Expectations, Learning and Macroeconomic Persistence

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Abstract

Monetary DGSE models under rational expectations typically require large degrees of features as habit formation in consumption and inflation indexation to match the inertia of macroeconomic variables.

This paper presents an estimated model that departs from rational expectations and nests learning by economic agents, habits, and indexation. Bayesian methods facilitate the joint estimation of the learning gain coefficient together with the 'deep' parameters of the economy.

The empirical results show that when learning replaces rational expectations, the estimated degrees of habits and indexation drop closer to zero, suggesting that persistence arises in the model economy mainly from expectations and learning.

Keywords: persistence, constant-gain learning, expectations, habit formation, inflation inertia, Bayesian econometrics, New-Keynesian model.

JEL classification: C11, D84, E30, E50, E52.

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1. Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a popular tool for the analysis of the monetary transmission mechanism.[1] These models are built under the hypothesis of *rational expectations* and assume intertemporal optimizing behavior by economic agents. Being derived from explicit microeconomic foundations, they facilitate policy evaluation in terms of the welfare of private agents. Unfortunately, the canonical monetary models with rational expectations often cannot match the observed behavior of macroeconomic variables, and, in particular, they fail to match the **persistence** of aggregate output and inflation.

Economists have therefore proposed a number of extensions to the standard framework by embedding potential sources of endogenous persistence. They have incorporated features such as habit formation in consumption, indexation to lagged inflation in price-setting, rule-of-thumb behavior, or various adjustment costs. Christiano, Eichenbaum, and Evans (2005) incorporate several of these extensions and can account for the inertia in the data. Smets and Wouters (2003, 2005) estimate similar models by Bayesian methods, incorporating a mix of frictions and persistent structural shocks, and obtain a remarkable fit of the data. Also, Boivin and Giannoni (2005) and Giannoni and Woodford (2003), in smaller models, but which still incorporate additional sources of persistence, derive impulse responses that approximate those derived from VARs.

The cited extensions essentially improve the empirical fit by adding lags in the model equations. Researchers estimating these rich models under the assumption of rational expectations typically find that substantial degrees of habit persistence and inflation indexation are supported by the data. Those additional sources of persistence appear, therefore, necessary to match the inertia of macroeconomic variables.

1.1. ***Contribution of the paper***

This paper suggests a different direction, by revisiting the expectations formation of the agents. The paper departs from the conventional rational expectations assumption. Agents in the model form expectations using correctly-specified economic models, but they do not have knowledge about the model parameters. They use historical data to learn those parameters over time, updating their beliefs through constant-gain learning. The paper then evaluates the potential for **learning** as a mechanism that can endogenously generate persistence in the economy and improve the fit of current monetary DSGE models. More in detail, the paper aims to disentangle the role of learning versus 'mechanical' sources of persistence,[2] such as habits and indexation, in generating persistence in macroeconomic variables.

The paper starts by taking an agnostic view. The model *nests* different sources of persistence: learning by private agents along with the 'mechanical' sources of persistence, such as habit formation in consumption and indexation to past inflation in price-setting, which are essential under rational expectations to account for the observed persistence. It is left to the data to disentangle the role of the various sources. The scope is to test whether those mechanical sources of persistence are still necessary to match the data when the assumption of rational expectations is relaxed in favor of learning.

The model is estimated using likelihood-based Bayesian methods. The econometric approach allows me to *jointly* estimate the coefficients describing agents' learning, such as the gain coefficient (indicating their learning speed), together with the 'deep' parameters of the economy. This strategy responds to a potential criticism of models with learning, in which the results might depend on the parameters that need to be chosen by the researcher. Here the learning speed is, instead, jointly estimated with the rest of the system.

In providing an empirical analysis of the importance of learning, the paper builds on previous literature on adaptive learning in macroeconomics. Not many studies have analyzed the empirical implications of adaptive learning. At the earlier stages, this literature was mainly theoretical and focused on convergence of the models to the Rational Expectations Equilibrium (REE).[3] More recently, a number of papers[4] have

[1] Clarida, Gali', and Gertler (1999), Goodfriend and King (1997), McCallum and Nelson (1999), and Woodford (2003) are standard examples describing dynamic general equilibrium models for monetary policy analysis.

