Asset Pricing with Commitment
Consumption in Heterogenous Agents Economy*

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Abstract

This paper studies portfolio choice and asset prices in a model with two consumption goods, one of which can only be adjusted at a cost. I ask whether this two goods model with adjustment costs can produce the low covariance between consumption and the risky asset returns observed in the real data, and thus explain the risk premium puzzle of Mehra and Prescott (1985). This paper finds that the adjustment cost lowers the covariance between consumption and excess returns. When the adjustment cost increases from 0 to 0.2, the covariance is reduced by about 60%. However, the ability in reducing the covariance is not monotonic in adjustment costs. For larger values of adjustment costs, an increase in the adjustment cost makes the covariance higher, not lower. Therefore, the model can only partly explain the low covariance between consumption and risky asset returns and risk premium puzzle.

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1 Introduction

It is well known that the standard consumption based asset pricing model fails to explain various patterns found in stock return data. This difficulty in explaining stock return data might be caused by many restrictive assumptions made by the standard CCAPM model. One of such assumptions is that there exists a single divisible consumption good which can be adjusted freely and instantly. Although it makes the model mathematically tractable, the assumption ignores both the interaction between different consumption goods and frictions in consumption goods markets. Consumption of different goods are clearly not perfect substitutes. For example, food and housing provide distinct utility to consumers. Furthermore, markets for various consumption goods may operate with frictions. Buying or selling a house incurs transaction costs, and these costs include brokage fees, taxes, time spent, etc.

I study portfolio choices and asset prices in a model with two consumption goods, one of which can only be adjusted at a cost, in a heterogenous agents economy. Can a two consumption goods model with adjustment costs reconcile the low covariance between consumption and risky asset returns with the high risk premium observed in the data without making the extreme preference assumption?

I follow Chetty and Szeidl (2005) and categorize consumption goods into commitment and non-commitment consumption. If the good can only be adjusted at a cost, it belongs to commitment consumption; otherwise, it is non commitment consumption. Commitment consumption is closely related to the notion of a durable good, but it is a broader concept. The adjustment of most durable goods involves transaction costs; for example, the brokage fee paid when buying a house. Therefore, they belong to commitment consumption. However, there are service plans which do not fall into the usual definition of durable goods but incur costs when adjusted, for example, cell phone contracts and life
Following Constantinides (1990), I take the returns processes, which is calibrated by observed asset return data, as given. Given these returns processes, I calculate optimal portfolio and consumption choices for each agent, then aggregate consumption across different agents. Lastly, I compare aggregated consumption processes implied by the model with the real data, and ask whether the covariance of model generated consumption with stock returns is low enough to match that observed in the data.

My primary finding is that adjustment costs lower the covariance between consumption and excess returns, and thus alleviates the risk premium puzzle. When the adjustment cost increases from 0 to 0.2, the covariance is reduced by about 60%. However, the ability in reducing the covariance is not monotonic in adjustment costs. For larger values of adjustment costs, the effect of adjustment costs in reducing the covariance is reversed. For those values, an increase in adjustment costs makes the covariance higher, not lower. Therefore, the effect of adjustment costs on the covariance between consumption and excess returns is not monotonic, and the adjustment model alone can not fully explain the risk premium puzzle.

In the presence of an adjustment cost, the agent will follow a \((y_1, y^*, y_2)\) strategy in adjusting her commitment consumption: She does not change her commitment level when the wealth to commitment ratio is between \(y_1\) and \(y_2\). When her wealth to commitment ratio reaches \(y_1\) or \(y_2\), she immediately changes her commitment level such that her new wealth to commitment ratio is \(y^*\). Since there exist an inactive region for commitment consumption, the agent will change her commitment consumption less frequently, therefore reduce the volatility of commitment consumption growth. The existence of the inactive region also reduces the correlation between commitment consumption and excess return. Because of these effects, the covariance between commitment consumption and excess return is reduced. However, the covariance between non-commitment consumption and
excess returns will be higher if there exists an adjustment cost in changing commitment consumption. This is because non-commitment consumption acts as the only buffer to wealth shocks, thus it is more responsive to wealth shock.

When there exists an adjustment cost, the share of wealth invested in the risky asset also has an interesting U-shaped pattern. The agent invests a lower percentage wealth in the risky asset when her wealth to commitment ratio is further away from the boundaries. On the other hand, the agent invests a higher percentage when her wealth to commitment ratio is close to the boundaries. Due to adjustment costs, a positive wealth change in the inactive region causes less utility gain, thus weaker incentive to invest. If the wealth change can induce an adjustment of commitment consumption, it has a higher effect on utility, thus stronger incentive. Because these opposite effects, the overall risk taking of the whole economy is not monotonic in adjustment costs. It will depend on the adjustment cost and the cross sectional distribution of wealth to commitment ratios. When values of adjustment costs are very high, an increase in the adjustment cost will increase the overall risk taking behavior of the economy. And a higher percentage of wealth invested in the risky assets means a higher mean and volatility for consumption growth.

