Supplementary Material for "Labor Market Dynamics under Long-Term Wage Contracting"

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Appendix A Details of Proofs

Proof of Propositions 3 and 4 Clearly the Pareto-efficient contract must have a constant wage throughout or one could improve firm value by smoothing consumption for the worker. The worker value is strictly increasing in the level of this wage, and the firm value strictly decreasing. Hence the Pareto-frontier is strictly decreasing.

A convex combination of two such contracts (with worker values V^1, V^2 and wages w^1, w^2) yields strictly higher value V^α for a consumer than the weighted average of the values that the contracts deliver on their own, $\alpha V^1 + (1-\alpha)V^2$, due to concave preferences. The wage level $\hat{w}$ that would yield the worker the latter value would hence be lower than the weighted average of the two wages, $\alpha w^1 + (1-\alpha)w^2$. The firm makes a strictly higher profit paying wage $\hat{w}$ than $\alpha w^1 + (1-\alpha)w^2$. Hence the frontier is strictly concave.

The firm and worker values are differentiable with respect to the constant contract wage and the derivatives equal $-\frac{1}{1-\beta(1-\delta)}$ and $\frac{u'(w)}{1-\beta(1-\delta)}$, both continuous. The derivative of the firm value with respect to the worker value $f_V^{FC}(V, z, V^u)$ is a ratio of the first derivative to the second, substituting in place of the wage the strictly increasing relationship between wage and worker value from $V = \frac{u(w)}{1-\beta(1-\delta)} + \beta\delta E V^u(z') + \dots$ As the derivative of the worker value with respect to the wage is strictly positive, the expression for $f_V^{FC}(V, z, V^u)$ is continuous. Hence the frontier is continuously differentiable. □

Proof of Proposition 5 (Unique Labor Market) Fix z, V^u and assume $f(V^u(z), z, V^u) > k$. One can then solve from $q(\theta)f(V, z, V^u) = k$ for $\Theta(V)$ as a continuously differentiable strictly decreasing function in the neighborhood of $V^u(z)$. Plugging this into the maximand $\mu(\Theta(V))(V - V^u)$ we arrive at a continuously differentiable function of V in this neighborhood. The derivative

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is

$$\mu(\theta)[1-\frac{\mu'(\theta)}{(-q'(\theta)\theta)}\frac{(-f_V(V,z,V^u))}{f(V,z,V^u)}(V-V^u)]. \tag{1}$$

At $V = V^u$ this equals $\mu(\theta) > 0$. The term in brackets is strictly decreasing from $V^u(z)$ to the point $\overline{V}(z)$ where $f(\overline{V}(z), z, V^u) = 0$. There exists a unique point where the derivative equals zero in between.

Because of the discrete time formulation we may need to truncate the matching function to guarantee probabilities between zero and one. This was ignored above. When V is low, $\Theta(V)$ may potentially be so high that $\mu(\Theta(V))$ is above one. For such values of V, we truncate $\mu(\Theta(V)) = 1$. In this case, when V is low, raising its value clearly raises the maximand because there is no effect of the matching probability falling at the same time. The optimum therefore has to lie above this truncation range. Similarly, as V increases eventually $q(\Theta(V))$ will equal one. This gives an upper bound on possible values of V. If the first order condition yields a value outside this feasible range, then we have a boundary solution. (The calibrations used will be such that this does not happen.)

Note that this result does not hinge on Cobb-Douglas preferences. A sufficient condition for the derivative to be strictly decreasing is that $\frac{\mu'(\theta)}{-q'(\theta)\theta} = -1 - (\frac{q'(\theta)\theta}{q(\theta)})^{-1}$ decreases weakly in θ. □

Proof of Proposition 6 (Equivalence with Bargaining) Maximizing the Nash product implies that wage contracts are on the efficient frontier $f(V, z, V^u)$. The necessary condition for optimality of the Nash product

$$\max_V (V - V^u(z))^\alpha f(V, z, V^u)^{1-\alpha}$$

coincides with that of the competitive search equilibrium (see proof of Proposition 5).

Proof of Proposition 8 See Thomas and Worrall (1988). □

The necessary conditions for an optimum are written with the help of the Lagrangian

$$\begin{aligned} L = {} & z - w + \beta \sum_{z'} \pi(z'|z)(1-\delta) f(V(z'), z', V^u) \\ & + \lambda\{u(w) + \beta \sum_{z'} \pi(z'|z)[(1-\delta)V(z') + \delta V^u(z')] - V\} \\ & + \sum_{z'} \pi(z'|z)\beta(1-\delta)\eta(z')(V(z') - V^u(z')) \\ & + \sum_{z'} \pi(z'|z)\beta(1-\delta)\psi(z') f(V(z'), z', V^u) \end{aligned}$$

as follows

$$L_w = -1 + \lambda u'(w) = 0$$

$$L_{V(z')} = \frac{\partial}{\partial V} f(V(z'), z', V^u)(1 + \psi(z')) + \lambda + \eta(z') = 0 \ \forall z'$$

$$\frac{\partial}{\partial V} f(V, z, V^u) = -\lambda$$

Therefore we have

$$\frac{\partial}{\partial V} f(V, z, V^u) = \frac{\partial}{\partial V} f(V(z'), z', V^u)(1 + \psi(z')) + \eta(z'). \tag{A.1}$$

Note that there is a strictly increasing mapping between the promised value and wage for a given state: $\frac{\partial}{\partial V} f(V, z, V^u) = -\frac{1}{u'(w)}$.

Proof of Proposition 10 If the worker's participation constraint binds for some future state z', then (A.1) and $\eta(z') > 0, \psi(z') = 0$ imply $\frac{\partial}{\partial V} f(V, z, V^u) > \frac{\partial}{\partial V} f(V(z'), z', V^u)$. Given the envelope condition, this implies $w'(z') > w$. If the firm's constraint binds, then (A.1) and $\psi(z') > 0, \eta(z') = 0$ imply $\frac{\partial}{\partial V} f(V, z, V^u) < \frac{\partial}{\partial V} f(V(z'), z', V^u)$. Given the envelope condition, this implies $w'(z') < w$. If neither constraint binds, $\eta(z') = \psi(z') = 0$ and $w'(z') = w$. Contract wages remain constant whenever possible, but the participation constraints restrict contract values to the interval $[V^u(z), \overline{V}(z, V^u)]$ for each z. □

References

Stokey, N. L., R. E. Lucas, and E. C. Prescott (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press.

Thomas, J., and T. Worrall (1988): "Self-Enforcing Wage Contracts," *Review of Economic Studies*, 55, 541–554.