DOES IT PAY TO GET A REVERSE MORTGAGE?

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Many of America's elderly are considering reverse mortgages as a way to relieve their financial pressures. These financial instruments let homeowners a) remain in their homes for as long as possible and b) borrow against their home equity at terms that include large fixed fees and high interest rates. Repayment of this borrowing is triggered by moving, is repaid out of house sale proceeds, and is capped by the value of those proceeds.

Reverse mortgagees who borrow sums that are large relative to their house values and remain in their homes for extended periods of time win this gamble. They enjoy the use of their homes and the borrowed money and simply hand over the keys when they exit their homes, be it vertically or horizontally, for the last time. Reverse mortgagees who borrow large sums and whose home exits occur early lose this gamble. Thus, reverse mortgages constitute the purchase of a no-exit annuity – an annuity that pays off in the form of the housing services of your current home provided you don't permanently exit your home. Since not exiting is partly conditioned on not dying, the no-exit annuity encompasses some longevity insurance. But it also introduces extra risk associated with exiting the home prior to death.

This paper uses single households from the Health and Retirement Study (HRS) data to study the economic gains or losses associated with taking out reverse mortgages. These data are examined within a dynamic structural life-cycle model featuring consumption, housing, and mobility decisions. These decisions are made in light of lifespan and mobility uncertainty. Model solution and estimation are based on the Mathematical Programming with Equilibrium Constraints approach.

We find that seniors are relatively high risk adverse and home equity is their most important component of precautionary savings. In addition, reverse mortgages are a very bad option for house-rich, but cash-poor households. For such households, taking out the standard reverse mortgage and borrowing the maximum permitted amount reduces expected utility, on average, to the same degree as a 120 percent loss in financial assets. For house-rich and cash-rich households, reverse mortgages raise expected utility, on average, to the same degree as a 47 percent increase in financial assets.

The intuition for these findings is that reverse mortgages have both risk mitigating and risk expanding properties. In particular, they ease the liquidity problem and provide a form of longevity insurance. However, they introduce a new risk: the moving risk. The moving risk together with the lack of diversification are the main causes of welfare losses for house-rich but cash-poor households.

KEYWORDS: Housing, Elderly Mobility, Precautionary Savings, Dynamic Discrete and Continuous Choices, Constrained optimization approach

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1 Introduction

This paper estimates a dynamic life-cycle model of retiree consumption, housing and moving decisions. We focus on the estimation of the elderly structural preference parameters using a subsample of single retirees from the Health and Retirement Study (HRS). We first study the optimal choice of consumption, housing and elderly mobility in the absence of a reverse mortgage. Then, we calculate the welfare gain for HRS respondents in taking out a reverse mortgage. We are motivated by two empirical evidences.

First, baby boomers started retiring in 2001, signing the beginning of an accelerated rate of growth in the number of retirees in the US. By 2030, one out of five people is projected to be 65 years or older. Despite this rapid augment in the retired population, the financial situation for future retirees remains uncertain. As a matter of fact, increases in the cost of living and in health care costs, curtailments in medical coverage and other employee benefits plans and cutbacks in Social Security benefits expose many of today's households at risk of having to adjust to a decreased standard of living in retirement. An analysis of their financial portfolio shows clearly that for most retirees their house is their major asset. More than 80 percent of older households own their homes (Munnel et al, 2007), which are worth approximately $4 trillion. Economists and policymakers look at these financial assets as a potential source of savings to finance consumption in the elderly. In the traditional life-cycle model that builds on Modigliani and Brumberg (1954), Ando and Modigliani(1963), and Friedman (1957) individuals save in order to smooth consumption over the course of their lives which is optimal under the standard assumption of concave utility. Specifically, households build savings during their working years and divest those savings to support consumption in retirement. However, when it comes to home equity, this pattern is not followed. Typically, older households do not divest home equity. Instead, homeownership rates remain stable until late in life and median home equity increases with age as older homeowners pay off mortgages and home value appreciates. Before the advent of the reverse mortgage, only two alternatives were available to older homeowners to divest their home equity. They could sell and move out or they could borrow against their house property taking a conventional loan, such as mortgage or home equity loan. Traditional loans have to be repaid, either through installments or on maturity and older homeowners are often neither eager nor able to incur in new monthly obligations. Therefore, selling and moving out represented the best way to cash in the savings locked up in residential property.

Second, in the 1990s, reverse mortgages became available and provided a new way to convert home equity into cash. A reverse mortgage is a financial instrument that allows borrowers to access the equity in their home that would otherwise not be liquid, by providing income while not requiring payment as long as the borrower lives in the same house. When the retiree moves out or dies, the reverse mortgage lender keeps the minimum between the house value and the outstanding debt. The amount of money that could be borrowed via a reverse mortgage generally depends on the borrower's age and the value of the home. The minimum age for almost all reverse mortgage programs is 62. A 2005 study by Stucki estimated the potential market at 13.2 million older households. However, at the end of 2007, only 265,234 federally insured reverse mortgages were issued (Department of Housing and Urban Development, 2007b). This represents about 1% of the 30.8 million households with at least one member age 62 and older in 2006 (U.S. Census Bureau,2006). Reverse mortgage was designed with house-rich but cash-poor people in mind, but they haven't bought it. Therefore, we ask the following policy questions: "Why house-rich but cash-poor households did not buy this financial instrument?" and, hence,"Does it pay to get a reverse mortgage?".

Assessing the potentiality of reverse mortgages requires jointly analyzing consumption, housing and moving decisions. This paper presents a structural dynamic model of these decisions. Moving for financial reasons is only one type of elderly mobility. Other types of elderly mobility include moving for assistance reasons, changes in marital status, health problem or services, climate or weather, desire to change neighborhoods or the location, shopping or other consumption services, and public transportation. In this paper we study a dynamic life-cycle model where retirees optimize over housing and consumption. They acquire housing services by either owning or renting the house they live in. We incorporate in the model a housing preference shock, hence retirees can change their housing tenure and value incurring in a one-period transaction cost. The main sources of income during retirement are social security, pensions and investment income. Given our focus on retirement, income from labor is not considered.

Statistical and demographic data on reverse mortgagees are not available, consequently we select a subsample of single retirees from the Health and Retirement Study that could represent a potential target segment for this financial instrument. Empirical evidence shows that retirees support their consumption with social security income and tend not to divest their home equity. The non-housing financial assets are a small fraction of the house value. Our subsample includes both discrete and continuous data. Therefore, in this paper we are able to extend the literature on discrete choice processes by including also continuous choices. There is an extensive literature focusing on the solution of discrete choice models, however both the theoretical literature and most of the subsequent applications focus on discrete decision processes. To our knowledge, this is one of the first attempts to estimate a dynamic structural model with both continuous and discrete choice and state variables.

We address the estimation difficulties by formulating the empirical dynamic life-cycle model as a Mathematical Problem with Equilibrium Constraints. The past decade has seen an enormous increase in computer speed and advances in algorithms and software for large-scale problems. These technological advances allows the modeler to solve realistic large-scale economic models. Nowadays, even though nonlinear large-scale and optimization problems arise in many economic applications, very few economic problems have been analyzed using mathematical programming approaches. This paper solves an economic policy question using cutting-edge methods in computational science and state-of-the-art software and represents an example of interaction between economics and computational science. We first present an application of the Mathematical Programming with Equilibrium Constraints (MPEC) approach to a life-cycle model and then we introduce the structural estimation. Our MPEC approach to finite-horizon dynamic programming problems allows us to simultaneously find the optimal consumption and other choice variables in each period. This approach, creating a link between past, current and future economic variables, could be used to solve any consumption saving problem of adequate complexity. In addition, the conventional approach to structural estimation consists in repeatedly taking a guess of the structural parameters and solving the dynamic programming problem until the log-likehood is maximized. This could be computationally very demanding. With the MPEC approach we formulate the dynamic programming problem and the structural estimation as a constrained optimization problem. The unique equilibrium that needs to be solved is the one associated with the optimal value of the structural parameters and other endogenous variables. While this paper constitutes the first application of the MPEC approach to an empirical structural model with finite horizon dynamic programming, this method can be used in a wide range of dynamic programming and structural estimation problems. Specifically, three main features of the MPEC approach could be appealing to any economist interested in solving complex models. First, the implementation of the MPEC approach is particularly easy. The researcher needs just to specify an objective function and a set constraints. Second, it is very fast. Our structural dynamic problem involves more than 70,000 variables and could be solved in an amount of time that ranges between 10 and 60 minutes. Third, we use state-of-art methods and software developed by computational scientists and mathematicians highly regarded for their robustness and efficiency.

This paper yields two main findings.

First, we obtain reasonable estimates for the structural preference parameters. Specifically, retirees are relatively high risk adverse and greatly value their house compared to consumption. This parameter configuration suggests that they prefer to make safe investments and save against unexpected shocks. The house is a safe and illiquid asset that would prevent a quick access to the resources accumulated in the working period. Therefore, home equity is the most important component of precautionary savings in retirement.

Second, our model explains why house-rich but cash-poor households have not bought reverse mortgages with issues related to the moving risk and the high up-front cost. Reverse mortgage provides liquidity and a form of longevity insurance, however, moving becomes a risky proposition. As a matter of fact, if homeowners move out, they have to repay the minimum between the house value and the outstanding debt. Both consumption and housing profiles are affected in the periods following the move. The moving risk and the lack of diversification generate welfare losses for house-rich but cash-poor households equal to a 120 percent decrease in their initial non-housing financial assets.

The findings that retirees' precautionary savings are locked in the house and that reverse mortgages are risky financial instruments together with the empirical evidence that retirees tend to not divest their home equity, finance their consumption mostly with social security income and tilt their financial portfolio towards safe assets might explain why, after almost twenty years from its first appearance, the reverse mortgage is still a niche product.

The structure of the paper is as follow. Section 2 contains the literature review. Section 3 explains the features of a reverse mortgage contract, evaluates the lender's expected gain and provides some empirical evidence about reverse mortgagees. Section 4 presents the household's life-cycle model. Section 5 describes the solution method. Section 6 illustrates the HRS data. Section 7 contains the results and the welfare analysis. Section 8 illustrates some policy experiments. Section 9 concludes.

2 Literature Review

This paper draws on three main strands on literatures: life-cycle and precautionary savings, housing and portfolio choice and discrete choice.

We build on the studies of life-cycle behavior in Kotlikoff and Summers (1981), Carroll and Summers (1991) and Kotlikoff et al. (2001). Hubbard, Skinner, and Zeldes (1994) and Carroll (1997) parameterize and simulate life-cycle consumption models with uncertainty. Gourinchas and Parker(2002) estimate a structural model of optimal life-cycle consumption expenditures in the presence of realistic labor income uncertainty. Cagetti(2003) structurally estimates a model of wealth accumulation over the life cycle. French (2005) estimates a life-cycle model of labour supply, retirement, and savings behavior in which future health status and wages are uncertain. Hubbard, Skinner, and Zeldes (1994), Palumbo (1999) and Hurd (1989) represent good attempts at modelling the consumer behavior after retirement. However, in these papers housing is not taken into account. Given the empirical evidence that for most retirees the house is their major asset, we extend this strand of literature analyzing the optimal consumption and housing choice for older households.

Specifically, we follow Cocco (2005) and Yao and Zhang (2005a,b) by explicitly modeling the housing decision and allowing households to derive utility from both housing and other consumption goods. They assume constant interest rates, do not consider bonds and incorporate a stochastic labor income stream. Campbell and Cocco (2003) study the optimal choice between a fixed-rate and an adjustable-rate mortgage. Types and determinants of elderly mobility have been studied in Meyer and Speare (1985).

Additionally, we build on the literature on discrete choice models. The framework was introduced by Rust (1987,1988) and then extended in Hotz and Miller (1993) and Aguirragabiria and Mira (2002). However, most of the theoretical papers and the empirical applications focus only on discrete choice processes. Given that our data involve both discrete and continuous choice we extend this literature by including also continuous choices.