This paper extends the seminal work of Grossman and Laroque (1990), who analyze optimal portfolio and consumption choices when it is costly to adjust the single consumption good. First, I study a two goods model, in which the interaction between different goods can be explored. Second, because Grossman and Laroque (1990) study the optimal policy of a single agent, their analysis does not have implications on the aggregate consumption and risk premium. In this paper, optimal consumption and asset demand are aggregated across heterogenous agents. Thus, I can explore the implications of the model in a macro perspective. Third, Grossman and Laroque (1990) do not characterize the long run cross-sectional distribution of wealth to commitment ratio, which is the state variable in the model. Following the conjecture of Grossman and Laroque (1990),
Marshall and Parekh (1999) ask whether adjustment costs in a single consumption good model can explain the empirical failure of CCAPM. They find that adjustment costs can reduce covariance between consumption and stock return significantly, but still fall short of matching the magnitude observed in the data.

The paper is related to recent development of asset pricing with multiple consumption goods. For example, Yogo (2006) studies two goods consumption based representative agent model in which utility is defined over durable and non-durable goods. The model can explain the cross-sectional and time variation in expected stock returns conditional on the existence of the “equity premium puzzle” (Mehra and Prescott (1985)). Piazzesi, Schneider and Tuzel (2007) consider a consumption-based asset pricing model where housing is explicitly modeled as a consumption goods. They find that the model delivers a simple explanation for the long-horizon predictability of excess stock returns. Both papers assume the adjustment in durable goods or housing is costless, and leave an important question unanswered, namely, the risk premium puzzle.

The paper is also related to Chetty and Szeidl(2007) and Flavin and Nakagawa (2008). They also solve a two goods model with adjustment cost, however, they do not focus on establishing the link between the adjustment cost model with the habit model. Chetty and Szeidl (2007) find that the aggregate behavior of an economy with adjustment costs and heterogenous agents is similar to the behavior of an economy with a representative agent with a habit based preference. The focus of my paper is to explain the equity premium puzzle, or the low covariance between consumption and stock returns observed in the data.

The remainder of the paper is organized as follows. Section 1 introduces the model and the optimal rules. Section 2 describes the method to solve the model. Section 3 characterizes individual and aggregate choices variables, and discuss the implication of the model on risk premium puzzle. Section 4 concludes the paper.
2 The Model

2.1 Setup

I consider an economy with a continuum of agents who have identical preferences over two consumption goods: commitment consumption (H) and non-commitment consumption (C). The agents differ only in their initial wealth to commitment ratio, and each agent maximizes expected time separable lifetime utility given by

\[ E \int_0^\infty e^{-\delta t} u(C_t, H_t), \]  \hspace{1cm} (1)

where \( \delta \) is the discount factor and the instantaneous utility at time \( t \) \( u(C_t, H_t) \) is given by CES function

\[ u(C_t, H_t) = \left( sC_t^\lambda + (1 - s)H_t^\lambda \right)^\frac{1}{1-\gamma}. \]  \hspace{1cm} (2)

The parameter \( s \), with \( s \in (0, 1) \), measures relative preference for commitments, and a smaller \( s \) implies that commitment consumption is more important. The parameter \( \lambda \), with \( \lambda \in (-\infty, 1) \), is related to the elasticity of substitution between the two consumption goods. As \( \lambda \) goes to 1, the two goods become perfect substitutes. On the other hand, the two goods become perfect complement when \( \lambda \) goes to \(-\infty\). If \( \lambda \) equals to zero, the utility function is Cobb-Douglas. The parameter \( \gamma \) measures the relative risk aversion to the consumption bundle of the two goods if there is no adjustment cost. The parameter \( \gamma \) is also related to the elasticity of intertemporal substitution.

The agent can invest her wealth into two assets: a risky stock and a riskless bond.
subject to the budget constraint,

\[ W_t = B_t + X_t, \]  

(3)

where \( B_t \) is the value of the agent’s holding of the risk-free asset and \( X_t \) is the value of the agent’s holding of the risky asset.

The values of the bond and the stock are governing by

\[ dB_t = r B_t dt \]

(4)

\[ dX_t = X_t[\mu + r)dt + \sigma dw_t], \]

(5)

where \( r \) is risk free rate, and \( \mu \) and \( \sigma \) are mean excess return and the standard deviation of the risky asset, respectively. The term \( w_t \) is a standard Wiener process. Therefore, the stock return is iid normal for any fixed interval.

If the commitment consumption is not adjusted at time \( t \), the law of motion for the wealth will be

\[ dW_t = B_t r dt + X_t((\mu + r)dt + \sigma dw_t) - C_t dt - H_t dt \]

(6)

\[ dW_t = W_t r dt + X_t(\mu dt + \sigma dw_t) - C_t dt - H_t dt. \]

(7)

If the agent decides to change her commitment consumption level at \( \tau \), she has to pay a transaction cost which is proportional to the commitment consumption level, and the proportional factor is \( k \). Thus, the wealth immediately after the change will be

\[ W_\tau = W_{\tau^-} - k H_{\tau^-}. \]

(8)

The structure of the adjustment cost is similar to Grossman and Laroque (1990). In their
paper, the adjustment cost, however, is proportional to the consumption stock instead of the flow defined in this paper. As proved in Marshall and Parekh (1999), the two specifications are essentially the same. Although this assumption is not suitable for every type of commitment consumption, it is reasonable for most of them. For example, the brokerage fee paid for buying a house usually is a fixed proportion of house value. As explained by Marshall and Parekh (1999), this assumption also has important implication on the aggregation. This feature precludes aggregation even with complete markets, therefore it is important to work in a heterogenous agent model.