[2] The paper refers to them as 'mechanical' since in the case of habits, researchers need to alter the consumers' utility function to imply dependence on lagged consumption, and in the case of indexation, they posit a rule to induce inertia through the assumption that a fraction of firms simply adjust prices automatically, according to the past observed inflation rate.

[3] Evans and Honkapohja (2001), Bullard and Mitra (2002), and Preston (2005) are examples that verify the learnability of the REE in monetary models.

[4] Branch et al. (2004), Bullard and Eusepi (2005), Orphanides and Williams (2005b), Primiceri (2003), Sargent (1999), and

employed learning to analyze the evolution of U.S. inflation and monetary policy. These papers share the use of learning as a tool that can help in understanding some particular historical episodes, which are often harder to explain under rational expectations.

The present paper tries, instead, to provide a more general empirical study of the effects of learning. Its scope is akin to the work by Williams (2003), who studies the implications of learning for persistence and volatility in simple calibrated real and monetary business cycle models. The present paper shares his scope of studying the effects of learning, but it exploits, instead, actual time series data. This allows me to verify if learning is supported by the empirical evidence and to compare the model with learning with alternative descriptions of the economy. The paper is also related to the recent work by Adam (2005), who likewise assumes that economic agents use simple econometric models to forecast macroeconomic variables and shows how deviations from rational expectations may strengthen the internal propagation mechanism of a simple business cycle model.

Similarly to recent empirical papers in macroeconomics,[5] this paper adopts Bayesian methods in the estimation. The techniques are similar to those used by Schorfheide (2000, 2005) and Lubik and Schorfheide (2004, 2005), among others. But Schorfheide (2000), as well as several papers that share the same techniques, estimate DSGE models under rational expectations.[6] The current paper, instead, provides the first example of the use of Bayesian methods to estimate a DSGE model with non-fully rational expectations and learning. This represents a methodological contribution of the paper. Bayesian methods are appealing in this context because they facilitate the joint estimation of the learning parameters together with the rest of the system.

A potential criticism of models with adaptive learning, also discussed in Marcet and Nicolini (2003), emphasizes the arbitrary choices, often available to the researcher, which render the model hardly falsifiable. Milani (2004a), for example, shows how estimates strongly vary over the range of possible gain coefficients. In the present paper, instead, the gain coefficient is also estimated, leaving less room for arbitrariness.

More generally, by estimating a DSGE model with learning, the paper provides an example of a 'Non-Rational Expectations Econometrics', which Ireland (2003) judged as missing from the branch of the literature that studies, usually theoretically, the impact of learning in macroeconomics.

1.1.1. ***Results***

The empirical results show that the essential role of mechanical sources of persistence (habits, indexation) in DSGE monetary models rests on the assumption of rational expectations. When agents are allowed to learn the true parameters of the economy over time, habits and indexation are no longer essential, being estimated at values close to zero in the data. This finding suggests that learning can represent an important source of persistence in the economy. Indeed, learning might represent a *single* mechanism capable of creating persistence, replacing the features needed in various sides of the conventional rational expectations model to improve its empirical properties. Furthermore, the posterior model probabilities show that the specification with learning fits better than the specification with rational expectations.

Sargent, Williams and Zha (2004), among others.

[5] An and Schorfheide (2006) provide a first review of this literature.

[6] Schorfheide (2005) assumes an incomplete information model in which agents need to update their beliefs about the inflation target using a Bayesian learning rule. In his model, however, agents still form fully-rational expectations.