Finally, we follow Judd and Su (2008) who applied the Mathematical Programming with Equilibrium constraints to estimate the Zucher bus model (Rust, 1987). In this paper we present the first application of the MPEC to an empirical structural model with finite horizon dynamic programming.

3 Reverse Mortgage

The reverse mortgage market was created in 1987 with the HUD (Department of Housing and Urban Development) program called Home Equity Conversion Mortgage (HECM). The United States Congress passed FHA (Federal Housing Administration) Reverse Mortgage Legislation, the Housing and Community Development Act of 1987, (S. 825) on December 22, 1987. President Ronald W. Reagan signed FHA Reverse Mortgage Legislation (S. 825) on February 5, 1988. The first FHA Reverse Mortgage was made to Marjorie Mason, of Fairway, KS by James B. Nutter & Company on October 19, 1989. In 1996 the Federal National Mortgage Association (Fannie Mae) created the Home Keeper reverse mortgage to address needs unsatisfied by the HECM program, such as individuals with higher property values, condominium owners, and seniors wishing to use a reverse mortgage to purchase a new home. These two reverse mortgages allow nearly every senior citizen to access the equity in her home without moving out or taking a conventional mortgage.

We briefly present the main features and requirements of the HECM senior reverse mortgage program.

Reverse mortgages are a special type of home equity loan that allow owners to convert some of the equity in their homes to cash. The loan does not have to be repaid as long as the borrower lives in the house. To be eligible for a reverse mortgage, a borrower must be 62 or older, own the home outright (or have a low loan balance) and have no other liens against the home. The retiree does not have to satisfy any credit or income requirements. The borrower can receive the proceeds in one of the following ways: a lump sum at the beginning, monthly payments until a fixed term or a life-long annuity, by establishing a credit-line with or without accrual of interest on the credit balance, or a combination of the aforementioned. There are no monthly or other payments to be made during the term of the loan, however a reverse mortgage accrues interest charges, beginning when the first payment is made to the borrower. When she dies or relocates, the minimum between the house value and the loan plus the cumulated interest has to be repaid. Even if the accumulated loan and interest exceed the realizable value of the house at disposal, the repayment is capped at that value only (nonrecourse loan). The amount of loan is a function of the age of the borrower and any co-applicant, the current value of the property and expected property appreciation rate, the current interest rate and interest rate volatility, closure and servicing costs and other specific features chosen.

A reverse mortgage is just one of several financial instruments that allow a homeowner to secure liquid funds against the equity in a house. In general, Home Equity Conversion Products could be useful to all those who are house-rich but cash-poor. [1] Conventional home equity loans are different from reverse mortgages in four main respects. First, they require the payment of interests and some principal before moving. Second, the maximum amount of money that can be borrowed is determined by several variables including credit history and income. Third, the failure to repay the loan or meet loan requirements may result in foreclosure. Fourth, the closing cost are generally lower.

In the early 1990s, projections of potential demand for reverse mortgages varied between 800,000 older households (Merrill, Finkel, and Kutty, 1993) and more than 11 million older households (Rasmussen et al., 1995). A more recent study (Stucki, 2005) estimated the potential market at 13.2 million older households. Moving from the potential market to the actual market, at the end of 2007, only 265,234 federally insured reverse mortgages were issued (Department of Housing and Urban Development, 2007b). This represents about 1% of the 30.8 million households with at least one member age 62 and older in 2006 (U.S. Census Bureau, 2006) and about 2% of the potential market as estimated by Stucki.

3.1 Lender's Perspective

We assume that the reverse mortgage borrower i chooses to receive the proceeds as a lump sum at the closure of the contract in time j.

The maximum amount that the household can initially borrow V_{it} is assumed to be a fraction of the house value and of the borrower's age. In general, the higher the age of the borrower, the larger is the amount that can borrowed.

$$V_{ij} = \kappa_i H_{ij} \tag{1}$$

At the closure of the contract, the retiree has to pay some up-front costs, which we denote as F. They are assumed to be a fraction of the house value plus some additional cash for closing costs f. Specifically, they include an origination fee that covers the lender's operating expenses (2% value of the house), an up-front mortgage insurance premium MIP (2% value of the house), an appraisal fee and certain other standard closing costs (about $2000-4000).

[1] The Home Equity Conversion Products include the following products, in addition to reverse mortgage. Home reversion / sale and lease back allows the homeowner to sell his house outright now, but she keeps the right to live in it for life for a nominal/reduced rent. The sale profits could be paid in a lump sum or as an annuity. The interest-only mortgage allows the borrower to get an immediate lump sum. She is required to make only interest payments during the tenure of the loan and the principal is due only on maturity or death or a permanent move or sale. The mortgage annuity/ home income enables the individual to use the loan amount buy a life annuity. The interest on the mortgage is deducted from the annuity and the balance is paid as periodic income. The principal is repaid on death or sale of the house. The shared appreciation mortgage provides loans at a below market interest rate. The lender obtains a pre-agreed share in any appreciation in the property value over the accumulated value of the loan The loan is due at death or moving or sale.

$$F = \lambda H_{ij} + f \tag{2}$$

Historical closing costs have been significantly large compared to the conventional home equity loan and this has been cited as one of the main motifs for the relative weakness of the demand. Part of the reason for the high up-front cost has been the MIP charged by the Federal Housing Authority (FHA, a subsidiary of HUD). In addition to the initial MIP, FHA charges an ongoing 0.5% annual premium on the loan balance. By charging MIPs, HUD insures the borrower against the risk of not being able to access her loan funds if the lender goes out of business. Additionally, it insures the lender against the risk that the resale value at termination is less than the outstanding loan. Therefore, since the FHA bears the risk of default, the up-front insurance premium paid to the FHA are significantly larger than the conventional insurance payments made on conventional loans. Until now, house price growth and rapid mobility have left FHA with small losses and substantial reserves.

Let $\overline{B}$ denote the cash available to borrower i at time j, after the payment of the up-front costs. $\overline{B}$ is the lender's initial cost.

A reverse mortgage accrues interest charges, beginning when the first payment is made to the borrower and then the interest is compounded annually. Let G_{it} be the outstanding debt at time t:

$$G_{it} = \overline{B} \sum_{j=1..t} (1 + i_D)^{t-j} \tag{3}$$

i_D is the nominal interest rate on reverse mortgage. In present value, the repayment in period t for household i is:

$$D_{it} = \frac{\min(H_{it}, G_{it})}{R^{t-j}} \tag{4}$$

If the borrower moves out of the house or dies at time t, that person would be required to repay the minimum between the house value and the outstanding debt.

Let $n_{i,t}$ household i's probability of being alive at time t and $m_{i,t}$ her probability of moving at time t. The expected gain for the lender is:

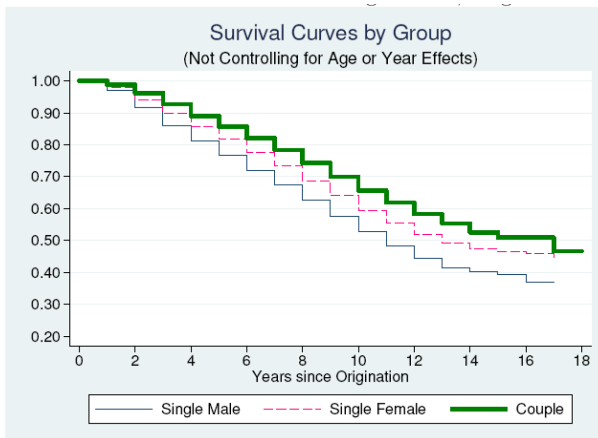
$$EGain_{j,i} = F + \sum_{t=j+1..T} n_{i,t-1}\{(1 - n_{i,t})(1 - m_{i,t}) + n_{i,t}m_{i,t}\}D_{it} \tag{5}$$

A simple calculation, without taking into account interest rate risk, house price risk and possibility of adverse selection, shows that a homeowner with a house value equals to \$100,000 could borrow about \$47,000, \$31,000 , or \$10,000 respectively if she closes a Monthly Adjusting HECM, a Annually Adjusting HECM, and a Fannie Mae HomeKeeper contract at age 62. This represents the actual cost for the lender. Given women survival probabilities and US mobility rate, the expected gain for the lender is about \$74,000, \$64,000, \$ 30,000.

3.2 Empirical Evidence on Reverse Mortgage Borrowers

Statistical and demographic data about reverse mortgage loan borrowers is not currently available. However, in December 2006, AARP conducted the first national survey of reverse mortgage borrowers and homeowners who had considered these loans but decided against them. We briefly summarize their findings.

Between 1993 and 2004, the median annual income of reverse mortgage borrowers increased from \$12,289 to \$18,240 (HUD, 2007b). The self-reported income data from the AARP Survey shows that a third of borrowers (33 percent) reported incomes of less than \$20,000, and nearly two-thirds (62 percent) reported incomes of less than \$30,000. According to census data, the median net worth among the general population of older households, excluding home equity, was \$23,369 in 2000; among households age 75 and older, median net worth was only \$19,025 (He et al., 2005). More than half of reverse mortgage borrowers in the AARP survey (54 percent) reported having less than \$25,000 in financial savings, but their average net worth is not available. On average, reverse mortgage borrowers are more likely to be house-rich than typical older homeowners. Close to half of reverse mortgage borrowers (46 percent) have homes worth \$100,000 to \$199,999, compared with only about one-third of general homeowners (34 percent). Average property values of HECM borrowers is \$142,000 in 2000, while the median house value is \$65,624 for households without HECM. More than half (57 percent) of HECM borrowers in 2000 are single women. Bowen et al (2008), using all the 18 years of HECM loan data, present the first systematic evidence on loan characteristics over time. Their data do not describe income, financial wealth and consumption of borrowers. Figure 1 and 2 are from Bowen et al (2008). Figure 1 presents the loan survival curves for single male, single female and couples. Figure 2 shows the termination hazard rates corresponding to the survival curves plotted in Figure 1. The inverse-U shape of these hazard rates implies that termination hazard is low in the years immediately after the closure of the contract and then increases with time. Additionally, if the loan has not been terminated within 10 years, termination hazard reduces with time. This suggests that if the borrower has not died or moved in the first 10 years of the contract, she will likely stay in the home for a very long time. Davidoff et al. (2007) shows that, empirically, reverse mortgagees have exited homes unusually rapidly. Only 66% for men and 62% for women of these loan terminations are attributed to death as opposed to payoff while alive.



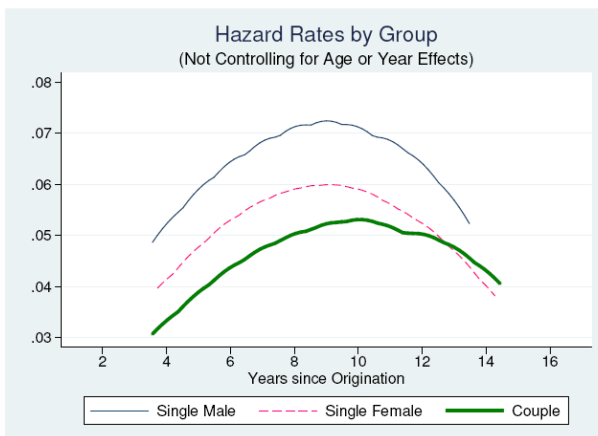


Figure 1: Survival Curves of HECM Loans for Single Males, Single Females, and Couples (Bowen et al.,2008)

Figure 2: Termination Hazard Rates of HECM Loans for Single Males, Single Females, and Couples (Bowen et al.,2008)

4 The Model

This section describes a model of post-retirement decision making. We consider the optimal consumption, housing and moving decisions for a single retiree. The individual dynamically chooses consumption, housing tenure and housing value. When she decides to move, i.e. change in housing tenure and house value, transaction costs are incurred.