Finally, the agent faces a solvency condition:

\[
W_t - kH_t \geq 0.
\]  
(9)

The individual’s problem can be summarized as follow: the agent chooses her consumption plans for commitment, \(\{H_t\}\), and non-commitment, \(\{C_t\}\), risky asset demand, \(\{X_t\}\), and optimal stopping rule, \(\tau\), to maximize her life time utility

\[
\max_{C_t, H_t, \tau} E \int_0^\infty e^{-\delta t} u(C_t, H_t) dt
\]  
(10)

subject to budget constraint (3), wealth accumulation (4) and (5), and solvency constraint (6).

### 2.2 Optimal Policies

To derive the optimal policy, I follow the strategy of Grossman and Laroque (1990). First, let \(V(W, H)\) be the maximized value of the objective in equation (1), starting from initial conditions \((W, H)\). In addition, let \(\tau\) be the first date at which it is optimal to adjust the level of commitment consumption and \(H^*\) be the new commitment consumption level
right after the adjustment. Then, \( V(W, H) \) satisfies

\[
V(W, H) = \sup_{H^*, \tau, X_t, C_t} \mathbb{E} \left[ \int_0^\tau e^{-\delta t} u(C_t, H) dt + e^{-\delta \tau} V(W_{\tau^-} - kH_{\tau^-}, H^*) \right].
\] (11)

It is easy to show that the value function \( V \) is increasing in \( W \), that is, \( V_W > 0 \). As for the other state variable, an increase in \( H \) will have two opposite effects. First, it increases the current utility, \( u(C_t, H) \); Second, it decreases the future utility at the time of adjustment, \( V(W_{\tau^-} - kH_{\tau^-}, H^*) \). When the level of commitment is very low relative to wealth, the first effect will dominate the second effect since the marginal utility \( u_H \) is very high. On the other hand, if the level of commitment is very high, the marginal effect on \( u \) will be very small and the second effect will dominate the first effect. Therefore, the value function is increasing in \( H \) for the low level of commitment, \( V_H > 0 \), and it is decreasing in \( H \) for the high level of commitment, \( V_H < 0 \). For example, living in a five million dollar house definitely increases your utility today relative to a half million dollar house, but it will be a huge financial burden if you lose your job in the future.

Since \( V \) is homogenous with degree \( 1 - \gamma \), we can reduce the dimension of the state space by one. Define

\[
x_t = \frac{X_t}{H},
\]

\[
c_t = \frac{C_t}{H}.
\]

Then

\[
H^{-(1-\gamma)}V(W, H) = \sup_{H^*, \tau, x_t, c_t} \mathbb{E} \left[ \int_0^\tau e^{-\delta t} u(c_t, 1) dt + e^{-\delta \tau} V\left(\frac{W_{\tau^-} - kH_{\tau^-}}{H}, \frac{H^*}{H}\right) \right] = V\left(\frac{W}{H}, 1\right).
\]
Define the new state variable \( y \) as

\[
y = \frac{W}{H} - k
\]  

and new functions \( h \) and \( M \) as

\[
h(y) = V(y + k, 1)
\]

\[
M = \sup_y (y + k)^{-(1-\gamma)}h(y).
\]

Note that

\[
M y_{\tau}^{1-\gamma} = \sup_y \left(\frac{y_{\tau}}{y + \gamma}\right)^{1-\gamma}V(y + \lambda, 1)
\]

\[
= \sup_y V(y_{\tau}, \frac{y_{\tau}}{y + \gamma})
\]

\[
= \sup_{H^*} V\left(\frac{W_{\tau}}{H} - k, \frac{H^*}{H}\right).
\]

Therefore, we can transform the original value function into a new value function \( h \),

\[
h(y) = \sup_{\tau, x_t, c_t} E \left[ \int_0^\tau e^{-\delta t} u(c_t, 1) dt + e^{-\delta \tau} M y_{\tau}^{1-\gamma} \right].
\]

From equation (5), we know that, between adjustments in commitment consumption, the evolution of the new state variable is

\[
dy_t = \frac{dW}{H} = [r(y_t + k) - 1] dt + x_t[\mu dt + \sigma d\omega_t] - c_t dt.
\]

