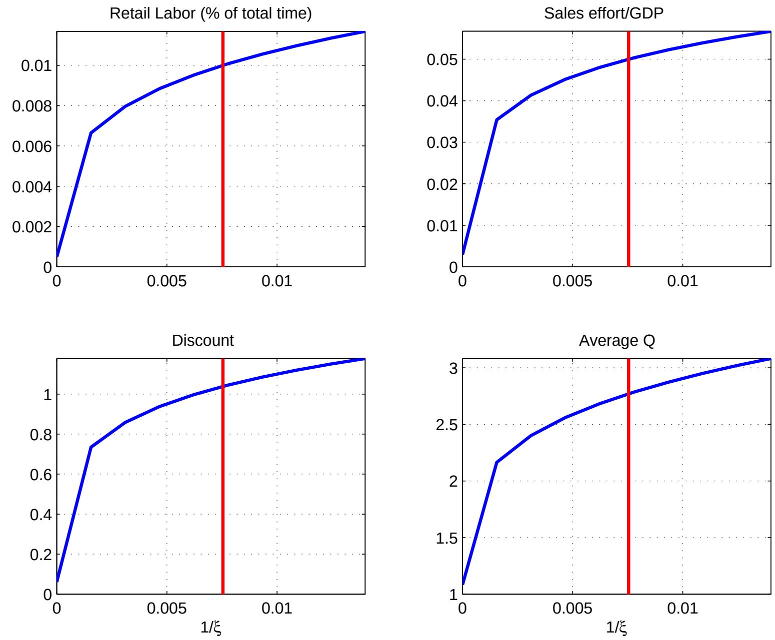


Figure 1: Impact of friction on steady-state

Notes: The figure plots the steady-state of the model varying the degree of search friction. As $1/\xi$ increases, the friction increases. The vertical line indicates the benchmark calibration of the search friction. Top left: share of time spent shopping. Top right: share of expenditures on sales effort in GDP. Bottom left: discount (recall that the monthly price is normalized to be one). Bottom right: average Q.

offer larger discounts to new customers. An existing customer base becomes increasingly valuable, and hence the value of Q rises above unity. In the baseline calibration, the discount offered is on average one, meaning that the producer offers the first month free of charge. The average Q is relatively high, about 2.7, implying that the value of the customer base is substantial. The value derives from the front-loaded costs of attracting customers: both the costs of sales effort as well as the discounts used. The present discounted value of future profits per customer (the excess of price over marginal cost) exactly compensates for these costs.

## 3.3 Differing Responses of Investment and Tobin's Q to Shocks

Customer capital can help explain (i) why Tobin's Q may not predict investment behavior well, and (ii) why a firm's cash-flow can have predictive power over Tobin's Q in an investment regression. To understand the mechanism, it is useful to look at the impulse response to a productivity shock. Figure 2 displays the responses of the key variables in the model, when the friction is small. In this case, the firm is relatively unconstrained by its customer base, and its growth following a positive shock is mainly limited by physical adjustment costs. An initial burst of discounts and increase in sales effort is enough to build up the customer base. Consequently, in response to the shock, both marginal and average Q rise, and comparing the impulse responses suggests that they are highly correlated. Note that the model implies that marginal Q and the investment rate I/K must be perfectly correlated, due to the first-order condition for investment (9).

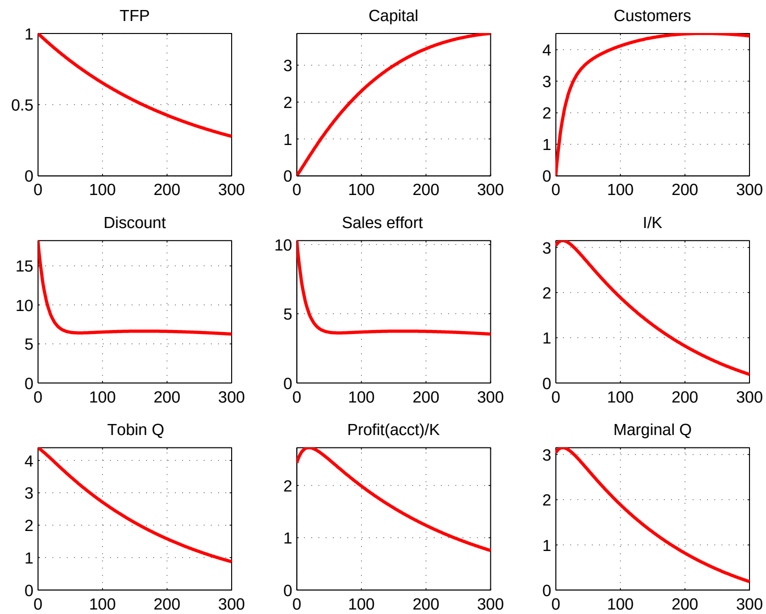


Figure 2: Responses to tfp-shock with low frictions

Notes: Impulse response of some model variables to a persistent technology shock, for the calibration with very small search frictions. The variables are all in % deviation from the nonstochastic steady-state. The period is a month.