4.0.1 Preferences

Individual i's plan is to maximize her expected lifetime utility at age t, $t = 64, ..., T$. T is set exogenously and equals 95. In each period she receives utility U_{it}, from non-durable consumption C_{it} and housing services H_{it}.

The within-period retiree's preference over consumption and housing services are represented by the Cobb-Douglas utility function:

$$U_{it}(C_{it}, d_{it}) = \frac{(C_{it}^{1-\omega} H_{it}^{\omega})^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it}) \tag{6}$$

where C_{it} denotes consumption, H_{it} the house value, ω measures the relative importance of housing services versus numeraire nondurable good consumption, γ is the coefficient of relative risk aversion. Let d_{it} be the discrete housing choice, described in next section.

We assume that $\varepsilon_{it}(d_{it})$ is independent across individuals and time. It is Extreme Value Type I distributed. It represents housing preference shock. Individuals move out of their home for several reasons, which are explained in detail in our survey. Some households move out for financial reasons, looking for a smaller or less expensive house. Others because they desire to live near or with their children or other relatives, for health problem, for climate or weather reasons, for reasons related to leisure activities or public transportation and for change in marital status. We model this unobserved utility from moving as housing preference shock.

When the individual dies, her terminal wealth, TW_{it}, is bequeathed according to a bequest function $b(TW_{it})$:

$$b(TW_{it}) = \theta_B \frac{TW_{it}^{1-\gamma}}{1-\gamma} \tag{7}$$

The idea behind the bequest function is that the retiree i will receive utility from the knowledge that if she dies at time t her heirs would receive the terminal wealth TW_{it}. A similar formulation was used by Carroll (2000b). The degree of altruism is given by the parameter θ_B. In the baseline case, we assume $\theta_B = 1$, that is the retiree has a strong bequest motive. Section 8 revisits the bequest motive.

4.1 Choice set

In each discrete period t, the household makes two joint and simultaneous decisions, a discrete housing decision and continuous consumption decision.

Housing is a discrete multi-stage choice. First, the household decides whether to move or stay in the house. The household that moves out makes the choice of owning or renting and the value of the new house. Consistent with our data, homeowners that move could not afford a larger house and renters are only allowed to rent a new house of any value.

First of all, the household makes the discrete choice of staying or moving out in period t:

$$d^1_{it} = \begin{cases} D^M_{it} = 1 & \text{if household } i \text{ moves out of her house in period } t \\ D^M_{it} = 0 & \text{otherwise} \end{cases}$$

Second, if she moves out of the house, she makes the binary choice of owning or renting a new house.

$$d^2_{it}|d^1_{it} = \begin{cases} D^O_{it} = 1 & \text{if household } i \text{ owns her house in period } t \\ D^O_{it} = 0 & \text{if household } i \text{ rents her house in period } t \end{cases}$$

The third stage decision over housing is the house value. To simplify the computation, we discretize the house value.

$$d^3_{it}|d^1_{it}, d^2_{it} = H_{it}$$

Therefore, the discrete choice set d_{it} is:

$$d_{it} = \{d^1_{it}, d^2_{it}, d^3_{it}\}$$

Let C_{it} be the continuous choice of consumption.

4.2 Housing Expenses

Per-period housing expenses ψ are a fraction of the market value of the house. We assume that ψ is constant across individuals with the same housing level and deterministic. They depends on D^O_i, the housing tenure indicator variable which is equal to one for homeowners and zero for renters. For homeowners, housing expenses represent a maintenance cost, sustained to keep the house at a constant quality. For renters, housing expenses represent the rental cost. For both homeowners and renters, the housing expenses are assumed to be a constant value over time, denoted by ψ^{own} and ψ^{rent} respectively.

$$\psi_{it} = [D^O_i \psi^{own} + (1 - D^O_i)\psi^{rent}]H^*_{it} \tag{8}$$

where $H^*_{it} = D^M_{it} H_{it} + (1 - D^M_{it}) H_{it-1}$.

If the retiree decides to sell her house at time t and move to another house, she pays (receives) the difference in owner-occupied housing wealth. In addition, she incurs a one-time transaction cost $\phi(D^O_{it})$. The cost of moving is:

$$M_{it} = D^M_{it} D^O_{it-1}[D^O_{it} H_{it} - H_{it-1} + H_{it}\phi(D^O_{it})] + D^M_{it}(1 - D^O_{it-1})(1 - D^O_{it}) H_{it} \phi^{rent} \tag{9}$$

The transaction cost equals a fraction ϕ^{own} (ϕ^{rent}) of the market value of the new house when the investor moves to an owner-occupied (a rental) house, i.e.

$$\phi(D^O_{it}) = [D^O_{it}\phi^{own} + (1 - D^O_{it})\phi^{rent}] \tag{10}$$

Typically we have larger moving costs for the case of a retiree that buys a new house, that is $\phi^{own} > \phi^{rent}$.

4.3 The Household's Problem

The state space in period t consists of variables that are observed by the agent and the econometrician X_{it} and by variables observed only by the agent $\varepsilon_{it}(d_{it})$.

$$X_{it} = \{A_{it}, H_{it-1}, D^O_{it-1}, Age_t\}$$

where A_{it} is household i's non-housing financial assets at time t, H_{it-1} the previous period house value, and D^O_{it-1} the previous period housing tenure.

The term $\varepsilon_{it}(d_{it})$ references a vector of unobserved utility components determined by the discrete alternative and it is Type I Extreme Value distributed.

The household maximizes the expected lifetime utility over consumption C_{it} and housing d_{it}:

$$V_t(X_{it}, \varepsilon_{it}) = \max_{d_{it}, C_{it}} E_t \left[\sum_{t=64}^{T} \beta^{t-64} (N(t-1,t) n_t U(C_{it}, d_{it}) | X_{it}, \varepsilon_{it}) + b(TW_{it}) \right] \tag{11}$$

s.t

$$A_{it+1} = RA_{it} + ss - C_{it} - \psi_{it} - M_{it} \tag{12}$$

$$C_{it} \geq C_{MIN} \tag{13}$$

where q_t denote the probability of being alive at age t conditional on being alive at age $(t-1)$, and let $N(t,j) = (1/n_j)\prod_{k=1}^{t} n_k$ denote the probability of living to age t, conditional on being alive at age j.

Eq. (12) denotes period t retiree i's budget constraint. Let ss denote the retiree's income, which includes social security, pension and other retiree benefits.

Eq. (13) defines the retiree i's constraints on consumption at age t.

The value function for period t is given by the following expression:

$$\begin{aligned} V_{it}(X_{it}, \varepsilon_{it}) &= \max\nolimits_{d_{it}, C_{it}} U_{it}(C_{it}, d_{it}) + \varepsilon_{it}(d_{it}) + \\ &\quad \beta(n_{t+1} E[V_{it+1}(A_{it+1}, H^*_{it}, D^O_{it}, \varepsilon_{it+1} | X_{it,} C_{it})] + b(TW_{it+1})) \\ &\quad \text{s.t.} \qquad (14) \\ A_{it+1} &= RA_{it} + ss - C_{it} - \psi_{it} - M_{it} \\ C_{it} &\geq C_{MIN} \end{aligned}$$

The computation of the optimal policy functions is complicated due to the presence of the vector $\varepsilon_{it}(d_{it})$. It enters nonlinearly in the unknown value function EV_{it+1}. Following Rust (1988), we introduce the additivity and the conditional independence assumptions thus EV_{t+1} does not depend on ε_{it}.

Therefore the Bellman equation can be rewritten as:

$$\begin{aligned} V_t(X_{it}, \varepsilon_{it}) &= \max_{d_{it}, C_{it}} [U(C_{it}, d_{it}) + \varepsilon_{it}(d_{it}) + \beta\eta_{it+1} EV(X_{it+1})] \qquad (15) \\ &= \max_{d_{it}} \left\{ \left[\max_{C_{it}} \{U(C_{it}, d_{it}) + \beta\eta_{it+1} V(X_{it+1}) | d_{it}\} \right] + \varepsilon_{it}(d_{it}) \right\} \end{aligned}$$

The solution of period t's problem could be divided in two parts. There is an inner maximization with respect to the continuous choice conditional on the discrete housing choice and then an outer maximization with respect to the multi-stage discrete choice.

We assume that there is a measurement error in consumption distributed as a normal with mean 0 and unknown variance σ^2. Given the observed realization of household choices and states $\{C_{it}, d_{it}, X_{it}\}$, the objective is to estimate the preferences denoted as $\theta = \{\gamma, \omega, \sigma\}$. We allow for heterogeneity in the state variables, X_{it} and ε_{it}, but not in preferences θ.

4.4 Inner Maximization

Let $r(X_{it}, d_{it})$ represent the indirect utility function associated to the discrete choice d_{it} :

$$r(X_{it}, d_{it}, \theta) = \max_{C_{it}} \{U(C_{it}, d_{it}) + \beta n_{it+1} V_{t+1}(X_{it+1}) | d_{it}\} \qquad (16)$$

This function has to be computed for each possible d_{it}, subject to the contemporaneous budget constraint and the constraint on consumption.

4.5 Outer Maximization

Under the assumption that $\varepsilon_{it}(d_{it})$ is distributed as a Type I Extreme Value error, the conditional choice probabilities are given by the following formula:

$$P(j|X_{it},\theta) = \frac{\exp\{r(X_{it},j,\theta)\}}{\sum_{k\in d_{it}(X_{it})}\exp\{r(X_{it},k,\theta)\}} \tag{17}$$

and $V_{t+1}(X_{it+1})$ is given by:

$$V_{t+1}(X_{it+1}) = \ln\left[\sum_{k\in d_t(X_t)}\exp\{r(X_{it},k,\theta)\}\right]$$

5 Solution Method

The solution method is innovative in three main respects. First, we present a constrained optimization approach to a life-cycle dynamic programming problem. Second, this is the first application of the MPEC approach to an empirical structural model with finite horizon dynamic programming. Third, we estimate the structural model including both discrete and continuous choices.

We describe the constrained optimization approach for a simple life-cycle consumption saving problem underlining its novelty with respect to the conventional approach. Specifically, the use of a mathematical programming language allows us to rewrite the DP and estimation problem as a nonlinear programming problem which involves the optimization of an objective function subject to linear equality and inequality constraints. We present the details for the full model in the Appendix.

5.1 Simple Life-Cycle Model

For ease of exposition, we assume that there is only one continuous state variable (wealth) and one continuous choice variable (consumption).

The backward solution from time T for true value functions is described as follows. The last period value function is known and equal to $V_T(W)$.

In periods $t = 1...(T-1)$ the Bellman equation is:

$$V_t(W) = \max_c \; u(c) + \beta V_{t+1}(RW - c)$$

Given V_{t+1}, the Bellman equation implies, for each wealth level W, three equations that determine the optimal consumption, c^*, $V_t(W)$, and $V_t'(W)$:

Euler Equation:

$$u_t'(c^*) - \beta R V_{t+1}'(RW - c^*) = 0$$

Bellman equation:

$$V_t(W) = u(c^*) + \beta R V_{t+1}(RW - c^*)$$

Envelope Condition:

$$V_t'(W) = \beta R V_{t+1}'(RW - c^*)$$

The backward solution from time T for approximate value functions requires several steps.

We choose a functional form and a finite grid of wealth levels. Let $W_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. We will use these grid points for approximating the value functions. Let $\Phi(W; a)$ be the function that we use to approximate the value functions, $V(W)$. If we assume that it is a seventh-order polynomial centered at $\overline{W}$, then

$$\Phi(W; a, \overline{W}) = \sum_{k=0}^{7} a_k (W - \overline{W})^k$$

The time t value function is approximated by

$$V_t\,(W) = \Phi(W; a_t, \overline{W}_t) = \sum_{k=0}^{7} a_{k+1,t} (W - \overline{W}_t)^k$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{W}$ on time. We will choose $\overline{W}_t = (W_t^{\max} + W_t^{\min})/2$, the period t average wealth. Note that $\overline{W}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, we will drop it as an explicit argument of Φ. So, $\Phi(W; a_t)$ will mean $\Phi(W; a_t, \overline{W}_t)$.