After the transformation, we can apply the same logic used by Grossman and Laroque (1990). The idea is that, for a fixed \( M \), control problem (15) is an optimal stopping problem in which the payoff of stopping at state \( y \) is just \( M y_{\tau}^{1-\gamma} \). Denote \( h(y; M) \) as the
optimal stopping problem with fixed $M$. Then, the original problem $h(y)$ is equivalent to $h(y; M^*)$, where $M^*$ is defined implicitly by a fixed point mapping,

$$M^* = \sup_y (y + k) h(y; M^*). \quad (17)$$

For a fixed $M$, we know it is not optimal to stop if the value of waiting, $h(y)$, is strictly greater than the value of stopping, $My^{1-\gamma}$. If the condition $h(y) > My^{1-\gamma}$ holds at time 0, the probability of durable good adjustment in the time interval $[0, \Delta t]$ is zero. Hence, $h(y; M)$ has to satisfy the Bellman equation,

$$h(y_0) = \sup_{x_t, c_t} E \left[ \int_0^{\Delta t} e^{-\delta t} u(c_t, 1) dt + e^{-\delta \Delta t} h(y_{\Delta t}) \right]. \quad (18)$$

By Taylor expansion and Itô’s lemma, we know that

$$\lim_{\Delta t \to 0} h(y_{\Delta t}) = h(y) + h'(y) dy + \frac{1}{2} h''(y)(dy)^2$$

$$= h(y) + h'(y)[(r(y + k) - 1 - c)dt + x(\mu dt + \sigma dw)] + \frac{1}{2} h''(y)x^2 \sigma^2 dt$$

and

$$\lim_{\Delta t \to 0} \int_0^{\Delta t} e^{-\delta t} u(c_t, 1) dt = u(c, 1) dt.$$
Therefore

\[
\begin{align*}
    h(y) &= \lim_{\Delta t \to 0} \sup_{x,t} E \left[ \int_0^{\Delta t} e^{-\delta t} u(c_t, 1) dt + e^{-\delta \Delta t} h(y_{\Delta t}) \right] \\
    &= \sup_{x,t} E[u(c_t, 1) dt + (1 - \delta \Delta t)h(y) + h'(y)[r(y + k) - 1 - c]dt] \\
    &\quad + x(\mu dt + \sigma dw) + \frac{1}{2}h''(y)x^2 \sigma^2 dt;
\end{align*}
\]

that is,

\[
\sup_{x,c} \left[ u(c, 1) + h'(y)[(r(y + k) - 1 - c) + x\mu] + \frac{1}{2}h''(y)x^2 \sigma^2 - \delta h(y) \right] = 0. \quad (19)
\]

First order conditions for \(x\) and \(c\) are

\[
\begin{align*}
    u'(c, 1) &= h'(y) \quad (20) \\
    h''(y)\sigma^2 x &= -h'(y)\mu. \quad (21)
\end{align*}
\]

Equation (20) is the envelope condition for non-commitment consumption. It says that the marginal utility of consuming one additional unit of non-commitment consumption relative to commitment is the same as the marginal utility of saving. Equation (21) implies that the percentage invested in risky asset is determined by the relative risk aversion, \( \frac{h'(y)}{h''(y + k)} \), average excess return, \( \mu \), and standard deviation, \( \sigma \), for the risky asset. It is given by

\[
\frac{x}{y + k} = -\frac{h'(y)}{h''(y + k)} \frac{\mu}{\sigma^2}.
\]

Since the relative risk aversion is state dependent, the share of the risky asset will not be constant. This feature differs from the standard model without adjustment cost where the
share of risky asset is a constant, but it is similar to existing model with state dependent risk aversion.

After characterizing the optimal policies for non-commitment consumption and portfolio, we need to find the optimal stopping rule for changing commitment level. When the agent decides to change her commitment level, she has to pay a transaction cost. On the other hand, it allows the agent to choose her commitment level optimally. Thus, the agent needs to compare the value before the adjustment, $V(W^0, H^0)$, with the value after the adjustment, $\max_H V(W^0 - kH^0, H)$.

Let $H^*$ be the optimal commitment level when the wealth to commitment ratio is $W^0 - kH^0$, and take Talor’s expansion for $V(W^0 - kH^0, H^*)$ around $V(W^0, H^0)$,

$$V(W^0 - kH^0, H^*) - V(W^0, H^0) \approx V_W(-kH^0) + V_H(H^* - H^0).$$

If the wealth to commitment ratio is very high, $V_H$ will be very low relative to $V_W$. The benefit of the adjustment, $V_H(H^* - H^0)$, will dominate the cost, $V_W(-kH^0)$. If the wealth to commitment ratio is very low, both $V_H$ and $H^* - H^0$ will be negative. The benefit of adjustment will again dominate the cost, and it is optimal to change. These properties imply that the agent will change her commitment level when the wealth to commitment ratio is either too high or too low.

As proved in Grossman and Laroque (1990), the agent follows a $(y_1, y_2)$ rule on her choice of commitment consumption, that is, if her wealth to commitment consumption ratio is between $(y_1, y_2)$, the agent chooses not to change her commitment level to avoid paying adjustment costs. On the other hand, if her wealth to commitment ratio falls outside of $(y_1, y_2)$ band, she optimally chooses a new level of commitment such that the new wealth to commitment ratio is $y^*$, which is inside of $(y_1, y_2)$ band. The idea is simple, why living in a small house if you are a billionaire, conversely, living in a mansion even if
you are unemployed does not make too much sense either.

