Lecture 8:
Mechanics of
Real Business Cycles:

R.G. King, C.I. Plosser and S.T. Rebelo, "Production, Growth and Business Cycles," 2 papers in *Journal of Monetary Economics*, 1988.

Another related paper: R.G. King and S.T. Rebelo, "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics* 1999

Questions posed by KPR

- What are the business cycle properties of the basic neoclassical model of capital accumulation, when it is augmented by productivity shocks?
- What are desirable strategies for computing linear approximation (or loglinear approximation) solutions to dynamic equilibrium models?
- Do these methods continue to work if there is a stochastic trend in productivity, i.e., there are permanent variations in the level of productivity
- If government policy or private market failure leads competitive equilibrium to be suboptimal, are entirely new methods necessary?

Recipe: Part A

1. Start with model which includes technical progress and describes a growing economy (depends on t via deterministic trend growth)
2. Restrict preferences and technology so that steady state growth is feasible
3. As in growth models, find a transformed economy which is stationary (i.e., in which the equations do not depend on t except as subscript)
4. Write down a discrete time Lagrangian
5. Find the FOCs and TVC for this economy
6. Find the stationary point of these FOCS (which surely satisfies the TVC)

Recipe: Part B

1. Linear/loglinear approximation
2. Solve the resulting linear RE model in general: {Y} depends on {X}
3. Calculate the solution under a particular driving process, obtaining a solution in state space form
4. Use this solution to calculate
   a. Comparative dynamics (impulse responses)
   b. Population moments
   c. Stochastic simulations
5. Use economic analysis to interpret solution: e.g., permanent income theory including Fisher's rule for consumption growth and real interest rate

Extensions of Recipe

- KPR88b: To suboptimal equilibria
- KPR88b: To permanent variations in productivity (stochastic trend productivity) motivated by unit root findings
- Later: to all sorts of real and monetary models, including New Keynesian models
- Now routine:
  - Write down FOCs and other conditions of equilibrium
  - Linearize around appropriately defined stationary point
  - Get dynamic outcomes using linear RE techniques

Recipe: Part A

- The original economy: production with deterministic technical progress expressible in labor augmenting form

$$Y_t = A_t F(K_t, X_t N_t) \quad and \quad X_{t+1} = \gamma X_t$$

$$\Rightarrow y_t = A_t F(k_t, N_t) \quad with\, y_t = \frac{Y_t}{X_t} and\, k_t = \frac{K_t}{X_t}$$

[form necessary for ss growth]

Consumption, Investment and Capital Accumulation

$$C_t + I_t = Y_t \ \ and \ \ K_{t+1} = (1-\delta)K_t + I_t$$

$$\Rightarrow c_t + i_t = y_t \ \ and \ \ \gamma k_{t+1} = (1-\delta)k_t + i_t$$

$$with\, c_t = \frac{C_t}{X_t} and\, i_t = \frac{I_t}{X_t}$$

$$- -$$

$$(1-\delta)\frac{K_t}{X_t} + \frac{I_t}{X_t} = \frac{K_{t+1}}{X_t} = \frac{K_{t+1}}{X_{t+1}}\frac{X_{t+1}}{X_t} = \gamma k_{t+1}$$

Labor and leisure

- Per-capita hours of work and leisure must be constant in steady state
- This comes despite fact that wage rate will grow due to technical progress

$$W_t = wX_t = wX_0\gamma^t$$

Preference restrictions necessary for ss growth to be optimal

- Need: constant consumption growth in face of constant ss real interest rate (implied by above)
- Need: constant work level in face of real wage growth
- Utility (general): u(C,L)

Preference restrictions
(sometimes called KPR utility)

- Intertemporal MRS: must be constant elasticity (as in permanent income model)
- MRS between work and leisure, must display invariance to growing consumption and wage rate (offsetting income and substitution effects)

$$u(c,L) = \frac{1}{1-\sigma} c^{1-\sigma} v(L) \; or \; u(c,L) = \log(c) + v(L)$$

Original and modified utility

For first utility function above

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) = (X_0)^{1-\sigma} \sum_{t=0}^{\infty} (\beta\gamma^{1-\sigma})^t u(c_t, L_t)$$

For both utility functions:

(i) X_0 affects welfare but not preferences (over c,L), so set X_0=1 for convenience and abstract from it