We would like to find coefficients a_t such that each time t Bellman equation, along with the Euler and envelope conditions, holds with the Φ approximation; that is, for each time $t < T - 2$, we want to find coefficients a_t such that for all W

$$\Phi(W; a_t) = \max_c \; u(c) + \beta \Phi(RW - c; a_{t+1})$$

and for time $t = T - 1$, we want to find coefficients a_t such that for all W

$$\Phi(W; a_t) = \max_c \; u(c) + \beta V_T(RW - c)$$

This is not possible unless the solution is a degree 7 polynomial. We need to approximately solve the Bellman equation. To this end, we need to specify the various errors that may arise in our approximation. We will consider three errors and one side condition.

First, at each time t and each $W_{i,t}$, the absolute value of the Euler equation if consumption is $c_{i,t}$, which we denote as $\lambda_{i,t}^e \geq 0$, satisfies the inequality

$$-\lambda_{i,t}^e \leq u'(c_{i,t}) - \beta R \Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_{i,t}^e$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $W_{i,t}$ with consumption $c_{i,t}$ is denoted by λ_t^b and satisfies

$$-\lambda_t^b \leq \Phi(W_{i,t}; a_t) - [u(c_{i,t}) + \beta\Phi(RW_{i,t} - c_{i,t}; a_{t+1})] \leq \lambda_t^b$$

Third, the envelope condition error, λ_t^{env}, satisfies

$$-\lambda_t^{env} \leq \Phi'(W_{i,t}; a_t) - \beta R\Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_t^{env}$$

where $\Phi'(x; a_t)$ is the derivative of $\Phi'(x; a_t)$ with respect to x.

Fourth, because the true value functions are concave, we want our approximate value functions to also be concave. Sometimes we will impose concavity of the approximate value functions on the $W_{i,t}$ grid with the secant condition

$$\Phi(W_{i,t}; a_t) \geq \Phi(W_{i-1,t}; a_t) + \frac{(\Phi(W_{i+1,t}; a_t) - \Phi(W_{i-1,t}; a_t))}{(W_{i+1,t} - W_{i-1,t})}(W_{i,t} - W_{i-1,t})$$

With these definitions, the constrained optimization approach to a life-cycle dynamic programming problem can be rewritten as:

$$\min_{a,c,\lambda} \sum_t \sum_i \lambda_{i,t}^e + \sum_t \lambda_t^b + \sum_t \lambda_t^{env} \tag{18}$$

subject to:

$$-\lambda_{i,t}^e \leq u'(c_{i,t}) - \beta R\Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_{i,t}^e$$

$$-\lambda_t^b \leq \Phi(W_{i,t}; a_t) - [u(c_{i,t}) + \beta\Phi(RW_{i,t} - c_{i,t}; a_{t+1})] \leq \lambda_t^b$$

$$-\lambda_t^{env} \leq \Phi'(W_{i,t}; a_t) - \beta R\Phi'(RW_{i,t} - c_{i,t}; a_{t+1}) \leq \lambda_t^{env}$$

where we choose the value function approximation parameters a, the consumption choices on the wealth grid, c, and the errors, $\lambda \geq 0$, so as to minimize the sum of errors. We may also add the concavity constraint if necessary to attain a concave value function approximation.

There are many variations on this theme. Standard value function iteration ignores the $\sum_t \lambda_t^{env}$ term and imposes $\lambda_{i,t}^e = 0$, both of which we could do here. A more general specification would be

$$\min_{a,c,\lambda} P^e\left(\sum_t \sum_i \lambda_{i,t}^e\right) + P^b\left(\sum_t \lambda_t^b\right) + P^{env}\left(\sum_t \lambda_t^{env}\right)$$

where the P^j parameters are penalty terms. Conventional value function iteration is $P^{env} = 0$ and P^e being "infinitely" larger than P^b.

This setup can be easily extended by including also discrete state variables. This would require to redefine both the a coefficients and the errors λ also over the grid points of the discrete state variables.

In the empirical part, we have continuous data on wealth and consumption. We assume that the measurement error in consumption is normally distributed with mean 0 and unknown variance σ^2. We can use the Euler Equation to recover the predicted value of consumption, denoted as c^{pred}. The probability that household n chooses consumption $c_{n,tp}$ in period tp is:

$$\Pr(c_{n,tp}|W_{n,tp}^{data}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c_{n,tp}^{data} - c_{n,tp}^{pred})^2}{2\sigma^2}}$$

Therefore the log-likelihood is given by:

$$L(\theta) = \sum_{i=1}^{N} \sum_{tp=1}^{TP} \Pr(c_{n,tp} \mid W_{n,tp}^{data}, \theta) \tag{19}$$

The constrained optimization approach to structural estimation with finite horizon dynamic programming is:

$$\begin{aligned} & Max \qquad L(\theta) - Penalty \cdot \Lambda \\ \text{subject to} \quad & : \\ & Euler\ Error \\ & Bellman\ Error \\ & Envelope\ Error \end{aligned} \tag{20}$$

where $\Lambda = \sum_t \sum_i \lambda_{i,t}^e + \sum_t \lambda_t^b + \sum_t \lambda_t^{env}$.

Therefore, the structural estimation of a life-cycle model simply becomes a problem of optimizing a function of many variables subject to a set of constraints. The inequality approach we use can be formulated as constraints in a nonlinear programming problem. To our knowledge, this is the only stable method for dynamic programming problems of this kind. Moreover, in most economic models the differentiability of the value function is key for the characterization, analysis, and computation of optimal solutions The constrained optimization approach to dynamic programming allows us to simultaneously obtain the approximation of the value function and of its derivative. By imposing the Envelope Condition in our set of constraints we get a sharper characterization of the optimal solution that is suitable for computation and we obtain an explicit expression for the derivative of the value function.

In our approach, given the last period value function, we find simultaneously consumption, savings and the other economic endogenous variables in each period. This approach, creating a link between past, current and future economic variables, allows us to obtain the only equilibrium that is associated with the optimal consumption and saving decision in each period. Given the enormous increase in computer speed and advances in algorithms and software for large-scale problems in the past decade, this technique permit us to keep track of the grid of possible values of state variables and is adequate for solving any consumption saving problem of reasonable complexity.

The traditional approach in estimating finite horizon dynamic structural models consists in taking a guess of the structural parameters, solving the dynamic programming, calculating the loglikelihood and repeating these steps until the loglikelihood is maximized. This can be computationally very demanding. We use the MPEC approach to solve our empirical model. This approach consists in formulating the life-cycle dynamic programming problem and the maximum likelihood estimation of the preferences as a constrained optimization problem. The idea behind the MPEC approach is to choose structural parameters and endogenous economic variables simultaneously and symmetrically, to solve the dynamic programming and the maximum likelihood problems in a one-step procedure. In several respects, the MPEC approach could not be considered completely new, since it is based on ideas and methods developed in statistics and econometrics literature. Nevertheless, the current econometric literature seems to consider this approach infeasible. Judd and Su (2008) argue that the constrained optimization approach is feasible if one uses standard methods in the mathematical programming literature. They apply the MPEC approach to the canonical Zucher bus repair model (Rust, 1987). We extend their approach, presenting the MPEC with finite horizon dynamic programming.

Fernandez-Villaverde et al. (2006) shows that economic dynamic models generally lack a closed-form solution, hence economists approximate the policy functions of the agents with numerical methods. It follows that the researcher can evaluate only an approximated likelihood associated with the approximated policy function, instead of the exact likelihood function. They argue that as the approximated policy function converges to the exact policy, the approximated likelihood also converges to the exact likelihood. Introducing the Envelope Condition we are able to get a good high order approximation not only of the value function but also of its derivative which is crucial for structural estimation.

Finally, given that our study involves both discrete and continuous choices and these data are present in our sample, we extend the existing literature on discrete choice including also continuous decisions in the model. The continuous state variable is the financial assets and the continuous choice variables is consumption. The discrete and discretized variables are the moving decision, the owning decision, and the housing level.

6 The Data

The Health ad Retirement Study (HRS) is a US national panel study which covers a wide range of topics. In particular, questions on family structure, employment status, demographic characteristics, housing, stocks, bonds, IRA, other financial assets, income, pension, social security, and benefits are relevant for our analysis. Questionnaires assessing individual activities and household patterns of consumption were mailed to a subsample of the Health and Retirement Study (HRS). The Consumption and Activities Mail Survey (CAMS), the survey including this information on consumption, was first conducted in 2001. The survey is conducted every two years.

We select a group of households that is the potential target segment for RM, according to estimates from the public policy perspective.

Our sample includes single, retired homeowners, 62 years old or older. Social security is the homeowners' main source of income. Pensions and earned interests on financial assets contribute much less as a source of per-period income. We eliminate all households with incomplete records or missing information about their consumption and financial situation for the years 2000, 2002, 2004. After these cuts were made, a sample of 175 single households observed for three consecutive periods remain.

Non-housing financial assets include stocks, bonds, saving accounts, mutual funds, individual retirement accounts (IRAs), other assets. It does not include the value of any real estate or business. Given that this target segment for RM has almost no debt, focusing on the total non-housing financial assets gives nearly identical results as focusing non-housing financial wealth. Consumption includes vehicles, washing machine, drier, dishwasher, television, computer, telephone, cable, internet, vehicle finance charges, vehicle insurance, health insurance, food and beverages, dining/drinking out, clothing and apparel, gasoline, vehicles, prescription and nonprescription medications, health care services, medical supplies, trip and vacations, tickets to movies, sorting events and performing arts, hobbies, contribution to religious, educational, charitable or political organizations, cash or gift to family, friends outside your household. Housing expenses for homeowners represent the maintenance cost incurred to keep the house at a constant quality, and for renters, represent the rental cost.

Table 1 shows the descriptive statistics for house value, financial assets, consumption, social security income and age for the first year in the panel.

Table 1. Descriptive Statistics

	Percentiles			Min	Max
	25%	50%	75%		
H	\$40,000	\$70,000	\$92,000	\$ 2,500	\$170,000
A	\$5,000	\$17,500	\$63,000	\$0	\$276,548
H/A	0.86	2.5	7.5	0.11	1500
C	\$6,270	\$9,774	\$15,090	\$800	\$84,380
ss	\$6,972	\$9,468	\$11,340	\$0	\$ 24.701
Age	69	74	79	64	84

Housing represents a significant proportion of the retirees' total assets. The median house value is \$70,000 and is 2.5 times the value of non-housing financial assets. Consumption seems to parallel social security income. The average per-period income is \$20,000.