We can summarize the optimal policies for consumption and portfolio as in the following Theorem:

**Theorem 1.** Let \( y = \frac{W}{H} - k \). There exist a triple constant \( \{y_1, y^*, y_2\} \), where \( y_1 < y^* < y_2 \) and function \( h(y) \) such that the optimal rules are

- **Stopping Rule:** Adjust the level of commitment if the wealth to commitment ratio reaches \( y_1 \) or \( y_2 \).

- **Commitment Consumption:** Change the new level of commitment such that \( y' = y^* \) if \( y \) reaches \( y_1 \) or \( y_2 \).

- **Non-Commitment Consumption:** \( u'(c^*, 1) = h'(y) \)

- **Risky Asset Demand:** \( x = \frac{h'(y) \mu}{h''(y) \sigma^2} \)

**Proof.** See Appendix.

\[ \square \]

### 3 Numerical Methods

#### 3.1 Individual Problem

An analytical solution for the optimal policies does not exist. Therefore, I rely on numerical methods to examine the problem. As discussed in the previous section, the value function \( h(y, M^*) \), where \( y \in (y_1, y_2) \), is defined by the ordinary differential equation (ODE).

\[
u(c^*, 1) + h'(y)[(r(y + k) - 1 - c^*) + x\mu] + \frac{1}{2}h''(y)(x^*)^2\sigma^2 - \delta h(y) = 0,
\]

where the optimal non-commitment consumption, \( c^* \), risky asset demand, \( x^* \) and \( M^* \) are defined in (20), (21) and (17), respectively. To characterize the optimal rules, we need to
find the inaction boundary \((y_1, y_2)\), the reset value, \(y^*\), and the value function \(h(y, M^*)\).

To find these values, I follow the method used by Marshall and Parekh (1999), and it can be summarized as the following:

- Choose an initial guess for \(M\) as \(M^0\), a constant.
- Set an initial guess for \(y_1\), such that \(h(y_1) = y_1^{1-\gamma}M\) and \(h'(y_1) = (1-\gamma)y_1^{-\gamma}M\).
- Solve the ODE forward by Euler or Runge-Kutta schemes.
- Find a value \(y_2 > Y_1\) at which boundary conditions are satisfied; if \(y_2\) cannot be found, choose a different value of \(y_1\).
- Evaluate the criterion function at \(M^0\),

\[
S(M) = [M - \max_y (y + k)^{1-\gamma}h(y; M)].
\]
- Search for \(M^*\) such that \(S(M^*) = 0\).

### 3.2 Aggregation of Heterogenous Agents

To analyze the behavior of aggregate consumption and risky asset demands, I need to aggregate individual choices across different agents. Again, I have to apply numerical methods to characterize the aggregate behavior of model variables and the long run cross sectional distribution of wealth to durable ratio. First, I approximate the continuous model by a discrete counterpart in which the agent is only allowed to make consumption and portfolio decisions at discrete time. Second, I simulate a long time series of exogenous asset returns defined by (4) and (5). Third, I draw a large number of agents, who differ only in their wealth to commitment ratios \(y\), from the long run cross sectional distribution of wealth to commitment ratio. Fourth, for each agent who optimally chooses her
consumption and portfolio, I keep track of her wealth, consumption, and asset demands over time. Lastly, I aggregate all variables across all agents. In the appendix, I describe how to obtain the long run cross sectional distribution of wealth to commitment ratio.

4 Calibration Results

4.1 Parameter Values

I calibrate the parameters of asset returns to quarterly U.S. data. In particular, I calibrate $\mu$ and $\sigma$ to match the mean and variance of returns for the CRSP value-weighted stock portfolio. The risk-free rate $r$ is calibrated to match the mean return of one-month Treasury bills. These moments imply $\mu = 0.06277$, $\sigma = 0.1505$, and $r = 0.00926$ in annual frequency.

I set the values of preference parameters $\{\gamma, \alpha, \delta, s\}$ to those obtained in previous studies for the baseline model, but I vary their values in robustness checks. In a baseline model, the elasticity of substitution $\alpha$ and the relative preference $s$ are set to those estimated by Yogo (2006) in his durable goods model. The risk aversion measure $\gamma$ is set to 5, and this implies relative risk aversion of 5 if there is no adjustment cost. The annual discounting factor $\delta$ is set to 0.04.

Lastly, it is hard to pin down the adjustment cost $k$, the central interest of this paper. Since previous studies do not provide much guidance on this subject, I choose a set of values ranging from 0.002 to 2 instead of setting one value. For some goods, the adjustment costs seem to be a very small portion of expenditure. On the other hand, it might be substantial. For example, the typical brokerage fee alone for buying a house is 5% of the house value. Assume the service flow generated by the house is 2.5%. The adjustment parameter $\lambda$ will be 2 which is the upper bound set in this paper.
I summarize parameter values for the base line model in the following table

[Insert Table 1]

4.2 Results for Individual Problem

In the commitment consumption model, the relative risk aversion as well as the percentage of wealth invested in the risky asset are state dependent. I plot shares of the risky asset as a function of the wealth to commitment ratio for different adjustment costs in Figure 1.