In contrast, Figure 3 plots the same responses for our benchmark calibration with a more frictional product market. The firm's growth is now more clearly constrained by the size of the customer base than by physical capital. Given the increased productivity the firm does not need to increase its capital stock immediately. Building up the customer base (to increase production and take advantage of the higher productivity) is time-consuming due to the convex costs of increasing sales effort. As a result, the size of the customer base, and hence also sales, evolve more slowly than in Figure 2. This constraint on how much the firm can sell affects investment, generating a hump-shaped response. The firm's profits are also hump-shaped, because the firm initially incurs high costs to expand its customer base, leading to increased profits only as the size of the firm grows with accumulating customers and capital. In contrast, the value of the firm and Tobin's Q increases immediately as the firm makes higher profits on existing customers and further is expected to add customers and increase profits later. The plots suggest that in the model, investment is more correlated with profits than with Tobin's Q, because investment and profits share the hump-shaped dynamics, which differ from the monotonic response of Tobin's Q.

A limitation of the model is that the friction reduces the overall sales volatility, as seen by comparing the scales of Figures 2 and 3. Note however that due to constant returns in production, the volatility of sales in the model tends to be high.

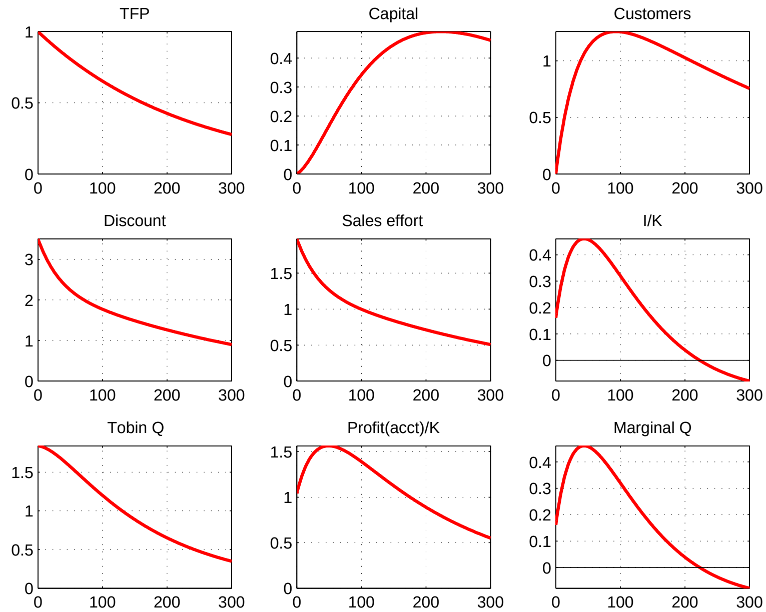


Figure 3: Responses to tfp-shock with high frictions

Notes: Impulse response of some model variables to a persistent technology shock, for the benchmark calibration. The variables are all in % deviation from the nonstochastic steady-state. The period is a month.

### 3.4 Regressing Investment on Tobin's Q and Cash Flow

We next study the implications of the model for investment regressions, examining whether it generates a low correlation of investment and Tobin's Q, and a "cash flow sensitivity puzzle." We simulate a balanced panel of firms from the model and run Q-regressions, as well as Q-regressions augmented with cash flows, in this simulated data. More precisely, we run the regression

$$\frac{I_{it}}{K_{it}} = b_0 + b_1 \frac{F_{it}}{K_{it+1}} + \varepsilon_{it}, \tag{14}$$

where $F_{it}$ is the end-of-period firm value, i.e. $F_{it} = \beta E_t V_{it+1}$, $V_{it}$ is the beginning of period firm value, and $K_{it+1}$ is the beginning of period $t+1$ capital stock. This is the exact regression specified by the Hayashi model, where the timing is driven by the one-period time-to-build.[23]

The regression augmented with cash flows is

$$\frac{I_{it}}{K_{it}} = b_0 + b_1 \frac{F_{it}}{K_{it+1}} + b_2 \frac{CF_{it}}{K_{it}} + \varepsilon_{it}, \tag{15}$$

where $CF_{it}$ is the cash flow, which we measure as output net of labor costs and sales costs (which are not counted as physical investment).[24]

[23]The regression differs slightly from the specifications used in empirical studies, which use a different timing.

[24]If we measure cash flows as profits net of labor costs only, the results are similar. Hence, our results do not seem to rely much on the fact that the investment in intangible capital is not measured.