(ii) modification of discount factor

Solving optimization problem

- Via Lagrangian

$$L = \sum_{t=0}^{\infty} (\beta^*)^t u(c_t, L_t)$$

$$+\sum_{t=0}^{\infty} (\beta^*)^t \lambda_t [A_t f(k_t, N_t) + (1-\delta)k_t - \gamma k_{t+1} - c_t]$$

$$+\sum_{t=0}^{\infty} (\beta^*)^t \omega_t [1 - N_t - L_t]$$

$$Concepts : (\beta^*)^t \lambda_t \; is\, shadow\; price\, of\;\; c, y$$

$$(\beta^*)^t \omega_t \; is\, shadow\; price\, of\;\; N, L$$

FOCs+TC

$$c_t : (\beta^*)^t [D_1 u(c_t, L_t) - \lambda_t] = 0$$

$$L_t : (\beta^*)^t [D_2 u(c_t, L_t) - \omega_t] = 0$$

$$N_t : (\beta^*)^t [-\omega_t + \lambda_t A_t D_2 f(k_t, N_t)] = 0$$

$$k_{t+1} : (\beta^*)^t [-\lambda_t + \beta^* \lambda_{t+1} (A_{t+1} D_1 f(k_{t+1}, N_{t+1}) + (1-\delta))] = 0$$

$$(\beta^*)^t \lambda_t : [A_t f(k_t, N_t) + (1-\delta) k_t - \gamma k_{t+1} - c_t] = 0$$

$$(\beta^*)^t \omega_t : [1 - N_t - L_t] = 0$$

$$TVC : \lim_{t \to \infty} (\beta^*)^t \lambda_t k_{t+1} = 0$$

Stationary point

- Constrained by

$$c:[D_1 u(c,L)-\lambda]=0$$

$$L:[D_2 u(c,L)-\omega]=0$$

$$N:-\omega+\lambda AD_2 f(k,N)=0$$

$$k:[-\lambda+\beta^{*}\lambda(AD_1 f(k,N)+(1-\delta))]=0$$

$$\lambda:[Af(k,N)+(1-\delta)k-\gamma k-c]=0$$

Recipe: Part B

- Linearization/loglinearization

$$\xi_{cc}\hat{c}_t + \xi_{cL}\hat{L}_t - \hat{\lambda}_t = 0$$

$$\xi_{Lc}\hat{c}_t + \xi_{LL}\hat{L}_t - \hat{\omega}_t = 0$$

$$-\hat{\omega}_t + \hat{A}_t + \xi_{nk}\hat{k}_t + \xi_{nk}\hat{k}_t = 0$$

$$N * \hat{N}_t + L * \hat{L}_t = 0$$

$$-\hat{\lambda}_t + \hat{\lambda}_{t+1} + \eta_A \hat{A}_{t+1} + \eta_k \hat{k}_{t+1} + \eta_N \hat{N}_{t+1} = 0$$

$$-[\hat{A}_t + s_N \hat{N}_t + s_k \hat{k}_{t+1}] + [s_c \hat{c}_t + s_i \phi \hat{k}_{t+1} - s_i(\phi - 1)\hat{k}_t] = 0$$

Solution of Linear Model

- Stage 1: Relate endogenous variables to sequences of exogenous variables (as in Blanchard-Kahn)

$$\hat{k}_{t+1} = \mu_1 \hat{k}_t + \psi_1 \hat{A}_t + \psi_2 \sum_{j=0}^{\infty} \mu_2^{-j} E_t \hat{A}_{t+j+1}$$

- Stage 2: Assuming driving process and evaluate discounted sums

$$\hat{A}_t = \rho \hat{A}_{t-1} + e_t$$

$$\hat{k}_{t+1} = \mu_1 \hat{k}_t + [\psi_1 + \psi_2 \frac{\rho}{1-(\rho/\mu_2)}]\hat{A}_t = \mu_1 \hat{k}_t + \pi_{kA} \hat{A}_t$$

$$Y_t = \Pi s_t \qquad\qquad s_t = M s_{t-1} + G e_t$$

- Linear difference system (Bellman)
- State space model (Hamilton)
- RE solution (from BK or KW)
- Used for calculations of responses to one-time shocks (impulse responses); stochastic simulations; calculations of moments…

$$\begin{bmatrix} y_t \\ c_t \\ i_t \\ n_t \\ \ldots \end{bmatrix} = \begin{bmatrix} \pi_{yk} & \pi_{ya} \\ & \\ & \\ & \\ & \end{bmatrix} \begin{bmatrix} k_t \\ a_t \end{bmatrix}$$

$$\begin{bmatrix} k_t \\ a_t \end{bmatrix} = \begin{bmatrix} \mu & \pi_{ka} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} k_{t-1} \\ a_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e_t$$

Substantive conclusions from KPR

- Productivity shocks (A) must be persistent to generate business cycle phenomena: these give rise to important wealth effects on consumption.
- Although labor is invariant to trend growth in productivity (X), it varies sharply in response to productivity shocks (A)
- Large productivity variations are necessary to produce substantial volatility in output [not stressed in paper, but implicit in graphs].

Do we need large productivity shocks?

- KR in *Handbook of Macroeconomics* review developments from 1988 to 1999 when there was:
  - A huge number of RBC studies
  - A rising concern that productivity shocks measured via Solow residual were “too big” and “too important” to RBC modeler conclusions

KR strategy

- Modified basic 1988 model to introduce several key features that had previously been studied separately
  - Labor choice on extensive margin (to work or not) rather than intensive margin (how many hours to work)
  - Varying utilization of capital

Implications

- Model economy:
  - Solow residual as poor proxy for actual productivity shock (utilization not observed)
  - High response to actual productivity shocks (so the implied shocks could be smaller)
  - Alternative method of extracting shocks