[Insert Figure 1]

The figure shows that the percentage of wealth invested in the risky asset is a U-shape as the wealth to commitment ratio increases. This is because the agent is more risk averse when her wealth to commitment ratio moves further away from the boundaries and less so when her wealth to commitment ratio is close to the boundaries. As the wealth to commitment ratio is further away from the boundaries, it is less likely that the commitment level will be changed in the next period. If consumption level of commitment goods is unchanged, the agent can only adjust her non commitment for any changes in wealth. This creates a distortion in consumption in the presence of adjustment costs which reduce the marginal utility of wealth. Consequently, it reduces the incentive to invest. Conversely, if a change in wealth induces commitment adjustments, it will have a larger impact on marginal utility. When the wealth to commitment ratio is close to the boundaries, it is more likely that the commitment level will be changed in the next period. Hence, the agent wants to invest more. The other interesting feature of the optimal investment policy is its asymmetry, that is, the share of wealth invested in the risky asset is higher on the upper boundary than the lower one. When the agent invests
the risky asset, it is likely that her wealth will decrease next period. Because this downside risk is more significant for the agent with low wealth to commitment ratio, it induces the agent hold less risky assets if her wealth to commitment ratio is quite low.

As the adjustment cost becomes larger, the variation of risky shares increases in the inactive region. When \( k = 0.002 \), the difference between the highest percentage to the lowest one is only 0.02, which is only about a 4% change. When \( k = 0.2 \), the difference becomes 0.35 and is about a 70% change. This big change in the variation of risky share is due to a higher adjustment cost. As adjustment cost increases, the distortion in consumption becomes more severe. Therefore, the incentive to invest less becomes stronger when the wealth to commitment ratio is further away from the boundaries, and so is the incentive to invest more when the wealth to commitment ratio is close to the boundaries.

In addition to describing how much to invest in the risky asset, the optimal rules for consumptions are also important policy variables. In Figure 2, I plot inactive regions in the commitment consumption for different adjustment costs. I also plot standard deviations of consumption growth rates implied by the model for different adjustment costs.

[Insert Figure 2]

In the commitment consumption model, the agent will change her commitment level infrequently because of adjustment costs. Even for a very small adjustment cost, the inactive region can be very wide. As the adjustment cost increases, the inactive region becomes wider but at a decreasing rate. Since the agent adjusts her commitment level less frequently, the standard deviation of commitment consumption growth becomes smaller as the adjustment cost increases. For non-commitment consumption, its growth rate becomes more volatile as the adjustment cost increases. This is because non-commitment consumption serves as a buffer for wealth shocks. When commitment consumption is
adjusted less frequently, the agent has to change her non-commitment consumption to compensate for the utility loss due to adjustment cost. If we define the consumption bundle as a function of commitment and non-commitment consumption, the standard deviation of its growth rate will not be a monotonic function of adjustment costs. It first decreases with adjustment costs due to the decreasing effect of commitment consumption. Then it increases as commitment consumption becomes more and more volatile.

4.3 Results for an Economy with Heterogenous Agents

As described in previous sections, I draw N agents with different wealth to commitment ratios, and then trace their wealth and choice variables for T periods. I aggregate consumptions, risky asset demand and wealth across all agents to form aggregated series. In the baseline model, I choose N to be 40000, and T to be 100000. A larger N or T only have insignificant effects on the results.

In Figure 3, I plot the long run cross sectional distribution for wealth to commitment ratio for economies with different adjustment costs.

[Insert Figure 3]

As the adjustment cost increases, the density function becomes more spread out. This has an implication on the demand of the risky assets for the whole economy. As a larger proportion of agents move close to the boundaries where the agent is likely to invest more, the demand of the risky assets for the whole economy becomes higher. On the other hand, a higher adjustment cost causes agents, who are away from the boundaries, more risk averse, which leads to a lower demand of the risky asset. When the adjustment cost is small, the latter effect dominates the former one. Conversely, the former effect dominates the latter one when the adjustment cost is high. Therefore, the overall consumption growth volatility is a monotonic function of adjustment costs.
In Table 2, I display the mean and standard deviation for growth rates for aggregated commitment consumption, non commitment consumption and the bundle of commitment and non-commitment consumption under different adjustment costs in Panel a and Panel b. In order to compare those statistics with those obtained from real data, I list moments reported by Marshall and Parekh (1999) in Panel c. Those statistics are calculated from real data for different consumption measures.