2. A Simple Model with Learning and Structural Sources of Persistence

The aggregate dynamics of the model is given by the following specification, nesting learning and structural sources of persistence as habit formation and inflation indexation[7]

$$\widetilde{x}_t = \widehat{E}_t\widetilde{x}_{t+1} - (1-\beta\eta)\sigma\left[i_t - \widehat{E}_t\pi_{t+1} - r_t^n\right] \tag{1}$$

$$\widetilde{\pi}_t = \xi_p\left[\omega x_t + \left[(1-\eta\beta)\sigma\right]^{-1}\widetilde{x}_t\right] + \beta\widehat{E}_t\widetilde{\pi}_{t+1} + u_t \tag{2}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left[\chi_\pi\pi_t + \chi_x x_t\right] + \varepsilon_t \tag{3}$$

where

$$\widetilde{\pi}_t \equiv \pi_t - \gamma\pi_{t-1} \tag{4}$$

$$\widetilde{x}_t \equiv (x_t - \eta x_{t-1}) - \beta\eta\widehat{E}\left(x_{t+1} - \eta x_t\right) \tag{5}$$

and where x_t denotes the output gap, π_t denotes inflation, i_t denotes the nominal interest rate, and r_t^n, u_t, and ε_t denote demand, supply, and monetary policy shocks. Equation (1) is the log-linearized Euler equation that arises from households' consumption decisions under (internal) habit formation; $\beta \in (0,1)$ is the household's discount factor, $\sigma > 0$ is the elasticity of intertemporal substitution of consumption in the absence of habits, and $0 \leq \eta \leq 1$ measures the degree of habit formation. Current output gap depends on lagged and expected one-period and two-period ahead output gaps, and on the *ex-ante* real interest rate. Equation (2) is the Phillips curve that arises from optimal Calvo price-setting, when firms that cannot re-optimize are allowed to follow an indexation rule, as proposed by Christiano, Eichenbaum, and Evans (2005). Coefficient ω denotes the elasticity of the marginal disutility of producing output with respect to an increase in output, ξ_p is a parameter that is inversely related to the degree of price stickiness, and $0 \leq \gamma \leq 1$ represents the degree of indexation to past inflation. Current inflation depends on lagged and one-period ahead inflation, and on current, lagged, and one-period-ahead output gap (with habit formation, in fact, the log marginal utility of real income entering the Phillips curve is a linear function of x_t and $\widetilde{x}_t$ rather than a linear function of x_t alone). Monetary policy is described by equation (3), which is a Taylor rule with partial adjustment, where ρ is the interest-rate smoothing term, and χ_π and χ_x are feedback coefficients to inflation and output gap.

In the model, $\widehat{E}_t$ indicates subjective (possibly non-rational) expectations, while the usual mathematical expectation operator E_t is left to denote model-consistent rational expectations.

The natural real interest rate and the cost-push shocks evolve according to univariate AR(1) processes

$$r_t^n = \phi^r r_{t-1}^n + \nu_t^r, \quad \nu_t^r \sim iid\left(0, \sigma_r^2\right) \tag{6}$$

$$u_t = \phi^u u_{t-1} + \nu_t^u, \quad \nu_t^u \sim iid\left(0, \sigma_u^2\right). \tag{7}$$

2.1. Expectations Formation: Constant-Gain Learning

As made clear by equations (1) and (2), agents need to form forecasts of future macroeconomic conditions. Following recent learning literature, the agents are assumed to behave as econometricians, employing an economic model and forming expectations from that model.

Agents estimate

$$Z_t = a_t + b_t Z_{t-1} + c_t u_t + d_t r_t^n + \varepsilon_t \tag{8}$$

using variables that appear in the Minimum State Variable (MSV) solution of the system under rational expectations (defining $Z_t \equiv [\pi_t, x_t, i_t]'$ and where a_t, b_t, c_t, d_t are coefficient vectors and matrices of appropriate dimensions). Therefore, the agents employ a correct model of the economy, but they do not have

[7] The reader is referred to Milani (2004b) for a full derivation of the model. As in most papers in the adaptive learning literature (see Evans and Honkapohja 2001 for a general treatment), the loglinearized equations are similar to those obtained under rational expectations, but with a different expectations operator. For a different approach of considering learning, see Preston (2004, 2005), who introduces learning directly from the primitive assumptions of multi-period decision problems. Preston's approach is followed in Milani (2004b), leading to similar estimation results.