Figure 3- 7 illustrates how consumption, social security, non-housing finan-

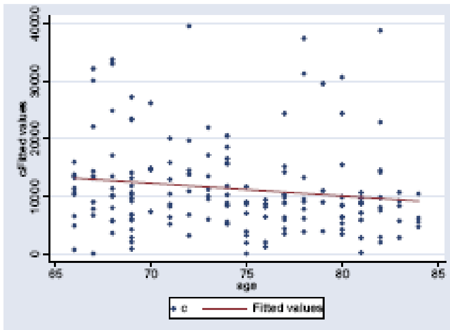


Figure 3: Consumption

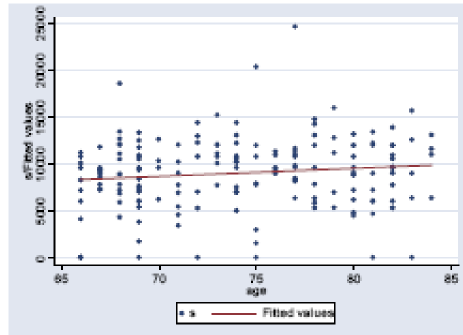


Figure 4: Social Security Income

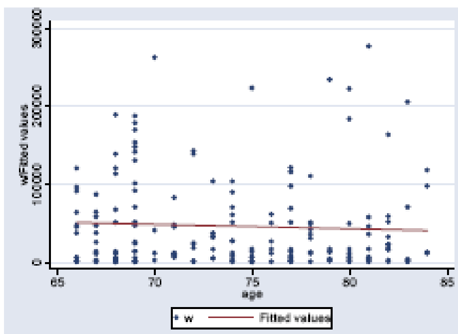


Figure 5: Non-housing financial assets

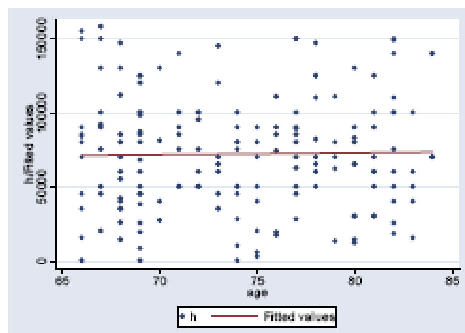


Figure 6: Housing

cial assets and housing vary with age. Near retirement, the average consumption exceed the average social security income, implying that social security income, pensions and liquid savings contribute to finance per-period expenses. As the retiree ages consumption decreases and is almost completely financed with Social Security income after age 75. The non-housing financial assets represent a small fraction of the house value and gradually reduce with age. Housing is constant over time, supporting the thesis that retirees tend not to divest their home equity.

Table 2 presents the composition of the financial portfolio.

Table 2 Financial Portfolio composition

	Percentiles			Min	Max
	25%	50%	75%		
Stocks	\$0	\$0	\$0	\$0	\$125,000
Chck	\$300	\$2500	\$9,000	\$0	\$100,000
Cds	\$0	\$0	\$4,000	\$0	\$200,000
Tran	\$700	\$4,000	\$8,500	\$0	\$30,000
Bonds	\$0	\$0	\$0	\$0	\$80,000
Ira	\$0	\$0	\$1000	\$0	\$137,000
Debt	\$0	\$0	\$0	\$0	\$7,000

For almost all the retirees in the sample, the financial portfolio does not contain risky assets. Retirees have most of their savings in checking and saving accounts and transportation. About 40% of the retirees have certificates of deposits and approximately 25% have IRAs. Less than 10% have stocks and about 5% have bonds.

In each period, about 10% of the households in our sample moves out of her house. Among those who moved, about 35% decide to rent a new house, while about 65% buy a new house. At the end of the three years of the panel, about 25% of the population moved and about 10% rented a new house. The moving decision does not appear to be strictly related with age. About 50% of the retirees moves near or with children or other relatives or friends. About 25% moves for financial reasons and the remaining 25% moves for health problem or services, weather or climate reasons, better location or retirement related area or other reasons.

7 Calibration and Results

The subjective discount rate is $\beta = 0.96$ and the real interest rate is $r = 0.04$. Following Yao and Zhang (2005a), the rental rate is $\psi^{rent} = 6\%$ and maintenance cost is $\psi^{own} = 1.5\%$. Transaction costs are $\phi^{own} = 6\%$ and $\phi^{rent} = 1\%$, respectively, when moving to an owner-occupied house and when moving to a rental house. In the baseline case we assume $\theta_B = 1$, that is the retiree has a strong bequest motive.

We estimate the parameter γ using a grid search approach. Given the parameter γ, we use the MPEC approach to estimate ω and σ. Table 3 presents the estimation results.

Table 3. Structural Estimation Results

Parameter	Variable	Estimate
γ	coefficient of relative risk aversion	3.87 (0.61)
ω	preference parameter over housing	0.85 (0.04)
σ	s.d. of measurement error in consumption	0.87 (0.07)

We find reasonable estimates of the preference parameters.

The coefficient of relative risk aversion is 3.87 and it is similar to other estimates that rely on different methodologies (see Auerbach and Kotlikoff (1987), Cagetti (2003) and French (2005)). According to the related literature, a small estimate of the coefficient of relative risk aversion means that households save little given their level of assets and their level of uncertainty. On the other side, more risk averse individuals would save more in order to buffer themselves against future risks. Our estimate of 3.87 implies a relatively high coefficient of risk aversion, suggesting that households have elevate levels of precautionary savings. In agreement with this result we obtain an estimate of the preference parameter over housing equal to 0.85. To our knowledge, there are no previous structural estimates of this parameter for retirees. Our estimate of ω is consistent with our sample data in which the retiree consumption is a small fraction compared to the house value.

These two estimates together, describing the elderly preferences over housing and consumption, can help understanding the retiree behavior. In particular, they show that retirees are relatively high risk adverse and that they significantly value their house as a safe and illiquid asset in which precautionary savings can be easily locked.

We compute the standard errors using a bootstrap procedure. This procedure is adequate in our case since it could reduce the downward bias of asymptotic standard errors in maximum likelihood estimation of non-linear systems. In addition, this procedure is feasible given that the MPEC approach allows us to solve the structural estimation problem in a short amount of time. Resampling was conducted by sampling with replacement across households as is standard practice in panel models. To obtain the global optimum we use different initial starting points. The optimal solution was not influenced by the initial starting points. In total the standard errors are calculated with 100 bootstraps.

7.1 Do Reverse Mortgages Pay?

A reverse mortgage is a loan against the retiree's home that does not have to be paid back for as long as the retiree lives there. We assume that the retiree chooses to receive the proceeds as a single lump sum of cash at the closure of the contract. Following the notation in section 2, let F be the up-front cost and $\overline{B}$ the cash available to retiree i at the closure of the contract. Let G_{it}^{RM} be the real outstanding debt at time t.

If the retiree decides to move out of the house, she has to repay the minimum between the value of the house and the accumulated debt plus a one-time transaction cost $\phi(D_{it}^O)$. The cost of moving is:

$$M_{it} = D_{it-1}^O D_{it}^M [D_{it}^O H_{it} - \max(0, H_{it-1} - G_{it}^{RM}) + H_{it}\phi(D_{it}^O)] \qquad (21)$$

The welfare gain from reverse mortgage is calculated as the percentage increase in the initial financial assets that makes the household without a reverse mortgage as well off in expected utility terms as with a reverse mortgage.

For each household in our sample, we calculate the expected lifetime utility from closing the reverse mortgage contract in the first year of our panel, namely year 2000. Then, we calculate the percentage increase in their initial financial assets that makes them as well off as with the reverse mortgage.

We explain our simulation results and we assess the validity of our model in predicting the retirees' behavior in light of the empirical evidence on reverse mortgagees.

We first introduce some notation. We define cash-poor households with little or no financial assets and few or no possessions. Specifically, let LA denote initial value of non-housing financial assets less than \$10,000, MA between \$10,000 and \$60,000 and HA greater than \$60,000. Cash-poor households belong to the LA group. It is worth noting that since the baseline per period income is \$20,000, all the households in our sample are income-poor. Hence, our definition of cash-poor refers to the non-housing financial assets.

We consider three house values. Let LH denote low house value (\$40,000), MH medium house value (\$80,000) and HH high house value (\$120,000). House-rich households belong to the HH subgroup.

Therefore, house-rich but cash-poor households have the highest house value (HH) and the lowest initial non-housing financial assets (LA) and are located in the right upper quadrant in the following tables. Reverse mortgages have been designed with these households in mind, but they have not bought this product in the past twenty years. The small fraction of reverse mortgage borrowers appear to belong to the (MA,HH) group, namely they have some financial assets and are house-rich.

Table 4 shows the median welfare gain as a function of the initial non-housing financial assets and house value. The number in parenthesis represents the median non-housing financial assets for each group.

Table 4 Median welfare gain (median non-housing financial assets)

	LH	MH	HH
LA	-59.10% (\$1,000)	-64.42% (\$2,000)	-120.4% (\$2,250)
MA	-27.43% (\$16,000)	29.5%(\$28,000)	24.07% (\$46,000)
HA	85.82% (\$120,000)	19.91% (\$103,000)	46.65% (\$135,250)

The common belief is that a reverse mortgage benefits those with resources tied up in home equity, those defined house-rich but cash-poor. This simulation shows otherwise. Specifically, our simulation shows that house-rich but cash-poor households experience the largest welfare loss from a reverse mortgage contract equal to a 120% decrease in their initial assets. Additionally, all cash-poor (LA) households and households with small house and low or medium financial assets experience a welfare loss. On the other side, all cash-rich (HA) households experience a welfare gain.

This simulation, highlighting the pros and the cons of the contract, could help understanding why the reverse mortgage is still a niche product after about twenty years from its first appearance.

A reverse mortgage provides liquidity and a form of longevity insurance. The retiree can cash in some of the saving locked in the house and would be able to experience higher levels of consumption than otherwise possible. Furthermore, she can live in the same house while alive, regardless of the amount of the outstanding debt. Reverse mortgages constitute the purchase of a no-exit annuity – an annuity that pays off in the form of the housing services of the current home provided the retiree does not permanently exit her home. Since not exiting is partly conditioned on not dying, the no-exit annuity encompasses some longevity insurance. However, closing this contract implies incurring in very high start-up costs and facing a new risk, the moving risk.[2] The high up-front costs significantly contributes to the welfare loss for households with small house. For example, a 62 years old household with a \$40,000 house can borrow about \$20,000. But the cash available at the closure of the contract, after the payment of about \$10,000 of up-front costs, is just nearly \$10,000. Moreover, a reverse mortgage is a risky financial instrument, that incorporates an unusual risk, the risk of moving and having to repay the cumulated debt. Our finding that the moving risk is a realistic and serious risk is supported by empirical evidence. Reverse mortgages should be appealing to homeowners that plan to remain in their home longer. However, empirical evidence shows that reverse mortgagees exit their home unusually rapidly, suggesting that an unexpected event happens and forces them to move out. If the household has to move, for any exogenous reason, her future well-being, ability to meet unforeseen costs, consumption profile and housing choices are significantly affected. This is specifically true for households with initial low financial assets, as a matter of fact some of the choices over consumption and housing, available before closing the reverse mortgage contract, are not affordable anymore after. Hence, the precautionary motive appears to be mostly concentrated among the wealth poor individuals. For wealth poor households closing a reverse mortgage contract would represent one of the top ten investing mistake, namely the lack of diversification. Rule of thumb, if someone puts all of her eggs in one basket she is taking a much greater risk than if she diversifies. The retiree with initial low financial assets has all her life-savings locked in the house, which is a safe asset under our specification of non-stochastic house price. If she closes a reverse mortgage contract, she reallocates all her saving into a risky financial instrument. While closing a reverse mortgage contract would prevent cash-poor households to diversify their investment, it would not prevent cash-rich households to spread their investment around. Consequently, the latter would not experience any welfare loss from the contract. Additionally, the welfare loss for cash-poor retirees comes from not assessing their own level of risk. Essentially, each retiree has to consider how much money she can comfortably afford to

[2] In this study, risks are conditions or events that could occur, and whose occurrence, if it does take place, has a harmful or negative effect. The moving risk is associated to a decrease in the expected life time utility.

lose in the worst scenario of the game. Cash-poor retirees would take a high risk investment when closing a reverse mortgage for which they could be not prepared if they have to move out.