[Insert Table 2]

As we can see from the table, the adjustment cost has a very small but positive effect on the average growth rate for consumptions. This is because the agent has an incentive to invest more in order to induce changes in commitment, and consequently a larger risky asset share leads to a higher growth rate of consumption. Adjustment costs reduce the standard deviation of commitment consumption growth. When $k$ increases from 0.002 to 0.2, the standard deviation of commitment consumption growth is reduced from 0.0345 to 0.0319, which is about a 8% reduction. For non-commitment consumption, the standard deviation of growth rate increases as the adjustment cost increases and is more sensitive to the change in the adjustment cost. The bigger the adjustment cost, the larger the distortion in consumption. This leads to the agent uses more non-commitment consumption to compensate for the loss in utility caused by the adjustment cost. Therefore, non-commitment consumption become more responsive to wealth shocks as the adjustment cost becomes bigger. For the bundle of both consumptions, the standard deviation is reduced from 0.002 to 0.2, which is about 10%. In short, although adjustment cost can reduce the volatility of consumption growth, the extent of the reduction is quite limited. A very high adjustment cost might even increase the standard deviation of consumption growth. There are two reasons for this non monotonic feature. First, non-commitment consumption serves as the only buffer for wealth shocks. Hence, it is more responsive to
the change in wealth. Second, a higher adjustment cost makes the agent less risk averse when the agent is close to the boundaries of inactive region. Besides, a higher adjustment cost also makes wealth of commitment ratios closer to the boundaries for a larger percentage of agents. Overall, the standard deviation increases for very high levels of adjustment cost.

The model does not perform very well when compared with moments obtained from real data. Although the model roughly matches the means of real data, the standard deviations generated by the model are too high relative to those from real data.

Next, I investigate whether the model can explain risk premium puzzle, or low covariance between consumption and risky asset returns. Following Marshall and Parekh (1999), I calculate the “theoretical risk premium” which is defined as

\[
RP = \frac{-\text{cov}((c_{bt+\Delta t}/c_{bt})^{-\gamma}, r_{t+1}^e)}{E((c_{bt+\Delta t}/c_{bt})^{-\gamma})}.
\]

As discussed in Marshall and Parekh (1999), the risk premium puzzle is related to the discrepancy between the theoretical risk premium from the standard model and that calculated from real consumption data. A good model should explain low covariance between consumption and excess return and low theoretical risk premium associated with consumption data. In Table 3, I display covariances between various consumption measures and excess returns, and the theoretical risk premium in Panel a. In Panel c, I present the theoretical risk premium calculated by Marshall and Parekh (1999) using observed consumption data.

[Insert Table 3]

Adjustment costs reduce the covariance between commitment consumption and excess returns at the expense of increasing the covariance between non-commitment consumption
and excess returns. Overall, the covariance between the consumption bundle and excess returns decreases as the adjustment cost increases for most of adjustment cost values. However, for very high level of adjustment costs, the covariance actually increases instead of decreasing. As for theoretical risk premium, the reduction of theoretical risk premium is more than 60% as the adjustment cost is increased from 0 to 0.2. But the effect is reversed as the adjustment cost increases even further. For high values of adjustment costs, implied theoretical risk premium even increases as adjustment cost becomes higher. Compared with the theoretical risk premium implied from data, the model can not fully explain the low values implied from the data. If we measure the theoretical risk premium using the commitment consumption as the consumption measure, the model can produce the low values implied from the real data. For other consumption measures, the theoretical risk premiums implied from the model are much higher than those implied by the data. However, one caveat is that there are considerable measure error in consumption data as explained by many previous studies. A manifestation of such problem is the negative risk premium produced from the data.

In summary, the adjustment model produces a lower covariance between consumption and excess return, and thus a lower theoretical risk premium. But the reduction in the covariance is not enough to fully explain the very low level implied from observed consumption data alone.

5 Conclusion

This paper studies portfolio choice and asset prices in a model with two consumption goods, one of which can only be adjusted at a cost. I ask whether this two goods model with adjustment costs can produce the low covariance between consumption and the risky asset returns observed in the real data, and thus explain the risk premium puzzle.
of Mehra and Prescott (1985). This paper finds that the adjustment cost lowers the covariance between consumption and excess returns. When the adjustment cost increases from 0 to 0.2, the covariance is reduced by about 60%. However, the ability in reducing the covariance is not monotonic in adjustment costs. For larger values of adjustment costs, an increase in the adjustment cost makes the covariance higher, not lower. Therefore, the model can only partly explain the low covariance between consumption and risky asset returns and risk premium puzzle.
Reference


Table 1. Baseline Parameter Values in Annual Frequency

This table presents the parameter values used in the calibration exercise. The values are in annual frequency.

<table>
<thead>
<tr>
<th>Preference</th>
<th>$\gamma$</th>
<th>$\alpha$</th>
<th>$s$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.43</td>
<td>0.2</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Returns</td>
<td>$\mu$</td>
<td>$r_f$</td>
<td>$\sigma$</td>
<td></td>
</tr>
<tr>
<td>0.06227</td>
<td>0.00926</td>
<td>0.1505</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>$k$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.002,2)</td>
<td></td>
<td></td>
<td></td>
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</tr>
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</table>
Table 2. Mean and Standard Deviation of Aggregate Consumption Growth

This table displays the mean and standard deviation for consumption growth rates for aggregated non-commitment consumption $c$, commitment consumption $h$ and consumption bundle of commitment and non-commitment $cb$. The consumption bundle is calculated as $(sC^\lambda_t + (1 - s)H^\lambda_t)^{\frac{1}{1-\gamma}}$. In Panel c, statistics calculated by Marshall and Parekh (1999) are reported. CEN=consumer expenditures on nondurables; CES=consumer expenditures on services excluding the service flow from owner-occupied housing; NDS=CEN+CES; YCD=service flow from the stock of consumer durable;CESH=the services flow from owner-occupied housing.