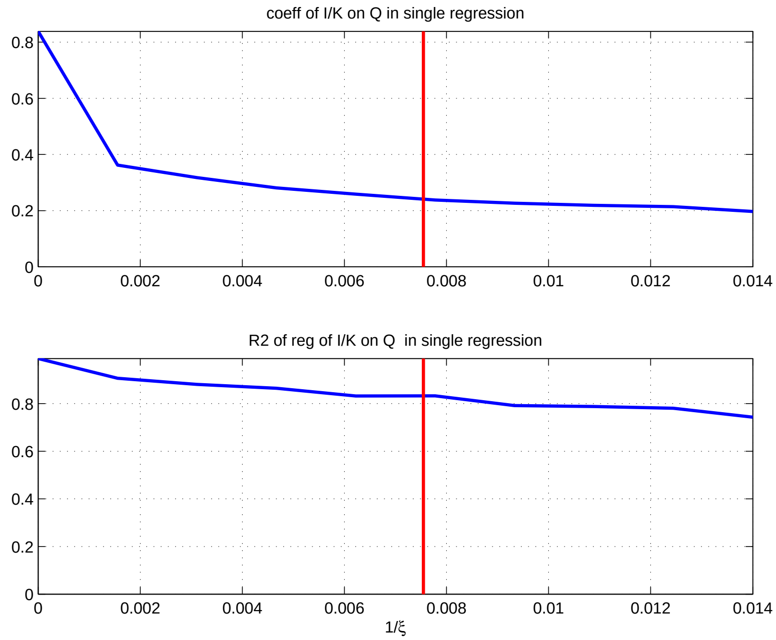


Figure 4: Impact of friction on Q-regression

Notes: This figure reports the coefficient on Q ($b_1$), and the $R^2$ from regression (14) on simulated data from the model, for different degrees of search friction. The benchmark calibration is indicated with the full vertical line.

Figure 4 presents the slope coefficient $b_1$ and $R^2$ from regression (14) in simulated data. As expected, as frictions diminish and the model approaches the Hayashi (1982) model, the Q-regression begins to work perfectly, with $R^2$ converging to one and the slope coefficient converging to the adjustment cost elasticity specified (one). However, with increasing frictions the slope coefficient falls quickly, and the regression $R^2$ also falls. This is qualitatively consistent with the standard finding that Q-regressions do not perform well in the data.

Figure 5 presents the slope coefficients $b_1$ on Q and $b_2$ on cash flow, and the regression $R^2$, from regression (15). Again, in the limiting case of no search friction, the Q-theory regression works perfectly.[25] As the friction increases, the coefficient on cash flow increases and the coefficient on Q falls, as does the $R^2$. For our benchmark calibration, the coefficient on Q is close to zero. These results are based on the complementarity of customer capital with physical capital, which implies that both investment and profits show a hump-shaped response to a shock to technology, while Tobin's Q responds on impact. Of course, the fact that cash flows matter does not reflect a financial constraint, as there are none in the model.

[25] In this limiting case, there is almost perfect collinearity between profits and investment, so we add a small measurement error to cash flow to be able to run the regression.

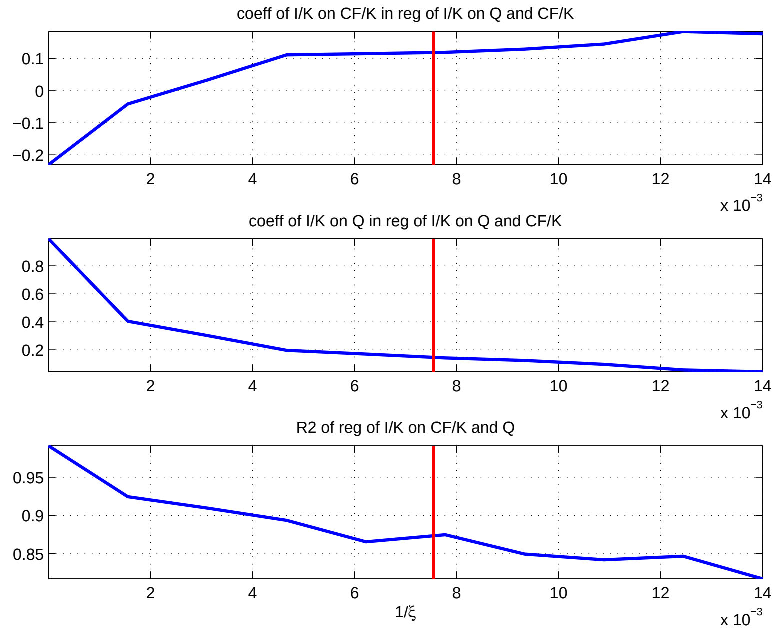


Figure 5: Impact of friction on Q-regression with cash flow

Notes: This figure reports the coefficients on Q ($b_1$) and cash flow ($b_2$) as well as the $R^2$ from regression (15) on simulated data from the model. The benchmark calibration is indicated with a vertical full line.