Hence, these findings could explain why the reverse mortgage market is still extremely small. A reverse mortgage is a risky financial product. Campbell and Viceira (2002) show that risky assets should be attractive to young households with many years until retirement and modest savings. Such households have large human wealth relative to financial wealth, and their human wealth is relatively safe; thus they should be willing to tilt their financial portfolios strongly toward risky assets. However, the attractiveness of risky investments diminishes later in life as human wealth declines and financial assets accumulate. Consistent with our data, the retirees' financial portfolio consists mostly of safe assets. The house is not only a safe asset, but it is also the main financial asset for the retirees. Closing a reverse mortgage, would imply moving all the savings invested in the illiquid safe asset into a liquid risky financial instrument. But this would contradict both the empirical evidence and most of personal finance recommendations according to which retirees should be much more cautious with their asset allocation than younger investors and should turn to safe investing.

8 Policy Experiments

The framework presented above allows for many possible policy experiments and extensions. In this section we choose the following four: reduction in current income, bequest motive, no up-front cost and no moving risk. These policy experiments allows us to better identify the risk expanding and the risk mitigating aspects of a reverse mortgage.

8.1 Reduction in Current Income

Increases in the living cost and in health care costs, curtailments in medical coverage and other employee benefit plans, cutbacks in Social Security benefits, and declining individual saving rates make it likely that many retirees will have to adjust to a decreased standard of living in their older years (Palmer, 1994). Reverse mortgages have been originally introduced as financial instruments able to relieve retirees from their financial pressure. In this subsection, we investigate the case of a 10% reduction in current income. The goal is to assess the importance of the liquidity insurance aspect of reverse mortgages. This policy experiment is particularly relevant because according to the AARP Survey, the median reverse mortgage borrower has a per period income less than $20,000.

In the model retirees are not allowed to borrow and therefore current consumption is limited by current resources. A reduction in per period income causes a decrease in current consumption. Reverse mortgages, augmenting the resources available to consumption, ease the liquidity problem and generate welfare gains larger than in the baseline case. The simulation shows that the group of households that experience the largest welfare gain are those in the middle right quadrant (MA,HH). Therefore, our simulation is consistent with the data on reverse mortgage borrowers, according to which the median borrower belong to the (MA,HH) subgroup. The moving risk and the lack of diversification still cause welfare losses for cash-poor households.

Table 5. Median Welfare Gain, 10% cut in current income

	LH	MH	HH
LA	-30.12% ($1,000)	-21.77% ($2,000)	-120.5% ($2,250)
MA	4.27% ($16,000)	184.28%($28,000)	208.30% ($46,000)
HA	93.62% ($120,000)	138.16% ($103,000)	23.19% ($135,250)

8.2 No Bequest Motive

Leaving a bequest is an important reason to save for many retirees. The baseline degree of altruism θ_B is assumed to be equal to 1, hence, the retiree has a strong bequest motive. Even though households often want to leave some bequests, in reality many families do not leave any. In this subsection we consider the case in which $\theta_B = 0$. The retiree does not receive any utility from leaving a bequest and would like to consume all her assets before she dies.

Comparing the retirees with and without bequest motive, two main aspects are worth noting. First, the welfare gain without a bequest motive always exceeds the gain in the baseline case. The interpretation of this finding comes from noting that, in the bequest model, if someone is certain to die with positive terminal wealth, a dollar decrease in consumption at time t will result in a dollar increase in bequests. Therefore, the retiree's utility of bequest comes from the utility the heirs would receive from the bequest. While in the bequest model the increase in the initial financial assets from closing a reverse mortgage would be partly consumed and partly bequeathed, it would be entirely consumed in the model without a bequest motive. In our specification, the retiree receives higher utility from her own consumption than from leaving a bequest, hence we are able to motivate the smaller welfare gain in the baseline case. The liquidity aspect of this financial instrument and the consequent ability to increase the per period consumption in the " not-moving" winning state become, in this scenario, more relevant. Second, similar to the baseline case, all cash-poor (LA) households and, particularly, house-rich but cash-poor retirees, experience a welfare loss from a reverse mortgage. The moving risk and the lack of diversification in the investments are the main causes this welfare loss.

Table 6. Median Welfare Gain, No Bequest

	LH	MH	HH
LA	-20.52% ($1,000)	-29.83% ($2,000)	-120.3% ($2,250)
MA	40% ($16,000)	189.52%($28,000)	26.74% ($46,000)
HA	102.9% ($120,000)	121.88% ($103,000)	64.04% ($135,250)

8.3 No Up-front Costs

According to the AARP Survey, many possible reasons could explain the reluctance of older homeowners to tap their home equity: aversion to debt, desire to leave a bequest, and the strategy of saving home equity as a last resort for major economic or health crises (Fisher et al., 2007). However, among homeowners who had enough interest to go through counseling but finally decided not to apply for a loan, high costs were cited most frequently (by 63 percent of non-applicants) as a reason for not applying for a reverse mortgage. In this subsection, we investigate the case in which the up-front costs are set equal to zero. Compared to the baseline case, the welfare gain is larger, nevertheless reverse mortgages remain a risky financial instrument unappealing for house-rich but cash-poor households.

Table 7. Median Welfare Gain, No up-front cost

	LH	MH	HH
LA	-39.64% ($1,000)	8.56% ($2,000)	-120.3% ($2,250)
MA	17.20% ($16,000)	170.65%($28,000)	218.83% ($46,000)
HA	102.68% ($120,000)	98.17% ($103,000)	71.36% ($135,250)

8.4 No Moving Risk

In this subsection we assume that the retiree does not face any moving risk and remain in her house while alive. In this scenario, reverse mortgages become safe assets. All the retirees experience a significant welfare gain from taking a reverse mortgage. Particularly, house-rich but cash-poor homeowners have the largest welfare gain equal to a seventy two time increase in their initial financial assets. This result can explain the rational behind reverse mortgage contracts. House-rich but cash-poor households can greatly benefit from the contract if they do not move out of their home. But, in the worst scenario of the game in which they have to move out, they experience the largest welfare losses. Intuitively, a reverse mortgage can be seen a gamble. Gambling involves a small stake for a large prize. The small stake is the initial up-front cost that the retiree has to pay to participate in the "reverse mortgage game". The big prize is the higher consumption that could be enjoyed if the retiree wins, namely if she does not move out. If the retiree moves out while alive, she loses the game and incur in a significant welfare loss. Gambling can make someone who is initially poor relatively rich, however luck plays an important role in this game.

Table 8. No Moving Risk

	LH	MH	HH
LA	3,550% ($1,000)	7,804% ($2,000)	71,732% ($2,250)
MA	243% ($16,000)	418%($28,000)	496% ($46,000)
HA	27.95% ($120,000)	115.18% ($103,000)	35.19% ($135,250)

9 Conclusion

This paper analyzes the retiree consumption, housing and mobility decisions and provides a plausible explanation for the existence of a niche reverse mortgage market after about twenty years from the first appearance of this financial instrument.

Retirees are relatively high risk adverse and home equity is the most important component of precautionary savings after retirement. Reverse mortgages provide liquidity and longevity insurance, but introduce a new risk, the moving risk. Closing this contract is risky especially for cash-poor households, as a matter the fact that if they have drawn down on their home equity through a reverse mortgage, their ability to meet unforeseen costs or move into alternative housing may be limited. Intuitively, a reverse mortgage can be seen a gamble. Gambling involves a small stake for a large prize. The small stake is the initial up-front cost that the retiree has to pay to participate in the "reverse mortgage game". The big prize is the higher consumption that could be enjoyed if the retiree wins, namely if she does not move out. If the retiree moves out while alive, she loses the game and incur in a significant welfare loss. Gambling can make someone who is initially poor relatively rich, however luck plays an important role in this game. These results underline the urgency for further policy analysis directed at designing safe and appealing financial instruments for the elderly which let them liquidate some of their home equity without incurring major risks.

The enormous increase in computer speed and advances in algorithms and software offer economists the possibility to analyze complex economic problems with simpler computer programs and greater precision. In this paper, we present the first application of the Mathematical Programming with Equilibrium Constraints approach to a structural estimation model with finite-horizon dynamic programming and continuous and discrete choices. This simple approach could be fruitfully extended to richer representations of life-cycle consumption saving behavior and structural estimation problems.

10 References

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11 Technical Appendix

MPEC with DP: Discrete and Continuous Choices

The panel data used in this study involves 3 years and about 175 individuals. The available data are both continuous and discrete.

The continuous data include data on consumption and non-housing financial assets. The discrete (or discretized) data are the individual's housing tenure (own-rent), her moving decision and her house value. We have additional data on the individuals' demographics, including age.

The MPEC with DP approach consists in solving simultaneously the dynamic programming problem and the maximum likelihood estimation of the preference parameters.

12 Dynamic Programming with Approximation of the Value Function

Life Cycle Model:

One continuous state variable: wealth

Two discrete state variables: previous period housing tenure and previous period house value

One continuous choice variable: consumption

Many discrete choices: Not Move(N), Move to House value h with housing tenure q (Mhq), where q ={Own,Rent}

12.1 Backward Solution from Time T for True Value Functions

In each period, the household chooses whether to stay in her house or to move out. If she moves out, she can either buy or rent a new house and she can choose her new house value. Let the subscripts d^N, d^{Mhq} denote respectively the decision not to move, the decision to move to house value h and housing tenure q. The housing tenure is a binary variable that takes value 1 if the household own the house

The last period value function is known and equal to $V_T(W,H,Q)$ where W is the household's financial wealth, H her previous period house value and Q her previous period housing tenure.

In periods $t = 1..(T-1)$ we define:

$$\begin{aligned}
V_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + ss; H, Q) + \varepsilon_t^N \\
V_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + ss; h, q) + \varepsilon_t^{Mqh}
\end{aligned}$$

where M is the transaction cost:

$$M = Q(H - qh + \phi^{own}qh + \phi^{rent}(1-q)h) + (1-Q)(1-q)\phi^{rent}h$$

and ψ is the per-period housing expense:

$$\psi = [Q\psi^{own} + (1-Q)\psi^{rent}]H + [q\psi^{own} + (1-q)\psi^{rent}]h$$

c_{d^N} and $c_{d^{Mqh}}$ are the consumption levels respectively if the individual does not move and if she moves to house value h choosing the housing tenure q. ss is the household's per-period income. η_{t+1} is her survival probability. ε_t^N and ε_t^{Mqh} are type I extreme value errors.

Following Rust, we assume that the additivity and the conditional indipendence assumptions hold.