<table>
<thead>
<tr>
<th>k</th>
<th>c</th>
<th>h</th>
<th>cb</th>
<th>c</th>
<th>h</th>
<th>cb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>0.0253</td>
<td>0.0121</td>
<td>0.0122</td>
<td>0.1115</td>
<td>0.0771</td>
<td>0.0771</td>
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<tr>
<td>0.004</td>
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<td>0.012</td>
<td>0.0122</td>
<td>0.1246</td>
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<td>0.0742</td>
</tr>
<tr>
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<td>0.0263</td>
<td>0.0121</td>
<td>0.0123</td>
<td>0.1383</td>
<td>0.0721</td>
<td>0.0713</td>
</tr>
<tr>
<td>0.2</td>
<td>0.029</td>
<td>0.0129</td>
<td>0.0131</td>
<td>0.1643</td>
<td>0.0697</td>
<td>0.0681</td>
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<tr>
<td>0.5</td>
<td>0.032</td>
<td>0.0138</td>
<td>0.0139</td>
<td>0.1769</td>
<td>0.0725</td>
<td>0.0702</td>
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Panel a: Annually

Panel b: Quarterly
<table>
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<th>h</th>
<th>cb</th>
<th>c</th>
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<th>cb</th>
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<tr>
<td>0.002</td>
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<td>0.003</td>
<td>0.003</td>
<td>0.072</td>
<td>0.0345</td>
<td>0.0341</td>
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<td>0.003</td>
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<td>0.0326</td>
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<td>0.003</td>
<td>0.0031</td>
<td>0.0808</td>
<td>0.0323</td>
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<td>0.0033</td>
<td>0.0891</td>
<td>0.0319</td>
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<tr>
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<td>0.008</td>
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<td>0.0036</td>
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Panel c: Observed Moments in Quarterly Real Data

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<tr>
<th>Consumption Measure</th>
<th>Mean</th>
<th>Standard Deviation</th>
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<tr>
<td>CEN</td>
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<tr>
<td>CES</td>
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<td>0.0059</td>
</tr>
<tr>
<td>NDS</td>
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<td>0.0056</td>
</tr>
<tr>
<td>YCD</td>
<td>0.00886</td>
<td>0.0050</td>
</tr>
<tr>
<td>CESH</td>
<td>0.00618</td>
<td>0.0043</td>
</tr>
</tbody>
</table>
Table 3. Covariance between consumption and excess return and Theoretical Risk Premium

This table presents covariance between consumption and excess return, and theoretical risk premium defined as in Marshall and Parekh (1999) for different adjustment costs $k$. In panel c, theoretical risk premiums are calculated by Marshall and Parekh (1999) using real data.

**Panel a: Model, Annually**

<table>
<thead>
<tr>
<th>k</th>
<th>c</th>
<th>h</th>
<th>cb</th>
<th>c</th>
<th>h</th>
<th>cb</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0062</td>
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</tr>
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</table>

**Panel b: Model, Quarterly**

<table>
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<tr>
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<th>h</th>
<th>cb</th>
<th>c</th>
<th>h</th>
<th>cb</th>
</tr>
</thead>
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<td>0.0018</td>
<td>0.0050</td>
</tr>
<tr>
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<td>0.0012</td>
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<td>0.0007</td>
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<tr>
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<td>0.0013</td>
<td>0.0359</td>
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<td>0.0081</td>
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</table>

**Panel c: Observed Data, Quarterly**
<table>
<thead>
<tr>
<th>Consumption Measure</th>
<th>RP</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEN</td>
<td>0.0015</td>
</tr>
<tr>
<td>CES</td>
<td>0.0017</td>
</tr>
<tr>
<td>NDS</td>
<td>0.0016</td>
</tr>
<tr>
<td>YCD</td>
<td>-0.0010</td>
</tr>
<tr>
<td>CESH</td>
<td>-0.0008</td>
</tr>
</tbody>
</table>
This figure plots the percentage of wealth invested in the risky asset as a function of the state variable $y$ for different adjustment costs $k$. The state variable $y$ is defined as the wealth to commitment ratio after the adjustment cost, $\frac{W}{H} - k$. 
Figure 2. Inactive Region and Standard Deviations of Consumption Growth

This figure plot the range of inactive regions as a function of adjustment costs, $k$. It also plot standard deviations of individual non-commitment consumption, commitment consumption and the consumption bundle as functions of adjustment costs. The standard deviations are in quarterly frequency.
Figure 3. Long Run Cross Sectional Distribution

This figure plots the long run cross sectional distribution of wealth to commitment ratios. For each simulation, I obtain a cross sectional distribution of wealth to commitment ratios for $N = 40000$ agents at $T = 100000$. The long run cross sectional distribution is obtained by averaging cross sectional distributions over 1000 simulations.