# 4 An Empirical Test

While the product market is somewhat frictional for all goods, some markets are likely to be more frictional than others. The model predicts that in those markets, the standard Q-regressions should work less well – the coefficient on Tobin's Q should be smaller, and cashflow have more explanatory power. It is difficult to separate out from the data those firms facing more frictional markets from those facing less frictional markets, to test this theory, however. On the other hand, the theory also predicts that if we select a subset of firms that are likely to face higher frictions than the average firm, the regressions should work less well for that group than average. Compustat provides data on the advertising costs of firms. We postulate that those firms that are spending a lot on advertising are doing so because they face frictional product markets that require more intensive sales effort.[26] Is it the case that the Q-regressions work less well for those firms than the average firm?

We use annual firm level data on investment, cash flows, and Tobin's Q from Compustat.[27] In addition, for each 2-digit industry we calculate the time series average of the ratio of total advertising expenses in the industry to total industry sales.[28] We sort industries into three groups: low, medium, and high average advertising - less than .5% of sales on average, between .5% and 2.5%,

[26] It may well be that for some firms sales efforts do not take the form of advertising, which means that we are capturing only a subset of the firms facing more frictional markets.

[27] See the data appendix for details of data construction.

[28] The firm-level advertising data is also from Compustat. Item data45.

greater than 2.5%. Dropping the medium group, we study the differences between the low and high advertising samples.

Table 2: Summary statistics for low and high advertising samples

| Median | Low advertising industries | High advertising industries |
|---|---|---|
| Q | 1.5 | 1.9 |
| PI/K | .35 | .50 |
| I/K | .20 | .20 |

Table 2 provides summary statistics for the two samples. Consistent with our model, Tobin's Q and the profit rate are higher in high advertising industries. The physical investment rates are similar, suggesting that the sort is not leading to substantial differences in production technologies across the groups (e.g., the industries have roughly the same rates of depreciation).

Table 3: Advertising expenditure and Q-regressions

| | Low Advertising sample | | | | High Advertising sample | | | |
|---|---|---|---|---|---|---|---|---|
| Coeff. | 1a | 2a | 3a | 4a | 1b | 2b | 3b | 4b |
| Q | .042 | .032 | .044 | .035 | .032 | .022 | .023 | .015 |
| s.e. | .002 | .002 | .001 | .001 | .002 | .002 | .001 | .001 |
| CF/K | – | .061 | – | .065 | – | .080 | – | .082 |
| s.e. | – | .002 | – | .002 | – | .003 | – | .003 |
| $R^2$ | .37 | .40 | .05 | .12 | .37 | .41 | .04 | .11 |
| N$\overline{T}$ | 21,366 | | | | 10,761 | | | |
| Time effects | y | y | y | y | y | y | y | y |
| Fixed effects | y | y | n | n | y | y | n | n |

Table 3 presents the results from the regressions[29]

$$\frac{I_{i,t}}{K_{i,t-1}} = b_0 + b_1 Q_{i,t-1} + f_t + d_i + \epsilon_{i,t},$$

$$\frac{I_{i,t}}{K_{i,t-1}} = b_0 + b_1 Q_{i,t-1} + b_2 \frac{CF_{i,t-1}}{K_{i,t-1}} + f_t + d_i + \epsilon_{i,t},$$

for each of the two samples, with or without firm fixed effects. Consistent with our theory, in industries with high advertising: (i) the coefficient on Q is significantly lower, and (ii) the coefficient on cash flow significantly higher. Q-theory works less well in the subset of firms likely to face above average frictions in the product market. These results appear to be robust to changes in the definition of Q, changes in the timing and specification of the regressions (levels vs. logs), as well as the exclusion of firm fixed effects.

[29] Note that we follow the standard timing of investment regressions in the empirical literature, i.e. we use the lagged values of the profit rate and Tobin Q. This is in contrast to the model regressions which were with the timing which is correct in the model.

## 5 Conclusions and Future Work

This paper developed a model of production and investment in the face of frictional product markets, which can rationalize some failures of the neoclassical Q-theory. Compustat data confirm that in industries with above average advertising expenses, Q-theory works less well than for the average firm.

In future work, it may be interesting to explore the model's hump-shaped dynamics in the context of aggregate shocks. Marketing and advertising expenses appear investment-like in that they are strongly procyclical and volatile. It may also be interesting to consider the implications for expected returns and the value premium, i.e. the negative correlation between Q and expected returns. If customer capital is easier to adjust than physical capital, then firms with a large customer base – and hence high Q – may have lower expected stock returns.

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