To simplify the notation, we introduce the following expressions, which are evaluated at the optimal consumption level:

$$\begin{aligned}
\widehat{V}_{d^N,t} &= u(c^*_{d^N}, H) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^N} - \psi + ss; H, Q) \\
\widehat{V}_{d^{Mhq},t} &= u(c^*_{d^{Mhq}}, h) + \beta\eta_{t+1}V_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + ss; h, q)
\end{aligned}$$

The extreme value assumption on ε_t implies that we can reduce the dimensionality of the dynamic programming problem. The Bellman equation is given by the following closed form solution:

$$\begin{aligned}
V_t(W,H,Q) &= \Pr(N|W,H,Q)\cdot\widehat{V}_{d^N,t} + E(\varepsilon_t^N|N) \\
&\quad + \sum_h\sum_q\{\Pr(Mhq|W,H,Q)\cdot\widehat{V}_{d^{Mhq},t} + E(\varepsilon_t^{Mhq}|M)\} \\
&= \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h\sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}
\end{aligned}$$

Given V_{t+1}, the Bellman equation implies, for each wealth level W, three set of equations that determine the optimal consumption, $c^*_{d^N}$,$c^*_{d^{Mhq}}$, $V_t(W,H,Q)$, and $V'_t(W,H,Q)$

Euler Equations:

$$\begin{aligned}
u'(c^*_{d^N}, H) - \beta\eta_{t+1}RV'_{t+1}(RW - c^*_{d^N} - \psi + ss; H, Q) &= 0 \\
u'(c^*_{d^{Mhq}}, h) - \beta\eta_{t+1}RV'_{t+1}(RW - c^*_{d^{Mhq}} - \psi - M + ss; h, q) &= 0
\end{aligned}$$

Envelope Condition:

$$V'_t(W,H,Q) = \Pr(N|W,H,Q)\cdot\widehat{V}'_{d^N,t} + \sum_h\sum_q \Pr(Mhq|W,H,Q)\cdot\widehat{V}'_{d^{Mhq},t}$$

Bellman equation:

$$V_t(W,H,Q) = \ln\left\{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})\right\}$$

The time $t = 1..(T-1)$ probabilities of not moving and moving to house value h with housing tenure q are:

$$\Pr(N|W,H,Q) = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^N,t})}{\exp(V_t(W,H,Q))}$$

$$\Pr(Mhq|W,H,Q) = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(\widehat{V}_{d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{d^{Mhq},t})} = \frac{\exp(\widehat{V}_{d^{Mhq},t})}{\exp(V_t(W,H,Q))}$$

12.2 Backward Solution from Time T for Approximate Value Functions

Let $\Phi(W,H,Q;a)$ and $\Phi_d(W,H,Q;b)$ be the functions that we use to approximate respectively the value functions, $V(W,H,Q)$. and the policy functions $c_d^*(W,H,Q)$, with $d=\{d^N,d^{Mhq}\}$. If we assume that they are a seventh-order polynomials centered at $\overline{W}$, then

$$\Phi(W,H,Q;a,\overline{W})=\sum_{k=0}^{7}a_{k,H,Q}(W-\overline{W})^k$$

The time t value function is approximated by

$$V_t\,(W,H,Q)=\Phi(W,H,Q;a_t,\overline{W}_t)=\sum_{k=0}^{7}a_{k+1,H,Q,t}(W-\overline{W}_t)^k$$

The time t policy functions are approximated by

$$c_{d,t}^*\,(W,H,Q)=\Phi(W,H,Q;b_{d,t},\overline{W}_t)=\sum_{k=0}^{7}b_{k+1,H,Q,d,t}(W-\overline{W}_t)^k$$

where the dependence of the value function on time is represented by the dependence of the a coefficients and the center $\overline{W}$ on time and the dependence of the policy functions on time is represented by the dependence of the b coefficients and the center $\overline{W}$.

We will choose $\overline{W}_t=(W_t^{\max}+W_t^{\min})/2$, the period t average wealth. Note that $\overline{W}_t$ is a parameter and does not change during the dynamic programming solution method. Therefore, we will drop it as an explicit argument of Φ. So, $\Phi(W,H,Q;a_t)$ will mean $\Phi(W,H,Q;a_t,\overline{W}_t)$.

We would like to find coefficients a_t and $b_{d,t}$ such that each time t Bellman equation, along with the Euler and envelope conditions, holds with the Φ approximation; that is, for each time $t<T-2$, we want to find coefficients a_t such that for all W

$$\Phi(W,H,Q;a_t)=\ln\left\{\exp(\widehat{V}_{d^N,t})+\sum_h\sum_q\exp(\widehat{V}_{d^{Mhq},t})\right\}$$

where

$$\begin{aligned}\widehat{V}_{d^N,t} &= u(c_{d^N}^*,H)+\beta\eta_{t+1}\Phi_{t+1}(RW-c_{d^N}^*-\psi+ss;H,Q;a_{t+1})\\ \widehat{V}_{d^{Mhq},t} &= u(c_{d^{Mhq}}^*,h)+\beta\eta_{t+1}\Phi_{t+1}(RW-c_{d^{Mhq}}^*-\psi-M+ss;h,q;a_{t+1})\end{aligned}$$

and for time $t=T-1$, we want to find coefficients a_t given that

$$\begin{aligned}\widehat{V}_{d^N,T-1} &= u(c_{d^N}^*,H)+\beta\eta_T V_T(RW-c_{d^N}^*-\psi+ss;H,Q)\\ \widehat{V}_{d^{Mhq},T-1} &= u(c_{d^{Mhq}}^*,h)+\beta\eta_T V_T(RW-c_{d^{Mhq}}^*-\psi-M+ss;h,q)\end{aligned}$$

This is not possible unless the solution is a degree 7 polynomial. We need to approximately solve the Bellman equation. To this end, we define various errors.

First, we create a finite grid of wealth levels we will use for approximating the value functions. Let $W_{i,t}$ be grid point i in the time t grid. The choice of grids is governed by considerations from approximation theory. Then we create a grid of house values. Let $H_{j,t}$ be grid point j in the time t grid.

Next we need to specify the various errors that may arise in our approximation. We will consider three errors and one side condition.

First, at each time t and each $W_{i,t}$ and each previous period house value $H_{j,t-1}$ and housing tenure Q_{t-1}, the absolute value of the Euler equations if consumption is respectively $c^*_{i,j,d^N,t}$ and $c^*_{i,j,Q,d^{Mhq},t}$, which we denote as $\lambda^e_{i,j,Q,t} \geq 0$, satisfies the inequality

$$-\lambda^e_{i,j,Q,t} \leq u'(c^*_{i,j,d^N,t}, H_{j,t-1}) - \beta\eta_{t+1}R\Phi'(RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss; H_{j,t-1}, Q_{t-1}; a_{t+1}) \leq \lambda^e_{i,j,Q,t}$$

$$-\lambda^e_{i,j,Q,t} \leq u'(c^*_{i,j,d^{Mhq},t}, H_t)$$

$$-\beta\eta_{t+1}R\Phi'(RW_{i,t} - c^*_{i,j,d^{Mhq},t} - \psi - M + ss; H_t, Q_t; a_{t+1}) \leq \lambda^e_{i,j,Q,t}$$

where $\Phi'(x; a_{t+1})$ is the derivative of $\Phi(x; a_{t+1})$ with respect to x.

Second, the Bellman equation error at $W_{i,t}$ with consumption $c_{i,j,d^N,t}$ and $c_{i,j,d^{Mhq},t}$is denoted by $\lambda^b_{j,Q,t}$ and satisfies

$$-\lambda^b_{j,Q,t} \leq \Phi(W_{i,t}, H_{j,t-1}, Q_{t-1}; a_t) - \ln\left\{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^b_{j,Q,t}$$

where

$$\begin{aligned}
\widehat{V}_{i,j,d^N,t} &= u(c^*_{i,j,d^N,t}, H_{j,t-1}) + \beta\eta_{t+1}\Phi(RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss; H_{j,t-1}, Q_{t-1}; a_{t+1}) \\
\widehat{V}_{i,j,d^{Mhq},t} &= u(c^*_{i,j,d^{Mhq},t}, H_t) + \beta\eta_{t+1}\Phi(RW - c^*_{i,j,d^{Mhq},t} - \psi - M + ss; H_t, Q_t; a_{t+1})
\end{aligned}$$

Third, the envelope condition errors, λ^{env}_t, satisfies

$$-\lambda^{env}_{j,Q,t} \leq \Phi'(W_{i,t}, H_{j,t-1}, Q_{t-1}; a_t) - \{f_{i,j,d^N,t} \cdot \Phi'(RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss; H_{j,t-1}, Q_{t-1}; a_{t+1})$$

$$+ \sum_h \sum_q [f_{i,j,d^{Mhq},t} \cdot \Phi'(RW_{i,t} - c_{i,j,d^{Mhq},t} - \psi^{Mhq} - M + ss; H_t, Q_t; a_{t+1})]\} \leq \lambda^{env}_{j,Q,t}$$

where $\Phi'(x;a_t)$ is the derivative of $\Phi'(x;a_t)$ with respect to x and

$$f_{i,j,d,t} = \Pr(d|W_{i,t},H_{j,t},Q_t) = \frac{\exp(\widehat{V}_{i,j,d,t})}{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})}$$

Fourth, we introduce the policy function errors:

$$-\lambda^{cons}_{i,j,Q,d,t} \leq \Phi(W_{i,t},H_{j,t},Q_t;b_t) - c^*_{i,j,d,t}(W_{i,t},H_{j,t},Q_t) \leq \lambda^{cons}_{i,j,Q,d,t}$$

12.3 Empirical Part

In the theorical DP part we obtain the coefficients used in the approximation of the value function.

In this part, for any individual data of financial wealth, initial period house value and age, we calculate the predicted consumption and probabilities of moving. First, the individual makes the housing decision $d^H_{n,tp}$, with $d^H = \{d^N, d^{Mhq}\}$, then she makes her consumption decision.

Let $c^{pred}_{n,tp}$ and $c^{data}_{n,tp}$ denote respectively the predicted and the true value of consumption for household n at time tp.

For any given discrete choice on housing $d^H_{n,tp}$, using the real data on consumption, we calculate the measurement error:

$$\Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(c^{data}_{n,tp}-c^{pred}_{n,tp})^2}{2\sigma^2}}$$

The probability for the discrete choice on housing is given by:

$$\Pr(d^H_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{e^{V_{d,n,tp}}}{\sum_m e^{Vm,n,tp}}$$

Therefore the joint probability of making the discrete housing choice $d^H_{n,t}$ and the continuous consumption choice $c_{n,t}$ is given by:

$$\Pr(d^H_{n,tp}, c_{n,tp}|W^{data,}_{n,tp} H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) =$$

$$\Pr(d^H_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) * \Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1})$$

The Log-Likelihood is given by:

$$\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d^H_{n,tp}, c_{n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, \theta)$$

where N denotes the number of individuals in the sample and TP the number of time periods in the panel data.

12.4 MPEC

With these definitions,let

$$\Lambda = \sum_t \sum_i \sum_j \sum_Q \lambda^e_{i,j,Q,t} + \sum_t \sum_j \sum_Q \lambda^b_{j,Q,t} + \sum_t \sum_j \sum_Q \lambda^{env}_{j,Q,t} + \sum_t \sum_i \sum_j \sum_Q \sum_d \lambda^{cons}_{i,j,Q,d,t}$$

and let P be a penalty parameter.

The MPEC approach to the estimation of the preference paramenters is:

$$\underset{\theta,a,c}{Max} \mathcal{L}(\theta) - P\Lambda$$

subject to:

Bellman error:

$$-\lambda^b_{j,Q,t} \leq \Phi(W_{i,t}; a_t) - \ln \left\{ \exp(\widehat{V}_{i,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,d^{Mhq},t}) \right\} \leq \lambda^b_{j,Q,t}$$

Euler error

$$\begin{aligned} -\lambda^e_{i,j,Q,t} &\leq u_{c;i,j,d^N,t} + \beta \Phi^+_{W;i,j,d^N,t} \leq \lambda^e_{i,j,Q,t} \\ -\lambda^e_{i,j,Q,t} &\leq u_{c;i,j,d^{Mhq},t} + \beta \Phi^+_{W;i,j,d^{Mhq},t} \leq \lambda^e_{i,j,Q,t} \end{aligned}$$

Envelope error

$$-\lambda^{env}_{j,Q,t} \leq \Phi_{W;i,d^N,t} - \{ f_{i,d^N,t} \cdot \Phi^+_{W;i,j,d^N,t} + \sum_h \sum_q [f_{i,j,d^{Mhq},t} \cdot \Phi^+_{W;i,j,,d^{Mhq},t}] \} \leq \lambda^{env}_{j,Q,t}$$

Policy function error

$$-\lambda^{cons}_{i,j,Q,d,t} \leq \Phi(W_{i,t}, H_{j,t}, Q_t; b_{d,t}) - c^*_{i,j,d,t}(W_{i,t}, H_{j,t}, Q_t) \leq \lambda^{cons}_{i,j,Q,d,t}$$

The probability of decision d :

$$f_{i,j,d,t} = \frac{\exp(\widehat{V}_{i,j,d,t})}{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})}$$

12.5 AMPL

12.5.1 Backward Solution from Time T for Approximate Value Functions in AMPL

In order to formulate this problem in AMPL, we need to list every quantity that is computed.

The time-specific wealth grids $W_{i,t}$ are fixed. We discretize the house value.

The parameters are

$$W_{i,t}, \beta, \eta_t, R, \psi^{own}, \psi^{rent}, \phi^{own}, \phi^{rent}, \theta_B$$

The basic variables of interest are

$$\begin{array}{l} c_{i,j,d^N,t},\ c_{i,j,d^{Mhq},t} \\ a_{k,j,Q,t}, b_{k,j,Q,d,t} \\ \lambda^e_{i,j,Q,t}, \lambda^b_{j,Q,t}, \lambda^{env}_{j,Q,t}, \lambda^{cons}_{i,j,Q,d,t} \end{array}$$

AMPL does not allow procedure programming; therefore, we need to define other variables to represent quantities defined in terms of other variables. We first need

$$\begin{aligned} u_{i,j,d^N,t} &\equiv u\left(c^*_{i,j,d^N,t}, H_{j,t-1}\right) \\ u_{c;i,j,d^N,t} &\equiv u'\left(c^*_{i,j,d^N,t}, H_{j,t-1}\right) \\ W^+_{i,j,d^N,t} &\equiv RW_{i,t} - c^*_{i,j,d^N,t} - \psi + ss \\ f_{i,j,d^N,t} &= \Pr(N|W_{i,t}, H_{j,t-1}, Q_{t-1}) \end{aligned}$$

$$\begin{aligned} u_{i,j,d^{Mhq},t} &\equiv u\left(c^*_{i,j,d^{Mhq},t}, H_t\right) \\ u_{c;i,j,d^{Mhq},t} &\equiv u'\left(c^*_{i,j,d^{Mhq},t}, H_t\right) \\ W^+_{i,j,d^{Mhq},t} &\equiv RW_{i,t} - c^*_{i,j,d^{Mh},t} - \psi - M + ss \\ f_{i,j,d^{Mhq},t} &= \Pr(Mhq|W_{i,t}, H_{j,t-1}, Q_{t-1}) \end{aligned}$$

We next use those variables to build more variables

$$\begin{aligned} \Phi_{i,j,Q,t} &\equiv \Phi(W_{i,t}, H_{j,t-1}, Q_{t-1}; a_t) \\ \Phi_{W;i,j,Q,t} &\equiv \Phi'(W_{i,t}, H_{j,t-1}, Q_{t-1}; a_t) \\ \Phi^+_{i,j,d^{MQ},t} &\equiv \Phi(W^+_{i,j,d^{NM},t}, H_{j,t-1}, Q_{t-1}; a_{t+1}) \\ \Phi^+_{W;i,d^N,t} &\equiv \Phi'(W^+_{i,j,d^{NM},t}, H_{j,t-1}, Q_{t-1}; a_{t+1}) \\ \Phi^+_{i,j,d^{Mhq},t} &\equiv \Phi(W^+_{i,j,d^{Mh},t}, H_{j,t}, Q_t; a_{t+1}) \\ \Phi^+_{W;i,j,d^{Mhq},t} &\equiv \Phi'(W^+_{i,j,d^{Mhq},t}, H_{j,t}, Q_t; a_{t+1}) \\ \Psi_{i,j,d,t} &\equiv \Phi(W_{i,t}, H_{j,t-1}, Q_{t-1}; b_{d,t}) \end{aligned}$$

With these variables defined, the Bellman equation error inequality becomes

$$-\lambda^b_{j,Q,t} \leq \Phi_{i,j,Q,d,t} - \ln\left\{\exp(\widehat{V}_{i,j,d^N,t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})\right\} \leq \lambda^b_{j,Q,t}$$

where

$$\begin{aligned}
\widehat{V}_{i,j,d^N,t} &= u_{i,j,d^N,t} + \beta\eta_{t+1}\Phi^+_{i,j,d^N,t} \\
\widehat{V}_{i,j,d^{Mhq},t} &= u_{i,j,d^{Mhq},t} + \beta\eta_{t+1}\Phi^+_{i,j,d^{Mhq},t}
\end{aligned}$$

the Euler equation error inequalities become

$$\begin{aligned}
-\lambda^e_{i,j,Q,t} &\leq u_{c;i,j,d^N,t} + \beta\Phi^+_{W;i,j,d^N,t} \leq \lambda^e_{i,j,Q,t} \\
-\lambda^e_{i,j,Q,t} &\leq u_{c;i,j,d^{Mhq},t} + \beta\Phi^+_{W;i,j,d^{Mhq},t} \leq \lambda^e_{i,j,Q,t}
\end{aligned}$$

and the envelope error inequality becomes

$$-\lambda^{env}_{j,Q,t} \leq \Phi_{W;i,d^N,t} - \{f_{i,j,d^N,t}\cdot\Phi^+_{W;i,j,d^N,t} + \sum_h \sum_h [f_{i,j,d^{Mhq},t}\cdot\Phi^+_{W;i,j,,d^{Mhq},t}]\} \leq \lambda^{env}_{j,Q,t}$$

The probability of decision d :

$$f_{i,j,d,t} = \frac{\exp(\widehat{V}_{i,j,d,t})}{\exp(\widehat{V}_{i,j,d^{NM},t}) + \sum_h \sum_q \exp(\widehat{V}_{i,j,d^{Mhq},t})}$$

The policy function errors are

$$-\lambda^{cons}_{i,j,Q,d,t} \leq \Psi_{i,jQ,,d,t} - c^*_{i,j,Q,d,t} \leq \lambda^{cons}_{i,j,Q,d,t}$$

12.5.2 Empirical Part in AMPL

We consider individuals in our sample such that $Age^{data}_{n,tp} = 1..(T-2)$.

Let $W^{data}_{n,tp}$, $Age^{data}_{n,tp}$, $H^{data}_{n,tp-1}$ and $Q^{data}_{n,tp-1}$denote respectively the data on financial wealth, age, previous period house value and previous period housing tenure for household n in year tp in the panel data. Given these data, the variables of interest are:

$$\begin{aligned}
c^{pred}_{d^N,n,tp} &= \Psi_{d^{NM}}(W^{data}_{n,tp}, Age^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) \\
c^{pred}_{d^{Mhq},n,tp} &= \Psi_{d^{Mh}}(W^{data}_{n,tp}, Age^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1})
\end{aligned}$$

$$
\begin{aligned}
u^{pred}_{d^N,n,tp} &\equiv u\left(c^{pred}_{d^{NM},n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}\right)\\
u_{c;d^N,n,tp} &\equiv u'\left(c_{d^{NM},n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}\right)\\
W^{+}_{d^N,n,tp} &\equiv RW^{data}_{n,tp} - c^{pred}_{d^N,n,tp} - \psi(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) + ss\\
f^{pred}_{d^{NM},n,tp} &= \Pr(NM|W^{data}_{n,t}, Age^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1})
\end{aligned}
$$

$$
\begin{aligned}
u^{pred}_{d^{Mhq},n,tp} &\equiv u\left(c^{pred}_{d^{Mh},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}\right)\\
u_{c;d^{Mhq},n,tp} &\equiv u'\left(c_{d^{Mhq},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}\right)\\
W^{+}_{d^{Mhq},n,tp} &\equiv RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) - M(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp}) + ss\\
f^{pred}_{d^{Mhq},n,tp} &= \Pr(Mhq|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp})
\end{aligned}
$$

We next use those variables to build more variables

$$
\begin{aligned}
\Phi^{data}_{n,tp} &\equiv \Phi(W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}})\\
\Phi^{data}_{W;n,t} &\equiv (\Phi^{data})'(W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}})\\
\Phi^{+}_{d^N,n,tp} &\equiv \Phi^{data}(W^{+}_{d^{NM},n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})\\
\Phi^{+}_{W,d^N,n,tp} &\equiv \Phi'(W^{+}_{d^{NM},n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})\\
\Phi^{+}_{d^{Mhq},n,tp} &\equiv \Phi(W^{+}_{d^{M},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})\\
\Phi^{+}_{W;d^{Mhq},n,tp} &\equiv \Phi'(W^{+}_{d^{M},n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}, H^{data}_{n,tp-1}, H^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})
\end{aligned}
$$

$$
\widehat{V}^{pred}_{d^N,n,tp} = u(c^{pred}_{d^{NM},n,tp}, H^{data}_{n,tp}) + \beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{NM},n,tp} - \psi(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}) + ss; H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}; a_{Age^{data}_{n,tp}+1})
$$

$$
\widehat{V}^{pred}_{d^{Mhq},n,tp} = u(c^{pred}_{d^{Mhq},n,tp}, H^{choice}_{n,tp})
$$

$$
\begin{aligned}
&+\beta\Phi(RW^{data}_{n,tp} - c^{pred}_{d^{Mhq},n,tp} - \psi(H^{choice}_{n,tp}, Q^{choice}_{n,tp}) - M(H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{choice}_{n,tp})\\
&+ss; H^{data}_{n,tp-1}, H^{choice}_{n,tp}, Q^{data}_{n,tp-1}, Q^{choice}_{n,tp}; a_{Age^{data}_{n,tp}+1})
\end{aligned}
$$

The probabilities of not moving and moving are:

$$f^{pred}_{d^{NM},n,tp} = \Pr(H_{d^N,n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{NM},n,tp})}{\exp(\widehat{V}^{pred}_{d^N,n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}$$

$$f^{pred}_{d^{Mhq},n,tp} = \Pr(H_{d^{Mhq},n,tp}|W^{data}_{n,tp}, H^{data}_{n,tp-1}, Q^{data}_{n,tp-1}, Age^{data}_{n,tp}) = \frac{\exp(\widehat{V}^{pred}_{d^{Mh},n,tp})}{\exp(\widehat{V}^{pred}_{d^N,n,tp}) + \sum_h \sum_q \exp(\widehat{V}^{pred}_{d^{Mhq},n,tp})}$$

We introduce the following constraints concerning the measurement error in consumption:

$$\Pr(c_{n,t}|d^H_{n,tp}, W^{data}_{n,tp}, H^{data}_{n,tp}, Q^{data}_{n,tp}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(\frac{(c^{data}_{n,tp} - c^{pred}_{d,n,tp})^2}{2\sigma^2}$$

KNITRO Problem Characteristics
Objective goal: Maximize
Number of variables: 72746
bounded below: 23688
bounded above: 0
bounded below and above: 23521
fixed: 0
free: 25537
Number of constraints: 103488
linear equalities: 0
nonlinear equalities: 35280
linear inequalities: 35280
nonlinear inequalities: 32928
range: 0
Number of nonzeros in Jacobian: 960344
Number of nonzeros in Hessian: 287529

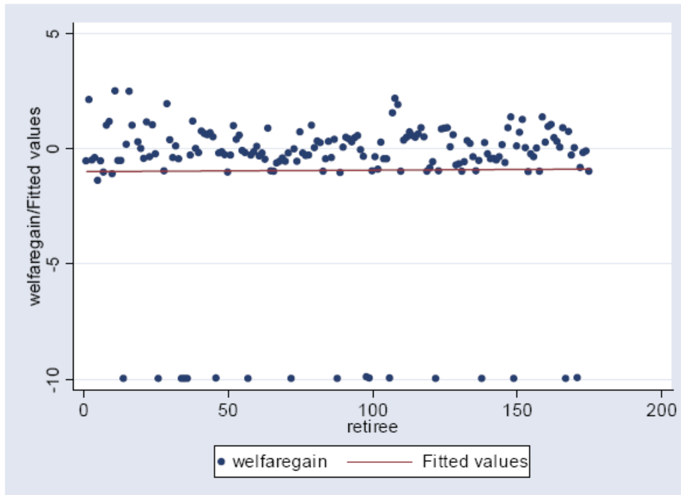

Figure 5: Welfare Gain from Reverse Mortgage

The following figure presents the welfare gain as a function of age.